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MAGNETIC TURBULENCE IN TOKAMAKS

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ABSTRACT -

From a discussion of the disruption process, it is concluded that this process plausibly consists of the onset of a fine grain turbulence. This turbulence must be able to produce the large values of the inductive electric field $-\frac{1}{c}\frac{\partial \Psi}{\partial t}$ which are associated with the reorganization of the poloidal flux $\Psi(r)$ and the current density I(r) on the magnetic surfaces of radius r. It is then plausible that the turbulence belongs to a class of "rippling" modes, in the presence of which the Ohm law takes the form

$$-\frac{1}{c}\frac{\partial \Psi}{\partial t} = \eta I - \frac{\partial}{r\partial r}\left(r K \frac{\partial I}{\partial r}\right)$$

The anomalous term $-\frac{1}{r\partial r}\left(rK\frac{\partial I}{\partial r}\right)$ may explain the experimental values of $-\frac{1}{c}\frac{\partial \Psi}{\partial t}$ for magnetic perturbations corresponding to a substantial radial ergodicity of the flux lines. The

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stability of the modes in the presence of such an ergodicity is accordingly considered. It is found that the modes may be unstable even in collisionless regime, the ergodicity playing a role similar to the resistivity to partially remove the M.H.D. constraint.

RESUME -

D'une discussion du processus de disruption, il ressort que ce processus consiste probablement en l'apparition d'une turbulence à grain fin. Cette turbulence doit être capable de produire les valeurs élevées du champ électrique inductif $-\frac{1}{c}\frac{\partial\Psi}{\partial t}$, associées à la réorganisation du flux poloidal $\Psi(r)$ et de la densité de courant I(r) sur la surface magnétique de rayon r . Il est alors plausible que la turbulence consiste en des modes "rippling" en présence desquels la loi d'Ohm prend la forme

$$-\frac{1}{c}\frac{\partial\Psi}{\partial t} = \eta I - \frac{\partial}{r\partial r}\left(r K \frac{\partial I}{\partial r}\right)$$

Le terme anormal $\frac{\partial}{r\partial r}\left(r K \frac{\partial I}{\partial r}\right)$ peut expliquer les valeurs constatées de $-\frac{1}{c}\frac{\partial\Psi}{\partial t}$ si la perturbation magnétique réalise une substantielle ergodicité radiale des lignes de flux. Il est montré que les modes sont instables en présence d'une telle ergodicité même en régime non collisionnel, l'ergodicité jouant un rôle analogue à celui de la résistivité pour atténuer la contrainte M.H.D.

I - DISCUSSION OF THE DISRUPTIVE PROCESS -

The disruptions associated with the surface $q = 1$ and the soft disruptions associated with the surface $q = 2$, as they are detected through soft X ray emission [1, 2, 3], consist of a sudden (50 - 200 μs) partial flattening of the temperature profile in a large domain on both sides of the resonant magnetic surface where the safety factor $q = 1$ or 2. They are generally preceded by the relatively slow onset of an oscillating structure which has been identified to magnetic islands in the case $q = 2$ [2, 4]. Owing to the strong analogy of the X ray signal (in space and time) in the case $q = 1$ and $q = 2$, this interpretation is also plausible for the disruptions $q = 1$. Eventually the oscillating structure persists after the disruption. Near the end of the disruptions $q = 2$ a very strong and short negative voltage pulse around the major axis appears [5]. Such a negative spike is absent in the case $q = 1$. This is not surprising owing to the fact that the effect of the disruptions $q = 1$, while significant relatively far from the resonant magnetic surface, does not reach the plasma edge. The presence of the voltage spike in the case $q = 2$ means a sudden variation of the poloidal flux embrassed by the magnetic surface at the plasma edge. It is likely that the poloidal flux $2\pi R\,\Psi(r)$ embrassed by the surface r (see Fig. (I)) varies by a quantity of the same order in the domain affected by the disruptions. Assuming that this variation takes place progressively during the disruption process, this means the existence of an inductive toroidal electril field $-\frac{1}{c}\frac{\partial \Psi}{\partial t}$ significantly larger than the normal value of the resistive effect ηJ. This conclusion is plausible also in the case of the disruptions $q = 1$ [6]. Assuming that the variation of $\Psi(r)$ during the regeneration periods between two disruptions (caused by the rotational part of the electric field $\eta(r)\ J(r)$) is cancelled out by the variation of $\Psi(r)$ caused by the disruptions, the value of $-\frac{1}{c}\frac{\partial\Psi}{\partial t}$ during the latter is $\gtrsim 3\eta J$. It must be noted that disruptions $q = 1$ may exist without magnetic activity on the surface $q = 2$ and vice versa, and therefore that the coupling between the two types of perturbation, however interesting, do not play a major role in the disruption mechanism.

The simplest idea to interpret the disruptions is that they consist of the development of the magnetic islands initially present near the resonant surfaces $q = 1$ or 2, which would invade in particular the domain inside the surface $q = 1$ or the domain outside the surface $q = 2$, while keeping roughly their topology. In the case of the disruption $q = 1$, this idea has been encouraged by the fact that the tearing mode which is at the origin of the magnetic island has a relatively large linear growth rate. In fact there are serious objections to this interpretation. For instance it is difficult to understand on this basis why the action of the disruptions decreases progressively with the distance from the resonant surface, or why the oscillating structure may persist after the disruption, or why the disruptions $q = 2$ induce a large voltage spike during a time which is short compared to duration of the disruption. The major objection is however that the assumed topology change of the flux lines seems impossible in the very short time of the disruptions. Let us consider for instance the case of a disruption $q = 1$, depicted on Fig. (2). Let Φ be the flux between the closed flux lines $\mathcal{C}$ (initially the helical neutral line of the magnetic island) and $\mathcal{B}$ (initially the magnetic axis). We have

$$-\frac{1}{c}\frac{d\Phi}{dt} = \int_{\mathcal{B}} E_{\parallel}\, d\ell - \int_{\mathcal{C}} E_{\parallel}\, d\ell$$

$$= 2\pi R \left(\eta_{\mathcal{B}} I_{\mathcal{B}} - \eta_{\mathcal{C}} I_{\mathcal{C}}\right) > 0 \qquad (1)$$

where $E_{\parallel}$ and ℓ are the component of the electric field and the abscissa along the lines $\mathcal{B}$ or $\mathcal{C}$, and $\eta_{\mathcal{B}}, \eta_{\mathcal{C}}, I_{\mathcal{B}}, I_{\mathcal{C}}$ are the values of the resistivity and the current density on $\mathcal{B}$ and $\mathcal{C}$. The line $\mathcal{B}$, which is reminiscent of the plasma center is likely to be hotter than the line $\mathcal{C}$, and therefore $\eta_{\mathcal{B}} < \eta_{\mathcal{C}}$. It then results from (1) that

$$\left|\frac{1}{c}\frac{d\Phi}{dt}\right| < 2\pi R\, \eta_{\mathcal{B}} \left|I_{\mathcal{B}} - I_{\mathcal{C}}\right| \qquad (2)$$

Before the disruption, the value of Φ is equal to the difference $\Phi_{tor} - \Phi_{pol}$ of the toroidal flux Φ_{tor} and the poloidal flux Φ_{pol} embrassed by the surface $q = 1$ of radius $r = r_1$. After the disruption, assumed to have a duration δt, the value of Φ is 0. We then obtain from (2), assuming a parabolic profile for the initial density current

$$\frac{1}{c}\frac{d\Phi}{dt} \simeq \frac{1}{\delta t}\,\frac{\pi r_1^2 B}{2c}\,(1 - q_0) < 2\pi R\, \eta \left|I_{\mathcal{B}} - I_{\mathcal{C}}\right| \qquad (3)$$

where q_0 is the initial value of q on the magnetic axis and B is the static field. On the other hand, the magnetic energy which is initially disponible for the process has the form $\frac{1}{2} L\left(I_0 - I_1\right)^2$ where I_0 and I_1 are the initial current density on the axis and the magnetic surface $q = 1$, and L is a coefficient of self induction. During the disruption, a magnetic energy $\frac{1}{2} L' \left|I_{\mathcal{B}} - I_{\mathcal{E}}\right|^2$ will appear, with the coefficient L' obviously of the same order as L. The energy conservation imposes that

$$\frac{1}{2} L \left|I_0 - I_1\right|^2 > \frac{1}{2} L' \left|I_{\mathcal{B}} - I_{\mathcal{E}}\right|^2$$

and it then results from (3), noting that $I_0 - I_1 \simeq 2 I (1 - q_0)$

$$\frac{1}{2c\,\delta t} \pi r_1^2 B < \left(2\pi R \eta I\right) \quad 2 \left(\frac{L}{L'}\right)^{1/2}$$

Typically, in the T.F.R. case, the L.H.S. term of this inequation arises to 200 volts, to be compared to a value of $2\pi R \eta I \simeq$ 2 volts. The process is therefore incompatible with a value $\frac{L}{L'} \sim 1$.

We may then consider as plausible that the disruptions are due to a fine grain turbulence. This turbulence must induce a reorganization of the poloidal flux $2\pi R \Psi(r)$ and the current density I(r) (now taken of course in the average at the scale of the turbulence), corresponding to an inductive electric field $-\frac{1}{c}\frac{\partial \Psi}{\partial t}$ larger than the normal value of ηJ. A first possibility is that the turbulence induces anomalous values of the resistivity η. However the disruptions take place for values of the temperature T, density n and current density which are by no means critical for the onset of such an anomaly. It is more likely that the anomalous field $-\frac{1}{c}\frac{\partial \Psi}{\partial t}$ is produced by an electromagnetic effect, namely an effect of the type $\frac{\delta V \times \delta B}{c}$ associated with a magnetic perturbation δB and a velocity perturbation δV both transverse to the static field. The turbulence may then be specified by a potential vector δA parallel to the static field B and an electrostatic potential $\delta \psi$ so that

$$\delta B = \nabla \times \delta A \quad ; \quad \delta A \parallel B \quad ; \quad \delta E = -\frac{1}{c}\frac{\partial \delta A}{\partial t} - \nabla \delta \psi \tag{4}$$

The electric field $\overline{E}_{\parallel}$ along the perturbed flux lines, averaged at the scale of the turbulence (symbol ——) is then given by

$$\overline{E}_{\parallel} = -\frac{1}{c}\frac{\partial \overline{\Psi}}{\partial t} - \overline{\nabla \delta\psi \cdot \frac{\delta B}{B}} = -\frac{1}{c}\frac{\partial \overline{\Psi}}{\partial t} - \mathrm{div}\left(\overline{\frac{\delta B}{B}\delta\psi}\right) \tag{5a}$$

As the vector $\overline{\delta B \delta\psi}$ may vary only in the radial direction along which inhomogeneity exists and as, in the average at the scale of the turbulence, we must have $\overline{E}_{\parallel} = \eta \overline{I}$, we obtain from (5a)

$$-\frac{1}{c}\frac{\partial \overline{\Psi}}{\partial t} = \eta \overline{I} + \frac{\partial}{r\partial r}\left(r\,\overline{\frac{\delta B_r}{B}\delta\psi}\right) \tag{5b}$$

an equation which may be considered as the new Ohm law in the presence of the turbulence. Unstable electromagnetic modes of the form (4) which exhibit a non vanishing value of $\overline{\delta B_r \delta\psi}$ are somewhat similar to the "rippling" modes which have been investigated by Furth, Killeen and Rossenbluth [9]. For convenience we will also call them "rippling" modes. Actually it is plausible that the turbulence we have in view consists of modes of this class. We will consider them in some detail in the next sections. We note simply here that if such a turbulence exists the angular deviation of the magnetic field produces a transverse electron current $\delta I \sim \bar{I}\frac{\delta B}{B}$ which builds up a charge density $-\int^t \mathrm{div}\, \delta I \; dt \sim -\int^t \frac{\partial \bar{I}}{\partial r} \frac{\delta B_r}{B} dt$. The latter is likely to be neutralized by a charge density $-\int^t \alpha\, \delta\psi\, dt$ corresponding to the presence of the potential $\delta\psi$, the quantity α being a proper constant. We then have $\delta\psi \sim -\frac{1}{\alpha}\frac{\partial \bar{I}}{\partial r}\frac{\delta B_r}{B}$ and the Ohm law (5b) becomes

$$-\frac{1}{c}\frac{\partial \bar{\Psi}}{\partial t} = \eta\, \bar{I} - \frac{\partial}{r\partial r}\left(r K \frac{\partial \bar{I}}{\partial r}\right) \quad ; \quad K \sim \frac{1}{\alpha}\, \frac{\overline{\delta B_r^2}}{B^2} \qquad (6a)$$

The turbulence liberates a fraction of the magnetic energy W associated with the poloidal field B_θ . We have in fact

$$-\frac{dW}{dt} = \iiint -\frac{1}{c}\frac{\partial \bar{\Psi}}{\partial t}\, \bar{I}\, d_3x = \iiint \eta (\bar{I})^2 d_3x + \iiint K\left(\frac{\partial \bar{I}}{\partial r}\right)^2 d_3x \qquad (6b)$$

The second term represents the power which is liberated by the turbulence and which is disponible to maintain its amplitude.

II - RIPPLING MODES IN THE LINEAR REGIME -

It is convenient to introduce, for a given applied electromagnetic perturbation

$$\begin{aligned} \delta E &= -\frac{1}{c}\frac{\partial \delta A}{\partial t} - \nabla \delta\psi \quad , \quad \delta B = \nabla \times \delta A \\ \delta A &= (a(x)\, \exp i\omega t + \text{Comp. Conj.}) + O(|a^2|, \dots) \\ \delta\psi &= (\psi(x)\, \exp i\omega t + \text{Comp. Conj.}) + O(|a^2|, \dots) \end{aligned} \qquad (7)$$

the following set of quantities

1°) The first order current and charge densities

$$\begin{aligned} \delta I &= j(x) \exp i\omega t + \text{Comp. Conj.} \\ \delta \rho &= \rho(x) \exp i\omega t + \text{Comp. Conj.} \end{aligned} \tag{8}$$

as they appear from the action of the field $\delta E, \delta B$ on each plasma species. The field $j(x)$, $\rho(x)$ results from the field $a(x)$, $\psi(x)$ by a linear transformation which depends analytically on the frequency ω.

2°) The bilinear form in the fields $a(x)$, $\psi(x)$ and $a^*(x)$, $\psi^*(x$ which is specified by

$$\mathcal{L}(\omega; a, \psi; a^*, \psi^*) = -\frac{1}{4\pi} \iiint |\nabla \times a|^2 \, d_3x + \iiint \left(\frac{j}{c} a^* - \rho \psi^*\right) d_3x \tag{9}$$

which again depends analytically on ω. It is readily verified that if the field a, ψ and the frequency ω represent a self consistent mode, the Maxwell equations (neglecting the displacement currents) which must be verified by a, ψ and ω are equivalent to stating that the form $\mathcal{L}$ in an extremum with respect to all variations of the field a^*, ψ^*. This implies that $\mathcal{L} = 0$, an equation which gives the frequency ω when the geometrical structure $a(x)$, $\psi(x)$ is known. Also, it may be verified that a field $\delta E, \delta B$ of the form (7), with ω real, assumed to be created by oscillating charges and currents independent of the plasma, provides a power W to the system consisting of the plasma and the magnetic field inside the boundary surface, given by

$$W = -2\omega \, \mathcal{I}m \left(\mathcal{L}(\omega; a, \psi; \; a^*, \psi^*) \right) \tag{10}$$

Let us first consider the case of a collisional plasma with negligible temperature, and a mode (See Fig. (1))

$$\begin{aligned} a(x) &= a(r) \; \exp\left(i\,\ell\theta + i\,m\,\varphi\right) \qquad ; \quad a \parallel B \\ \psi(x) &= \psi(r) \; \exp\left(i\,\ell\theta + i\,m\,\varphi\right) \end{aligned} \tag{11}$$

exhibiting along unperturbed flux lines the wave number $K_{\parallel} = \frac{1}{R}\left(m + \frac{\ell}{q(r)}\right)$. The ion contribution to j is then given by

$$j_{\parallel} = 0 \quad ; \quad j_{\perp} = -nec \, \frac{\nabla_{\perp}\psi \times B}{B^2} + \frac{\mathcal{E}}{4\pi}\left(-\frac{\partial \nabla_{\perp}\psi}{\partial t}\right) \tag{12}$$

where n is the plasma density, e is the electron charge, $\mathcal{E} = \frac{c^2}{C_A^2}$ and C_A is the Alven velocity associated with the static field. On the other hand the electron contribution to j is readily found to be

$$\begin{aligned} j_{\perp} &= I \, \frac{\nabla \times a}{B} + n\,e\,c \, \frac{\nabla_{\perp}\psi \times B}{B^2} \\ j_{\parallel} + \frac{\delta\eta}{\eta} I &= \left(-i K_{\parallel} \psi - \frac{1}{c} \, i\omega a\right) \frac{1}{\eta} \\ \frac{\delta\eta}{\eta} &= -\frac{\ell}{\omega r} \, \frac{c}{B} \, \frac{\partial I}{I \partial r} \; \psi \end{aligned} \tag{13}$$

Using the charge continuity equation $i\omega\rho + \operatorname{div} j = 0$ to calculate ρ, and noting that $|\nabla \times a| \approx |\nabla_{\perp} a|$, we obtain

$$\mathcal{L} = \iiint d_3x \Big(-\frac{1}{4\pi} |\nabla_\perp a|^2 + \frac{\varepsilon}{4\pi} |\nabla_\perp \psi|^2$$
$$+ \frac{1}{\eta i\omega} \left(\frac{i\omega}{c} a + i K_\parallel \psi \right) \left(-\frac{i\omega}{c} a^* - i K_\parallel \psi^* \right)$$
$$+ \frac{1}{\omega} \frac{\partial I}{\partial r} \frac{\ell}{r} \frac{1}{B} \left(a\psi^* + a^*\psi + \frac{c}{\omega} K_\parallel \psi\psi^* \right) \Big) \qquad (14)$$

By putting $\psi = \omega \psi'$ for ω real we transform $\mathcal{L}$ as

$$\mathcal{L}(\omega; a, \psi; a^*, \psi^*) = \Lambda(\omega; a, \psi'; a^*, \psi'^*)$$

The bilinear form Λ in $a, \psi'; a^*, \psi'^*$ may be analytically continued in the plane ω. The system of equations which determine the field a, ψ and the frequency ω is equivalent to stating that $\mathcal{L}$ is extremum for all variations of a^*, ψ^*. For ω real this system is equivalent to state that Λ is extremum for all variations of a^*, ψ'^*. This equivalence is valid in the complex plane ω if Λ has been analitically continued. For $i\omega = \gamma$ real we obtain

$$\Lambda = \iiint d_3x \Big(-\frac{1}{4\pi} |\nabla_\perp a|^2 - \frac{\varepsilon\gamma^2}{4\pi} |\nabla_\perp \psi'|^2$$
$$- \frac{\gamma}{\eta} |a + K_\parallel c \psi'|^2 + \frac{\partial I}{\partial r} \frac{\ell}{r} \frac{1}{B} \left(a\psi'^* + a^*\psi' + K_\parallel c \psi'^* \psi' \right) \Big) \qquad (15)$$

The form Λ is real in that case. Therefore, if considered as an hermitic form in a, ψ', it must be extremum for all variations of a, ψ'. It is then obvious that a field a, ψ' which makes $\Lambda > 0$ for a value of $\gamma > 0$ guarantees the existence of an unstable mode with a growth rate $> \gamma$.

The tearing modes corresponds to a trial filed a, ψ' satisfying the M.H.D. constraint $\psi' = \frac{1}{c k_\parallel} a$ outside a thin singular layer including the resonant magnetic surface r = r_s where $k_\parallel = 0$. Inside this layer we may take a = constant and interpolate the values of ψ'. The true rippling modes of FURTH et al. correspond to a = 0 and ψ' localized in a thin layer on one side of the resonant surface. The rippling modes we have in view correspond to a field $a, \psi' = \frac{i\hat{\psi}}{\gamma}$ symetric with respect to the position r = r_s, with the field a and localized in radial intervals δ and δ', respectively. Such a field is depicted on the figure (3). We then obtain from (15), assuming that $\frac{\ell}{r} \sim \frac{1}{\delta}$ and $\delta' < \delta$

$$\Lambda \sim -\frac{1}{4\pi} |a|^2 \frac{1}{\delta} - \frac{\varepsilon \gamma^2}{4\pi} |\psi'|^2 \frac{1}{\delta'}$$
$$- \frac{\gamma}{\eta} \left(\delta |a|^2 + \delta' k_\parallel^2 c^2 |\psi'|^2 \right)$$
$$+ \frac{\partial I}{\partial r} \frac{\ell}{r} \frac{1}{B} \left(a \psi'^* + a^* \psi' \right) \delta$$

where $k_\parallel \sim \frac{\ell}{r} \frac{\partial q}{q^2 \partial r} \delta'$. We will assume that $\frac{\partial I}{I \partial r} \sim \frac{\partial B_\theta}{B_\theta \partial r} \sim \frac{\partial q}{q \partial r} \sim \frac{1}{r}$. We then find $\Lambda > 0$ if

$$\delta' \sim r \left(\frac{\tau_A}{\tau_R}\right)^{1/2} ; \quad \delta \sim r \left(\frac{\tau_A}{\tau_R}\right)^{1/6} ; \quad \gamma \sim \frac{1}{\tau_R^{2/3} \, \tau_A^{1/3}} \qquad (16)$$

where $\tau_R = \frac{r^2}{\eta c^2}$, $\tau_A = \frac{r}{c_{A\theta}}$ and $c_{A\theta}$ is the Alfven velocity associated with the field B_θ . The new Ohm law (6a) is easily obtained from (5b) in the form

$$- \frac{1}{c} \frac{\partial \overline{\psi}}{\partial t} = \eta \bar{I} - \frac{\partial}{r \partial r} \left(r \kappa \frac{\partial \bar{I}}{\partial r} \right)$$

$$\kappa \sim \eta \, \frac{\overline{\delta B_r^2}}{B_\theta^2} \left(\frac{r}{\delta}\right)^2 r^2$$

A value of $\left|-\frac{1}{c}\frac{\partial\overline{\psi}}{\partial t}\right|$ significantly large than $\eta\overline{J}$ implies that

$$\frac{\delta B_r}{\delta} > \frac{B_\theta}{r} \qquad (17)$$

This condition means that the transverse magnetic perturbation δB is large compared to the transverse component of the equilibrium field which is created by the shear in a radial interval equal to the scale δ of the magnetic turbulence.

Let us now assume that the electrons have a finite temperature T and the collision frequency ν_c is small. Except in a relatively small layer near the resonance surface we have in that case

$$|k_\parallel v_{th}| > |\omega|,\ \nu_c \ ; \quad T = \frac{m v_{th}^2}{2} \qquad (18)$$

The electron contribution to j results from the integration of the Vlasov Equation. Neglecting the gradients of the temperature and of the density n, we obtain instead of (13)

$$\frac{1}{c} j_\parallel = \frac{\omega^2}{k_\parallel^2 c^2}\left(\frac{c k_\parallel \psi}{\omega} + a\right)\frac{n e^2}{T} - \frac{\ell}{r}\frac{1}{c}\frac{\partial I}{\partial r}\frac{1}{B}\frac{1}{k_\parallel} a \qquad (19)$$

$$j_\perp = I\frac{\nabla \times a}{B}$$

Assuming to simplify that the ion have no temperature so that the Eqs. (12) are still valid, it comes

$$\mathcal{L} = \iiint d_3x \left(\frac{ne^2}{T} \left(\psi + \frac{\omega a}{c k_\parallel} \right) \left(\psi^* + \frac{\omega a^*}{c k_\parallel} \right) - \frac{\ell}{r} \frac{1}{cB} \frac{\partial I}{\partial r} \frac{1}{k_\parallel} |a|^2 - \frac{1}{4\pi} |\nabla_\perp a|^2 + \frac{\varepsilon}{4\pi} |\nabla_\perp \psi|^2 \right) \qquad (20)$$

This expression of $\mathcal{L}$, valid in the domain where (18) is satisfied, does not contain the terms proportionnal to $\frac{\partial I}{\partial r} (a\psi^* + a^*\psi)$ which appear in the expression (14) and which are responsible for our rippling instability. In fact, in the domain where $k_\parallel$ is too small for the condition (18) to be satisfied, the expression of $\mathcal{L}$ changes progressively from (20) to join (14) in the small layer where $|k_\parallel v_{th}| < (|\omega| \nu_c)^{1/2}$. The thickness of this layer is larger than the scale δ' given by (16) if $\left|\frac{v_{th}}{qR}\right| < \left(\frac{\nu_c}{\tau_A}\right)^{1/2}$. It may be shown that our rippling modes then persist as in the collisional case depicted above.

III - STABILITY OF THE RIPPLING MODES IN THE PRESENCE OF THE TURBULENCE -

We have seen in the preceding section that the rippling modes are able to produce the large inductive electric fields $\left|\frac{1}{c}\frac{\partial \psi}{\partial t}\right| > \eta I$ characteristic of the disruptions if the magnetic perturbation satisfies the condition (17). It is then natural to ask the question : are our rippling modes still unstable in these conditions ? We may answer this question by the same technique as in the linear regime, by considering a perturbation $\delta E, \delta B$ of the form (7), calculating the response in current and charge density of the form (8) in the presence of the turbulence, then calculating the bilinear form $\mathcal{L}(\omega, a, \psi, a^*, \psi^*)$ specified by (9) which may be used in fact to determine the modes as in the linear case. To simplify the problem we will assume that the linear response of ions is still valid in the presence of the turbulence, focusing our attention on the electron response and neglecting again the density and temperature gradients. We will assume also that the condition (17) is largely satisfied so that the field a, ψ exibits a wave number $\mathcal{K}_{\parallel}$ $\mathcal{K}_{\parallel} \sim \frac{\ell}{r} \frac{\delta B}{B} \sim \frac{1}{\delta} \frac{\delta B}{B}$ along perturbed flux lines which is large. (The quantity $\mathcal{K}_{\parallel}$ must be of course distinguished from the linear value $k_{\parallel} = \left(m + \frac{\ell}{q(r)}\right) \frac{1}{R}\Big)$. We assume in particular that the condition $\mathcal{K}_{\parallel} V_{th} > |\omega|, V_c$, similar to (18) is satisfied and that

$$\mathcal{K}_{\parallel} \gg |k_{\parallel}| \quad \left(\text{if } |r - r_s| \lesssim \delta\right) \qquad (21a)$$

$$\left|\mathcal{K}_{\parallel} \psi\right| \gg \left|\frac{a\,\omega}{c}\right| \qquad (21b)$$

The condition (21b) may be verified a posteriori.

We first consider the electron contribution to $\mathcal{L}$ in the absence of a gradient $\frac{\partial \bar{I}}{\partial r}$ of the averaged current density $\bar{I}$. The electron response to the field a, ψ in that case essentially consists of reaching the new thermodynamical equilibrium along flux lines, in which, because of the condition (21b), the electrostatic potential plays the major role. This response then essentially consists of the charge density $\rho = -\frac{ne^2}{T}\psi$.
Taking into account the definition (9), the corresponding value

of $\mathcal{L}$ is found equal to $\iiint \frac{ne^2}{T} |\psi|^2 d_3x$. We now calculate the term of $\mathcal{L}$ which is proportionnal to $\frac{\partial I}{\partial r} a$. This needs the calculation of that part j' of the electron current response which is proportionnal to $\frac{\partial \bar{I}}{\partial r} a$. The current j' may be obtained by integration of the Vlasov equation, and in fact is transparent on (19). We have

$$j'_{\parallel} = \int_{flux\ line} \left(-i \frac{\ell}{r} \frac{\partial \bar{I}}{B \partial r} a\right) d\ell$$

$$j'_{\perp} = \frac{\nabla \times a}{B} \bar{I}$$

The presence of the turbulence makes the integrand in $\int_{flux\ line}$ a stationnary randon function with a small coherence leugth $\frac{1}{\mathcal{K}_{\parallel}}$. Therefore $j'_{\parallel}$ is the sum of a small value $< j'_{\parallel} > \sim -i \frac{\ell}{r} \frac{\partial \bar{I}}{B \partial r} \frac{1}{\mathcal{K}_{\parallel}} a$ coherent with the field a, ψ , and of a series of balistic terms. These terms are incoherent with the field a, ψ and have not to be known to calculate the expression of $\mathcal{L}$. Finally we retain only the current $< j'_{\parallel} >$ estimated above and the current $j_{\perp}$. Calculating through the charge continuity equation the corresponding charge density

$$-\frac{1}{i\omega} \left(i\, K_{\parallel} j'_{\parallel} + \operatorname{div} j'_{\perp} \right) = -\frac{1}{\omega} \left(O\left(\frac{K_{\parallel}}{\mathcal{K}_{\parallel}}\right) + 1 \right) \frac{\partial I}{B \partial r} \frac{\ell}{r} a$$

and using (21a), we obtain from the definition (9) the desired part of $\mathcal{L}$ proportionnal to $\frac{\partial \bar{I}}{\partial r} a$ in the form

$$\iiint \frac{\ell}{r} \frac{1}{\omega B} \frac{\partial \bar{I}}{\partial r} a \psi^* d_3x$$

To calculate the term of $\mathcal{L}$ which is proportionnal to $\frac{\partial \bar{I}}{\partial r}\, \psi$, we may use the fact that the power which is added to the plasma by a perturbation a, ψ, ω , with ω real, must tend to 0 when ω tends to 0, i.e., for a static perturbation. Taking into account (10), this means that $\omega\, \mathcal{I}m\,(\mathcal{L})$ tends to 0. This determines the desired part of $\mathcal{L}$ in the form

$$\iiint \frac{\ell}{r} \frac{1}{\omega B} \frac{\partial \bar{I}}{\partial r}\, \psi \; a^{*} \; d_3 x$$

It finally results from there arguments that the expression of $\mathcal{L}$ in the presence of a strong turbulence is given by

$$\mathcal{L} = \iiint d_3 x \left(\frac{ne^2}{T} |\psi|^2 - \frac{1}{4\pi} |\nabla_{\perp} a|^2 + \frac{\varepsilon}{4\pi} |\nabla_{\perp} \psi|^2 \right.$$
$$\left. + \frac{\ell}{r} \frac{1}{\omega B} \frac{\partial \bar{I}}{\partial r} (a\, \psi^{*} + \psi\, a^{*}) \right) \qquad (22)$$

The essential difference between the expressions (22) and (20) of $\mathcal{L}$ is that we have recovered in (22) the terms proportionnal to $\frac{\partial \bar{I}}{\partial r} (a\psi^{*} + a^{*}\psi)$ which appear in the collisional expression (14), and which are responsible for our rippling instability. It must be noted that the origin of this situation is the fact that the current $\dot{j}_{\parallel}'$ considered above has a small component coherent with the field a, ψ . This cancellation is fundamentally linked to the existence of balistic terms of $\dot{j}_{\parallel}'$ incoherent with the field a, ψ . In fact these terms necessarily generate torsionnal Alven type modes which represent a damping mechanism for the mode a, ψ . This damping effect is automatically taken into account by the variationnal technique of the form $\mathcal{L}$. In the present collisionless case, it partially removes the M.H.D. constraint and allows the onset of unstable rippling modes. Indeed, by putting $\psi = \omega\psi'$ and transforming the form $\mathcal{L}(\omega; a, \psi; a^* \psi^*)$ into $\Lambda(\omega; a, \psi'; a^*, \psi'^*)$ as explained in the § II, it is readily shown that unstable modes with a growth rate γ exist if there exists a trial field a, ψ' such that the hermitic form

$$\Lambda = \iiint d_3x \left(-\frac{ne^2}{T}|\psi'|^2\gamma^2 - \frac{1}{4\pi}|\nabla_\perp a|^2 - \frac{\varepsilon}{4\pi}|\nabla_\perp \psi'|^2\gamma^2 + \frac{\ell}{r}\frac{\partial \bar{I}}{\partial r}\left(a\psi'^* + a^*\psi'\right)\right)$$

takes a positive value. Unstable rippling modes with a and $\psi' = \frac{i\psi}{\gamma}$ localized symmetrically on both sides of the resonant surface in the same interval δ exist by taking

$$\gamma \sim \frac{c\lambda_D}{qRr} \sim \frac{\rho_{thi}}{r}\frac{1}{\tau_A}$$

$$\lambda_D^2 = \frac{T}{ne^2} \quad ; \quad \rho_{thi} = \frac{c}{eB}\, T^{1/2}\,(\text{ion mass})^{1/2}$$

The Ohm law then becomes

$$-\frac{1}{c}\frac{\partial \Psi}{\partial t} = \eta\,\bar{I} - \frac{\partial}{r\partial r}\left(r K \frac{\partial \bar{I}}{\partial r}\right)$$

$$K \sim \frac{qRr}{c}\,\lambda_D\,\frac{\overline{\delta B_r^2}}{B^2}$$

The poloidal magnetic energy liberated by unit time by the turbulence $-\frac{dW}{dt}$ is given by (6b). The scale δ must be large enough for the power $-\frac{dW}{dt}$ to be larger than the Joule losses $\sim \iiint \frac{|\delta B^2|}{\delta^2} \eta c^2 \, O_3 x$. This condition imposes a lower limit to δ, namely

$$\delta > r \left(\frac{\tau_A}{\tau_R}\right)^{1/2} \left(\frac{r}{\rho_{thi}}\right)^{1/2} = \delta_m$$

The turbulence produces an anomalous inductive field $-\frac{1}{c}\frac{\partial \overline{\Psi}}{\partial t} - \eta \overline{I}$ larger than $\eta \overline{I}$ if we have

$$\delta B > \delta_m \frac{B_\theta}{r}$$

IV - CONCLUDING REMARKS -

The turbulence could exist in the disruptive form, with $\delta B \gg \delta_m \frac{B_\theta}{r}$, and accordingly produce anomalous inductive field $-\frac{1}{c}\frac{\partial \overline{\Psi}}{\partial t} \gg \eta \overline{I}$. It could be triggered by the singularity of the slope $\frac{\partial \overline{I}}{\partial r}$ at the separatrix of the magnetic islands initially present on the resonant surface, and propagate explosively in the plasma as explained in [10].

It is possible also that the turbulence exists in the non disruptive state of the plasma at the minimum scale $\delta = \delta_m$ and the minimum level $\delta B \sim \delta_m \frac{B_\theta}{r}$. In that case the diffusion coefficient should roughly have a value $\sim \delta_m^2 \gamma \sim \eta c^2$, which is not inconsistent with experimental data. The new Ohm law (with now $-\partial \overline{\Psi}/c\partial t - \eta \overline{I}$ only a fraction of $\eta \overline{I}$) could stabilize the tearing modes in the linear range on the surface q = 2 by preventing the singular current layer to appear. A critical density, beyond which this stabilization is not effective enough, and soft or hard disruptions are triggered, could correspond to a critical ratio of the growth rate of the tearing mode on the surface $q = 2$ to the inverse scale time γ of the turbulence. The new Ohm Law should also result in anomalous skin effects and anomalous acceleration of Runaways. Such effects, if present, could be a proof of the presence of the turbulence.

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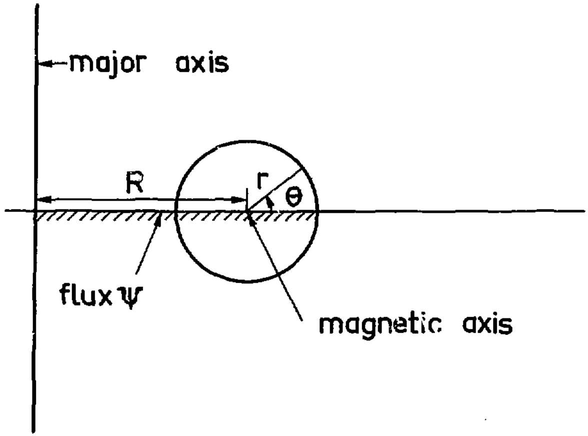
FIGURE CAPTIONS -

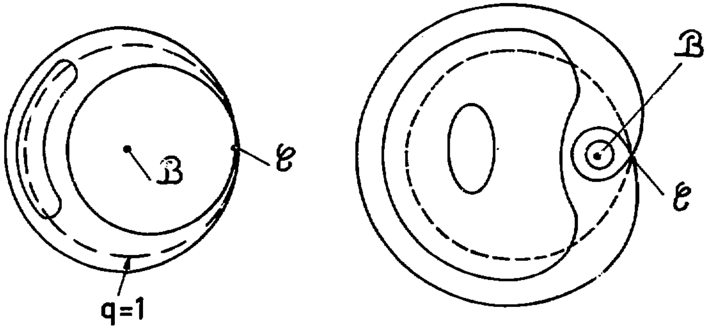
Figure (I) - Tokamak Geometry.

Figure (2) - Hypothetical disposition of magnetic island at the beginning (I) and the end (2) of a disruption $q = 1$.

Figure (3) - Trial field $a_1 \psi'$ for a rippling mode in the collisionnal case.



_ Fig. 1 _



(1) (2)

Fig. 2

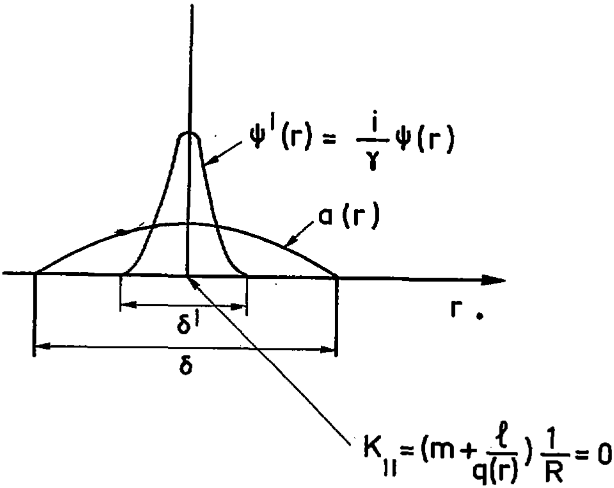

$\psi'(r) = \frac{i}{\gamma}\psi(r)$
$a(r)$
r
δ'
δ
$K_{\parallel} = (m + \frac{\ell}{q(r)})\frac{1}{R} = 0$

Fig. 3