IN7700788

INIS-mf--3875

INTERACTION OF A NEUTRAL GAS WITH A MOVING MAGNETIZED PLASMA - A THEORY

by

R. K. Varma
Physical Research Laboratory
Ahmedabad - 380 009
INDIA

1N7700788

INTERACTION OF A NEUTRAL GAS WITH A MOVING MAGNETIZED PLASMA - A THEORY

R.K. VARMA
Physical Research Laboratory
Ahmedabad 380009, INDIA.

Abstract

A theory for the interaction of a moving magnetized plasma with a neutral gas is developed here. It is shown that whereas the relative terminal velocity attained after the active phase of interaction is roughly given by $V_{term} = (2e\phi_{in}/M_n)^{1/2}$ (ϕ_{in} is the ionization potential of the neutral gas, and M_n is its mass) it is argued that a small plasma neutral gas interaction will not ensue unless the initial plasma velocity exceeds a certain threshold velocity given roughly by $V_{min} = (2e\phi_{in}/M_i)^{1/2}$ (M_i is the mass of the plasma ion). The "Alfven critical velocity" is thus to be identified with the terminal velocity V_{term} . The observed rapid deceleration of plasma is explained as being due to the sharing of the plasma momentum and energy by the freshly ionized stationary neutral atoms that are added to the plasma stream while a part of the energy is used up in heating the electrons which in turn ionize the neutral atoms. A model for the acceleration of electrons is given which explains both the triggering of the interaction and subsequent acceleration to the observed electron energies of 100 eV or more. It also

explains the observed threshold with respect to the neutral density. Further, since the acceleration occurs predominantly along the magnetic field in the model, it is also consistent with the observed polarization of the He I 4686 Å line. The model thus explains most of the observations in a natural way.

1. Introduction

In connection with his theory of "Hetegonic" processes Alfven (1954, 1960) has proposed that when a neutral gas moves through a magnetized plasma it is abruptly ionized if the relative velocity exceeds the value V_c given by

$$\frac{1}{2} M_n V_c^2 = e\,\phi_{ion} \qquad (1)$$

where M_n is the mass of the neutral atom and ϕ_{ion} is its ionization potential. Having become thus ionized it is then, according to Alfven, stopped by the magnetic field. The velocity V_c is known as the "Alfven critical velocity" and the above hypothesis is referred to as the "Alfven critical velocity hypothesis".

This hypothesis has an intuitive appeal from the point of view of energetics : In the situation considered above (with the plasma at rest and the neutral gas moving through it) the energy is contained in the motion of the neutrals. If the kinetic energy $\frac{1}{2} M_n V^2$ of the neutral atom can somehow be utilized for the ionization of the latter, then Eq. (1) would give the minimum velocity for the ionization to occur, provided there is no loss to the thermal component of the system.

As already pointed out by Alfven (1960) there exists the difficulty regarding the actual mechanism of transfer of energy from the neutral mass motion to the bound electrons of the neutral atoms so that the ionization could occur. There is, of course, the possibility of ionization by direct impact of the atoms with the plasma ions, but as discussed by Sherman (1973), and as will be discussed later here, the rate of ionization due to this process is far too low to account for the observations. Since electrons are much more efficient in ionizing the atoms than ions of the same energy, one should look for the transfer of energy via the electrons and therefore for a mechanism for energizing the electrons at the expense of the neutral mass motion. It appears, therefore, that the process of ionization in the above phenomenon would have to be a more complicated plasma physical process rather than the simple ion-neutral impact ionization.

Besides this difficulty, there is another basic difficulty, however, which we now discuss : If we assume the Galilean invariance of the above physical phenomenon and therefore of the relation (1), then the same relation should hold if we transform to the frame of the initial velocity of the neutral gas. In this frame the neutral gas will be at rest while the magnetized plasma moves through it. (Such a situation can be more easily obtained in the laboratory and has indeed been investigated by Danielsson (1970)). However, since the energy is now contained in the plasma ion motion (the plasma ion mass M_i being, in general, different from the neutral atomic mass), the relation (1) for the critical velocity is not at all intuitively obvious. It would appear that one should rather write the following relation instead

$$\frac{1}{2} M_i V^2 = e\,\phi_{ion} \qquad (2)$$

since one would argue that the kinetic energy is now contained in the plasma ion and this is to be utilized for the ionization of the neutral atom. We shall discuss the meaning and significance of the above two relations in the following sections of the paper.

There are a number of experiments which, in one form or another, exhibit the so-called "critical velocity phenomena". A good review of these has been given by L. Danielsson (1973). But the most direct experiment to study this phenomena has been performed by Danielsson (1970) and by Danielsson and Brenning (1975). The experimental results show that when a magnetized plasma moves through a neutral gas at rest, with an initial velocity <u>very much</u> exceeding that determined by Eq. (1), it gets decelerated and attains a <u>terminal</u> velocity equal to that determined by Eq. (1). The experiments have not established, however, that the terminal velocity is also the <u>threshold</u> velocity for the interaction between the plasma and the neutral gas to occur. In fact there seems to be an indication in the experiments that an active interaction does not occur until the initial relative velocity $V_0 \gtrsim 2V_c^{(1)}$ where $V_c^{(1)}$ is the value as determined from Eq. (1).

In this paper we present a theory of the interaction of a neutral gas with a moving magnetized plasma with particular reference to the direct interaction experiment of Danielsson. We show that the velocity given by Eq. (1) is essentially the relative terminal velocity attained after the interaction is over. On the other hand, the threshold relative velocity required for an active interaction between the plasma and the neutral gas to occur is shown to be given approximately by Eq. (2). For the gases used in the Danielsson experiment (H-plasma and neutral He) $V_c^{(2)} = 2V_c^{(1)}$, where $V_c^{(2)}$ is the value determined from Eq. (2). As noted above, there is an indication to this effect in the results of the Danielsson experiment. But more experiments must be carried out to establish this fact.

In Sec. 2, we review briefly the important observations recorded in the experiments of Danielsson (1970) and of Danielsson and Brenning (1975). In the subsequent sections we show how we can explain most of these observations on the basis of our model.

2. A Review of the Experiments of Danielsson and Danielsson and Brenning.

We present here the most important results of the experiments of Danielsson (1970) and of Danielsson and Brenning (1975). These experiments will, hereafter, be referred to as I and II respectively.

A magnetized hydrogen plasma of density $n_i \sim 3\times10^{11}$ cm^{-3} (stated to be $10^{11} - 10^{12}$ cm^{-3} in II) moving with a velocity of about 3.8×10^5 m sec^{-1} in I (and $4{-}5\times10^5$ m sec^{-1} in II with corresponding initial proton energy $\sim$ 800 - 1300 eV) across a magnetic field of 0.1 - 0.5 Vs m^{-2} (kept at a magnetic field of 0.1 - 0.5 Vs m^{-2} constant value of 0.18 Vs m^{-2} in II and results reported for the values 0.18 and 0.36 Vs m^{-2} in I) was allowed to interact with a stationary neutral He gas (density $\sim$ 0.2 - 1 $\times$ 10^{14} cm^{-3})

The velocity of the plasma was measured 1 cm. behind the centre of the gas cloud as reported in I (no such precise position is mentioned in II). The following observations were recorded:

A(i) For very low plasma velocity V, the plasma is not retarded, but for V $>$ 40 km sec^{-1}, the relative retardation increased with V. As reported in II, the V X B probe signal at the rear part of the gas cloud exhibits a constant velocity plateau after an initial transient part at a high velocity. The plateau is always very close to, but usually below the "critical velocity" $V_c^{(1)}$.

A(ii) The entire retardation to roughly one - tenth of the initial velocity takes place over distances of the order of a few centimeters (4 - 5 cms.). This corresponds to a time interval of about 0.2 μ sec. as calculated with a mean velocity of roughly 2×10^7 cms. sec^{-1}.

A(iii) The polarization electric field E related roughly to the plasma velocity across the magnetic field through $\underset{\sim}{V} = c\, \underline{E} \times \underline{B} / B^2$, was also found to decrease from its upstream value corresponding to the initial plasma velocity to a low downstream value over distances of a couple of centimeters.

B(i) Before the interaction the plasma electron temperature in the plasma gun region was reported in I to be $\sim$ 5 eV, (5-10 eV in II with an upper limit of 20 eV as it reaches the interaction region in the absence of the neutral gas) and the proton energy $\sim$ 840 eV corresponding to the velocity V = 4×10^7 cms. sec^{-1} (800 - 1300 eV in II). After the interaction, the electron temperature was found to be about 100 eV. (The value of 85 eV reported earlier in I is considered to be conservative in II). A significant part of the heating, according to II, has been formed to occur within a short distance in the upstream part of the gas cloud.

B(ii) Various probes, absolute intensity and 4 mm microwave measurements in I indicated that a substantial number of He^+ ions were produced in the interaction. Active particle analysis made in II using the further stripping of A^+ to A^{++} to measure the electron temperature and density gives an electron density $n_e \sim 2 \times 10^{12}$ with an uncertainty of ± 80%. This is about seven times the original electron density in the plasma.

C(i) No effect was observed on the retardation of the plasma by increasing the neutral density beyond 5×10^{13} cm^{-3}. On the other hand, retardation quickly disappeared when the gas density was reduced below 2.10^{13} cm^{-3}.

C(ii) Though magnetic field is necessary for retardation, doubling its strength from 0.18 to 0.36 V sec. m^{-2} had only a little effect.

D(i) The experimental points on $V_{z=1}$ vs V_o plot (V_o being the initial plasma velocity, and $V_{z=1}$ the velocity measured 1 cm behind the centre of the gas cloud) all lie above $V_{z=1} = V_c^{(1)}$ line in I. (In II they are reported to lie close to, though usually below, the line at a suitable point behind the centre). One of the points actually lies roughly on the $V_{z=1} = V_c^{(2)}$ line ($V_c^{(1)}$ is the value determined from Eq. (2)).

D(ii) Points corresponding to larger values of V_o lie closer to the $V_{z=1} = V_c^{(1)}$ line indicating that perhaps the interaction is more complete for larger values of V_o.

E(i) The He I 4686 Å line is found to be partially polarized with respect to the electric vector parallel and perpendicular to the magnetic field. The degree of polarization is estimated to be +0.07 for $^{1}D - {}^{1}P$ transitions. This is a new observation reported in II. The interpretation of the observed polarization is that the ionization by electron impact occurs more by electrons moving along the field than by those moving perpendicular to it implying that the electron acceleration takes place along the magnetic field.

3. The Model : General Considerations.

Several theories have been given from time to time to explain this and other experiments. A good review of these is given by Sherman (1973), who has also discussed the difficulties of the various theories in explaining the essential features of the experiment. A critique of these theories, therefore, need not be given here. It must be mentioned, however, that most of the theories reviewed were given before the Danielsson direct interaction experiment was carried out and therefore cannot be expected to apply to this particular experimental sotuation. We shall be concerned here primarily with a theory for this direct interaction experiment, which gives more details of the process of interaction. A specific theory can thus be attempted against this detailed information.

As already mentioned in Sec. 1, we distinguish the two velocities in connection with the interaction:

(i) A minimum (threshold) relative velocity V_{min} (or plasma velocity when the neutrals are at rest) between the plasma and the neutral gas required before the interaction can take place.

(ii) The terminal velocity V_{term} (as measured, say 1 cm. behind the centre of the gas cloud) that the plasma attains after the completion of the interaction, when the initial velocity V_o exceeds the minimum needed for the interaction. It is not at all clear, a priori, that these velocities are equal. We in fact show using simple energy balance considerations in Sec. 5 that V_{min} is given roughly by Eq. (2). The terminal velocity, V_{term}, on the other hand, is shown in Sec. 4 to be approximately given by Eq. (1).

An adequate theory of the interaction of the neutral gas with a moving magnetized plasma must satisfactorily explain the following three associated observations on the observed time scales.

1. heating of the electrons to the required degree on a time scale $\sim$ 0.12 μ sec, since the heating has been found to occur (as reported in II) within a short distance in the upstream part of the gas cloud.

2. Ionisation of the neutral gas to the extent observed.

3. retardation of the plasma stream over distances of 4 - 5 cms. to the observed terminal velocity.

As the plasma comes into contact with the neutral gas the most important "classical" modes of interaction between the plasma ions and the neutral atoms are the following:

(i) The plasma ions losing energy and momentum to the neutrals through elastic collisions.

(ii) Charged exchange collisions between the plasma H^+ ions and the neutral He atoms.

(iii) Inelastic collisions resulting in the excitation and ionization of the helium atoms.

Table 1 gives the collision frequencies and mean free paths for the last two of these processes for the neutral gas density of $N.10^{14}$ cm^{-3}. For the plasma velocity of 4×10^7 cms. sec^{-1}, the time of passage of the plasma through the neutral gas cloud of dimensions $\sim$ 10 cms. is $\sim$ 0.25 μ sec. which is much shorter than the collision times 20 - 30 μ sec even at the neutral density 10^{14} cm^{-3} for these collisions. Similarly, the mean free paths for these collisions $\sim$ 400 - 700 cms. are much longer compared with the dimensions of the gas cloud $\sim$ 10 cm. It is thus clear that these processes involving direct impact of the ions with the neutral atoms can explain neither the deceleration of the plasma nor the ionization of the neutral atoms.

The electrons, on the other hand, have mean free paths for momentum transfer and ionization collisions with the neutral gas atoms (density $\approx 10^{14}$ cm^{-3}) which are comparable with the dimensions of the region of interaction (see Table 1). In fact, since the electrons move through the neutral gas with the E X B drift velocity, the mean free path along the plasma drift motion is $\lambda_{eff} \lesssim 1$ cm. for the typical electron energies of 20 - 100 eV in the system. These electron-neutral collisions are thus the only "classical" collisions that would occur in the system of the dimensions involved, and could thus be considered as providing a triggering mechanism for the interaction. Also since electrons of a given energy are much more efficient in ionizing the neutrals compared to protons of the same energy we shall consider the electrons to be the basic ionizing agency in explaining the observed ionization of the neutrals in the experiment.

3.1 An Outline of the Model

We give here an outline of the model which we discuss in detail in subsequent sections. The model is able to explain most of the observations relating to the experiment in a very natural way.

It will be observed that the three associated observations, (i) acceleration of the electrons, (ii) ionization of neutral helium, and (iii) the deceleration of the plasma are causally related in that order. The deceleration of the plasma occurs as the freshly ionized stationary helium atoms are added to the plasma stream and share its momentum. Thus the ionization of helium atoms is necessary for deceleration, and this in turn requires the acceleration of electrons to appropriate energies for the purpose.

The observed electron energies are, of course, of the order of 100 eV in the experiment [Observation B (ii)] and further, observation E (i) implies that the energization of electrons is anisotropic, being more along the magnetic field than across it. Acceleration mechanisms which not only account for this feature but also other characteristics listed in Sec. 2, have to be found.

A completely ionized plasma moving across a magnetic field B (such as the one obtained in the Danielssen experiment) has a polarization electric field E in the laboratory frame such that

$$\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{V} \times \underset{\sim}{B} = 0 \tag{3}$$

The velocity V is essentially the electric field drift velocity $\underset{\sim}{V} = c\,\underset{\sim}{E} \times \underset{\sim}{B}/B^2$, and the magnetic field is assumed to be large enough for this approximation to be valid. The electrostatic energy per unit volume in the plasma is $W_E = \epsilon E^2/8\pi$, where $\epsilon = 1 + 4\pi n M_i c^2/B^2$ is the low frequency dielectric constant of the plasma and is assumed to be large $\epsilon \gg 1$. We then have

$$W_E = \left(4\pi n M_i c^2/B^2\right)\frac{E^2}{8\pi} = \frac{1}{2}\, n M_i \left(cE/B\right)^2 \qquad (4)$$

Since cE/B is the magnitude of the plasma velocity $\underset{\sim}{V}$ and nM_i is its mass density W_E is essentially the kinetic energy of the mass motion of the plasma per unit volume. This electrostatic field energy W_E is a source of energy (around 3.10^{15} eV cm^{-3}, and about 900 eV per proton) which can be tapped for the energization of electrons so that they can in turn ionize the helium atoms.

One may point out here that the plasma energy is almost entirely contained in the massive ions ($\sim$900 eV per protons as against $\lesssim$ 10 eV per electron) and as such the acceleration of electrons must necessarily be at the expense of the energy of the protons, which is the same thing as the electrostatic field energy according to Eq. (4). One has thus to find out mechanisms to transfer the energy from the protons to the electrons through some electric fields which are manifestations of the collective behaviour of the plasma.

It is shown in Sec. 6 that the magnetic field plays a crucial role in producing such space charge electric field because of the widely different gyroperiods of the electrons and the ions. An acceleration mechanism for the electrons is presented which accelerates the electrons very efficiently along the field lines to the required energies and over a short distance within the gas cloud. It thus appears that the acceleration along the field lines is already adequate. However, since the observed polarization of the helium line is $\sim 0.07\%$, the acceleration normal to the field does seem to occur to some extent.

One way for the electrons to gain energy normal to the magnetic field is for them to gain energy directly from the polarization electric by moving antiparallel to it (which direction is normal to the magnetic field). However, so long as the plasma is collisionless and free from electrostatic fluctuations, the electrons will not be able to move across the magnetic field. The collisions of the electrons with the neutrals will, however, enable them to move across the field. But as is shown in the Appendix, for the initial electron energy of $\sim$ 10 eV and a neutral density $n_n = 5 \times 10^{13}$ cm^{-3}, the electron neutral collision frequency ν_{en} falls short by a factor of about 120 to explain the observed electron energies of 100 eV gained in a time $\sim 0.1\,\mu$sec, assuming that the entire energy is gained in this manner. This is, of course, in conformity with the observation E (i) which favours acceleration along the field line over that across it. Nevertheless, since some acceleration across the field line does occur an enhanced collision frequency ν^* for the electrons has to be invoked resulting probably from field fluctuations arising out of an instability. The acceleration along the field line must, however, be considered as the primary acceleration process.

4. Dynamics of Plasma-Neutral Gas Interaction

We shall now consider the dynamics of the plasma neutral gas interaction. In the following analysis we shall assume that an enhanced electron collision frequency ν^* exists which causes an enhanced transport of electrons across the electric field. We shall also presuppose the acceleration mechanism along the field lines, described in Sec. 6. The resulting energetic electrons will then cause ionization of the neutral atoms.

In this section we shall find an expression for the terminal velocity and show that it is indeed related to the neutral gas atomic mass M_n as is required by Eq. (1).

4.1. Terminal Velocity.

To determine the expression for the terminal velocity we shall use the equation of continuity and equations of momentum and energy balance for the plasma in interaction with the neutral gas. To derive these equations we start from the Boltzmann equations for the various species of particles with the B.G.K. type of collision terms (Bhatnagar et al. (1954), Gross and Krook (1956), Bhatnagar (1961)) for simplicity. We consider only two kinds of ions, the original hydrogen plasma ions, and the ions from the ionized neutral (helium) gas, though a generalatization to many ions case is straight-forward.

Let f_i and f_n^+ denote respectively the distribution functions for these two species and let f_e be the electron distribution function, and f_n the distribution function for the neutral gas. The respective Boltzmann equations are then:

$$M_i\left(\frac{\partial f_i}{\partial t} + \underset{\sim}{v}\cdot\frac{\partial f_i}{\partial x}\right) + e\left(\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{v}\times\underset{\sim}{B}\right)\cdot\frac{\partial f_i}{\partial \underset{\sim}{v}}$$

$$= -\nu_{in} M_i f_i \qquad \text{(5a)}$$

$$M_n\left(\frac{\partial f_n^+}{\partial t} + \underset{\sim}{v}\cdot\frac{\partial f_n^+}{\partial \underset{\sim}{X}}\right) + e\left(\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{v}\times\underset{\sim}{B}\right)\cdot\frac{\partial f_n^+}{\partial \underset{\sim}{v}}$$

$$= \nu_{in}\frac{n_i}{n_n} M_n f_n + Q\frac{n_e}{n_n} M_n f_n \qquad \text{(5b)}$$

$$m_e\left(\frac{\partial f_e}{\partial t} + \underset{\sim}{v}\cdot\frac{\partial f_e}{\partial \underset{\sim}{x}}\right) - e\left(\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{v}\times\underset{\sim}{B}\right)\cdot\frac{\partial f_e}{\partial \underset{\sim}{v}}$$

$$= m_e\left(\frac{\delta f}{\delta t}\right)_{ion} - m_e\nu_{en}\left(f_e - f_{el}^{n}\right) - m_e\nu^*\left(f_e - f_{el}^{*}\right) \tag{5c}$$

where M_i, M_n and m_e are respectively the plasma ion, neutral gas ion, and electron masses, and n_i, n_n, and n_e are the respective number densities; ν_{in} represents the charge exchange collision frequency of the ions with the neutrals and is taken here to be velocity independent for simplicity. The collision term on the right hand side of (5a) clearly represents the loss of H^+ ions, mass M_i, due to charge exchange collisions, whereas the first collision term on the right of Eq. (5b) represents the gain of He^+ ions, mass M_n due to these collisions. The second term on the right of Eq. (5b) represents the production of He^+ ions due to ionization by electron impact with the rate Q. Finally, the collision terms on the right of the electron Boltzmann equation are as follows: The first term represents the rate of change of f_e due to ionization. The second represents the change of f_e due to collisions with the neutrals as a result of which the electron distribution will relax to a locally Maxwellian distribution f_{el}^{n}, with a mean velocity equal to that of the neutral gas. The third term similarly represents the relaxation of f_e to another locally Maxwellian distribution f_e^* with a zero mean velocity, as a consequence of collisions with the electric field fluctuations.

The rate of ionization Q is assumed to be solely due to the energetic electrons since electrons are known to have a much greater ionization efficiency compared with the ions with the same energy (von Engel (1955)); and the maximum of the ionization probability comes from electrons with energy roughly equal to twice the ionization energy of the neutrals. The term $(\delta f_e/\delta t)$ which represents the rate of change of f_e due to ionization is thus obviously related to Q through

$$n_e Q = \int d^3v \left(\delta f_e/\delta t\right)_{ion}. \tag{6}$$

The epxression for $(\delta f_e/\delta t)_{ion}$ itself is given by

$$\begin{aligned}\left(\delta f_e/\delta t\right)_{ion} = -\int d\underset{\sim}{v}' d\underset{\sim}{v}'' \Big[&\Theta\left(\tfrac{1}{2} m v^2 - e\phi_{ion}\right) f_e(\underset{\sim}{v}) \\ &\frac{n_n v}{2v'}\, g\left(\underset{\sim}{v}, \underset{\sim}{v}', \underset{\sim}{v}''\right) \delta\left(v^2 - v'^2 - v''^2 - 2e\phi_{ion}/m_e\right) \\ -\tfrac{1}{2} n_n\, &\Theta\left(\tfrac{1}{2} m_e v^2 - e\phi_{ion}\right)\left\{ g\left(\underset{\sim}{v}', \underset{\sim}{v}, \underset{\sim}{v}''\right) + g\left(\underset{\sim}{v}', \underset{\sim}{v}'', \underset{\sim}{v}\right)\right\} \\ &f_e(\underset{\sim}{v}')\, \delta\left(v'^2 - v^2 - v''^2 - 2e\phi_{ion}/m_e\right)\Big].\end{aligned} \tag{7}$$

where $g(\underset{\sim}{v}, \underset{\sim}{v}', \underset{\sim}{v}'')$ represents the probability that an electron with velocity $\underset{\sim}{v}$ ionizes the atom with the scattered electron having the velocity $\underset{\sim}{v}'$ and the ejected electron, the velocity $\underset{\sim}{v}''$ (Rudge and Seaton (1965), Rudge (1968)). The conservation of energy in the process is given by

$$\frac{1}{2} m v^2 = \frac{1}{2} m v'^2 + \frac{1}{2} m v''^2 + e\phi_{in} . \tag{8}$$

Thus the δ-functions in the integrands take care of the energy conservation while the Θ functions are the Heaviside step functions which are such that

$$\begin{aligned} \Theta\left(\tfrac{1}{2} m v^2 - e\phi_{in}\right) &= 1 \quad \text{for } \tfrac{1}{2} m v^2 \geqslant e\phi_{in} \\ &= 0 \quad \text{for } \tfrac{1}{2} m v^2 < e\phi_{in} \end{aligned} \tag{9}$$

They describe the fact that no ionization is possible unless $\frac{1}{2} m v^2 \geqslant e\phi_{in}$. The first term in the expression (7) represents the loss of an electron with the energy $\frac{1}{2} m v^2 \geqslant e\phi_{in}$ while the other two terms represent respectively the emergence of the scattered and the ejected electrons at the velocity $\underset{\sim}{v}$ in the interval $d\underset{\sim}{v}$. These two electrons are, as expected, symmetric in $\underset{\sim}{v}'$ and $\underset{\sim}{v}''$ since there is no way of distinguishing them.

Moment Equations.

To discuss the dynamics of the plasma as the freshly ionised helium ions are added to it during the process of interaction. We shall obtain one-fluid equations for the hydrogen and helium plasma considered as one fluid. To do this we start from the composite Boltzmann equation obtained by adding Eqns. (5a), (5b) and (5c). This gives

$$\left(\frac{\partial}{\partial t} + \underset{\sim}{v}\cdot\frac{\partial}{\partial \underset{\sim}{x}}\right)\sum_j M_j f_j + \left(\underset{\sim}{E} + \frac{1}{c}\,\underset{\sim}{v}\times\underset{\sim}{B}\right)\cdot\frac{\partial}{\partial \underset{\sim}{v}}\sum_j e_j f_j$$

$$= -\nu_{in}\left(M_i f_i - \frac{n_i}{n_n} M_n f_n\right) + Q M_n \frac{n_e}{n_n} f_n$$

$$- \nu_{en} m_e\left(f_e - \frac{n_e}{n_n} f_{el}^{(n)}\right) - \nu^* m_e\left(f_e - f_{el}^{*}\right)$$

$$+ m_e\left(\delta f_e/\delta t\right)_{ion} \qquad (10)$$

The sum on the left hand side includes the sum over the electrons and all the species of ions in the system. We note that the first two terms on the left hand side are entirely inertial : $\sum_j M_j f_j$ represents mass density per unit phase volume. The last two terms are electromagnetic : $\sum_j e_j f_j$ represents charge density per unit phase volume. The various moments of Eq. (10) are obtained below:

Zeroeth moment gives:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \underset{\sim}{V} \right) = -\nu_{in} n_i \left(M_i \underset{\sim}{V_i} - M_n \underset{\sim}{V_n} \right) + Q \left(n_e m_i + n_e M_n \right) . \tag{11}$$

where

$$\begin{aligned} \rho &= \sum_j M_j \int d^3 v \, f_j \\ \rho \underset{\sim}{V} &= \sum M_j \int d^3 v \, \underset{\sim}{v} \, f_j \end{aligned} \tag{12}$$

are the mass and momentum densities of the composite plasma and where the definition (6) for Q has been made use of.

First moment gives the equation for the momentum balance

$$\begin{aligned} &\frac{\partial}{\partial t} \left(\rho \underset{\sim}{V} \right) + \nabla \cdot \left[\rho \underset{\sim}{V} \underset{\sim}{V} + \overleftrightarrow{P} \right] - \left(\sigma \underset{\sim}{E} + \frac{1}{c} \underset{\sim}{J} \times \underset{\sim}{B} \right) \\ &= -\nu_{in} n_i \left(M_i V_i - M_n V_n \right) + Q n_e M_n \underset{\sim}{V_n} \\ &\quad - \nu_{en} m_e n_e \left(\underset{\sim}{V_e} - \underset{\sim}{V_n} \right) - \nu^* n_e m_e \underset{\sim}{V_e} \\ &\quad - Q n_e m_e \left(\underset{\sim}{V_e} - 2 \underset{\sim}{V_n} \right) . \end{aligned} \tag{13}$$

where $\underset{\sim}{V_i}$, $\underset{\sim}{V_e}$ and $\underset{\sim}{V_n}$ are respectively the fluid velocities of the original ion species (hydrogen), electrons and neutrals.

The ionizing electrons and the electrons ejected during the process of ionisation are assumed to emerge isotropically in the centre of mass frame which is essentially the frame of the neutral atom, involved in the collision. Thus the fluid velocity of the emerging electrons , the mean velocity with the function $f_{ei}^{(n)}$ will be essentially $\underset{\sim}{V}_n$. The locally Maxwellian velocity distribution f_{ei}^{*} is, however, assumed to be centered around zero mean velocity.

The pressure tensor $\overleftrightarrow{P}$ is defined by

$$\overleftrightarrow{P} = \int d^3v \left(\underset{\sim}{v} - \underset{\sim}{V}\right)\left(\underset{\sim}{v} - \underset{\sim}{V}\right) \sum_j M_j f_j \tag{14}$$

and charge and current densities σ and $\underset{\sim}{j}$ by

$$\begin{aligned} \sigma &= \int d^3v \sum_j e_j f_j \\ \underset{\sim}{j} &= \int d^3v \sum_j e_j f_j \underset{\sim}{v} \end{aligned} \tag{15}$$

Expanding the left hand side of Eq. (13) and using the equation of continuity (11) we obtain:

$$\begin{aligned} \rho\left(\frac{\partial \underset{\sim}{V}}{\partial t} + \underset{\sim}{V}\cdot\nabla\underset{\sim}{V}\right) + \nabla\cdot\overleftrightarrow{P} - \frac{1}{c}\underset{\sim}{J}\times\underset{\sim}{B} &= -n_e M_n Q\left(\underset{\sim}{V} - \underset{\sim}{V}_n\right) \\ &+ \nu_{in} n_i\left[M_i\left(\underset{\sim}{V} - \underset{\sim}{V}_i\right) - M_n\left(\underset{\sim}{V} - \underset{\sim}{V}_n\right)\right] \\ &- n_e m_e\left[\nu_{en}\left(V_e - \underset{\sim}{V}_n\right) + \nu^* \underset{\sim}{V}_e + Q\left(\underset{\sim}{V}_e - 2\underset{\sim}{V}_n\right)\right] \end{aligned} \tag{16}$$

where we have made use of the charge neutrality $\sigma \approx 0$.

Similarly the second moment gives the equation for the energy balance:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho V^2 + \frac{1}{2}\rho \overline{C^2}\right) + \nabla\cdot\left\{\frac{1}{2}\rho(V^2 + \overline{C^2})\underset{\sim}{V}\right.$$

$$\left. + \rho \underset{\sim}{V}\cdot\overline{\underset{\sim}{C}\underset{\sim}{C}} + \frac{1}{2}\rho\overline{C^2\underset{\sim}{C}}\right\} - \underset{\sim}{J}\cdot\underset{\sim}{E} =$$

$$-\nu_{in} n_i\left[\frac{1}{2}M_i\left(V_i^2 + \overline{C_i^2}\right) - \frac{1}{2}M_n\left(V_n^2 + \overline{C_n^2}\right)\right]$$

$$+\frac{1}{2}Q n_e M_n\left(V_n^2 + \overline{C_n^2}\right) - \frac{1}{2}\nu_{en} m_e n_e\left[\left(V_e^2 + \overline{C_e^2}\right)\right.$$

$$\left. - \left(V_n^2 + \overline{C_e^{(n)2}}\right)\right] - \frac{1}{2}\nu^* n_e m_e\left[\left(V_e^2 + \overline{C_e^2}\right) - \overline{C_{eL}^2}\right]$$

$$- Q n_e e \phi_{ion} . \qquad (17)$$

where $\overline{C_e^2}$, $\overline{C_e^{(n)2}}$ and $\overline{C_{eL}^2}$ are mean squared thermal velocities corresponding to the distributions f_e $f_{eL}^{(n)}$ and f_{eL}^* respectively. The energy conservation in elastic collisions with the neutrals and with the electric field fluctuations in the collision model used make the third and fourth terms on the right hand side of Eq. (17) vanish.

Expanding the right hand side of (17) and making use of Eqs. (11) and (16) gives:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho \overline{C^2}\right) + \nabla\cdot\left\{\frac{1}{2}\rho \overline{C^2}\,\underset{\sim}{V} + \underset{\sim}{q}\right\} + \overrightarrow{P} : \nabla \underset{\sim}{V}$$

$$- \underset{\sim}{J}\cdot\left(\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{V}\times\underset{\sim}{B}\right) = -\nu_{in}\, n_i \left\{\frac{1}{2} M_i\left[\left(\underset{\sim}{V}-\underset{\sim}{V_i}\right)^2 + \overline{C_i^2}\right]\right.$$

$$\left. - \frac{1}{2} M_n\left[\left(\underset{\sim}{V}-\underset{\sim}{V_n}\right)^2 + \overline{C_n^2}\right]\right\}$$

$$+ Q n_e\left\{\frac{1}{2} M_n\left[\left(\underset{\sim}{V}-\underset{\sim}{V_n}\right)^2 + \overline{C_n^2}\right] - e\phi_{ion} + m_e \underset{\sim}{V}\cdot \underset{\sim}{V_e}\right\}$$

$$+ n_e m_e\left(\nu_{en} + \nu^*\right)\underset{\sim}{V}\cdot\underset{\sim}{V_e}$$

$$- \nu_{en}\, n_e m_e\, \underset{\sim}{V}\cdot\underset{\sim}{V_n}\,. \qquad (18)$$

where $\underset{\sim}{q}$ is the heat flux vector given by

$$\underset{\sim}{q} = \frac{1}{2}\rho\,\overline{C^2\underset{\sim}{C}}\,. \qquad (19)$$

The quantity $\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{V}\times\underset{\sim}{B}$ in the Eq. (18) can be substituted for from the generalized Ohm's law which can be obtained from the Boltzmann equations (5a) and (5c) as:

$$
\begin{aligned}
\underset{\sim}{E} + \frac{1}{c}\underset{\sim}{V} \times \underset{\sim}{B} = {} & \frac{1}{n_e e c}\, \underset{\sim}{J} \times \underset{\sim}{B} - \frac{1}{n_e e} \nabla \cdot \overleftrightarrow{P} \\
& + \frac{m_e}{n_e e^2} \frac{\partial \underset{\sim}{J}}{\partial t} + \frac{m_e}{n_e e^2}\left(\nu_{en} + \nu^* + Q \right) \underset{\sim}{J} \\
& - \frac{m_e}{e}\left(\nu_{en} + \nu^* + Q - \nu_{in} \right) \underset{\sim}{V} \\
& + \frac{m_e}{n_e e^2}\left[n_e\left(Q + \nu_{en} \right) - n_i \nu_{in} \right] \underset{\sim}{V}_n
\end{aligned} \tag{20}
$$

Using Eq. (20) in Eq. (18) we get:

$$
\begin{aligned}
& \frac{\partial}{\partial t}\left(\frac{1}{2}\rho C^2 \right) + \nabla \cdot \left\{ \frac{1}{2}\rho C^2 \underset{\sim}{V} + \underset{\sim}{q} \right\} + \overleftrightarrow{P} : \nabla \underset{\sim}{V} = \\
& \nu_{in} n_i \left\{ \frac{1}{2} M_n \left[\left(\underset{\sim}{V} - \underset{\sim}{V}_n \right)^2 + \overline{C_n^2} \right] - \frac{1}{2} M_i \left[\left(\underset{\sim}{V} - \underset{\sim}{V}_i \right)^2 + \overline{C_i^2} \right] \right\} \\
& + Q n_e \left\{ \frac{1}{2} M_n \left[\left(\underset{\sim}{V} - \underset{\sim}{V}_n \right)^2 + \overline{C_n^2} \right] \right. \\
& \left. + m_e \underset{\sim}{V} \cdot \underset{\sim}{V}_e - e \phi_{ion} \right\}
\end{aligned} \tag{21}
$$

where the terms $\underset{\sim}{J} \cdot \underset{\sim}{V}$, $\underset{\sim}{J} \cdot \underset{\sim}{V}_n$ and $\underset{\sim}{J} \cdot (\nabla \cdot \overleftrightarrow{P})$ drop out since in this problem the current can only be perpendicular to $\underset{\sim}{V}$, $\underset{\sim}{V}_n$ and $\nabla \cdot \overleftrightarrow{P}$, for otherwise it would imply a charge separation in the direction of flow which cannot be sustained. Also terms small in the mass ratios m_e/M_i , m_e/M_n have been neglected.

Consider now the terms on the right hand side of Eq. (21). The first term involves ν_{in} the $H^+ - He$ charge exchange collision frequency which, as is seen from Table 1, is at least one order of magnitude smaller compared to Q, the coefficient of the second term. The rate of change of plasma thermal energy is thus essentially determined by the second term on the right hand side. The heating will accordingly continue and the plasma will decelerate until

$$\frac{1}{2} M_n \left[\left(\underset{\sim}{V} - \underset{\sim}{V}_n \right)^2 + \bar{C}_n^2 \right] = e \phi_{ion} - m_e \underset{\sim}{V} \cdot \underset{\sim}{V}_e \tag{22}$$

Since the thermal energy derives from the deceleration of the plasma, Eq. (22) gives the limiting or terminal velocity of the plasma. If we assume the neutral gas to be cold, $C_n = 0$, the small electron term $m_e \underset{\sim}{V} \cdot \underset{\sim}{V}_e$, we get the relation

$$\frac{1}{2} M_n \left(\underset{\sim}{V} - \underset{\sim}{V}_n \right)^2 = e \phi_{ion} \tag{23}$$

for the terminal velocity. This relation is identical with the Alfven relation (1). We have thus demonstrated that Eq. (1) or (23) gives the terminal velocity of the plasma after the interaction is complete. This is indeed what is shown by the experiments.

5. The Critical (threshold) Velocity for Interaction.

As discussed in Sec. 3.1, the critical velocity for the interaction to initiate is not the same as the terminal velocity discussed in the preceding section. We noted that the ionization of the neutrals by electron impact plays a crucial role in the interaction leading to the subsequent deceleration of the plasma. Accordingly, the interaction will not ensue until the energy gain by the electrons for a given velocity of the plasma (relative to the neutrals) exceeds the ionization energy of the neutrals. Since the only source of energy for the electrons to gain energy from is the kinetic energy of the plasma mass motion, or equivalently the electrostatic field energy $W_E = \epsilon E^2/8\pi$ $\epsilon = 4\pi n M_i c^2/B^2$ (See Sec. 3.1) there would exist a minimum plasma velocity below which ionization will not occur and the interaction described in Sec. 4 will not take place. We determine this minimum velocity in this section.

For the purpose of this analysis we add the second moment equations (the energy balance equations) of the Boltzmann equations (5a) and (5c). Since we are considering conditions for ionization the electron production due to ionization $(\delta f_e/\delta t)_{ion}$ will not contribute when the electron energy is below the threshold. If we substitute for $\underset{\sim}{J}$ from the Maxwell equation

$$\underset{\sim}{J} = -\frac{1}{4\pi}\frac{\partial \underset{\sim}{E}}{\partial t} + \frac{c}{4\pi}\,\mathrm{curl}\,\underset{\sim}{B}$$

$$\approx -\frac{1}{4\pi}\frac{\partial \underset{\sim}{E}}{\partial t} \tag{24}$$

assuming no appreciable self magnetic field to develop in the plasma, then we get

$$\frac{\partial}{\partial t}\left\{\sum \frac{1}{2}\rho_j\left(V_j^2+\overline{C_j^2}\right) + \frac{1}{8\pi}E^2\right\}$$

$$+\nabla\cdot\left\{\sum_j\left[\frac{1}{2}\rho_j\left(V_j^2+\overline{C_j^2}\right)\underset{\sim}{V_j} + \rho_j \underset{\sim}{V_j}\cdot\overline{\underset{\sim}{C_j}\underset{\sim}{C_j}}\right.\right.$$

$$\left.\left.+\frac{1}{2}\rho_j\overline{C_j^2\underset{\sim}{C_j}} + \underset{\sim}{q_j}\right]\right\} + \sum_j \overleftrightarrow{P}:\nabla\underset{\sim}{V_j}$$

$$= -\frac{1}{2}\nu_{in}\,n_i M_i\left(V_i^2+C_i^2\right) \tag{25}$$

Since the electron and ion motion are essentially the EXB drifts the last term on the left hand side which represents compression vanishes, because the EXB motion is divergence free. If we now integrate Eq. (34) over the whole space we get

$$\frac{d}{dt}\int d\vec{x}\left\{\sum_j \frac{1}{2}\rho_j\left(V_j^2+\overline{C_j^2}\right)+\frac{1}{8\pi}E^2\right\}$$

$$=-\frac{1}{2}\nu_{in}M_i\int d^3x\; n_i\left(V_i^2+\overline{C_i^2}\right) \tag{26}$$

Furthermore, since $V_i \simeq V_e = cE\times B/B^2$, then

$$\frac{1}{2}\rho_i V_i^2+\frac{1}{2}\rho_e V_e^2+\frac{1}{8\pi}E^2$$

$$=\left[1+4\pi(n_iM_i+n_em_e)c^2/B^2\right]\frac{E^2}{8\pi}$$

$$=\epsilon E^2/8\pi\,. \tag{27}$$

where ϵ is the dielectric constant of the plasma. Thus, as already stated in Sec. 3.1, expression (27) represents the electrostatic field energy density of the plasma. When $\epsilon \gg 1$, which is true in most cases including the present one, most of the field energy is contained in the $E \times B$ drift of the ions, Eq. (26) may then be written as

$$\frac{d}{dt}\int d\vec{x}\left\{\sum_j \frac{1}{2}\rho_j\overline{C_j^2}+\epsilon E^2/8\pi\right\}$$

$$=-\nu_{in}\int d\vec{x}\left(\frac{1}{2}\rho_i\overline{C_i^2}+\epsilon E^2/8\pi\right) \tag{28}$$

The integral on the left hand side represents the total thermal and field energy of the plasma, whereas the right hand side represents its loss to the neutrals due predominantly to ion-neutral charge exchange collisions. In the absence of the latter Eq. (28) then represents the conservation of the total plasma energy including the electron and ion thermal energies, the kinetic energy of mass motions and the field energy.

The energization of the electrons during the process of interaction has necessarily to come from the kinetic energy of the plasma ions. If , as the most favourable case, we assume that (i) the energy is lost from the ion motion and gained entirely by the electrons as the thermal energy without any being taken up by the ions as their thermal energy. (ii) The complete transfer of energy takes place over a very short time in the region of interaction (the actual mechanism not being specified or relevant for our prupose here). Then comparing the energy per unit volume before and after the discharge, we get from the conservation of energy, Eq. (28), (neglecting the loss term as being small)

$$\frac{1}{2}\left(n_i M_i + n_e m_e\right) V_0^2 + \frac{1}{2} n_i M_i \overline{C_{io}^2} + \frac{1}{2} n_e m_e \overline{C_{eo}^2}$$

$$= \frac{1}{2} n_e m_e C_{ef}^2 + \frac{1}{2} n_i M_i \overline{C_{i0}^2} \qquad (29)$$

where the subscript 'zero' refers to the initial values and 'f' to the final values.

Now for a Maxwellian distribution with temperature , the number of electrons per unit volume n', having the energy greater than the ionization energy $e\phi_{ion}$ is

$$n' = n_0 \exp\left[-e\phi_{ion}/T_e\right] \qquad (30)$$

Thus for a significant ionization by electron impact to occur one must have $T_e > e\phi_{ion}$. Using this in Eq. (29) we obtain+

$$\frac{1}{2} M_i V_0^2 > \frac{3}{2}\left(e\phi_{ion} - T_{e0}\right) \qquad (31)$$

as the condition on the initial plasma velocity V_0 so that significant ionization will occur for the subsequent interaction to take place actively. If we neglect the initial electron temperature T_{e0} then Eq. (31) is, except for a numerical factor, the same as the relation (2). In fact, because of this numerical factor, Eq. (31) gives a somewhat higher threshold than that given by Eq. (2). Furthermore, because of the assumptions (i) and (ii) made above and because of the loss term neglected in Eq. (28) to arrive at Eq. (29), Eq. (31) actually provides a lower limit on the threshold velocity V_0.

+ This assumes isotropic acceleration of the electrons; since we replace $\frac{1}{2} m C_{if}^2$ by $3/2\, T_e$ (which should be $\frac{1}{2} T_e$ or T_e for accelerations in one or two dimensions).

We thus see that Eq. (31) is the one that gives the critical velocity (lower limit) for interaction rather than Eq. (1) which gives the terminal velocity attained after the interaction. In fact, the threshold velocity V_c in the case of the present experiment is about at least twice as large as the terminal velocity as determined from Eq. (1) or (23). The experiment indeed shows that only for $V_0 \gg V_c^{(1)}$ do the experimental points lie closer to the $V_{z=l} = V_c^{(1)}$ line. For smaller plasma velocities (but still greater than $V_c^{(2)}$) the departure is large. One of the experimental points, for $V_0 = 1.7 \times 10^7$ cm sec^{-1} in fact, corresponds to $V_{z=l} \simeq$ [illegible] $V_c^{(1)}$, and thus rather falls on $V_{z=l} = V_c^{(2)}$ line. Recall that $V_c^{(1)}$ and $V_c^{(2)}$ refer to values as determined from Eqs. (1) and (2) respectively. This can be explained in terms of our model by saying that the interaction does not take place unless exceeds V_c , or more correctly the threshold velocity as determined from Eq. (31) and not strongly unless $V_0 \gg V_c^{(2)}$ [illegible] $V_c^{(1)}$. Only when it does so, does the terminal velocity have a value lying somewhere close to the Alfven value $V_c^{(1)}$.

Thus the results of the Danielsson experiment do seem to support the existence of a threshold velocity for the interaction, the lower limit of which is given by Eq. (31). No attempt has, however, been made to measure this threshold explicitly in the experiment.

6. Mechanism for the heating of electrons

The ionisation of the neutral He atoms, as is clear from the analysis in Sec. 5, plays a crucial role in the interaction process leading to the deceleration of the plasma, and the observed short deceleration time requires a high rate of ionisation. As pointed out earlier such high ionisation rates can arise only from ionisation by electron impact.
For a significant ionisation to occur the electrons must be heated to at least 24.5 eV, the ionisation potential for He. In fact, to be able to explain the experimental observations the electron must be heated to about 100 eV over the interaction length of 4.5 cms.

Various kinds of proposals have been made to heat the electrons to the required temperature, e.g. Lehnert's (1966) space charge theory where the electrons are supposed to gain energy from the space charge, or Sherman's (1970), (1972), (1973) version of the Buneman two stream instability where the electric field fluctuations are supposed to accelerate the electrons. There are some basic difficulties with both of these processes some of which are discussed by Sherman (1973).

The important basic question in this connection is what could be the basic triggering mechanism for the interaction which could lead subsequently to the acceleration of electrons to observed energies, with the acceleration along the magnetic field dominating that normal to it. Again, a glance at Table 1 shows that for the initial state of the plasma (proton energy $\sim$ 1000 eV, electron energy $\sim$ 10 eV) the ionization frecueny by electron impact still dominates all other frequencies. The proton - He charge exchange collisions will result in but a small deceleration of the plasma but no ionization. The combined ionization produced by both the electron and proton impact at the initial energies is the only effect that the initial phase of interaction would produce and must be considered as the basic triggering mechanism.

We describe here a simple model whereby making use of this initial ionization of helium we are able to accelerate electrons along the field lines to high enough energies to cause further rapid ionization of helium.

Let the plasma front PP' in Fig. 1 penetrate the neutral gas across its boundary NN'. As a result of the encounter a fraction of the helium atoms in the region $NN'P'P$ get ionized. The newly created electrons from the ionization quickly acquire the $E \times B$ drift velocity in about an electron gyroperiod $\sim$ 0.20 nsec. (in a magnetic field B = 1800 G), while the massive helium ions with a gyroperiod of $\sim$ 1.5 μsec remain almost immobile over the entire interaction time of $\sim$ 0.2 μsec. As a consequence of this, the plasma front keeps on sweeping the negative charge of the electrons created by ionization while the helium ions remain at their original positions. The latter yield a positive charge density which is maximum at the face NN$'$ of the neutral gas and which falls to zero at PP$'$. Likewise the electrons swept by the plasma front PP$'$ yield a negative charge density which falls to zero at NN$'$. Both these densities would of course, increase linearly with time as the plasma penetrates into the gas cloud. A strong polarization of charges with increasing charge density is thus produced.

It may be pointed out that though charge exchange collisions do not produce any new electrons, they do give rise to an immobile heavy He ion and thus contribute to the polarization charge density in a similar manner. All the three processes - e-He, p-He impact ionization, e-He and p-He charge exchange collision- should therefore be taken into account in calculating the total charge density.

We shall, for simplicity, consider the plasma velocity to remain constant as it penetrates the neutral gas. For a given penetration distance this will contribute a lower charge density compared to when the plasma velocity decreases, and will thus be somewhat pessimistic for the acceleration mechanism that we are considering here.

Let z represent the axial coordinate of a point in the tube as measured from NN', and let r denote its radial coordinate as measured from the axis. Then if z_o is the depth of penetration of the plasma at a certain time (distance between

NN' and PP', the radius of the tube, and V the assumed constant velocity, then the charge densities are given by

$$\sigma_+(r,z) = Q_T n_0 e(z_0 - z)\left(1-\beta r^2/r_0^2\right)/V$$
$$\sigma_-(r,z) = -Q\, n_0 e\, z\left(1-\beta r^2/r_0^2\right)/V \qquad (32)$$

which for $\beta \lesssim 1$ is defined in the range $0 \leqslant z \leqslant z_0$, $0 \leqslant r \leqslant r_0$. The radial variation of the charge density is taken to be of the form $(1-\beta r^2/r_0^2)$, and n_0 is its value on the axis, and Q_T denotes the total frequency

$$Q_T = Q + \nu_{in}^{ex} + \nu_{in}^{ion} \qquad (33)$$

Q and ν_{in}^{ion} being respectively the electron and ion impact in ionization frequencies, and ν_{in}^{ex} the charge exchange collision frequency. For the initial state of the plasma which we are here concerned with, namely with proton energy $\sim$ 1000 eV and electron energy $\sim$ 10eV (probably somewhat higher), and a neutral density $n_n = N.10^{14}\,cm^{-3}$

$$Q_T = (1.6 + 0.5 + 0.3)\times 10^5 N \text{ sec}^{-1}$$
$$= 2.4\times 10^5 N \text{ sec}^{-1} \qquad (33)$$

Now a charge distribution such as given by Eq. (32) has its electrostatic field energy which can be calculated. Whereas the charges cannot move across the field lines in planes parallel to NN' or PP' the field energy can lower itself by redistribution of charges through the motion of electrons along the field lines. In this process the field energy so released will go into the kinetic energy of the electrons imparted to them along the magnetic field. We thus have a simple process for the acceleration of electrons along the field lines. We shall next obtain a numerical estimate of the average energy gained per electron in this process at the points of maximum charge density $z = 0$ or $z = z_0$.

To find the energy one has to find out the potential distribution Φ, using the Poisson equation.

$$\nabla^2\Phi = -4\pi e n_0 Q_T\left(1-\beta r^2/r_0^2\right)\left(z_0-2z\right)/V \tag{34}$$

where the charge density of Eq. (32) has been used. The solution with the boundary condition that Φ vanishes at infinity is given by

$$\Phi = \frac{n_0 e Q}{V}\int d^3r'\left(z_0-2z\right)\left(1-\beta\frac{r^2}{r_0^2}\right)G\left(r,z|r',z'\right) \tag{35}$$

where the Green function $G(r,z|r',z')$ for the problem is given by

$$G\left(r,z|r',z'\right) = \frac{1}{\pi\sqrt{r\,r'}}\,Q_{-1/2}\left\{\left[r^2+r'^2+(z-z')^2\right]/2rr'\right\} \tag{36}$$

where $Q_{-1/2}$ is the Legendre function of order $-\frac{1}{2}$. The electrostatic energy of the charge distribution per unit length of the cylindrical column at a point z is given by

$$W = \frac{1}{2}\int_0^{r_c} 2\pi r\,dr\,\sigma(r,z)\,\Phi(r,z) \qquad (37)$$

or

$$W = 2\pi^2\left(\frac{Q_T n_o e}{V}\right)^2 r_o^6 \int_0^{\rho_c}\rho\,d\rho\,(\zeta_o-2\zeta)(1-\beta\rho^2)$$
$$\int_0^{\rho_c}\rho'\,d\rho'\,(\zeta_o-2\zeta')(1-\beta\rho'^2)\,G(\rho,\zeta|\rho',\zeta')\,d\zeta'$$
$$= \pi^2\left(\frac{Q_T n_o e}{V}\right)^2 r_o^6\, I(\zeta_o\,\zeta,\beta) \qquad (38)$$

where $I(\zeta_o,\zeta\,\beta)$ is the integral

$$I(\zeta_o\zeta\beta) = 2\int_0^{\rho_c}\rho\,d\rho\,(\zeta_o-2\zeta)(1-\beta\rho^2)\int_0^{\rho_c}\rho'\,d\rho'$$
$$\int_c^{\zeta_o} d\zeta'(\zeta_o-2\zeta')(1-\beta\rho'^2)\,G(\rho,\zeta|\rho'\zeta') \qquad (39)$$

and where

$$\rho = r/r_o,\ \zeta = z/r_o,\ \zeta_o = z_o/r_o,\ \rho' = r'/r_o,\ \zeta' = z'/r_o \qquad (40)$$

and

$$\rho_c = 1 \quad \text{for} \quad \beta \leq 1$$
$$\rho_c = \frac{1}{\sqrt{\beta}} \quad \text{for} \quad \beta > 1. \qquad (41)$$

The integral $I(\zeta_o,\zeta,\beta)$ has been evaluated numerically on a computer (IBM 360-44) for various values of the parameters. It will, of course, be largest at $\zeta = 0$ and $\zeta = \zeta_0$, where the charge density is largest.

Now if the charge distribution is to lower its energy by the flow of electrons along the field lines, the distribution of charges will get modified, while the electrons take up as kinetic energy the energy released in the process. The charge distribution will now be different along and perpendicular to the magnetic field. Assume that it is given by

$$\tilde{\sigma}(\underset{\sim}{r}) = \frac{Q_T \tilde{n}_0 e}{V}\left(z_0 - 2z\right)\left(1 - \beta\, x^2/r_0^2 - \beta'\, y^2/r_0^2\right) \qquad (42)$$

which for $\beta, \beta' \leq 1$ is defined in the region
$0 \leq z \leq z_0, \quad 0 \leq x \leq r_0, \quad 0 \leq y \leq r_0$
and the magnetic field is assumed to be along the y-direction. The potential $\tilde{\Phi}$ due to this charge distribution is given by

$$\tilde{\Phi}(\underset{\sim}{r}) = Q_T \tilde{n}_0 e \int d^3 r' \, \frac{(z' - 2z_0)}{|\underset{\sim}{r} - \underset{\sim}{r}'|}\left(1 - \beta\, x'^2/r_0^2 - \beta'\, y'^2/r_0^2\right) \qquad (43)$$

and the energy of this distribution per unit length of the cylindrical column at z is

$$\tilde{W} = \frac{1}{2}\int dx\,dy\; \tilde{\sigma}(\underset{\sim}{r})\,\tilde{\Phi}(\underset{\sim}{r})$$

$$= \pi^2 \left(\frac{Q_T \tilde{n}_o e}{V} \right)^2 r_o^6 \, \tilde{I}(\zeta_o, \zeta, \beta, \beta') . \quad (44)$$

Now if the distribution $\sigma(r, z)$ with the energy goes to $\tilde{\sigma}(x, y, z)$ with the energy $\tilde{W}$, the total number of electrons per unit length of the cylinder must be the same. Thus, we have the condition (assuming, $\beta, \beta' \leq 1$),

$$n_o \int_{-r_o}^{r_o} dx \int_{-(1-x^2)^{1/2}}^{(1-x^2)^{1/2}} dy \left[1 - \beta (x^2 + y^2)/r_o^2 \right]$$

$$= \tilde{n}_o \int_{-r_o}^{r_o} dx \int_{-(1-x^2)^{1/2}}^{(1-x^2)^{1/2}} dy \left[1 - \beta \frac{x^2}{r_o^2} - \beta' \frac{y^2}{r_o^2} \right] , \quad (45)$$

which gives

$$n_o \left(1 - \tfrac{1}{2} \beta \right) = \tilde{n}_o \left[1 - \tfrac{1}{4} (\beta + \beta') \right] . \quad (46)$$

Thus the difference in energy between the two distributions is

$$\delta W = W(\beta) - \tilde{W}(\beta, \beta') = \pi^2 \left(\frac{Q_T n_o e}{V} \right)^2 r_o^6$$

$$\left\{ I(\zeta_o \zeta, \beta) - 4 \left(\frac{2-\beta+}{4-\beta-\beta'} \right)^2 \tilde{I}(\zeta_o, \zeta, \beta, \beta') \right\} \quad (47)$$

where (46) has been used to substitute for $\tilde{n}_o$.
The average energy gained per electron $\delta \mathcal{E}$ is obtained by dividing δW by the number of electrons per unit length

which is $\pi r_0^2 n_0 (1-\frac{1}{2}\beta)$.

$$\delta\mathcal{E} = \delta W / \pi r_0^2 n_0 (1-\tfrac{1}{2}\beta)$$

$$= \frac{\pi n_0 r_0^4}{(1-\frac{1}{2}\beta)} \left(\frac{Q_T e}{V}\right)^2 \Big\{ I(\zeta_0 \zeta, \beta)$$

$$- 4 \left(\frac{2-\beta}{4-\beta-\beta'}\right)^2 \tilde{I}(\zeta_0 \zeta, \beta, \beta') \Big\} \qquad (48)$$

The maximum value for $\delta\mathcal{E}$ with respect to ζ will obviously come from $\zeta = 0$ or $\zeta = \zeta_0$, where the charge density is largest (positive at $\zeta = 0$, and negative at $\zeta = \zeta_0$). Both I and $\tilde{I}$ have been evaluated on the computer for a few values of β and β' and of ζ_0 for $\zeta = 0$. These are tabulated in Tables 2 and 3.

We now calculate $\delta\mathcal{E}$ for the parameters of the experiment viz., $n_0 = 3 \times 10^{11}$ cm^{-3}, $Q_T = 2.4 \times 10^5$N Sec^{-1} (see Eq. (33)), $V = 4 \times 10^7$ cms sec^{-1}, $r_0 = 5$ cms.

$$\pi n_0 r_0^4 \left(\frac{Q_T e}{V}\right)^2 = 3 \cdot 3 \times 10^3 N^2 \ eV. \qquad (49)$$

Since $\zeta_0 = z_0/r_0$, $\zeta_0 = 1$ corresponds to the penetration of the plasma into neutral gas cloud by $z_0 = 5$ cms. Table 4 gives $\delta\mathcal{E}$ for $\zeta_0 = 1$ at $\zeta = 0$, for different values of β, β' .

We see from the Table that for $\beta' = 0$, the energy gain per electron is almost as large as 100 eV for the neutral density $n_n = 10^{14}$ cm^{-3}. To this we may also add the initial energy of the electrons of about 10 eV. The electrons energized to the extent of 100 eV will cause rapid ionization of the helium and a strong interaction will ensue resulting in the deceleration of the plasma. For $n_n = 0.5 \times 10^{14}$ cm^{-3}, the energy gain of 24 eV is already almost equal to the ionization energy of helium and with the initial electron energy of 10 eV, the total electron energy of 34 eV will be enough to cause subcritical ionization of helium and consequent deceleration of plasma. But at $n_n = 0.2 \times 10^{14}$ cm^{-3} the energy gain per electron has fallen to a low value of 3.8 eV. Thus the total energy gain of $10 + 3.8 \approx 14$ eV is already substantial and the interaction would be shut off below this value of the neutral density. On the other hand, for neutral density 5×10^{13} cm^{-3} at which the total electron energy 10 $10 + 24 = 34$ eV is well above the ionization potential of helium no very significant change in the ionization rate Q and the consequent retardation of the plasma is likely to occur. These consequences of the model are in more or less complete agreement with the observation C(i) of Sec. 2.

Electron Heating Perpendicular to the Magnetic Field

We have so far considered electron heating only parallel to the magnetic field. As shown in the Appendix the electron-neutral collision frequency for the parameters of the experiment is by a factor of 120 too small to explain the electron acceleration by the polarization electric field. An instability like that considered by Sherman (1973) and Raadu (1975) and/or any other possible instabilities may provide electrostatic fluctuations for the electrons to have an enhanced collision frequency ν^*. As a result of this ν^*, the electrons can move across the magnetic field at a rapid enough rate to obtain substantial energy from the electric field in a time of the order of the observed interaction time. A high enough level of electric field fluctuations will be required for the purpose.

We do not, however, want to undertake this calculation here, but only wish to remark that heating perpendicular to the magnetic field would indeed occur, and would add to the heating along the field lines discussed above. The final electron temperature for a given neutral gas density may thus be higher than that given in Table 4. Furthermore the "perpendicular heating" should occur only in the later stage of the interaction, that is not before the instabilities have had time to develop to high levels. This aspect has not yet been analyzed in the experiment.

7. Comparison with the Experiment

We shall now summarize the essential features of the model and see how they explain the various experimental observations.

(i) Electron heating and ionization of helium

As we saw in the last section, the model yields an energy gain per electron of the order of 100 eV for a neutral density of 10^{14} cm^{-3} for the "parallel heating" alone. This can be further enhanced by the "perpendicular heating" so that the final electron temperature may be still higher. This is in good agreement with the observations [B (i) of Sec. 2].

These electron energies would give rise to a rapid ionization of the helium atoms. The rise time of the emission of the helium lines would be related essentially to the ionization rate Q and would be given by Q^{-1} = $(2.5 \times 10^6)^{-1}$ = 0.4 μsec corresponding to the electron energy of 100 eV and neutral density of 10^{14} cm^{-3}. This agrees well with the reported rise time of < 0.5 μsec.

(ii) The Polarization of the He I line 4686 Å

Since the acceleration of electrons along the field lines is the primary acceleration process in the model with the "perpendicular acceleration" coming possibly at a later stage, the observation on the polarization of the line emission is naturally accounted for [E (i) of sec. 2]. We have, however, made no attempt to make quantitative estimate of the degree of polarization because that will require the knowledge of the extent of "perpendicular acceleration". The latter depends on the level of electrostatic fluctuations generated in the system, and we do not yet know the types of instabilities involved and their saturation levels.

(iii) Region of Acceleration

According to the observation B (i) reported in II a significant part of the heating is found to occur within a short distance in the upstream part of the gas cloud. According to the acceleration mechanism given in Sec. 6, the acceleration should be largest at the front of the gas cloud (where the positive charge density is largest) and at the head of the plasma (where the negative charge density is largest). These then are the regions where a significant part of the heating should occur. In particular, the positive charge density region may, in fact, be considered to being "within a short distance in the upstream part of the gas cloud," so that the experimentally observed region [illegible] acceleration more or less coincides with acceleration region in the model.

(iv) <u>Retardation to the terminal velocity and the retardation time</u>

As discussed in Sec. 4 the ionization of the helium atoms gives rise to the deceleration of the plasma which, according to Eq. (21) and the discussion following it, attains a "terminal velocity" given by Eq. (23). This agrees very well with the observed value of the "terminal velocity" (Refer to A (i) and D (i) of Sec. 2).

To explain the observed deceleration of the plasma with time in detail one would have to solve Eqs. (16), (21) and other relevant equations for the velocity as a function of time as an initial value problem. We defer that to future work. We can, however, estimate the characteristic deceleration time from Eq. (16) as given approximately by $\tau_{dec} = \left[Q M_n (n_i + n_n^+) / (n_i M_i + n_n M_n) \right]^{-1}$. If $n_n^+ \ll n_i$ corresponding to the early phase of interaction when the number of helium ions produced is small, $\tau_{dec} = \frac{1}{4} Q^{-1}$. With $Q = 2.5 \times 10^6$ sec.$^{-1}$ (for electron energy of 100 eV and $n_n = 10^{14}$ cm^{-3}) $\tau_{dec} \approx 0.1 \mu$ sec, while $n_n^+ = n_i$ gives $\tau_{dec} = \frac{5}{8} Q^{-1} = 0.25 \mu s$, and $n_n^+ \gg n_i$ gives $\tau_{dec} = 0.4 \mu s$. These values may be compared with the characteristic deceleration time $\sim 0.1 \mu s$ as determined from the experimentally observed velocity deceleration curve. It may be concluded that perhaps a large number of helium ions are not produced in the main deceleration phase which is characterised by a time of 0.1 μsec. That the observed rise time of H_α emission, 0.5 μ and the ionization time = 0.4 μ are both greater than = 0.1 μ appears to be in agreement with the above conclusion.

(v) Effect of the Neutral Gas Density

The effect of neutral gas density on the interaction is already discussed in detail in Sec. 6. The consequencies of the model are found to be in agreement with the observation C (i) of Sec. 2.

(vi) Effect of the Magnetic Field

As is implicit in the entire discussion the magnetic field plays a crucial role in the process of interaction, in particular in the acceleration mechanism for electrons. It is the magnetic field which imparts an essentially electromagnetic character to the process of interaction, so that through such a collective plasma process the interaction is able to proceed at as fast a rate as it does. In its absence, the only possible interaction is via the binary collisions which have been shown to be too slow to be effective.

The precise manner in which the magnetic field acts is that it leads to the setting up of a polarization electric field to enable the plasma to move across it with the E X B drift velocity. The plasma kinetic energy can then (as has been shown in Sec. 5) be considered as an electrostatic field energy which can be discharged resulting in the "perpendicular heating" of the electrons.

Similarly, the process of "parallel acceleration", of electrons given in Sec. 6 also crucially depends on the magnetic field, for it is the latter which, by virtue of the widely different eletron and ion gyro-periods in it, make the charge separation possible in the manner discussed. However, once we have a sufficiently strong magnetic field to fulfil the above-mentioned role, an increase in its value by say a factor of 2 may not produce any significant effect on the retardation. This is indeed the content of observation C(ii).

(vii) Electron Multiplication factor

Active particle analysis carried out in II (Observation B (ii) of Sec. 2) gives a post-interaction electron density $n_e \approx 2 \times 10^{12}$ cm^{-3} with an uncertainty of $\pm$ 80%. This is about seven times the original electron density in the plasma. Since the ionization arises essentially from electron impact we can obtain the final density n_f using $n_f = n_o \exp(Q\tau)$,

where n_c is the initial density and Q is the ionization rate by electron impact. From Table 1, $Q = 2.5 \times 10^{6}$ Sec^{-1}

for $n_n = 10^{14}$ cm^{-3} and the electron energy of 100 eV. Thus for an interaction time $\sim 0.2\mu$ sec. the electron multiplication factor is $\exp(\nu\tau)$ $\exp(0.50) = 1.65$. This is about a factor 4 too small compared to the reported experimental value of 7. Since the value of the ionization frequency ν used is almost close to the maximum (which is 2.8×10^6 sec^{-1} for $n_n = 10^{14}$ cm^{-3}) it is difficult to see how one can get a multiplication factor as large as 7 in an interaction time of 0.2μ sec. unless the measurements correspond to a total lapse of time of the order of 0.8μ sec. or twice as large, for the neutral density $n_n = 0.5 \times 10^{14}$ cm^{-3}.

(viii) The effect of the initial plasma velocity on the interaction

We next come to the observations D(i) and D(ii) of sec. 2 which refer to the effect of the initial plasma velocity on the interaction.

According to the analysis in sec. 4, the deceleration of the plasma to terminal velocity occurs essentially because of the ionization of helium atoms and consequent sharing of the plasma momentum by the freshly ionized stationary helium ions. A complete deceleration to the terminal velocity (indicative of a "strong interaction") would thus crucially depend on the rapid ionization of the neutral atoms, which in turn is determined by the acceleration of the electrons to high enough energies.

The analysis in sec. 5, however, shows that there exists a threshold plasma velocity given by Eq. (31) below which the electrons could not be accelerated to high enough energies to ionize the neutrals, and as such the deceleration of the plasma would not occur. As already discussed in detail in sec. 5, experimental results given in D(ii) and D(ii) do indeed show that to be the case; and thus point to the existence of a threshold velocity though this aspect has not been explicitly investigated in the experiment.

Finally we again wish to emphasize that a clear distinction must be made between the "terminal velocity" and the threshold velocity for interaction.

Appendix

The perpendicular acceleration and the anomalous collision frequency

As has been discussed earlier, the kinetic energy of the plasma resides also as the total field energy of the polarization electric field that is associated with the motion of the plasma across the magnetic field. If the electrons can move across the magnetic field to gain energy from the electric field then they can suffer a perpendicular acceleration at the expense of the field energy.

In a completely collisionless plasma in a magnetic field the electrons cannot move across the magnetic field unless they undergo collisions with either other particles or with electric field fluctuations. When the plasma in the experiment encounters the neutral gas the electrons can move across the magnetic field on colliding with the neutral atoms. The resulting cross field drift of the electrons which is known as the Pederson drift is given by

$$\underset{\sim}{V_P} = (\Omega_e \tau_e)\left[1 + (\Omega_e \tau_e)^2\right]^{-1} c\, \underset{\sim}{E}/B \qquad \text{(A1)}$$

where $\Omega_e = eB/m_e c$ is the electron gyrofrequency, and τ_e the collision time of the electrons with the neutrals, or the electric field fluctuations or both. The energy gain by the electrons per unit time is given by

$$\frac{d\mathcal{E}}{dt} = |\Omega_e \tau_e| \left[1 + (\Omega_e \tau_e)\right]^{\cdots} \cdots$$

$$\approx m_e \nu_e \left(\frac{cE}{B}\right)^2 \tag{A2}$$

where $\Omega_e \tau_e \gg 1$ is assumed. Using the experimental value of $V_o = cE/B = 4 \times 10^7$ cms sec^{-1}, in the above relation, the energy gain $\Delta\mathcal{E}$ in a time 0.10 μsec (say first half of the total retardation time) is obtained from the above relation as

$$\Delta\mathcal{E} = \cdot 1 \times 10^{-7} \; \nu_e \quad eV \tag{A3}$$

The minimum energy for the ionization of helium is its ionization potential 24.5 eV. Hence the minimum ν_e for an electron to gain 24.5 eV in a time 0.10 μsec is

$$\nu_e^{min} \quad 10^8 \; sec^{-1} \tag{A4}$$

The electron-He collision frequency ν_{en} is given by (Akasofu and Chapman, 1972)

$$\nu_{en} = 4.6 \times 10^{-10} \, n(He) \, T_e^{1/2} (°K) \quad sec^{-1} \tag{A5}$$

For an initial electron temperature $\sim$ 10 eV, and a neutral density $n(He) = 5 \times 10^{13}$ cm^{-3} this gives

$$\nu_{en} = 8 \times 10^6 \quad sec^{-1} \tag{A5}$$

This falls by a factor of about 30 short of that required for the minimum energy needed for the ionization and by a factor of about 120 short of that needed for the observed energies of about 100 eV in the experiment.

Thus mere electron neutral collisions would not suffice to account for the observed electron energies if this were the sole acceleration mechanism. Such an acceleration mechanism even if it sufficed would deposit energy in the perpendicular component only and would not, therefore, account for the polarization measurement. Such a "perpendicular acceleration" will probably occur along with the "parallel acceleration" of sec. 6 in the later phase of the interaction.

Some instabilities (see for instance Sherman (1973), Raadu (1975)) may arise in this phase with high enough level of electrostatic fluctuations so that the anomalous collision frequency ν^* resulting therefrom yields a perpendicular acceleration required to explain the observations. No information is, however, available on the relation between percentage anisotropy in the velocity distribution and the percentage polariation; so no quantitative estimates of the perpendicular energy can be made.

Akasofu, S. and Chapman, S., 1972, Solar Terrestrial Physics, (Oxford University Press), p. 132.

Alfven, H., 1954, The Origin of the Solar System, Clarendon Press, Oxford, Chapter III.

Alfven, H., 1960, Rev. Mod. Phys. 32, 710.

Bhatnagar P.L., Gross E.P., and G. Krook, 1954, Phys. Rev. 94, 511.

Bhatnagar, P.L., 1962, Zeits. Astrophys., 54, 234.

Danielsson, L., 1970, Phys. Fluids 13, 2288.

Danielsson, L., 1973, Astrophys. Space Sci. 24, 459.

Danielsson, L., and Brenning, 1975, Phys. Fluids 18, 661.

Engel, A von : 1955, Ionized Gases, Clarendon Press, Oxford, p. 49.

Freeman, R.L., and Jones E.M., 1974, Culham Laboratory Report CLM-R 137.

Gross, E.P., and G. Krook, 1956, Phys. Rev. 102, 593.

Lehnert, B., 1967, Phys. Fluids, 10, 2216.

Raadu, M.A., 1975, Royal Institute of Technology, Stockholm Report 7 RITA-EPP-75-28.

Rudge M.R.H. and Seaton, M.J., 1965, Proc. Roy. Soc. A283, 262, 1965.

Rudge M.R.H., 1968, Rev. Mod. Phys. 40, 564.

Sherman J.C., 1969, Royal Institute of Technology, Stockholm Report, TRITA-EPP-69-29.

Sherman J.C., 1972, Nobel Symposium No. 21, p. 315
(Almqvist and Wicksell, Uppsalas).

Sherman J.C., 1973, Astrophys. Space Sci. 24, 487.

TABLE 1

Neutral density $n_n = N.10^{14}$ cm^{-3} $\qquad$ $V = 2 \times 10^7$ cm. sec.$^{-1}$

Type of collisions	Temperature T.eV.	Collision freq. sec.$^{-1}$	Mean free path λ (cm.)	λ_{eff} (cm.)
H^+-He charge Exchange	1000	5×10^4	4×10^2	
H^+-He impact ionization	1000	3×10^4	6.6×10^2	
Electron He momentum transfer	5	1.1×10^7	11.4	1.82
	25	2.6×10^7	11.4	0.71
	100	5.2×10^7	11.4	0.36
Electron He impact ionization	10	1.6×10^5		$1.25 \times 10^{[illegible]}$
	25	7.9×10^5		25
	100	2.5×10^6		8
	200	2.8×10^6 (max)		7

TABLE 2

I

ζ_0 \ $\beta = \beta'$	0.5	0.75	1.0
0.5	0.1343×10^{-1}	0.5311×10^{-1}	0.1439
1.0	0.7000×10^{-2}	0.2940×10^{-1}	0.7850×10^{-1}

TABLE 3

I

ζ_0 \ β'	0.0	0.2	0.5
0.5	0.1260×10^{-1}	0.1125×10^{-1}	0.9439×10^{-2}
1.0	0.1440	0.1286	0.1076

TABLE 4

N \ β'	0.5	0.2	0.0
N = 1	63.6	84.4	95.7 eV
N = 0.5	15.9	21.1	24.0 eV
N = 0.2	2.7	3.4	3.8 eV

Caption for Table 1 :

Collision frequencies and mean free paths for various types of collisions.

Foot note to Table 1 :

The values in the table have been obtained from the curves of reaction rates compiled by Freeman and Jones (1974).

Caption for Table 2 :

The values of the Integral I for different values of ζ_0 and $\beta = \beta'$.

Caption for Table 3 :

The values of the Integral I for $\beta = 1$ and for different values of β' and ζ_0 .

Caption for Table 4 :

The values of $\delta\mathcal{E}$ for $\beta = 1$, $\zeta_0 = 1$, $n_n = N.10^{14}$ cm^{-3} and for different values of β' .

Caption for Fig. 1. :

The plasma front PP' moves with velocity V across a magnetic field B into a neutral gas (shown with dots) with NN' being its front edge (idealized to be sharp). The region NPP'N' is then the region of interaction, at a given time. E is the polarization electric field.