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F. KOECHLIN A. SAMAIN

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ASSOCIATION EURATOM CEA

92260 Fontenay aux Roses (France)

DEPARTEMENT DE PHYSIQUE DU PLASMA ET DE LA FUSION CONTROLEE

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F. KOECHLIN - A. SAMAIN

ASSOCIATION EURATOM-CEA SUR LA FUSION
Département de Physique du Plasma et de la Fusion Contrôlée
Centre d'Etudes Nucléaires
Boite Postale n° 6, 92260 FONTENAY-AUX-ROSES (FRANCE)

ABSTRACT.

The drift modes energized by trapped electrons are discussed in the case where the azimuthal transverse wave number is of the order of the inverse ion thermal Larmor radius. For the usual values of the shear, the parallel wave number is then larger than 1/qR in the major part of the radial interval where the mode escapes ion Landau damping. The time during which the trapped electrons remain coherent with the mode is reduced and the Kadomtsev dissipative mechanism is less efficient. The critical shear for the onset of the instability and the induced electron energy transport coefficient at a given level of the density fluctuation are estimated.

RESUME.

Les modes de dérive rendus instables par les électrons piégés sont étudiés dans le cas où leur nombre d'onde azimuthal est de l'ordre de l'inverse du rayon de Larmor des ions. Pour des valeurs normales du cisaillement magnétique, le nombre d'onde le long du champ est alors supérieur à 1/qR dans la plus grande partie de l'intervalle radial où le mode échappe à l'amortissement par effet Landau des ions. Le temps pendant lequel les électrons piégés peuvent être cohérents avec le mode est réduit et le mécanisme de dissipation de Kadomtsev est moins efficace. Le cisaillement critique qui déclenche l'instabilité et le coefficient de transport pour l'énergie des électrons induit par le mode sont estimés.

I - INTRODUCTION.

In this note, we discuss the drift modes energized by trapped electrons in Tokamaks when the azimuthal transverse wave number K_θ and the thermal Larmor radius of hydrogen ions ρ_{thi} verify $K_\theta \rho_{thi} \sim 1$. Our motivation is to check if the density fluctuations which have been recently detected by the microwave scattering technique [1], [2], and which have in fact a wave number $K_\theta \sim \rho_{thi}^{-1}$, could consist of drift modes, and, assuming it is the case, if the electron energy flux resulting from the balance for electrons can be justified by the measured levels of the density fluctuations. The microwave diffusion technique gives the spectrum in frequency and transverse wave numbers of the electron density fluctuation $\delta n(\vec{x}, t)$. Some hypothesis must be made to estimate the transport coefficients from the experimental data. It is natural of course to assume that the turbulence originates from the conventionnal drift modes driven unstable by electrons. We then have

$$\frac{-e \quad \delta\psi(\vec{x}, t)}{T} \simeq \frac{\delta n(\vec{x}, t)}{n} \qquad (1)$$

where $\delta\psi(\vec{x}, t)$ is the fluctuating electric potential and n, $T = m\, V_{th}^2 /2$ are the density and the temperature of electrons. (charge e and mass m ; $\rho_{th} = V_{th} \; | eB/mC |^{-1}$). We make a more stringent assumption, namely that the turbulence consists of modes which approximatively retain their structure in the linear range. In this scheme the modes are stabilized at a saturation level by some non-linear mechanism allowing the hydrogen ions to become resonant [3] . Due to the large value of the scale time involved in the unperturbed parallel motion of electrons (namely the transit time $(K_{/\!/} \; V_{th})^{-1}$ or the trapping time γ_{Eth}), such a mechanism is not likely to affect the resonance of electrons. We may then calculate the irreversible action of the turbulence on electrons, e.g. the transport coefficients or the power density exchanged with the modes, by a 2^{d} order perturbation theory. (The transport coefficients for ion energy cannot be calculated, however, without stating the exact nature of the non linear process which saturates the turbulence).

Let us consider a simple mode specified by a potential of the form (see Fig. (1))

$$\delta\psi(\vec{x},t) = \psi_{+}(\vec{x}) \exp i\omega t + c.c. = \psi_{\ell}(r) \exp i(M\varphi + \ell\theta + \omega t) + c.c. \tag{2}$$

where φ is the angular coordinate around the major axis, θ is defined by $\theta = \varphi/q(r)$ ($q(r) = r B_{\varphi} / RB_{\theta}$) along a flux line on a magnetic surface of radius r and $\theta = 0$ in the equatorial plane. The mode may exist in the radial interval $\mathcal{D}_{\ell}$ where the parallel wave number $K_{\parallel\ell}(r) = |R^{-1}(M + \ell/q(r))|$ is small enough to prevent ion Landau damping, i.e. where $K_{\parallel\ell}(r) < |\omega| / V_{thi}$. Usually the electron density n(r), the temperature of hydrogen ions $T_i(r) = m_i V_{thi}^2/2$ and the safety factor q(r) satisfy $T_i \sim T$, $\partial n/n\partial r \sim \partial T_i/T_i\partial r \sim \partial q/q\partial r \sim r^{-1}$. We then have $\omega \sim K_\theta V_{thi}\rho_{thi}/r$, with $K_\theta \sim \ell/r$. The interval $\mathcal{D}_{\ell}$ is centered at the radius $r = r_\ell$ such that

$$R K_{\parallel}(r_\ell) = M + \ell/q(r_\ell) = 0 \tag{3a}$$

and is specified by

$$|r - r_\ell| < \Delta \sim \rho_{thi}\frac{L_s}{r} \sim \rho_{thi}\frac{qR}{r} \tag{3b}$$

where

$$L_s^{-1} = \left|\frac{r}{R}\frac{\partial}{\partial r}\left(\frac{1}{q}\right)\right| \sim \frac{1}{qR}$$

The next mode exists in the interval $\mathcal{D}_{\ell+1}$ centered at $r = r_{\ell+1}$ and

$$\rho = |r_{\ell+1} - r_\ell| = \frac{L_s}{K_\theta qR} \sim \frac{1}{K_\theta} \tag{3c}$$

We assume that $K_\theta \rho_{thi} \sim 1$, so that $\rho \sim \rho_{thi} \sim (r/qR)\,\Delta$. The consequence is that the neighbouring modes strongly overlap. On the other hand

$$K_{\parallel\ell}(r) = K_\theta \frac{r - r_\ell}{L_s} = \frac{r - r_\ell}{\rho} \frac{1}{qR}$$

This yelds $K_{\parallel\ell}(r)\; qR \sim 1$ for $(r - r_\ell) \sim \rho$, and $K_{\parallel\ell}(r)\, qR \sim qR/r$ when $|r - r_\ell| \sim \Delta$. Thus we have $K_{\parallel\ell}(r)\; qR \gg 1$ over tne major part of $\mathcal{D}_\ell$.

Let us consider the set of electrons with an energy $\mathcal{E} = mV^2/2$ on a magnetic surface labelled by r. Some of them are trapped ; let the frequencies

$$\omega_g = \omega_{g\,th} \frac{\mathcal{E}}{T} \quad ; \quad \omega_{g\,th} = \frac{c}{eB_\theta} \frac{mV_{th}^2}{2R^2} \tag{4 a}$$

$$\omega_b = \omega_{b\,th} \left(\frac{\mathcal{E}}{T}\right)^{1/2} \quad ; \quad \omega_{b\,th} = \frac{V_{th}}{qR\sqrt{2}} \left(\frac{r}{R}\right)^{1/2} \tag{4 b}$$

$$\gamma_\mathcal{E} = \gamma_{\mathcal{E}th} \left(\frac{\mathcal{E}}{T}\right)^{-3/2} \quad ; \quad \gamma_{\mathcal{E}th} = \frac{2\pi n e^4 (1 + Z_{eff}) \log \Lambda}{m^2 V_{th}^3} \frac{R}{r} \tag{4 c}$$

be their precession frequency around the major axis, their bounce frequency between the magnetic mirrors and their collision rate for detrapping. The mode may irreversibly transfer a power W to the circulating electrons of this set by Landau resonance and to the trapped electrons by a bounce resonance of the type $\omega + p\omega_b + M\omega_g = 0$, with $p \neq 0$, or by the dissipative Kadomtsev mechanism $(p = 0)$. (As $M\omega_{gth}/\omega_{bth} \sim K_\theta \rho_{th}\, q (R/r)^{-1/2} \sim (m/m_i)^{1/2} q (R/r)^{1/2} \ll 1$, we may neglect $M\omega_g$ with respect to ω_b).

The Landau resonance with the circulating electrons is active for small enough values of $K_{\parallel\ell}(r)$, such that the frequency gap $K_{\parallel\ell}(r)\; V (r/R)^{1/2}$ associated with the trapped electrons is smaller than either the mode frequency ω or the

Landau resonance broadening $\delta\omega$. This is the case when

$$K_{\parallel}\ell(r) \; qR < \frac{|\omega|}{\omega_b} \quad \text{or} \quad K_{\parallel}\ell(r)\, qR < \frac{\gamma_E}{\omega_b}$$

if we take into account [4] that $\delta\omega \sim (K_{\parallel}\ell(r)\ V)^{2/3}\ \nu_c^{1/3}$ where $\nu_c \sim \gamma_E r/R$ is the collision rate for 90° deflection of the considered electrons. Of course the Landau resonance occurs only if $\nu_c < K_{\parallel}\ell(r)\ V$, i.e., $K_{\parallel}\ell(r)\ qR > (r/R)^{3/2}\,\gamma_E/\omega_b$
Also, we must have $|\omega| = |K_{\parallel}\ell(r)\ V_{\parallel}| < K_{\parallel}\ell(r)\ V$, i.e., $K_{\parallel}\ell(r)\, qR > (r/R)^{1/2}|\omega|/\omega_b$.
The Landau resonance is also active for large values of $K_{\parallel}\ell(r)$ such that the circulating and trapped electrons can hardly be distinguished by the wave. This occurs when the parallel velocity change δV experienced by a trapped electron due to the mirror effect over a distance $\sim 2\pi/K_{\parallel}\ell(r)$ is small compared to the width of the Landau resonance $\delta\omega/K_{\parallel}\ell(r)$. As $\delta V \sim (r/R)^{1/2}\,V/(K_{\parallel}\ell(r)\ qR)$ this condition writes $K_{\parallel}\ell(r)\, qR > (\omega_b/\gamma_E)^{1/2}$.

It is readily shown (see appendix I) that when the magnetic field, the profiles n(r) and T(r) and the values of M and ω are given, the power W_L transfered to the electrons of the considered set by the Landau effect, when it is active, varies as $W_L \sim A\ |\psi_\ell(r)|^2/(K_{\parallel}\ell(r)\ V)$. It is also shown in the appendix I that the bounce resonances $\omega + p\,\omega_b + M\omega_g = 0$ with $p \neq 0$ are only active if

$$|\omega| \gtrsim \omega_b \quad \text{and} \quad \frac{|\omega|}{\omega_b} \lesssim K_{\parallel}\ell(r)\ qR \lesssim \left(\frac{\omega}{\gamma_E}\right)^{1/2} \qquad (5)$$

The corresponding transferred power W_b to the considered set is then given by $W_b \sim A\ |\psi_\ell(r)|^2\ (r/R)^{1/2}/(\omega_b\ K_{\parallel}\ell(r)\ qR)$,i.e., $W_b \sim W_L$. Therefore the power $W_L + W_b$, and generally the transport coefficients associated with the Landau and the bounce resonances, may be approximatively calculated as due to the Landau mechanism, if we include the case (5) in the domain of activity of the latter. This domain then becomes

$$K_{\parallel}\ell(r)\, qR > \mathrm{Sup}\left(\left(\frac{r}{R}\right)^{1/2}\frac{|\omega|}{\omega_b}\ ,\left(\frac{r}{R}\right)^{3/2}\ \frac{\gamma_E}{\omega_b}\right)$$

$$\text{and}\quad \left[\ K_{\parallel}\ell(r)\, qR < \mathrm{Sup}\left(\frac{|\omega|}{\omega_b},\ \frac{\gamma_E}{\omega_b}\right)\ \text{or}\quad K_{\parallel}\ell(r)\, qR > \left(\frac{\omega_b}{\gamma_E}\right)^{1/2}\right.$$

$$\left.\text{or}\quad |\omega| > \omega_b\ \right] \qquad (6)$$

This simplification is justified by the fact that in practise the power $W_L + W_b$ appears to be small compared the power W_K transfered to the considered set by the Kadomtsev mechanism. This means that the effect of the Landau and bounce resonances is comparatively small and that we may content ourselves with a rough estimation of this effect.

As compared to the powers W_L and W_b, the power W_K varies as $W_K \sim A\ \tau^{-1}\ (\omega'^2 + \tau^{-2})^{-1}\ (r/R)^{1/2}\ |<\psi_\ell(r)>|^2$ where $\omega' = \omega + M\,\omega_g$, $|<\psi_\ell(r)>|$ is the bounce averaged value of $\psi_\ell(r)\, \exp i(\ell\theta + M\varphi)$ and τ is the time during which a trapped electron remains in phase with the mode. On a magnetic surface where $K_{\parallel}\ell(r) \lesssim 1/qR$, as generally considered [5, 6], we have $|<\psi_\ell(r)>| \sim |\psi_\ell(r)|$ and τ is of the order of the time which is necessary for the amplitude λ of the bounce motion of a trapped electron to increase beyond qR, i.e. $\tau \sim \gamma_E^{-1}$. We have in that case $W_K \sim A(r/R)^{1/2}\,|\psi_\ell(r)|^2\ \gamma_E\,/(\omega'^2 + \gamma_E^2)$. If $K_{\parallel}\ell(r) \gg 1/qR$, we have $|<\psi_\ell(r)>| \sim |J_0(K_{\parallel}\ell(r)\,\lambda)\ \psi_\ell(r)| \sim |\psi_\ell(r)|\ (K_{\parallel}\ell(r)\ qR)^{-1/2}$. Also, the time τ is now the time necessary for the amplitude of the bounce motion to vary under the influence of collisions by a quantity $\sim 1/K_{\parallel}\ell(r)$ rather than $\sim$ qR. Therefore we have [7]

$$\tau \sim \gamma_E^{-1}\ \frac{1}{(K_{\parallel}\ell(r)\, qR)^2} \qquad (7)$$

and finally

$$W_K \sim A\ \frac{\gamma_E\ (K_{\parallel}\ell(r)\, qR)^2}{\omega'^2 + \gamma_E^2\,(K_{\parallel}\ell(r)\, qR)^2}\ \frac{1}{K_{\parallel}\ell(r)\, qR}\ \left(\frac{r}{R}\right)^{1/2}$$

It then appears that the power W_K is dominant compared to $W_L + W_b$ only if we have

$$|\omega'| = |\omega + M\omega_g| < \omega_b$$

$$\text{and} \quad \mathrm{Sup}\left(\frac{|\omega'|}{\omega_b}, \frac{\gamma_E}{\omega_b}\right) < K_{\parallel}\ell(r)\, qR < \left(\frac{\omega_b}{\gamma_E}\right)^{1/2} \qquad (8)$$

and the Kadomtsev mechanism has only to be considered in that case. Assume that $\gamma_E < \omega_b$ and $|\omega'| \sim |\omega|$. If $|\omega| < \gamma_E$, the Kadomtsev mechanism takes place for $\gamma_E/\omega_b < K_{\parallel}\ell(r)qR < (\omega_b/\gamma_E)^{1/2}$ but in fact, owing to the strong decrease of $W_K \sim A(r/R)^{1/2} |\psi_\ell(r)|^2/(\gamma_E (K_{\parallel}\ell(r)\, qR)^3)$ for $K_{\parallel}\ell(r)\, qR \gg 1$, it is only active for $K_{\parallel}\ell(r) qR \lesssim 1$. (See Fig. (2a)). If $|\omega| > \gamma_E$ the Kadomtsev mechanism takes place for $|\omega|/\omega_b < K_{\parallel}\ell(r)\, qR < (\omega_b/\gamma_E)^{1/2}$, but is essentielly active for values of $K_{\parallel}\ell(r)\, qR \sim (|\omega|/\gamma_E)^{1/2}$ corresponding to $\omega\tau \sim 1$. (See fig. (2b)).

The growth rate of the drift modes in the linear range and the averaged induced transport coefficient at a given level of the fluctuating potential are the sum of the contributions of the Landau and the Kadomtsev mechanisms, under conditions (6) and (8), respectively. The Landau contribution is proportionnal to the integral $I = \int dr\, |\psi_\ell(r)|^2 / K_{\parallel}\ell(r)$ over the radial interval where it takes place. The trapping effect simply reduces this contribution by reducing I. In this article we focuse our attention on the Kadomtsev mechanism. We estimate the critical shear $(\partial q/q\, \partial r)_c$ (given by (36)) for marginal stability and the averaged flux Γ_E (given by (37) and (43)) of electron energy across the magnetic surfaces due to this mechanism, for a mode of the following form, more realistic than the form (2)

$$\delta\psi(\vec{x}, t) = \exp i\omega t\ \exp iM\varphi\ \ \psi(r,\theta) + c.c. \qquad (9a)$$

$$\psi(r,\theta) = \sum_\ell \psi_\ell(r)\ \exp i\ell\theta \qquad (9b)$$

We compare the èstimated values of Γ_E associated with the measured level of density fluctuations to the values consistent with the electron energy balance, in the case of the TFR expriment.

II - PRINCIPLE OF THE CALCULATIONS.

When acting on a given particle species p (ions or electrons), the potential $\delta\psi$ given by (9a) produces a charge density $\delta\rho_p$ of the form

$$\delta\rho_p(\vec{x}, t) = \exp i\omega t \;\; \exp iM\varphi \;\; \rho_p(\omega, r, \theta) \qquad (10\,a)$$

where $\rho_p(\omega, r, \theta)$ is a linear fonctionnal of $\psi(r, \theta)$. It is convenient to consider the bilinear form in $\psi(r, \theta)$ and $\psi^*(r, \theta)$ defined by [8]

$$\mathcal{L}(\omega, M, \psi, \psi^*) = \sum_p \mathcal{L}_p(\omega, M, \psi, \psi^*)$$
$$\mathcal{L}_p(\omega, M, \psi, \psi^*) = - \iiint \rho_p(\omega, r, \theta) \;\; \psi^*(r, \theta) \;\; d_3x \qquad (10\,b)$$

We obviously have $\mathcal{L}(\omega, M, \psi, \psi^*) = 0$ when ω, M and ψ correspond effectively to a self consistent mode. The frequency ω may be determined by this equation when the geometrical structure of the mode, i.e. M and $\psi(r, \theta)$, is known. The fonction $\psi(r, \theta)$ may be determined by expressing that the fonctionnal $\mathcal{L}$ is an extremum with respect to ψ^*, a condition obviously equivalent to the equation $\sum_p \rho_p = 0$. The power W_p which is irreversibly transferred by the mode to the species p, i.e. the time averaged quantity

$$W_p = \iiint d_3x \; \left(\psi(r, \theta) \;\; \exp i(\bar{\omega} t + M\varphi) + c.c.\right)\left(-i\,\bar{\omega}\,\rho_p \exp i(\omega t + M\varphi) + c.c.\right)^*$$

where $\bar{\omega} = Re(\omega) \; \# \; \omega$, is given by

$$W_p = -2\bar{\omega} \; \mathrm{Im} \left(\mathcal{L}_p \left(\bar{\omega}, M, \psi, \psi^* \right) \right)$$

The angular momentum P_p around the major axis which is irreversibly transfered per unit time to this species has the value

$$\Gamma_p = -\frac{M}{\bar{\omega}} W_p = 2M \; \mathrm{Im} \left(\mathcal{L}_p \left(\bar{\omega}, M, \psi, \psi^* \right) \right)$$

As explained above, we will admit that these formulae are valid for electrons even when the modes form a stationnary turbulence. In fact, the quantity $\mathrm{Im}(\mathcal{L}_{p=e})$ for electrons will appear in the form of an integral over the radius of the magnetic surface and the energy $\mathcal{E}$ of the particles

$$\mathrm{Im} \left(\mathcal{L}_e \left(\bar{\omega}, M, \psi, \psi^* \right) \right) = \iint dr \, d\mathcal{E} \;\; L(r, \mathcal{E}) \qquad (11)$$

It is easily proved that the momentum which is transfered per unit time from the mode to the particles in the range dr $d\mathcal{E}$ is equal to $2M \, L(r, \mathcal{E}) \, dr \, d\mathcal{E}$. This momentum must be cancelled by the Laplace forces associated with a radial motion of these particles across the magnetic field . It results that the average fluxes Γ and $\Gamma_{\mathcal{E}}$ of electrons and electron energy across the magnetic surface r are given by :

$$\Gamma = \frac{c}{e} 2M \int L(r, \mathcal{E}) \;\; d\mathcal{E} \;\; 2\pi \frac{dr}{d\phi} \;\; \frac{1}{S} \simeq -\frac{c}{e B_\theta} \; \frac{2M}{R} \int L(r, \mathcal{E}) \, d\mathcal{E} \, \frac{1}{2\pi R 2\pi r}$$

$$\Gamma_{\mathcal{E}} = \frac{c}{e} 2M \int L(r, \mathcal{E}) \;\; \mathcal{E} \, d\mathcal{E} \, 2\pi \frac{dr}{d\phi} \;\; \frac{1}{S} \simeq -\frac{c}{e B_\theta} \; \frac{2M}{R} \int L(r, \mathcal{E}) \, \mathcal{E} \, d\mathcal{E} \, \frac{1}{2\pi R 2\pi r} \qquad (12)$$

where $d\phi$ is the poloidal flux between the magnetic surfaces r and r+dr, of area S.

III - CALCULATION OF THE FUNCTIONNAL $\mathcal{L}(\omega, M, \psi, \psi^*)$.

Assuming that the phase velocity $\omega / k_{\parallel} \rho(r) = \omega R / (M + \ell/q(r))$ of each component $\psi_\ell(r) \; \exp i(\ell\theta + M\varphi + \omega t)$ of the potential $\delta\psi$ (see Eq. (9b)) is larger than the thermal ion velocities, the charge density for the ion species is readily calculated by the usual integration along the unperturbed trajectories [9]. The functionnal $\sum_{p=\text{ion species}} \mathcal{L}_p$ reduces in fact to the term $\mathcal{L}_{p=i}$ corresponding to hydrogen ions if we have

$$n \gg \sum_{p'} n_{p'} Z_{p'} \; ; \; \frac{n}{m_i} \gg \sum_{p} \frac{n_{p'} Z_{p'}^2}{m_{p'}}$$

$$n\, m_i \gg \sum_{p'} m_{p'} \cdot n_{p'}$$

where p' labels the impurity species (density $n_{p'}$, charge $Z_{p'}|e|$ and mass $m_{p'}$). We then obtain

$$\sum_{p=\text{ion species}} \mathcal{L}_p(\omega, M, \psi, \psi^*) \simeq \mathcal{L}_i(\omega, M, \psi, \psi^*)$$
$$= \int \frac{n e^2}{T_i} \sum_{\ell} \left(1 - \alpha - \beta' \frac{V_{thi}^2}{\omega^2 R^2} \left(M + \frac{\ell}{q(r)} \right)^2 |\psi_\ell(r)|^2 + \gamma \left| \frac{\partial \psi_\ell(r)}{\partial r} \right|^2 \rho_{thi}^2 \right) 2\pi R \; 2\pi r \, dr \qquad (13\,a)$$

where, assuming to simplify that $\frac{\partial T_i}{\partial r} = 0$

$$\alpha = 2\beta' = \frac{\omega + M\omega_{di}}{\omega} \; \bar{\alpha}(s^2)$$
$$\gamma = - \frac{\omega + M\omega_{di}}{\omega} \; \frac{d\bar{\alpha}(s^2)}{ds^2} \qquad (13\,b)$$
$$\bar{\alpha}(s^2) = \exp\left(-\frac{s^2}{2}\right) J_0\left(\frac{s^2}{2}\right) ; \quad s = k_\theta \rho_{thi} ; \quad k_\theta = \left|\frac{\ell}{r}\right| \approx \left|\frac{M q(r)}{r}\right|$$
$$\rho_{thi} = \frac{V_{thi}}{|eB/m_i c|} \; ; \quad \omega_{di} = - \frac{c}{|e| B_\theta R} \, T_i \, \frac{\partial n}{n \partial r}$$

We have omitted in the expression of $\mathcal{L}_i$ cross terms of the form $\int (n e^2/T_i) \; O(r/R) \; (\psi_\ell \psi_{\ell+1}^* + \psi_{\ell+1} \psi_\ell^*) \, 2\pi R \, 2\pi r \, dr$ which, in our opinion, play a minor role in what follows. By assumption each $\psi_\ell(r)$ is localized in the interval $\mathcal{D}_\ell$ specified by (3).

To calculate the electron contribution $\mathcal{L}_{p=e}$ to the functionnal $\mathcal{L}$, we put as usual [5] the distribution function $\mathcal{F}$ for electrons in the form

$$\mathcal{F} = F - \frac{e\delta\psi}{T} F + \left(\sum_{\ell} g_{\ell}(\vec{v}, r, \theta) \ \exp(i\omega t + i m\varphi) + c.c. \right)$$

where F is the local Maxwellian and each term ℓ in $\sum_{\ell}$ is the linear response to the potential $\psi_{\ell}(r) \exp i\ell\theta \cdot \exp(i\omega t + i m\varphi)$
The functions $g_{\ell}(\vec{v}, r, \theta)$ verify in the trapped domain the Fokker Planck equation

$$i\omega \, g_{\ell} \exp i m\varphi + \frac{d(g_{\ell} \exp i m\varphi)}{dt} - i(\omega + m\omega_d) \frac{eF}{T} \psi_{\ell}(r) \exp(i\ell\theta + i m\varphi)$$
$$= C(g_{\ell} \exp(i m\varphi)) \qquad (14)$$

where d/dt is taken along unperturbed trajectories and

$$\omega_d = \frac{-c}{eB_{\theta}R} \, T\left(\frac{\partial n}{n\partial r} + \left(\frac{\mathcal{E}}{T} - \frac{3}{2}\right)\frac{\partial T}{T\partial r}\right) \qquad (\mathcal{E} = mv^2/2)$$

The charge density $\rho_{p=e}$ and the functionnal $\mathcal{L}_{p=e}$ specified by (10a) and (10b) are given by

$$\rho_e = \iiint d_3v \left(F \frac{-e^2\psi(r,\theta)}{T} + \sum_{\ell} e \, g_{\ell}(\vec{v}, r, \theta) \right)$$

$$\mathcal{L}_e = \int \frac{ne^2}{T} \sum_{\ell} |\psi_{\ell}(r)|^2 \, 2\pi R \, 2\pi r \, dr - \mathcal{L}' \qquad (15a)$$

$$\mathcal{L}' = \int \dots \sum_{\ell} \sum_{\ell'} e \, g_{\ell}(\vec{v}, r, \theta) \left(\psi_{\ell'}(r) \exp i\ell'\theta\right)^{*} d_3v \, d_3x . \qquad (15b)$$

Again we have omitted in the expression of $\mathcal{L}_e$ cross terms $\sim \int \frac{ne^2}{T} O\left(\frac{r}{R}\right) \left(\psi_{\ell} \psi^{*}_{\ell+1} + \psi^{*}_{\ell} \psi_{\ell+1}\right) 2\pi R \, 2\pi r \, dr$.

We describe the phase space for trapped electrons by the magnetic moment $\mu = (mv_{\perp}^2/2B)(1 + O(r/R)) = (\mathcal{E}/B)(1 + O(r/R))$, the radius r of the magnetic surface, the angular coordinate $\bar{\varphi}$ around the major axis which specify the trace in the equatorial plane of the flux line where the particle stands, the amplitude λ of the bounce motion along flux lines and the phase φ_b of this bounce motion. For values of $|\theta| < \pi/2$, i.e. $\lambda < \pi q R/2$, we may write

$$\left.\begin{aligned} \varphi &= \bar{\varphi} + \frac{\lambda}{R} \sin \varphi_b \\ \theta &= \frac{\lambda}{qR} \sin \varphi_b \\ V_{\parallel} &= \lambda \omega_b \cos \varphi_b \end{aligned}\right| \qquad (16a)$$

$$\lambda^2 = \frac{V_{\parallel}^2}{\omega_b^2} + \theta^2 q^2 R^2 \qquad (16b)$$

The time derivatives $d\varphi_b/dt = \omega_b$ and $d\bar{\phi}/dt = \omega_d$ are the bounce and the precessionnal frequencies given by (4a) and (4b). The constant of motion λ is a function of r, μ and $V_{\parallel}$, namely of r, μ and (see Fig. (1))

$$\mathcal{E}' = \frac{1}{2} m v^2 - \mu B_0 \left(1 - \frac{r}{R_0}\right) = \frac{1}{2} m V_{\parallel}^2 + 2\mu B \frac{r}{R} \sin^2 \frac{\theta}{2} \qquad (16c)$$

For $|\theta| < \pi/2$ we have $\mathcal{E}' = m \omega_b^2 \lambda^2/2$. We may write the variables $\exp i(\ell\theta + M\varphi)$, $g_\ell(\vec{V}, r, \theta) \exp iM\varphi$ and $C(g_\ell \exp iM\varphi)$ in the trapped domain

$$\exp i(\ell\theta + M\varphi) = \exp iM\bar{\varphi} \sum_p S_{\ell p}(\mu, r, \lambda) \exp ip\varphi_b \qquad (17a)$$

$$g_\ell(\vec{V}, r, \theta) \exp iM\varphi = \exp iM\bar{\varphi} \sum_p g_{\ell p}(\mu, r, \lambda) \exp ip\varphi_b \qquad (17b)$$

$$C(g_\ell \exp iM\varphi) = \exp iM\bar{\varphi} \sum_p C_{\ell p}(\mu, r, \lambda) \exp ip\varphi_b \qquad (17c)$$

The Fokker Planck equation (14) becomes

$$i(\omega' + p\omega_b) g_{\ell p} - i(\omega + M\omega_d) \frac{eF}{T} \varphi_\ell(r) S_{\ell p} = C_{\ell p} \qquad (18)$$

where $\omega' = \omega + M\omega_g$

The Kadomtsev mechanism has to be considered when the conditions (8) are satisfied. We anticipate that these conditions imply that the variable $g_\ell\ (\vec{V},r,\theta)$. expi$M\varphi$ inside the trapped domain is dominated by the term p = 0 of the series (17b) and is larger than in the circulating domain. In these conditions the variable g_ℓ expi$M\varphi \simeq g_{\ell o}(\mu,r,\lambda)$ expi$M\bar{\varphi}$ at a given position, i.e. for given r, $\bar{\varphi}$, and θ, is more localized in the $V_{\parallel}$ direction than in the $V_\perp$ direction, and we may state

$$C\left(g_\ell \ \exp i M\varphi\right) = C\left(g_{\ell o}(\mu,r,\lambda)\ \exp i M\bar{\varphi}\right)$$

$$= \frac{1}{2}\langle\delta V_{\parallel}^2\rangle \ \frac{\partial^2}{\partial V_{\parallel}^2}\left(g_{\ell o}(\mu,r,\lambda)\right)\exp i M\bar{\varphi} \qquad (19)$$

where $\langle\delta V_{\parallel}^2\rangle$ is the diffusion coefficient in the $V_{\parallel}$ direction, which for trapped electrons is readily found to be

$$\langle\delta V_{\parallel}^2\rangle = \frac{1}{2}\ \frac{8\pi n e^4\,(Z_{eff}+1)\,\mathcal{L}og\,\Lambda}{m^2\,V}$$

and where of course the variable $g_{\ell o}(\mu,r,\lambda)$ is considered as a function of $V_\perp$, $V_{\parallel}$, r, θ.

For values of $\lambda < \pi q R/2$ we may use (16b) to obtain

$$\frac{\partial}{\partial V_{\parallel}}\left(g_{\ell o}\right) = \frac{\partial g_{\ell o}}{\partial\lambda}\quad\frac{V_{\parallel}}{\omega_b^2\lambda}$$

$$\frac{\partial^2}{\partial V_{\parallel}^2}\left(g_{\ell o}\right) = \left(\frac{\partial^2 g_{\ell o}}{\partial\lambda^2}\left(\frac{V_{\parallel}}{\omega_b\lambda}\right)^2 + \frac{\partial g_{\ell o}}{\partial\lambda}\ \frac{1}{\lambda} - \frac{\partial g_{\ell o}}{\partial\lambda}\ \frac{V_{\parallel}^2}{\omega_b^2\lambda^3}\right)\frac{1}{\omega_b^2}$$

where $\partial g_{\ell o}/\partial\lambda$ and $\partial^2 g_{\ell o}/\partial\lambda^2$ are the normal partial derivatives of the function $g_{\ell o}(\mu,r,\lambda)$. We obtain from (16a) the expression of $\partial^2 g_{\ell o}/\partial V_{\parallel}^2$ as a function of μ, r, λ, φ_b

$$\frac{\partial^2 g_{\ell o}}{\partial V_{\parallel}^2} = \left(\frac{\partial^2 g_{\ell o}}{\partial \lambda^2} \cos^2 \varphi_b + \frac{\partial g_{\ell o}}{\partial \lambda} \frac{1}{\lambda} \left(1 - \cos^2 \varphi_b \right)\right) \frac{1}{\omega_b^2}$$

and it then results from (19) that the variable $C_{\ell o}(\mu, r, \lambda)$ specified by (17c) is given by

$$C_{\ell o}(\mu, r, \lambda) = \frac{1}{4} \frac{\langle \delta V_{\parallel}^2 \rangle}{\omega_b^2} \left(\frac{\partial^2 g_{\ell o}}{\partial \lambda^2} + \frac{\partial g_{\ell o}}{\partial \lambda} \frac{1}{\lambda} \right)$$

On the other hand substituting the values of φ and θ given by (16a) in the phase factor $\exp i \ell \theta \; \exp i M \varphi$ we obtain the variable $S_{\ell o}(\mu, r, \lambda)$ specified by (17a)

$$S_{\ell o} = J_0 \left(K_{\parallel \ell}(r) \; \lambda \right) \qquad \left(K_{\parallel \ell}(r) = \frac{1}{R} \left(M + \ell / q(r) \right) \right) \tag{20}$$

The equation (18) for p = 0 may be written

$$i \omega' g_{\ell o} - i \left(\omega + M \omega_d \right) \frac{eF}{T} \psi_\ell(r) \; J_0 \left(K_{\parallel \ell}(r) \lambda \right) =$$
$$\frac{\langle \delta V_{\parallel}^2 \rangle}{4 \, \omega_b^2} \left(\frac{\partial^2 g_{\ell o}}{\partial \lambda^2} + \frac{1}{\lambda} \frac{\partial g_{\ell o}}{\partial \lambda} \right)$$

This equation accept as a solution

$$g_{\ell o}(\mu, r, \lambda) = J_0 \left(K_{\parallel \ell}(r) \; \lambda \right) \frac{eF}{T} \; \psi_\ell(r) \; \frac{i(\omega + M \omega_d)}{i\omega' + \gamma_E \left(K_{\parallel \ell}(r) \, q R \right)^2} \tag{21}$$

where

$$\gamma_E = \frac{1}{4} \frac{\langle \delta V_{\parallel}^2 \rangle}{\omega_b^2 \, q^2 R^2} = \frac{1}{2} \frac{\langle \delta V_{\parallel}^2 \rangle}{V^2} \frac{R}{r}$$

is given by (4c). This solution is acceptable for values of $K_{\parallel \ell}(r) \, q R \gg 1$ because it is small in that case for $\lambda > \pi q R / 2$: this is consistent with the use of (16) and with the condition that $g_\ell(\vec{V}, r, \theta) \exp i M \varphi \simeq g_{\ell o}(\mu, r, \lambda) \exp i M \bar{\varphi} \simeq 0$

at the fronteer of the trapped domain.

For $K_{\parallel \ell}(r)\, q\, R \ll 1$, the variable $g_{\ell 0}(\mu, r, \lambda)$ consistent with this constraint may be approximated as

$$g_{\ell 0}(\mu, r, \lambda) = A(\mu, r)\,(\varepsilon' - \varepsilon'_0)$$

where $\varepsilon'_0 = 2\mu B r/R$ is the value of the variable ε' specified by (16c) at the fronteer of the trapped domain. The variable $A(\mu, r)$ may be adjusted by multiplying both sides ef Eq. (18) for $p = 0$ by $(g_{\ell 0}(\mu, r, \lambda))^*$ and integrating over $V_\parallel$ and θ in the trapped domain, for given μ and r. Noting that $S_{\ell 0} = 1$ in the present case, we obtain

$$i(\omega + M\omega_g)\,|A(\mu, r)|^2 \iint (\varepsilon' - \varepsilon'_0)^2\, dV_\parallel\, d\theta$$

$$-i(\omega + M\omega_g)\,\frac{eF}{T}\,\psi_\ell(r)\,A^*(\mu, r)\iint(\varepsilon' - \varepsilon'_0)\, dV_\parallel\, d\theta =$$

$$\iint C_{\ell 0}\, dV_\parallel\, d\theta\; g_{\ell 0}^* = \iint C_{\ell 0}\exp iM\bar{\varphi}\,\left(g_{\ell 0}\exp iM\bar{\varphi}\right)^* dV_\parallel\, d\theta \tag{22}$$

The last integral have the same value if we substitute the variable $C(g_\ell \exp iM\varphi)$ to its bounce average $C_{\ell 0}\exp iM\bar{\varphi}$. It then comes, using (19)

$$\iint C_{\ell 0}\, g_{\ell 0}^*\, dV_\parallel\, d\theta = \frac{1}{2}\langle \delta V_\parallel^2\rangle \iint \frac{\partial^2 g_{\ell 0}}{\partial V_\parallel^2}\, g_{\ell 0}^*\, d\theta\, dV_\parallel =$$

$$-\frac{1}{2}\langle \delta V_\parallel^2\rangle \iint\left(\frac{\partial g_{\ell 0}}{\partial V_\parallel}\right)^2 d\theta\, dV_\parallel = -\frac{1}{2}\langle \delta V_\parallel^2\rangle\, |A(\mu, r)|^2 .$$

$$\iint 2m\,(\varepsilon' - \varepsilon'_0)\,\sin^2\frac{\theta}{2}\, dV_\parallel\, d\theta$$

where we have used the fact that (see Eq. (16c))

$$\frac{\partial g_{\ell 0}}{\partial V_\parallel} = A(\mu, r)\,\frac{\partial(\varepsilon' - \varepsilon'_0)}{\partial V_\parallel} = A(\mu, r)\left(2m\left(\varepsilon' - \varepsilon'_0 \sin^2\frac{\theta}{2}\right)\right)^{1/2}$$

The Eq. (22) becomes

$$A(\mu, r)\left(i\omega'\left(\iint(\varepsilon'-\varepsilon'_o)^2\, dv_\parallel\, d\theta\right) + m\langle\delta V_\parallel^2\rangle\left(\iint\left(\varepsilon'-\varepsilon'_o \sin^2\frac{\theta}{2}\right) dv_\parallel\, d\theta\right)\right)$$

$$= i\left(\omega + M\omega_d\right)\frac{eF}{T}\ \psi_\ell(r)\ \left(\iint (\varepsilon'-\varepsilon'_o)\ dv_\parallel\ d\theta\right.$$

The integrals $\iint dv_\parallel\, d\theta$ are easily calculated replacing $dv_\parallel\, d\theta$ by $d\varepsilon'\, d\theta / mV_\parallel = d\varepsilon'\, d\theta / \left(2m\left(\varepsilon - \varepsilon'_o \sin^2\theta/2\right)\right)^{1/2}$.
We obtain finally, for $K_{\parallel\ell}(r)\ qR \ll 1$

$$g_{\ell o}(\mu, r, \lambda) = \frac{\varepsilon'_o - \varepsilon'}{\varepsilon'_o}\ \frac{eF}{T}\ \psi_\ell(r)\ \frac{i\ \omega + M\omega_d}{0{,}64\ i\omega' + \gamma_E} \qquad (23a)$$

It will be noticed that for $\lambda < nqR/2$

$$\frac{\varepsilon'_o - \varepsilon'}{\varepsilon'_o} = 1 - \frac{1}{4}\left(\frac{v_\parallel^2}{\omega_b^2} + q^2R^2\,\theta^2\right)\frac{1}{q^2R^2} = 1 - \frac{1}{4}\frac{\lambda^2}{q^2R^2} \simeq J_o\left(\frac{\lambda}{qR}\right) \qquad (23b)$$

The expression (21) of $g_{\ell o}$ valid for $K_{\parallel\ell}(r)\ qR \gg 1$ joins the expression (23) valid for $K_{\parallel\ell}(r)\ qR \ll 1$ when $K_{\parallel\ell}(r)\ qR \sim 1$. These expressions are therefore valid for $K_{\parallel\ell}(r) qR > 1$ and $K_{\parallel\ell}(r)\ qR < 1$, respectively.

Outside the trapped domain, the variable $g_\ell(\vec{v}, r, \theta)$ is of the order of

$$g_{\ell c} \sim \frac{e\,\psi_\ell(r)}{T}\ F\ \frac{\omega + M\omega_d}{\omega + O\left(K_{\parallel\ell}(r)\ V_\perp\ (r/R)^{1/2}\right)}$$

Inside the trapped domain, the terms $g_{\ell p\neq o}$ are of the order of

$$g_{\ell p\neq o} \sim \frac{e\,\psi_\ell(r)}{T}\ F\ \frac{\omega + M\omega_d}{\omega + O(\omega_b)}$$

It is readily verified that the conditions $g_{\ell o} > g_{\ell p\neq o}$ and $g_{\ell o} > g_{\ell c}$ are equivalent to the conditions (8).

The functionnal $\mathcal{L}'$ specified by (15b), with each $g_\ell(\vec{v}, r, \theta)\ \exp i M\varphi = g_{\ell o}(\mu, r, \lambda) \exp i M\bar{\varphi}$ proportionnal to $\psi_\ell(r)$ has the form

$$\mathcal{L}' = \sum_{\ell,\ell'} \int \frac{ne^2}{T} A_{\ell\ell'}(r)\ \psi_\ell(r)\ \psi^*_{\ell'}(r)\quad 2\pi R\ 2\pi r\ dr \qquad (24)$$

where

$$A_{\ell\ell'}(r) = \iiint_{dr} d_3x \iiint d_3V\ \frac{e\, g_{\ell o}(\mu, r, \lambda)\ \exp iM\bar{\varphi}}{\psi_\ell(r)\quad ne^2/T}\ \frac{1}{2\pi R\, 2\pi r\, dr}\ (\exp i\ell'\theta \exp iM\varphi)^*$$

and the space integral $\iiint_{dr}$ is performed between the magnetic surfaces r and r + dr. We may replace the variable $\exp i\ell'\theta\quad \exp iM\varphi$ by its bounce average $S_{\ell' o}(\mu, r, \lambda)\ \exp i M\bar{\varphi}$ specified by (17a), obtaining

$$A_{\ell\ell'}(r) = \iiint_{dr} d_3x \iiint d_3V\ \frac{e\, g_{\ell o}(\mu, r, \lambda)}{\psi_\ell(r)\quad ne^2/T}\ \left(S_{\ell' o}(\mu, r, \lambda)\right)^*\ \frac{1}{2\pi R\ 2\pi r\ dr}$$

For values $\ell = \ell'$ such that $K_\parallel \ell(r) \ll 1/qR$, we have $S_{\ell o} = 1$ and $g_{\ell o}$ is given by (23). We may calculate $A_{\ell\ell}$ in that case replacing $d_3x\ d_3V$ by

$$2\pi R\ 2\pi r\ dr\quad 2\pi\ V_\perp\ dV_\perp\ \frac{d\theta}{2\pi}\ \frac{2\, d\mathcal{E}'}{\left(2m(\mathcal{E}' - \mathcal{E}'_o\ \sin^2\theta/2\ )\right)^{1/2}}$$

(with $-\pi < \theta < \pi$ and $0 < \mathcal{E}' < \mathcal{E}'_o = 2\mu B\, r/R$). We obtain

$$A_{\ell\ell}(r) = \frac{16}{9\pi^{3/2}} \int_0^\infty \frac{\omega + M\,\omega_d}{0{,}64\,\omega' - i\,\gamma_{\mathcal{E}}}\quad \exp\left(-\frac{\mathcal{E}}{T}\right)\ \frac{d\mathcal{E}}{T}\ \left(\frac{\mathcal{E}}{T}\right)^{1/2} \left(\frac{2r}{R}\right)^{1/2} \qquad (26)$$

In fact the integrand in (25) is localized inside the trapped domain, and a reasonable estimation of $A_{\ell\ell'}$ may be obtained by reducing the integration to the region $|\theta| < \pi/2$. Using the expressions (20) and (21), (23) of $S_{\ell' o}$ and $g_{\ell o}$, and replacing $d_3x\ d_3V$ by

$$2\pi R\ 2\pi r\ dr\quad 2\pi V_\perp^2\ dV_\perp \left(\frac{r}{R}\right)^{3/2} \frac{\lambda\, d\lambda}{\sqrt{2}\ q^2 R^2}$$

(with λ in the interval $(0, \pi q R/2)$, we obtain

$$A_{\ell\ell'}(r) = \int \frac{\omega + M\omega_d}{\omega' - i\gamma_E x_\ell^2} \exp\left(-\frac{\mathcal{E}}{T}\right) \frac{d\mathcal{E}}{T} \left(\frac{\mathcal{E}}{T}\right)^{1/2}$$

$$\frac{1}{2\sqrt{\pi}} \left(\frac{2r}{R}\right)^{1/2} \int_0^{\pi/2} J_0(x_\ell \theta_0) \; J_0\left((M q(r) + \ell')\theta_0\right) \; \theta_0 \, d\theta_0 \qquad (27)$$

$$x_\ell = |M q(r) + \ell| \qquad \text{if} \quad |M q(r) + \ell| > 1.5$$

$$x_\ell = 1 \qquad \text{if} \quad |M q(r) + \ell| < 1.5$$

For $|M q(r) + \ell| = k_{\parallel \ell}(r) \, q R \ll 1$ and $\ell = \ell'$, we recover nearly the expression of $A_{\ell\ell}(r)$ given by (26), justifying the use of (27) for small values of $|M q(r) + \ell|$ or $|M q(r) + \ell'|$. We will accordingly retain the expression (27) for $A_{\ell\ell'}(r)$, with of course the constraints on $\mathcal{E}$ and ℓ imposed by (8).

It results from (27) that the quantities $A_{\ell\ell'}(r)$ are non null and have essentially the same sign for values of the difference $|\ell - \ell'|$ of order of unity. For fixed ℓ, ℓ', the functions $A_{\ell\ell'}(r)$ is localized in an interval near the radii r_ℓ, $r_{\ell'}$ (specified by (3a))

$$|r - r_{\ell'}| \sim |r - r_\ell| < \rho'$$

where the quantity $|M q(r) + \ell'| \sim |M q r + \ell|$ is either $\lesssim 1$ or $\sim (|\omega'| / \gamma_{Eth})^{1/2}$. Therefore

$$\rho' \lesssim \quad \mathrm{Sup}\left(\rho \, , \; \rho \left(\frac{\omega'}{\gamma_{Eth}}\right)^{1/2}\right) \qquad (28)$$

where $\rho \sim \rho_{thi}$ is defined by (3b). We note that

$$\sum_{\ell,\ell'} A_{\ell\ell'}(r) = \sum_\ell \int_0^\infty \frac{\omega + M\omega_d}{\omega' - i\gamma_E x_\ell^2} \exp\left(-\frac{\mathcal{E}}{T}\right) \frac{d\mathcal{E}}{T} \left(\frac{\mathcal{E}}{T}\right)^{1/2} \frac{1}{2\sqrt{\pi}} \left(\frac{2r}{R}\right)^{1/2}$$

$$\frac{1}{x_\ell} \left(\int_0^{\frac{\pi}{2}} J_0(x_\ell \theta_0) \; x_\ell \, d\theta_0\right) \int_{-\infty}^{+\infty} J_0(\alpha \theta_0) \; \theta_0 \, d\alpha$$

where ℓ and ε verify (8). We replace

$$\int_0^{\pi/2} J_o(x_\ell\, \theta_o)\, x_\ell\, d\theta_o \quad \text{where } x_\ell \geq 1 \quad \text{by} \int_0^\infty J_o(u)\, du = 1.$$

and split the summation $\sum_\ell$ in the form

$$\sum_{\ell,\, x_\ell < 1.5} + 2 \int_{x_\ell > 1.5} dx_\ell$$

We then obtain

$$\mathrm{Im}\left(\sum_{\ell,\ell'} A_{\ell\ell'}(r)\right) = \int_0^\infty a(r,\varepsilon)\, \frac{d\varepsilon}{T} \qquad (29\ a)$$

Where

$$a(r,\varepsilon) = (\omega + M\omega_d)\, \exp\left(-\frac{\varepsilon}{T}\right)\left(\frac{\varepsilon}{T}\right)^{1/2} \frac{1}{\sqrt{\pi}}\left(\frac{2r}{R}\right)^{1/2} I\left(\frac{\omega_b}{\gamma_\varepsilon}\right) I\left(\frac{\omega_b}{\omega'}\right)\cdot$$

$$\left(3\,\frac{\gamma_\varepsilon}{\omega'^2 + \gamma_\varepsilon^2} + \frac{1}{\omega'}\left(\mathrm{Arc\,tg}\left(\frac{\omega_b}{\omega'}\right) - \mathrm{Arc\,tg}\left((1.5)^2\,\frac{\gamma_\varepsilon}{\omega'}\right)\right)\right) \qquad (29\ b)$$

$$I(x) = 0 \quad \text{if} \quad |x| < 1 \quad ; \quad I(x) = 1 \quad \text{if} \quad |x| > 1.$$

Let us note also that

$$\mathrm{Im}\left(\int_{-\infty}^{+\infty} \sum_\ell A_{\ell\ell'}(r)\, dr\right) = \mathrm{Im}\left(\sum_{\ell\ell'} A_{\ell\ell'}(r_{\ell'})\right) \frac{1}{|M\, \partial q(r_{\ell'})/\partial r|}$$

$$= \frac{1}{|M\, \partial q(r_{\ell'})/\partial r|} \int_0^\infty a(r_{\ell'},\varepsilon)\, \frac{d\varepsilon}{T} \qquad (30)$$

IV - STABILITY OF THE MODE.

The potential $\psi(r,\theta) = \sum_\ell \psi_\ell(r)$ exp$i\ell\theta$ may be determined by expressing that the functionnal $\mathcal{L}(\omega, m, \psi, \psi^*)$ is an extremum with respect to ψ^* (r,θ). This means that the functionnal $\mathcal{L}_i + \mathcal{L}_e$ as it is given by (13), (15a) and (24), is an extemum with respect to each ψ_ℓ^* (r). It results that

$$\left(\frac{T_i}{T} + 1 - \alpha - \beta' \frac{V_{thi}^2}{\omega^2 R^2}\left(\frac{\ell}{q^2}\frac{\partial q}{\partial r}\right)^2 (r - r_\ell)^2\right) \psi_\ell(r) - \gamma \rho_{thi}^2 \frac{\partial^2 \psi_\ell(r)}{\partial r^2} = \frac{T_i}{T} \sum_{\ell'} A_{\ell'\ell}(r)\ \psi_{\ell'}(r) \qquad (31)$$

We assume that the solution of the set (31) may be sought in the form

$$\psi_\ell(r) = f(r - r_\ell) \quad \exp(i\ell\delta)$$

where the function f(r') (inside the interval $|r'| < \Delta$ specified by (3)) is approximatively constant for $|r'| < \Delta'$, and the range Δ' is larger than the range ρ' of the function $\sum_{\ell'} A_{\ell\ell'}(r_\ell + r')$. It results that in the summation $\sum_{\ell'}$, the functions $\psi_{\ell'}(r)$ are approximatively equal to $f(0)\ \exp i\ell'\delta$. The system (31) then becomes

$$\left(\frac{T_i}{T} + 1 - \alpha - \beta' \frac{V_{thi}^2}{\omega^2 R^2}\left(\frac{\ell}{q^2}\frac{\partial q}{\partial r}\right)^2 r'^2\right) f(r') - \gamma \rho_{thi}^2 \frac{\partial^2 f(r')}{\partial r'^2} = f(0)\ \frac{T_i}{T} \sum_{\ell'} A_{\ell'\ell}(r_\ell + r')\ \exp i(\ell' - \ell)\delta \qquad (32)$$

This equation has the usual form of the equation specifying the radial structure of drift waves [10], with the term responsible for amplification in the R.H.S. The real part of this term plays a negligible role in the destabilization process. Its imaginary part is maximum, and the mode is the most unstable, if the phase shift δ gives the maximum value to the quantity $\mathrm{Im} \sum_{\ell'} A_{\ell'\ell}(r_\ell + r')\ \exp i(\ell' - \ell)\delta$. As the useful values

of $A_{\ell\ell'}$ have the same sign, this is the case if $\delta = 0$, i.e. if all the components $\psi_\ell(r)\ \exp i\ell\theta$ of the potential $\psi(r,\theta)$ are in phase in the equatorial plane $\theta = 0$.
The R.H.S. of (32) is then equal to $(T_i/T)\ F(0)\ \sum_{\ell'} A_{\ell'\ell}(r_\ell + r')$ and is localized in the interval $|r'| < \rho'$.

For $|r'| > \rho'$, the R.H.S. of (32) vanishes and the solutions of this equation may be related to the hypergeometric function, as explained in the Appendix II. We must select the solution which is consistent with total absorption at r' = $\pm\Delta$. This means that f(r') behaves for $|r'| \to \infty$ as $(\exp i\, b\, r'^2/2)/|r'|^{1/2}$ where

$$b = \left(\frac{\beta'}{\gamma}\right)^{1/2} \frac{1}{\rho_{Hi}} \left|\frac{\ell}{q^2}\frac{\partial q}{\partial r}\right| \frac{V_{Hi}}{\omega R} \sim \frac{r}{qR}\frac{1}{\rho_{Hi}^2} \qquad (33\,a)$$

This solution (see Appendix II) may be considered as constant for values of r' in the interval

$$|r'| < \Delta' = \frac{1}{b^{1/2}} \sim \left(\frac{qR}{r}\right)^{1/2} \rho_{Hi} \qquad (33\,b)$$

Taking into account (28), the range Δ' is larger than ρ' as long as $\omega/\gamma_{Eth} < O(qR/r)$, a condition which is satisfied in present experiments. If the mode is weakly unstable, the function F (r') decreases outside this interval (for large values of r') according to the law

$$F(r') = \sqrt{2\pi}\ \ F(0)\ \left|\frac{\Delta'}{r'}\right|^{1/2}$$

For small values of $|r'| < \Delta'$, it exhibits a logarithmic derivative

$$-\left(\frac{\partial F}{F\,\partial r'}\right)_{r'=-0} = \left(\frac{\partial F}{F\,\partial r'}\right)_{r'=+0} = -b^{1/2}\exp\left(\frac{i\pi}{4}\right) 2\left(\frac{\Gamma\left(\frac{3}{4}+\frac{1}{4}\,i\,d\right)}{\Gamma\left(\frac{1}{4}+\frac{1}{4}\,i\,d\right)}\right)^{*}$$

(34a)

where

$$d = -\left(1 + \frac{T_i}{T} - \alpha\right) \frac{1}{b\,\gamma\,\rho_{thi}^2} \sim -\left(1 + \frac{T_i}{T} - \alpha\right) \frac{qR}{r} \qquad (34b)$$

If the mode has a strong growth rate or if it has reached a saturation level in presence of a non linear damping effect, the function f(r') may still be considered as constant for $|r'| < \Delta'$. However it could decrease in that case more rapidly outside this interval. In fact we will not use the structure of f(r') for $|r'| > \Delta'$.

The function f(r') selected above for $|r'| > \rho'$, must join smoothly through the interval $|r'| < \rho'$ where the R.H.S. of (32) is finite. This means that the logarithmic derivatives given by (34a) must be consistent with the equation

$$-\gamma\,\rho_{thi}^2 \left(\left(\frac{\partial f}{f\,\partial r'}\right)_{r'=+0} - \left(\frac{\partial f}{f\,\partial r'}\right)_{r'=-0} \right)$$
$$= \frac{T_i}{T}\; Im\left(\int_{-\infty}^{+\infty} \sum_{\ell'} A_{\ell'\ell}(r)\, dr \right)$$

obtained by integrating the two sides of (32) over the interval $|r'| < \rho'$. Using (34a) it results that

$$\gamma\,\rho_{thi}^2 \quad b^{1/2}\; exp\left(-i\frac{\pi}{4}\right)\; 4 \left(\frac{\Gamma\left(\frac{3}{4} + \frac{1}{4}\,i d\right)}{\Gamma\left(\frac{1}{4} + \frac{1}{4}\,i d\right)} \right)^{*} =$$
$$\frac{T_i}{T}\; Im\left(\int_{-\infty}^{+\infty} \sum_{\ell'} A_{\ell'\ell}(r)\, dr \right)$$

This equation determines the frequency ω of the mode. The marginal stability is obtained for $\Pi_d/4 \sim 1$, i.e., taking into account (34b)

$$1 + \frac{T_i}{T} - \alpha \simeq 0 \qquad (35)$$

and for

$$\frac{T_i}{T}\, \mathrm{Im}\left(\int_{-\infty}^{+\infty} \sum_{\ell'} A_{\ell'\ell}(r)\, dr\right) = -\gamma\, \rho_{thi}^{2}\, b^{-1/2} \frac{4}{\sqrt{\pi}}$$

We calculate $\mathrm{Im} \int_{-\infty}^{+\infty} \sum_{\ell'} A_{\ell'\ell}(r)\, dr$ from (30) and (29b), retaining the term which is proportionnal to $\partial T/\partial r$ only.

Using also (33a), we obtain the critical shear which stabilize the Kadomtsev mechanism

$$\left(\frac{\partial q}{q\partial r}\right)_c = \left(\frac{\partial T}{T\partial r}\right)^{2/3} \left(\frac{1}{r}\right)^{1/3} \left(\frac{\omega r}{k_\theta \rho_{th} v_{th}}\right)^{1/3} \frac{1}{\gamma^{1/2} \beta'^{\,1/6}}$$
$$\left(\frac{m}{m_i}\right)^{1/3} \frac{qR}{r}\; Z\left(\frac{|\omega|}{\omega_{bth}}, \frac{\gamma_{\varepsilon th}}{\omega_{bth}}, \frac{M\,\omega_{qth}}{\omega_{bth}}\right) \qquad (36)$$

where the function $Z(x,y,z)$ is given on Fig. (3) and γ, β', ω may be estimated from (13b) and (35).

V - FLUX OF ENERGY ACROSS THE MAGNETIC SURFACES.

The imaginary part of the functionnal $\mathcal{L}_e(\omega, M, \psi, \psi^*)$ as it is given by (15) (24), with the functions $\psi_\ell(r)$ equal to the function $f(r-r_\ell)$ considered above, takes the value

$$\mathrm{Im}(\mathcal{L}_e) = -\int \frac{ne^2}{T}\, 2\pi R\; 2\pi r\;\; \mathrm{Im}\left(A_{\ell\ell'}(r)\;\; f(r-r_\ell)\;\; f^*(r-r_{\ell'})\right) dr$$

Assuming again that the radial range ρ' around $r_\ell \sim r_{\ell'}$ where the function $A_{\ell\ell'}(r)$ is localized is smaller than the range Δ' where $f(r-r_\ell) = f(r-r_{\ell'}) = f(0)$ we obtain

$$\operatorname{Im} \mathcal{L}_e = -\int \frac{ne^2}{T} \, 2\pi R \, 2\pi r \, \operatorname{Im}\Big(\sum_{\ell\ell'} A_{\ell\ell'}(r)\Big) \, |f(0)|^2 \, dr$$

and using (29a)

$$\operatorname{Im} \mathcal{L}_e = \iint L(r,\varepsilon) \, dr \, d\varepsilon$$

$$L(r,\varepsilon) = -\frac{ne^2}{T} \, 2\pi R \, 2\pi r \, a(r,\varepsilon) \, \frac{1}{T} \, |f(0)|^2$$

where $a(r, \varepsilon)$ is given by (29b). The average radial flux of electron energy induced by the mode through the Kadomtsev mechanism is then given by (12). We obtain the term proportional to $\frac{\partial T}{\partial r}$ in the form

$$\Gamma_E = -n \, \mathcal{K}_k \, \frac{\partial T}{\partial r}$$

$$\mathcal{K}_k = \frac{e^2 |f(0)|^2}{T^2} \, k_\theta^2 \, \rho_{th}^2 \, v_{th}^2 \left(\frac{r}{R}\right)^{1/2} \frac{1}{\gamma_{Eth}} \, Z'\left(\frac{|\omega|}{\omega_{bth}}, \frac{\gamma_{Eth}}{\omega_{bth}}, \frac{M\omega_{gth}}{\omega_{bth}}\right) \qquad (37)$$

where the function Z' (x,y,z) is given on the Fig. (4). The contribution of the Landau and bounce resonances to Γ_E, taking into account the limitations (6), verifies

$$\Gamma_E = -n \, \mathcal{K}_L \, \frac{\partial T}{\partial r}$$

$$\mathcal{K}_L < 3 \, \frac{e^2 |f(0)|^2}{T^2} \, k_\theta^2 \, \rho_{th}^2 \, v_{th}^2 \, \frac{qR}{v_{th}} \qquad (38)$$

A crucial point is to estimate $\overline{f(0)^2}$ from the measured level of density fluctuations $\overline{\delta n^2(r,\theta)}$ at radius r ang angle θ, associated to the mode. We have from (9) and (1) :

$$\frac{\overline{\delta n^2\, r, \theta}}{n^2(r)} = \frac{2e^2}{T^2} \left| \psi(r,\theta) \right|^2 \qquad (39)$$

We assume that at each radius r, the number of components $\psi_\ell(r)$ of $\psi(r,\theta)$ which are effectively in phase is equal to a number N. We may write

$$\left|\psi(r,\theta)\right|^2 = \sum_{\ell = \dots, -N, 0, N, 2N, \dots} \left| f(r - r_\ell) + f(r - r_{\ell+1}) \exp i\theta + \cdots \right.$$

$$\left. + f\left(r - r_{\ell+N-1}\right) \exp i(N-1)\theta \right|^2$$

The quantities $f(r - r_\ell)$, ..., $f(r - r_{\ell+N-1})$ have approximatively the same value and are phase shifted by the same angle $\eta(r - r_\ell)$ specified by

$$\eta(r') \simeq b r' (r_{\ell+1} - r_\ell) = b r' \rho \simeq \frac{\rho r'}{\Delta'^2}$$

if we have

$$r_{\ell+N} - r_\ell = N\rho \lesssim 2\Delta'.$$

We obtain in these conditions

$$\left|\psi(r,\theta)\right|^2 = \int \frac{dr'}{N\rho} \left|f(r')\right|^2 \left| \sum_{p=1}^{N} \exp ip\left(\theta + \eta(r')\right)\right|^2$$

$$= \int \frac{dr'}{N\rho} \left|f(r')\right|^2 \left| \frac{\sin N\left(\theta + \eta(r')\right)/2}{\sin\left(\theta + \eta(r')\right)/2} \right|^2 \qquad (40)$$

The angle $\eta(r')$ is smaller than 1 for $|r'| < \Delta'^2/\rho \sim \Delta'^2/\rho_{thi} \sim \Delta$ i.e. in the major part of the interval $r' < \Delta$ where f(r') exists. For large values of θ, $\theta \sim \pi/2$ say, we may therefore neglect $\eta(r')$ in (40). Replacing $\sin^2 N\theta/2$ by 1/2, we obtain for such angles

$$\left|\psi(r,\theta)\right|^2 = \frac{1}{N\rho}\,\frac{1}{\sin^2(\theta/2)}\int_0^{\Delta}\left|F(r')\right|^2 dr' \qquad (41)$$

On the other hand, the angle $N\eta(r')/2$ is smaller than 1 in the interval $r' < \Delta'$ where f(r') $\simeq$ F (o). It then results f_om (40) that

$$\left|\psi(r,0)\right|^2 > \frac{2\Delta'}{N\rho}\; N^2 \left|F(0)\right|^2 \qquad (42)$$

We obtain from (41) and (42) the upper limit of N

$$N < \mathcal{B}^{1/2}\left(\frac{\int_0^{\Delta}\left|F(r')\right|^2 dr'}{\Delta'\;\left|F(0)\right|^2}\right)^{1/2}$$

where $\mathcal{B} = \left|\psi(r,0)\right|^2 / \left|\psi(r,\pi/2)\right|^2$ is the balooning effect exhibited by the mode. We finilly obtain from (41) and (39)

$$\frac{e^2\left|F(0)\right|^2}{T^2} < \mathcal{B}^{1/2}\,\frac{\rho}{4\Delta'}\,\frac{\overline{\delta n^2(r,\pi/2)}}{n^2} \qquad (43)$$

$$\mathcal{B} = \frac{\overline{\delta n^2(r,0)}}{\overline{\delta n^2(r,\pi/2)}}$$

where ρ and Δ' may be estimated from (4),(33, and (13b). The condition $N\rho < \Delta'$ is fulfilled if

$$\mathcal{B}^{1/2} \lesssim \frac{\Delta'}{\rho}$$

VI - APPLI_CATION TO THE TFR EXPERIMENT.

The microwave scattering experiment [1, 11] in the TFR device has allowed an esti mation of k_θ and $\delta n^2(r,\pi/2)$ at the radii 10 and 15 cm (limiter radius=20 cm). The balooning coefficient $\mathcal{B}$ has not been measured. We will assume that $\mathcal{B} < 10$. an upper limit which is consistent with the measurements made on ATC [2]. In these conditions, we obtain from (37) and (43)

$$\mathcal{K}_{K} < 800 \ cm^2/sec \quad at \quad r = 10 \ cm$$

$$\mathcal{K}_{K} < 80 \ cm^2/sec \quad at \quad r = 15 \ cm$$

The values of $\mathcal{K}_L$ corresponding to the Landau effect (see Eq. (38)) are smaller at least by a factor 3 at 10 cm and is equivalent at 15 cm.

The energy conduction coefficient deduced from the over all balance are $\simeq 2 \div 3 \ 10^3 \ cm^2/sec$. Therefore, our calculations and the measurements made in T.F.R. show that, specially at 15 cm, the drift waves hardly explain the anomalous conduction of electron energy (even if a substantial balooning is present). These calculations, however, has been made assuming that the turbulent modes approximatively retain their structure in the linear regime and that the electron transport coefficients may be calculated by a second order perturbation theory.

APPENDIX I.

The power W may be calculated as the limit for Im $\omega \to -0$ of the quantity

$$W = \int_{\mathcal{D}} \cdots \; (F + \delta F) \; \frac{d(\frac{1}{2} m V^2 + e\,\delta\psi)}{dt} \; d_3x \; d_3V =$$

$$\int_{\mathcal{D}} \cdots \; \delta F \, \frac{\partial e\,\delta\psi}{\partial t} \, d_3x \, d_3V = 2\omega \, \mathrm{Im} \left(\int_{\mathcal{D}} \cdots \; F_+ \; e\psi_+^* \; d_3x \; d_3V \right) \qquad (44)$$

where $\delta F(\vec{x}, \vec{v}, t) = F_+(\vec{x}, \vec{v}) \exp i\omega t + c.c.$ is the perturbation of the equilibrium distribution function F and $\mathcal{D}$ is the domain of phase space where the particles of the considered set are localized. For circulating particles, the variable $\delta\psi(\vec{x}, t)$ varies along an unperturbed trajectory as $e\psi_\ell(r) \exp i(\omega t + K_{\parallel \ell}(r) V_\parallel t)$ (see Eq. (2)). Assuming to simplify that the equilibrium distribution function F corresponds to thermodynamical equilibrium at temperature T, the Vlasov equation $\frac{dF}{dt} = -\frac{d\delta F}{dt}$ implies that

$$-\frac{F}{T} \; \frac{d\frac{1}{2} m V^2}{dt} = \frac{F}{T} \left(\frac{d\, e\,\delta\psi}{dt} - \frac{\partial e\,\delta\psi}{\partial t} \right) = -\frac{d\,\delta F}{dt}$$

so that

$$F_+ = -\frac{F}{T} \; e\,\psi_+ + \frac{F}{T} \; \frac{e\,\psi_+ \; \omega}{\omega + K_{\parallel \ell}(r) \; V_\parallel}$$

(If F corresponds to a confined plasma, the frequency ω must be replaced by $\omega + M\omega_d$ where ω_d is the diamagnetic frequency) Substituting in (44), we obtain the power $W = W_L$ due to the Landau effect, when it is active

$$W_L = \int_{\mathcal{D}} \cdots \; 2\omega^2 \left(\frac{F}{T} \right)^2 e^2 \; |\psi_\ell(r)|^2 \; d_3x \, d_3V$$

$$\pi \; \delta\left(\omega + K_{\parallel \ell}(r) \; V_\parallel\right)$$

As the quantity $K_{\parallel}\ell(r)\, V_{\parallel}$ varies by a quantity $\sim K_{\parallel}\ell$ (r) V in $\mathcal{D}$, we have

$$W_L = \left(\int_{\mathcal{D}} \cdots \; 2\pi\omega^2 \frac{F}{T} e^2 \, d_3x \, d_3V \right) O\left(\frac{|\psi_\ell(r)|^2}{K_{\parallel}\ell(r)\, V} \right)$$

For trapped particles, we have along an unpertubed trajectory $e\psi_+ = e\psi_\ell(r) \sum_p \alpha_p \exp i(p\omega_b + M\omega_g)t$ where the coefficients α_p, the bounce frequency ω_b and the precession frequency ω_g are constant. We now obtain from the Vlasov Equation the value of F_+ along this trajectory in the form

$$F_+ = -\frac{F}{T} e\psi_+ + \frac{F}{T} e\psi_\ell(r) \sum_p \frac{\omega \; \alpha_p}{\omega + p\omega_b + M\omega_g} \exp i(p\omega_b + M\omega_g)t$$

We note that the value of the integral $\int_{\mathcal{D}} \cdots F_+ \; e\psi_+^*$. d_3 x d_3 V is equal to $\int_{\mathcal{D}} \cdots \langle F_+ \; e\psi_+^* \rangle_{xv} \; d3x \; d3V$ where $\langle \; \rangle_{xv}$ means the time averaged value along the unperturbed trajectory passing through x, v. We then obtain from (44) the power $W = W_b$ due to collisionless resonances of the type $\omega + p\omega_b + M\omega_g = 0$ with $p \neq 0$

$$W_b = \int_{\mathcal{D}} \cdots \; 2\omega^2 \frac{F}{T} e^2 |\psi_\ell(r)|^2 \sum_p |\alpha_p|^2 \pi\, \delta(\omega + p\omega_b + M\omega_g) \, d_3x \, d_3V \tag{45}$$

Except for very few particles near the circulating domain, for which $\omega_b \simeq 0$, the frequency ω_b has a value of the order that given by (4b) for particles which bounce near the equatorial plane. The condition $p \neq 0$ means that W_B exists only if $|\omega| \gtrsim \omega_b \; (\gg M\omega_g)$ and that the sum $\sum_p$ in (45) extends over values of $|p| \sim |\omega| / \omega_b$. An estimation of the coefficients α_p may be obtained by assuming that the particles have a sinusoidal motion of amplitude $\lambda \sim q$ R along flux lines, namely that

$$\ell\theta + M\varphi = \left(\frac{\ell}{q(r)} - M \right) \frac{\lambda}{R} \sin \omega_b t + M\omega_g t$$

It results that $|\alpha_p|^2 \sim |J_p(k_\parallel \ell(r)\, \lambda)|^2$.
Therefore the power W exists only if $k_\parallel \ell(r)\, qR \gtrsim p \sim |\omega|/\omega_b$

We then have $|\alpha_p|^2 \sim 1/k_\parallel \ell(r)\, qR$. We further note that the frequency ω_b varies however inside $\mathcal{D}$ by a quantity $\sim \omega_b$ so that $\delta(\omega + p\omega_b)$ in (45) may be replaced approximatively by $1/p\,\omega_b$. We finally obtain

$$W_b \simeq \Big(\int_{\mathcal{D}} \cdots \; 2\pi\, \omega^2 \frac{F}{T}\; e^2 \; d_3x\, d_3V$$

$$O\Big(\Big(\frac{r}{R}\Big)^{-1/2} \frac{1}{\omega_b} \; \frac{|\psi_\ell(r)|^2}{k_\parallel \ell(r)\, qR}\Big) \qquad (46)$$

The term p = o in the L.H.S. of (45) gives the power W_K associated with the Kadomtsev mechanism, replacing (intuitively) the function $\pi\delta(\omega + M\omega_g)$ by $\tau^{-1}/((\omega + M\omega_g)^2 + \tau^{-2})$, where τ is the time during which the particle motion remains coherent with the wave in spite of the effect of collisions. We thus obtain

$$W_K = \Big(\int_{\mathcal{D}} \cdots \; 2\pi\, \omega^2 \; \frac{F}{T} \; e^2 \; d_3x\, d_3V$$

$$\Big(\frac{r}{R}\Big)^{1/2} \frac{|\langle\psi_\ell(r)\rangle|^2 \; \tau^{-1}}{(\omega + M\omega_g)^2 + \tau^{-2}} \Big)$$

where $|\langle\psi_\ell(r)\rangle| \simeq |J_0\,(k_\parallel \ell(r)\, q \quad)\; \psi_\ell(r)|$ is the bounce averaged value of $\psi_\ell(\vec{x})$. In the presence of collisions, the Dirac function $\pi\,\delta(\omega + p\omega_b + M\omega_g)$ which appears in (4 5) must also be replaced by $(\tau^{-1}/((\omega + p\omega_b + M\omega_g)^2 + \tau^{-2})$. This adds a further condition for the estimation (46) to be valid, namely $|\omega| > \tau^{-1}$. Taking into account the estimation (7), this condition writes $k_\parallel \ell(r)\, qR > (|\omega|/\gamma_E)^{1/2}$.

APPENDIX II.

The general solution of (32) with the R.H.S. equal to 0 writes [12]

$$f(r') = a_e \, \Psi_e(x) + a_o \, \Psi_o(x)$$

where $x = r' b^{1/2}$ and the functions $\Psi_e(x)$, $\Psi_o(x)$ behave for $x \to 0$

$$\Psi_e(x) = 1 - \frac{1}{2} \, d \, x^2 + \dots$$

$$\Psi_o(x) = x - \frac{1}{6} \, d \, x^2 + \dots$$

and for $x \to \infty$

$$\Psi_e(x) = \frac{2 \, \Gamma(1/2) \, \exp(-\pi d/8)}{\left|\Gamma\left(\frac{1}{4} + \frac{1}{4} i d\right)\right| \, x^{1/2}} \cos\left(\frac{1}{2} x^2 + \frac{1}{2} a \log x - \frac{\pi}{8} - \sigma(d)\right)$$

$$\Psi_o(x) = \frac{2 \, \Gamma(3/2) \, \exp(-\pi d/8)}{\left|\Gamma\left(\frac{1}{4} + \frac{1}{4} i d\right)\right| \, x^{1/2}} \cos\left(\frac{1}{2} x^2 + \frac{1}{2} a \log x - \frac{3\pi}{8} - \tau(d)\right)$$

$$\sigma(d) = \operatorname{Arg} \Gamma\left(\frac{1}{4} + \frac{1}{4} i d\right) \; ; \; \tau(d) = \operatorname{Arg} \left(\frac{3}{4} + \frac{1}{4} i d\right)$$

For the function f(r') to behave as $\exp i x^2/2$ for $r' \to \infty$, we must take

$$a_e = \frac{\Gamma(3/2)}{\left|\Gamma\left(\frac{3}{4} + \frac{1}{4} i d\right)\right|} \exp i \left(\frac{3\pi}{8} + \tau(d)\right)$$

$$a_o = \frac{\Gamma(1/2)}{\left|\Gamma\left(\frac{1}{4} + \frac{1}{4} i d\right)\right|} \exp i \left(\frac{\pi}{8} + \sigma(d)\right)$$

The value of $\left(\partial f / f \partial r'\right)_{r'=+0} = b^{1/2} \, a_o / a_e$ is then given by (34a).

For $r' \to \infty$ we obtain,

$$\left| \frac{F(r')}{F(0)} \right| = \frac{2\,\Gamma(1/2)\ \exp -\pi d/8}{\Gamma\left(\frac{1}{4} + \frac{1}{4}\, i d\right)} \ \frac{1}{|x|^{1/2}}$$

Assuming that $d \simeq 1$, according to the condition (35), we have $|F(r') / F(0)| \simeq \frac{1}{|x|^{1/2}} \sqrt{2\pi}\ |d|^{1/4}$

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FIGURE CAPTIONS

Fig. (1)

TOKAMAK GEOMETRY

Fig. (2)

Power W irreversibly transfered by a potential $\psi_\ell(r)\ \exp i(\ell\theta + M\varphi + \omega t)$ to the electrons of energy on the magnetic surface r. Thick line : Kadomtsev mechanism ; thin line: Landau mechanism. $x_1 = \mathrm{Sup}\left((r/R)^{1/2}|\omega|/\omega_b\ ,\ (r/R)^{3/2}\ \gamma_E/\omega_b\right)$.
a/ $|\omega| < \gamma_E$; b/ $|\omega| > \gamma_E$.

Fig. (3)

Values of the function $Z(x,y,z)$ which appears in Eq. (36). ($x = |\omega|/\omega_{bth}$; $y = \gamma_{Eth}/\omega_{bth}$; $z = M\omega_{gth}/\omega_{bth}$)

Fig. (4)

Values of the function $Z'(x,y,z)$ which appears in Eq. (37). ($x = |\omega|/\omega_{bth}$; $y = \gamma_{Eth}/\omega_{bth}$; $z = M\omega_{gth}/\omega_{bth}$)

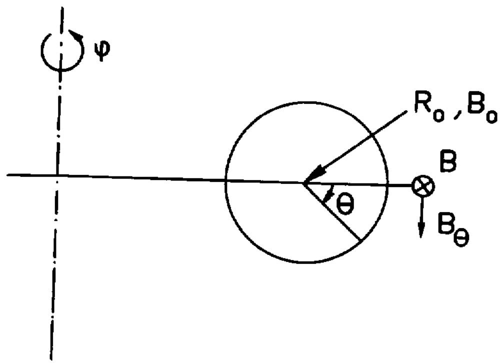
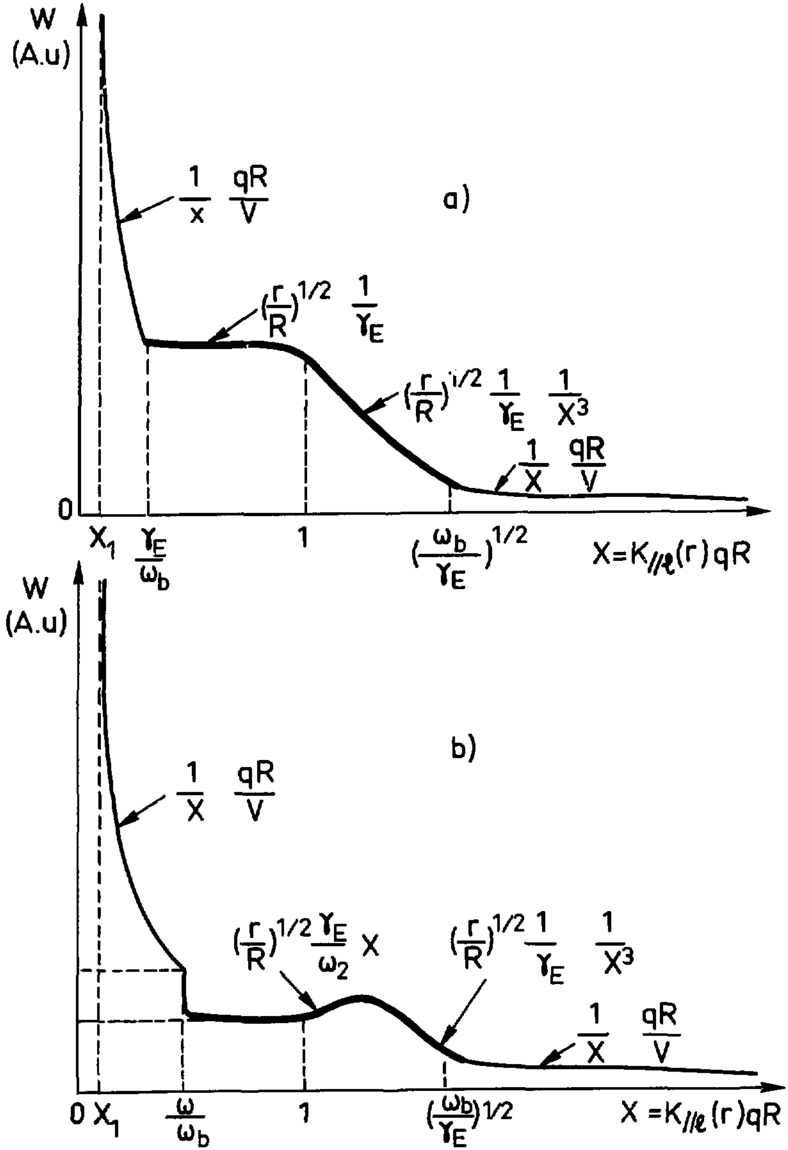


Fig. 1



W
(A.u)
a)
$\frac{1}{x}\ \frac{qR}{V}$
$(\frac{r}{R})^{1/2}\ \frac{1}{\gamma_E}$
$(\frac{r}{R})^{1/2}\ \frac{1}{\gamma_E}\ \frac{1}{X^3}$
$\frac{1}{X}\ \frac{qR}{V}$
0
X_1
$\frac{\gamma_E}{\omega_b}$
1
$(\frac{\omega_b}{\gamma_E})^{1/2}$
$X = K_{//\ell}(r)\,qR$
W
(A.u)
b)
$\frac{1}{X}\ \frac{qR}{V}$
$(\frac{r}{R})^{1/2}\frac{\gamma_E}{\omega_2}\ X$
$(\frac{r}{R})^{1/2}\frac{1}{\gamma_E}\ \frac{1}{X^3}$
$\frac{1}{X}\ \frac{qR}{V}$
0
X_1
$\frac{\omega}{\omega_b}$
1
$(\frac{\omega_b}{\gamma_E})^{1/2}$
$X = K_{//\ell}(r)\,qR$

Fig.2

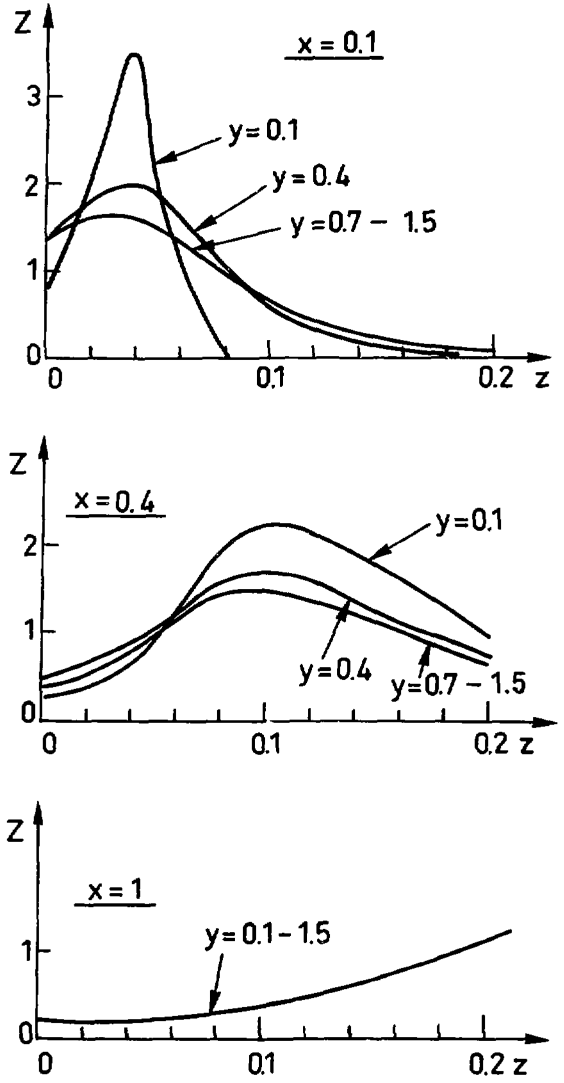
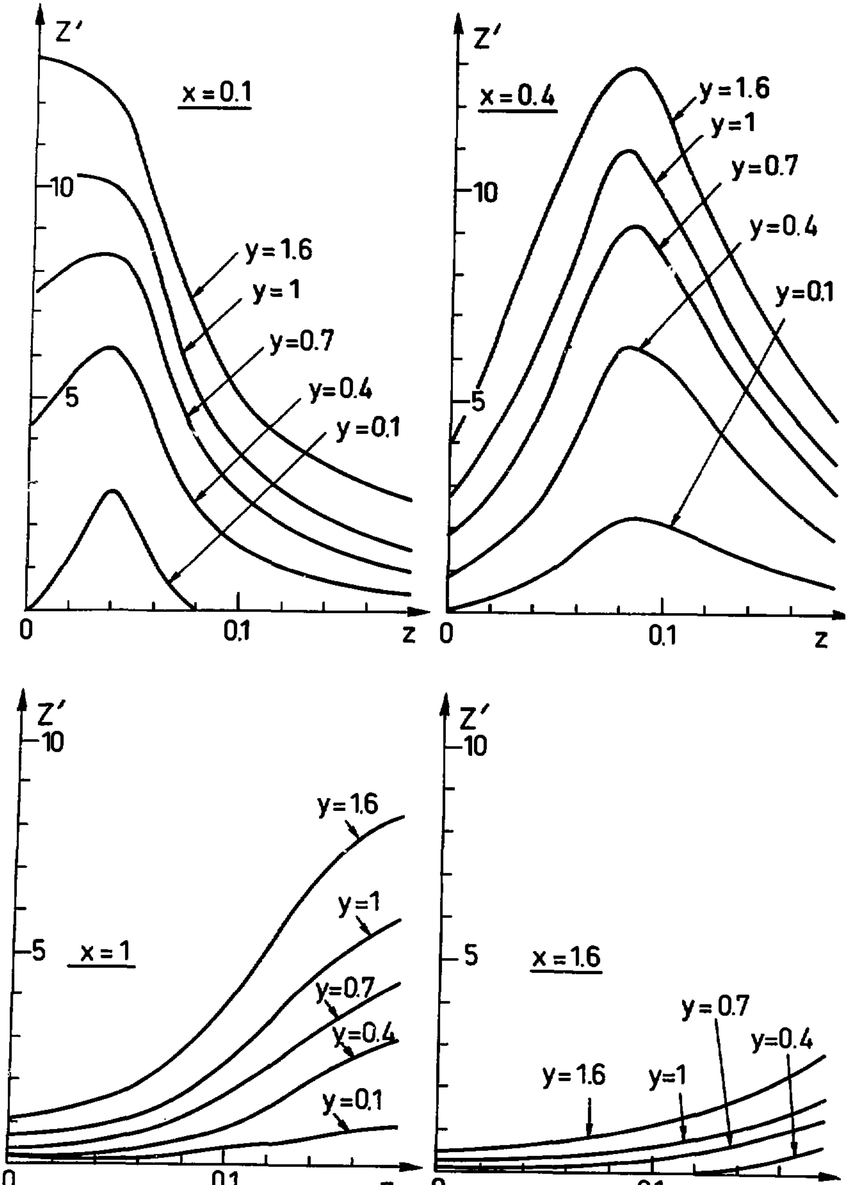

Z
x = 0.1
3
2
1
0
y= 0.1
y = 0.4
y = 0.7 – 1.5
0
0.1
0.2
z
Z
x = 0.4
2
1
0
y = 0.1
y = 0.4
y = 0.7 – 1.5
0
0.1
0.2
z
Z
x = 1
1
0
y = 0.1 – 1.5
0
0.1
0.2
z

Fig. 3



Z′
x = 0.1
10
5
y = 1.6
y = 1
y = 0.7
y = 0.4
y = 0.1
0
0.1
z
Z′
x = 0.4
10
5
y = 1.6
y = 1
y = 0.7
y = 0.4
y = 0.1
0
0.1
z
Z′
10
5
x = 1
y = 1.6
y = 1
y = 0.7
y = 0.4
y = 0.1
0
0.1
z
Z′
10
5
x = 1.6
y = 1.6
y = 1
y = 0.7
y = 0.4
0
0.1
z

_ Fig. 4 _