COMMENT ON THE PROCEDURE FOR CALCULATING 9j-SYMBOLS

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Reference [1] contains the texts of programs for calculating 9j-symbols written in ALGOL-60 and FORTRAN-IV. Practical use of these programs has revealed certain limitations of the 9j-symbol calculation procedure forming part of the ALGOL program; in this program, the 9j-symbols are calculated as the sum of the products of three Racah coefficients, the summation index being able in the general case to assume either whole or semi-whole values. In the ALGOL text in Ref. [1], on the other hand, the variant uses only whole values of the summation index. Generally speaking, in those cases where the summation index assumes semi-whole values the initial 9j-symbol may - by line or column transpositions - be reduced to a form corresponding to whole values of the summation index. Let us consider, for example, a 9j-symbol whose j parameters are semi-whole and whose L parameters are whole - i.e.

$$\begin{Bmatrix} j_1 & j_2 & L_{12} \\ j_3 & j_4 & L_{34} \\ L_{13} & L_{24} & L \end{Bmatrix} = \sum_{K} (2K+1) W[j_1 j_3 L L_{24}; L_{13} K] W[j_2 L_{24} L_{34} j_3; j_4 K] W[j_1 j_2 L L_{34}; L_{12} K] . \quad (1)$$

Here, the limits of variation of summation index k are determined by the conditions

$$\max \begin{bmatrix} |j_1 - L| \\ |j_2 - L_{34}| \\ |j_3 - L_{24}| \end{bmatrix} \leqslant K \leqslant \min \begin{bmatrix} j_1 + L \\ j_2 + L_{34} \\ j_3 + L_{24} \end{bmatrix} . \quad (2)$$

It follows that the index k assumes semi-whole values. On the basis of symmetry properties, the 9j-symbol determined by formula (1) can be written in the form

$$\begin{Bmatrix} j_1 & j_2 & L_{12} \\ j_3 & j_4 & L_{34} \\ L_{13} & L_{24} & L \end{Bmatrix} = \begin{Bmatrix} L_{12} & j_1 & j_2 \\ L_{34} & j_3 & j_4 \\ L & L_{13} & L_{24} \end{Bmatrix} = \sum_{K_1} (2K_1+1) W[L_{12} L_{34} L_{24} L_{13}; L K_1] \times \quad (3)$$

$$\times W[j_1 L_{13} j_4 L_{34}; j_3 K_1] W[L_{12} j_1 L_{24} j_4; j_2 K_1] .$$

In this formula, summation index k_1 determined by the conditions

$$\max \begin{bmatrix} |L_{12}-L_{24}| \\ |L_{13}-L_{34}| \\ |j_1-j_4| \end{bmatrix} \leq K_1 \leq \min \begin{bmatrix} L_{12}+L_{24} \\ L_{13}+L_{34} \\ j_1+j_4 \end{bmatrix} , \tag{4}$$

assumes (as can be seen) whole-number values.

It should be noted that, when one uses the procedure of 9j-symbols for calculating the coefficients of transformation $A\{jj \to LS\}$ from the jj coupling scheme to the LS coupling scheme which are frequently encountered in nuclear calculations, the index k assumes whole values and hence in this case the procedure described in Ref. [1] gives correct results. The same applies to 9j-symbols all of whose parameters assume whole values.

Having in mind the same general case, one should regard the summation index k as a variable of the real type and hence introduce changes into the text of the 9j-symbol procedure. The corrected text of this procedure, in which the designations from Ref. [1] have been retained as far as possible, has the form

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'REAL''PROCEDURE'J9(A,B,C,D,E,F,G,H,J);'VALUE'A,B,C,D,E,F,G,H,J;
'REAL'A,B,C,D,E,F,G,H,J;
'BEGIN''REAL'P,Q,R,K,KM,S;
S:=0;
P:=ABS(A-J);Q:=ABS(D-H);R:=ABS(B-F);
K:=P;'IF'K<Q'THEN'K:=Q;'IF'K<R'THEN'K:=R;
P:=A+J;Q:=D+H;R:=B+F;KM:=P;'IF'KM>Q'THEN'KM:=Q;
'IF'KM>R'THEN'KM:=R;KM:=KM+0.001;
'FOR'K:=K'STEP'1'UNTIL'KM'DO'
S:=S+(2*K+1)*W(A,D,J,H,G,K)*W(B,H,F,D,E,K)*
W(A,B,J,F,C,K);J9:=S 'END'J9;
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Otherwise the ALGOL program for calculating 9j-symbols presented in Ref. [1] remains unchanged.

REFERENCE

[1] ARTAMONOV, S.A., KHARITONOV, Yu.I., Preprint LIYaF-116, Leningrad (1974).