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G. Dattoli, G. Matone and D. Prosperi: HADRON POLARIZABILITIES AND QUARK MODELS. -

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1. - INTRODUCTION.

Aim of the present work is to give an estimate of the electric ($\bar{\alpha}$) and magnetic ($\bar{\beta}$) polarizabilities of hadrons, namely nucleons and π(K) mesons, in the framework of the most widespread accepted quark models.

The proton polarizabilities can be obtained by Compton scattering experiments at low energy where the differential cross section can be expressed in terms of the following expansion[1,2]:

$$\left(\frac{d\sigma}{d\Omega}\right)_p = \left(\frac{d\sigma}{d\Omega}\right)_o - \frac{e^2}{M_p}\,\omega^2 \left\{\bar{\alpha}_p(1+\cos^2\theta) + 2\bar{\beta}_p \cos\theta\left[1-3\,\frac{\omega}{M_p}\,(1-\cos\theta)\right]\right\} + O(\omega^4), \qquad (1)$$

ω and M_p being the photon energy and the proton mass respectively. Moreover $(d\sigma/d\Omega)_o$ is the cross section for the proton thought as structureless and the second term is a structure correction depending on the above mentioned polarizabilities. Several experimental data are now available[3,4,5].

In a similar way, Compton scattering experiments on deuterium can give information on the polarizabilities of the neutron[6].

For the electric polarizabilities of $\pi^{\pm}$ and $K^{\pm}$ mesons, experimental information can be obtained by precise energy measurements of the x-rays emitted by the corresponding exotic atoms[7,8,9]. At present only upper limits for $\bar{\alpha}_{\pi^{\pm}}$ and $\bar{\alpha}_{K^{\pm}}$ can be extracted from the available experimental data[9].

A detailed comparison between theoretical estimates and experimental determinations of the polarizabilities will be presented in the last section of the present paper.

2. - THEORETICAL EXPRESSIONS FOR THE POLARIZABILITIES.

Expressions for the electric and magnetic polarizabilities of bound systems (elementary particles, nuclei...) have been given by many authors[10, 11] in the following form ($\hbar = c = 1$) :

$$\bar{\alpha} = \frac{2}{3} \sum_n \frac{|\langle o|\vec{d}|n\rangle|^2}{E_n - E_o} + \frac{(Ze)^2}{M} \frac{\langle r^2_{ch}\rangle}{3} + O(\frac{1}{M^3}) \quad , \quad \bar{\beta} = \frac{2}{3} \sum_n \frac{|\langle o|\vec{m}|n\rangle|^2}{E_n - E_o} - \beta^D + O(\frac{1}{M^3}) , \quad (2)$$

where $\vec{d}$ and $\vec{m}$ are the usual electric and magnetic dipole moments, $|n\rangle$ is the complete set of excited states of energy E_n, E_o and M are the ground state energy of the system and its mass respectively. Moreover $\langle r^2_{ch}\rangle$ is the charge mean square radius and $(Ze/M)^2 \langle r^2_{ch}\rangle / 3$ is the retardation term due to the finite size of the system.

Finally, β^D is the diamagnetic polarizability whose explicit expression cannot be obtained in a model independent way, due to the lack of a satisfactory expression for the seagull term appearing in the scattering amplitude. A preliminary estimate of β^D for nucleons has been recently given by Ragsa[12, 13] who reduced it to a sum of contributions due to different nucleon states, retaining only the lowest ones (N (938), Δ(1236)).

An alternative approach may be obtained in the framework of a non-relativistic quark model where the quark-quark interaction is supposed to be nice, i.e. free of velocity dependent contributions, and exchange current effects[14, 15] are neglected. Thus in strict analogy with the "nuclear approach", the expression of β^D for a generic hadron H can be reduced to

$$\beta^D_H = \frac{1}{2M_H} \langle H|\vec{d}^2|H\rangle + \frac{e^2}{6M_q} \langle H|\Sigma i \; \hat{e}^2_i r^2_i|H\rangle , \quad (3)$$

where Mq is the effective quark mass, $\hat{e}_i$ is the charge operator for the quark - i and r_i is its distance from the center of mass of the bound system. Fûrthermore $|H\rangle$ stands for the hadron ground state.

By introducing an average value for the denominators ($E_n - E_o$) and by using the completeness relation $\sum_n |n\rangle\langle n| = 1$, eq. (2) can be rewritten in the form

$$\bar{\alpha} = \frac{2}{3} \frac{\langle H|\vec{d}^2|H\rangle}{\mathcal{E}_E} + \frac{(Ze)^2}{M_H} \frac{\langle r^2_{ch}\rangle}{3} + \dots \quad , \quad \bar{\beta} = \frac{2}{3} \frac{\langle H|\vec{m}^2|H\rangle}{\mathcal{E}_M} - \beta^D_H + \dots , \quad (4)$$

where $\mathcal{E}_E$ and $\mathcal{E}_M$ label, respectively, the average electric and magnetic denominators.

For $\vec{d}$ and $\vec{m}$ we will adopt the usual following forms:

$$\vec{d} = e \, \Sigma i \; \hat{e}_i \, r_i \qquad \vec{m} = \mu_p \, \Sigma i \; \hat{\alpha}_i \vec{\sigma}_i \quad (5)$$

where μ_p is the proton magnetic moment and $\hat{\alpha}_i$ are model dependent operators.

By neglecting orbital contributions to $\vec{m}$ we implicitely assume all quark to have $l = 0$, that is an $(s)^A$ quark configurations, where A is the number of quarks in the hadron.

Furthermore, since in all examined models $\langle H|\sum_{i\neq j} e_i \, e_j|H\rangle = 0$, we can write

$$\langle H|\vec{d}^2|H\rangle = e^2 \langle r^2_{ch}\rangle \langle H|\Sigma i \; \hat{e}^2_i|H\rangle = e^2 \langle r^2_{ch}\rangle \, \varrho^2_H , \quad (6)$$

where $\varrho^2_H = \langle H|\Sigma i \; \hat{e}^2_i|H\rangle$ is the mean square charge of the quarks.

In the same way, let us define

$$\langle H| \vec{m}^2 |H\rangle = \mu^2_P \, \tau^2_H , \quad (7)$$

where:

$$\tau_H^2 = \langle H|\Sigma i \quad \hat{a}_i^2 \,|\, H\rangle + \langle H\big|\Sigma i{\neq}j \quad (\hat{a}_i \quad \hat{a}_j) \quad \vec{\sigma} i \quad \vec{\sigma} j \,|\, H\rangle \,. \tag{8}$$

By neglecting terms of the order of $(1/M_H)^3$, from eqs. (4) and (6) one has :

$$\bar{\alpha}_H = \frac{1}{3} \quad e^2 \langle r_{ch}^2 \rangle \left\{ \frac{2\varrho_H^2}{\mathcal{E}_E} + \frac{Z^2}{M_H} \right\} . \tag{9}$$

Finally, by assuming all quarks to have $M_q = M_H/A$, from eqs. (4)(7) and (8) one obtains :

$$\bar{\beta}_H = 2\frac{\mu_P^2}{\mathcal{E}_M} \quad \tau_H^2 - \frac{e^2}{M_H} \langle r_{ch}^2 \rangle \quad \varrho_H^2 \quad (1+\frac{A}{3}) \,. \tag{10}$$

3. MODEL PREDICTIONS.

To analize how $\bar{\alpha}_H$ and $\bar{\beta}_H$ depend on different assumptionson the hadron structure, let us fix our attention on different quark models.For models (3. 1)to(3. 3)we assume the ordinary magnetic dipole operator$(\hat{a}_i \sigma_i \; \hat{e}_i)$; on the contrary, models(3. 4)are characterized by anomalous magnetic moments.

Results obtained for the physical quantities ϱ_H^2 and τ_H^2 defined in the previos section are reported in Table I . Finally, the details of the discussed models and the assumed wave functions are reported in Appendix.

3. 1. - SU (3), SU (4) AND HAN-NAMBU MODELS (16, 17, 18).

In the usual SU (3) model[16], baryons are composed of three ordinary Gell-Mann quarks with fractional charges. Nucleons are assumed to belong to the representation (8_1). In the same model mesons are composed of a quark and an antiquark; π and K mesons are assumed to belong to the representation (8).

As far as SU (4) is concerned[17], in addition to the usual Gell-Mann quarks, we have a charmed quark. Baryons are again composed of three ordinary quarks and mesons of a quark-antiquark pair. The wave functions of nucleons, π and K mesons are identical to those of SU (6) and, therefore, they give the same ϱ_H^2 and τ_H^2 values.

The Han-Nambu model (18), introduced to avoid the need for fractional charges, is a scheme with nine quarks. It requires asymmetry enlarged to SU (3)' ⊗ SU (3)''. Nucleons, π and K mesons belong to a singlet of SU (3)''.

3. 2. - GELL-MAN COLOURED MODEL AND OTHER NINE QUARK MODELS

The Gell-Mann coloured model [19] is a model whose components are a set of three undistinguishable triplets of quarks. They are the blue,red and dark repetition of the usual SU (3) quarks.

From a groupal point of view, this model can be associated with either U (3) ⊗ SU (3)(*), or with SU (3) ⊗ SO(3)(o) in the sense that such anidentification maybe irrelevant. The proof of this last

(*) In this case it is completely isomorphic to the Han-Nambu model.

(o) In this case it is isomorphic to a model by Tati (20). Let us note, from now on, that this last model gives for $\varrho_{P,N}^2$ and $\tau_{P,N}^2$ values too large for a satisfactory reproduction of experimental data on $\bar{\alpha}_{P,N}$ and $\bar{\beta}_{P,N}$.

statement was given in ref. (21) on the basis of the Ohnuki-Kamefuchi theorem[22].

It can be shown that the values of ϱ^2_H and τ^2_H given from this last model are identical to those of SU(6). This not surprising fact can be explained noting that the electromagnetic current is a colour singlet.

The Greenberg model[23] is strictly related to the previous one and was introduced to account for the Pauli principle. The quarks of the model, called paraquarks, consist of a single SU(3) triplet of parafermions of order 3; if the Green component fields are to be taken as independent fields, then the model contains nine quarks[21]. Again in ref. (21) it has been shown that Gell-Mann coloured quarks and the Greenberg paraquarks of order 3 are fully equivalent, this ensuring that the results which could be obtained from the Greenberg model for ϱ^2_H and τ^2_H are identical to those of SU(6).

Let us now discuss in this connection, other two model related to the Han-Nambu one: the SUB(24) and Myamoto (25) models. The first of these two last models is composed by three sets of fundamental triplets of fermions, denoted by U_i, S_i, B_i. They have integer charges and posses an additive quantum number called again charm as in SU(4); for the other quantum numbers of these particles see ref. (26). This model gives for ϱ^2_H and τ^2_H results identical to those of Han-Nambu one, being possible to establish a biunivocal correspondence between the S_i, U_i, B_i quarks and those (p_i, n_i, λ_i) of the Han-Nambu scheme, furthermore the charge matrix of the SUB model is the transposed of the Han-Nambu one.

As to the Myamoto model, we have again a model composed by nine particles with integer charges. The group symmetry is now SU(9), but it can be shown that $SU(3)' \otimes SU(3)''$ (which is subgroup of SU(9)) is sufficient to evaluate the electromagnetic effects. Furthermore, being the charges of the Myamoto model identical to those of the Han-Nambu one, we can find a complete equivalence between the e. m. effects calculated by the Han-Nambu and Myamoto models. This ensures that ϱ^2_H and τ^2_H are identical in the two schemes.

To summarize: The Han-Nambu, SUB and Myamoto models lead for ϱ^2_H and τ^2_H to identical values, as well as SU(6), SU(8), Gell-Mann coloured and Greenberg models.

3. 3. - THE QUARK-DIQUARK MODEL

We have discussed so far models where a baryon is composed of three quarks. Let us now consider the so called "quark-di[illegible]" model where a baryon is composed by two particles: a boson and a fermion. It can be described as follows:

a) It contains invariance under SU(3) to which quarks belong;
b) Furthermore it has objects called diquarks which are contained in a 21-dimensional representation.

In this framework the nucleons belong to a 56-dimensional representation. As far as mesons are concerned, their wave functions are identical to those of SU(6) and therefore $\varrho^2_{\pi\pm}$ and $\varrho^2_{K\pm}$ are identical to those calculated by SU(3).

3. 4. - QUARKS WITH ANOMALOUS MAGNETIC MOMENTS: THE SINGLET-TRIPLET (26) AND FRANKLIN (23) MODELS.

Let us now consider the singlet-triplet model of ref. (26). This model is a scheme with four components, three belonging to a triplet of SU(3) with spin 1/2 and one belonging to a singlet of SU(3) with spin 0.

In this model a baryon is composed of four particles: three (u_1, u_2, u_3) belong to a triplet with zero baryonic number, and the fourth (u_0) is a singlet. With the triplet one constructs octets and declupets as in SU(3), while the singlet gives to the baryons the correct baryonic number. Nucleons and π (K) mesons belong to $8_I \otimes (1)$ and 8 representations respectively.

As far as τ^2_H is concerned, we must note that the magnetic moments of the constituent quarks are completely anomalous.

It is possible to evaluate the a_1 and a_2 coefficients of the magnetic dipole operator of the quarks μ_1 and μ_2 by using the experimental results for neutron and proton magnetic moments. The numerical results are(26):

$$a_1 = 0.66 \;, \qquad\qquad a_2 = 0.35 \;.$$

For $\tau^2_{P,N}$ one obtains:

$$\tau^2_P = \frac{1}{6}\left[4\,(4\,a_1^2 + a_2^2 - 4\,a_2\,a_1) + 3\,a_2^2\right] ,$$

$$\tau^2_N = \frac{1}{6}\left[4\,(4\,a_2^2 + a_1^2 - 4\,a_2\,a_1) + 3\,a_1^2\right] . \qquad (11)$$

Another model of quarks with anomalous magnetic moments is that of ref. (28). The constituent quarks of this model are the usual quarks (p, n, λ) of SU(6), but in addition they possess a further degree of freedom: a "hidden" H-spin of magnitude 1. All known baryon states are assumed to be a singlet of the H-spin, and this account for the Pauli principle. For $\varrho^2_{P,N}$ we obtain the usual SU(6) values, while for $\tau^2_{P,N}$ we have:

$$\tau^2_P = \frac{1}{3}\Big|\,2\,(4\,a_1^2 - 4\,a_2\,a_1 + a_2^2) + a_2^2\,\Big| ,$$

$$\tau^2_N = \frac{1}{3}\left[2\,(4\,a_2^2 - 4\,a_2\,a_1 + a_1^2) + a_1^2\right] . \qquad (12)$$

The a_1, a_2 coefficients are identical to the previous ones (see ref. 26).

TABLE I - Values of ϱ^2_H and τ^2_H for nucleons and π(K) meson calculated by means of various quark models (see text for details).

	SU (6) model (16)	SU (3)' ⊗ SU (3)" model (18)	Quark-Diquark model (27)	Singlet-Triplet model (26)	Franklin model (28)
ϱ^2_P	1	$\frac{5}{3}$	1	3	1
ϱ^2_N	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{11}{25}$	2	$\frac{2}{3}$
$\varrho^2_{\pi^\pm}$	$\frac{5}{9}$	1	$\frac{5}{9}$	1	$\frac{5}{9}$
$\varrho^2_{K^\pm}$	$\frac{5}{9}$	1	$\frac{5}{9}$	1	$\frac{5}{9}$
τ^2_P	$\frac{17}{9}$	$\frac{25}{9}$	$\frac{13}{9}$	1, 93	1, 62
τ^2_N	$\frac{4}{3}$	$\frac{20}{9}$	$\frac{35}{34}$	1, 28	1, 38

4. - NUMERICAL RESULTS

The main problem to be solved before arriving to a reliable estimate of eqs (2) is the choice of the denominators $\mathcal{E}_E$ and $\mathcal{E}_M$.

As far as $\mathcal{E}_E$ for nucleons is concerned, it seems natural to assume that many relevant $0 \rightarrow n$ transitions lead to excitation states belonging to the multiplet $(8_1, \pi = -)$. Moreover, the M1 sum rules are dominated by the $0 \rightarrow \Delta(1236)$ transition. Other relevant M1 transitions could lead to levels whose excitation energy ranges from 500 to 1000 MeV.
Then we choose:

$$\mathcal{E}_E = 700 \text{ MeV} \quad , \quad \mathcal{E}_M = 400 \text{ MeV}.$$

For π and K mesons we expect the mean $0^- \rightarrow 1^+$ transitions, to lie at energies $\mathcal{E}_E \sim 10^3$ MeV.

Table II shows the results for electric polarizabilities, Table III a) the results for magnetic polarizabilities assuming the "nuclear approach" for β^D (11), Table III b) the results with the Ragusa's estimates for the diamagnetic contributions (12, 13):

$$\beta_P^D = 4.7 \times 10^{-4} \text{ fm}^3 \quad ; \quad \beta_N^D = 2.6 \times 10^{-4} \text{fm}^3.$$

5. - COMPARISON WITH EXPERIMENTAL RESULTS

By fitting with eq. (1) the available experimental cross section for (γ- P) elastic scattering (3, 4, 5), one obtains(29) the following results:

$$\bar{\alpha}_P = (12.4 \pm 2.5) \times 10^{-4} \text{ fm}^3 ,$$
$$\bar{\beta}_P = (1.8 \pm 2.8) \times 10^{-4} \text{ fm}^3. \qquad (13)$$

In the fitting procedure the following constraint was imposed:

$$\bar{\alpha}_P + \bar{\beta} \text{ [illegible] } \pm 0.\text{[illegible]}) \times 10^{-4} \text{ fm}^3 , \qquad (14)$$

It was obtained by Damashek and Gilmann(30) who numerically estimated the well known forward sum rule $\bar{\alpha}_P + \bar{\beta}_P = \sigma_{-2}/2\pi^2$.

On the other hand, as suggested in a recent paper by Baranov(31), at energies higher than 50 - 60 MeV eq. (1) cannot be considered as fully satisfactory. In principle in this energy range the interference of the π^0 - meson pole with the Born term must be also taken into account. After the introduction of pole-term in the fit, both values (13) remain practically unchanged.

Moreover from deuteron photoproduction data one obtains for the neutron(29):

$$\bar{\alpha}_N + \bar{\beta}_N \simeq \bar{\alpha}_P + \bar{\beta}_P . \qquad (15)$$

On the other hand no precise measurements of $\bar{\alpha}_N$ and $\bar{\beta}_N$ are now available.

Finally, the study of the x -ray transitions in mesic atoms allows only to obtain upper limits for pion and Kaon electric polarizabilities. In an experiment aiming to obtain an accurate measurement of the Kaon mass, Backenstoss et al(19) obtained:

$$\bar{\alpha}_{K^-} \lesssim 200 \times 10^{-4} \text{ fm}^3. \qquad (16)$$

TABLE II - Electric polarizabilities for nucleons, $\pi^{\pm}$ and $K^{\pm}$ mesons according to eqs. (2. 3) and (2. 5).

	SU(3) model (16)	SU(3)'⊗ SU(3)" model (18)	Quark-Diquark model (27)	Singlet-Triplet model (26)	Franklin model (28)
$\bar{\alpha}_P \times 10^4 fm.^3$	12. 2	18. 2	12. 2	30. 5	12. 2
$\bar{\alpha}_N \times 10^4 fm.^3$	5. 9	11. 9	4. 0	17. 8	5. 9
$\bar{\alpha}_{\pi^{\pm}} \times 10^4 fm.^3$	25. 8	28. 6	25. 8	28. 6	25. 8
$\bar{\alpha}_K \times 10^4 fm.^3$	9. 8	12. 6	9. 8	12. 6	9. 8

TABLE III - Magnetic polarizabilities for nucleons. In Table III(a) the "nuclear estimate" for $\beta^D_{N,P}$ has been adopted (eq. 2. 9). Table III(b) showes results obtained by inserting in eq. (2. 1) the Ragusa's estimate for $\beta^D_{P,N}$ (12, 13).

		SU(6) model (16)	SU(6)'⊗ SU(3)" model (18)	Quark-Diquark model (27)	Singlet-Triplet model (19)	Franklin model (28)
a)	$\bar{\beta}_P \times 10^4 fm.^3$	1. 8	0. 6	0. 7		
	$\bar{\beta}_N \times 10^4 fm.^3$	1. 6	0. 5	0. 3		
b)	$\bar{\beta}_P \times 10^4 fm.^3$	7. 3	12. 5	4. 3	7. 26	5. 34
	$\bar{\beta}_N \times 10^4 fm.^3$	5. 7	11. 2	1. 4	5. 33	5. 95

Let us now conclude with some remarks. By comparing Table II with the experimental data we are lead to conclude that:

a) Due to the uncertenties in the choice of δ_E, we cannot clearly select any model with respect to the other ones, on the basis of its predictions for $\bar{\alpha}_P$. Only the singlet-triplet model, characterized by a particularly high value of Q^2_P, can be ruled out (only for $\delta_E \sim 2 \div 2,5$ GeV this model reproduces experimetal values for $\bar{\alpha}_P$). In addition, it should be noted that all sets of models equivalent in the sense explained at the end of sect.(3. 2) cannot in any way be distinguished in this context.

b) In all the examined models we obtain $\bar{\alpha}_N = (1/3 \div 2/3)\bar{\alpha}_P$, while data now available seem to suggest $\bar{\alpha}_N \sim \bar{\alpha}_P$ (6, 29).

c) The experimental upper limits for $\alpha_{\pi^\pm}$ and $\alpha_{K^\pm}$ are too high by one order of magnitude with respect to present extimates, and therefore better experimental determinations of x-ray energies are seriously needed.

Comments on Table III can be summarized as follows:

a) In part a) the "singlet-triplet" and Franklyn model results have not been reported. The original assumption of no exchange current effects among quarks (nuclear approach) is in fact inconsistent with the strong anomalous magnetic moments of the quarks.

b) Values of a) are lower than b), and seem to be in better agreement with experimental results in the proton case. This suggests that Ragusa's extimate of β^D is probably underestimated.

c) In the SU(6) - SU(8) and in the colour model we have: $\bar{\alpha}_P + \bar{\beta}_P \cong 14 \times 10^{-4}$ fm^3 in remarkable agreement with the Damashek and Gilman sum-rule. But, on the other hand, we obtain: $\bar{\alpha}_P + \bar{\beta}_P / \bar{\alpha}_N + \bar{\beta}_N \sim 1.5 \div 2.$

APPENDIX

SU (6) - SU (8)

If one takes into account the spin for the SU (3) quarks of section (3. 1) baryons belong to the following Krönecker product of SU (6):

$$(2,3)\otimes(2,3)\otimes(2,3) = (2,1)\oplus(2,8_1)\oplus(2,8_2)\oplus(2,10)\oplus(4,1)\oplus(4,8_1)\oplus \oplus(4,8_2)\oplus(4,10). \qquad \text{(A. 1)}$$

Proton and neutron belong to the multiplet $(2, 8_1)$ and their wave functions are given by:

$$|P\uparrow\rangle = \frac{1}{\sqrt{18}}\Big[\,2|p\uparrow n\downarrow p\uparrow\rangle + 2|p\uparrow p\uparrow n\downarrow\rangle + 2|n\downarrow p\uparrow p\uparrow\rangle - |p\uparrow p\downarrow n\uparrow\rangle - |p\uparrow n\uparrow p\downarrow\rangle - |p\downarrow n\uparrow p\uparrow\rangle -$$
$$- |n\uparrow p\downarrow p\uparrow\rangle - |n\uparrow p\uparrow p\downarrow\rangle - |p\downarrow p\uparrow n\uparrow\rangle\Big],$$

$$|N\uparrow\rangle = \frac{1}{\sqrt{18}}\Big[-2|n\uparrow p\downarrow n\uparrow\rangle - 2|n\uparrow n\uparrow p\downarrow\rangle - 2|p\downarrow n\uparrow n\uparrow\rangle + |p\uparrow n\downarrow n\uparrow\rangle + |n\uparrow p\uparrow n\downarrow\rangle + |n\uparrow n\downarrow p\uparrow\rangle +$$
$$+ |p\uparrow n\uparrow n\downarrow\rangle + |n\downarrow p\uparrow n\uparrow\rangle + |n\downarrow n\uparrow p\uparrow\rangle\Big]. \qquad \text{(A. 2)}$$

Moreover both π and K mesons belong to the multiplet (1, 8) of the Krönecker product:

$$(2,3)\otimes(2,3^*) = (1,1)\oplus(3,1)\oplus(1,8)\oplus(3,8)\,. \qquad \text{(A. 3)}$$

Their wave functions are:

$$|\pi^+\rangle = \frac{|p\uparrow\bar{n}\downarrow\rangle - |p\downarrow\bar{n}\uparrow\rangle}{\sqrt{2}}\,, \qquad |\pi^-\rangle = \frac{|p\uparrow n\downarrow\rangle - |\bar{p}\downarrow n\uparrow\rangle}{\sqrt{2}}\,.$$

$$|K^+\rangle = \frac{|p\uparrow\bar{\lambda}\downarrow\rangle - |p\downarrow\bar{\lambda}\uparrow\rangle}{\sqrt{2}}\,, \qquad |K^-\rangle = \frac{|\bar{p}\uparrow\lambda\downarrow\rangle - |\bar{p}\downarrow\lambda\uparrow\rangle}{\sqrt{2}}\,. \qquad \text{(A. 4)}$$

Including the spin, the SU(4) quarks of the GIM model (17), are described by SU (8); it furnishes wave functions for nucleons and π(K) mesons identical to those of SU (6).

HAN-NAMBU MODEL

In the Han-Nambu model[18] the usual SU (3) triplet of quarks is substituted by nine quarks

$$\begin{pmatrix} p_1 & p_2 & p_3 \\ n_1 & n_2 & n_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{pmatrix}, \qquad \text{with integer charges} \qquad \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{pmatrix}.$$

The model symmetry is larger than SU (3) and is given by SU (3)' x SU(3)''; where SU(3)' acts on p, n, λ labels, while SU (3)'' acts on indices (1, 2, 3). The quarks are assumed to belong to the fundamental multiplet $(3, 3^*)$ so that baryons belong to the multiplet $(8_1, 1)$ of the Krönecker product:

$$(3,3^*)\otimes(3,3^*)\otimes(3,3^*) = (1,1)\oplus(8_1,1)\oplus(8_2,1)\oplus(10,1)\oplus(1,8_1^*)\oplus(8_1,8_1^*)\oplus$$
$$\oplus(8_2,8_1^*)\oplus(10,8_1^*)\oplus(1,8_2^*)\oplus(8_1,8_2^*)\oplus(8_2,8_2^*)\oplus(10,8_2^*)\oplus(1,10^*)\oplus \qquad (A.5)$$
$$\oplus(8_1,10^*)\oplus(8_2,10^*)\oplus(10,10^*).$$

By taking into account that the nucleon wave functions must be completely antisymmetric into the SU (3)'' components and with mixed symmetry in the SU (3)', we have:

$$|P\rangle = \frac{1}{6}\Big[2|p_1p_2n_3\rangle - 2|p_2p_1n_3\rangle + 2|p_2p_3n_1\rangle - 2|p_1p_3n_2\rangle + 2|p_3p_1n_2\rangle - 2|p_3p_2n_1\rangle -$$
$$-|p_1n_2p_3\rangle + |p_2n_1p_3\rangle - |p_2n_3p_1\rangle + |p_1n_3p_2\rangle - |p_3n_1p_2\rangle + |p_3n_2p_1\rangle - |n_1p_2p_3\rangle +$$
$$+|n_2p_1p_3\rangle - |n_2p_3p_1\rangle + |n_1p_3p_2\rangle - |n_3p_1p_2\rangle + |n_3p_2p_1\rangle\Big] ,$$

(A. 6)

$$|N\rangle = \frac{1}{6}\Big[-2|n_1n_2p_3\rangle + 2|n_2n_1p_3\rangle - 2|n_2n_3p_1\rangle + 2|n_1n_3p_2\rangle - 2|n_3n_1p_2\rangle + 2|n_3n_2p_1\rangle +$$
$$+|n_1p_2n_3\rangle - |n_2p_1n_3\rangle + |n_2p_3n_1\rangle - |n_1p_3n_2\rangle + |n_3p_1n_2\rangle - |n_3p_2n_1\rangle + |p_1n_2n_3\rangle -$$
$$-|p_2n_1n_3\rangle + |p_2n_3n_1\rangle - |p_1n_3n_2\rangle + |p_3n_1n_2\rangle - |p_3n_2n_1\rangle\Big].$$

π and K mesons belong to the (8, 1) multiplet of the Krönecker product

$$(3^*,3)\otimes(3,3^*) = (1,1)\oplus(1,8)\oplus(8,1)\oplus(8,8) \qquad (A.7)$$

and their wave functions are:

$$|\pi^+\rangle = \frac{|p_1\bar{n}_1\rangle + |p_2\bar{n}_2\rangle + |p_3\bar{n}_3\rangle}{\sqrt{3}} , \qquad |\pi^-\rangle = \frac{|\bar{p}_1n_1\rangle + |\bar{p}_2n_2\rangle + |\bar{p}_3n_3\rangle}{\sqrt{3}} ,$$

(A. 8)

$$|K^+\rangle = \frac{|p_1\bar{\lambda}_1\rangle + |p_2\bar{\lambda}_2\rangle + |p_3\bar{\lambda}_3\rangle}{\sqrt{3}} , \qquad |K^-\rangle = \frac{|\bar{p}_1\lambda_1\rangle + |\bar{p}_2\lambda_2\rangle + |\bar{p}_3\lambda_3\rangle}{\sqrt{3}} .$$

Finally, to calculate τ^2 we are forced to enlarge the symmetry using SU (6)' x SU (3)'', so that the nucleons belong to the multiplet $(2, 8_1, 1)$ of the Krönecker product

$$(2,3,3^*)\otimes(2,3,3^*)\otimes(2,3,3^*) = (2\otimes2\otimes2,\ 3\otimes3\otimes3,\ 3^*\otimes3^*\otimes3^*) =$$
$$= (2,1,1)\oplus(2,1,8_1^*)\oplus(2,1,8_2^*)\oplus(2,1,10^*)\oplus(2,8_1,1)\oplus(2,8_1,8_1^*)\oplus$$
$$\oplus(2,8_1,8_2^*)\oplus(2,8_1,10^*)\oplus(2,8_2,1)\oplus(2,8_2,8_1^*)\oplus(2,8_2,8_2^*)\oplus(2,8_2,10^*)\oplus \qquad (A.9)$$
$$\oplus(2,10,1)\oplus(2,10,8_1^*)\oplus(2,10,8_2^*)\oplus(2,10,10^*)\oplus(4,1,1)\oplus(4,1,8_1^*)\oplus$$
$$\oplus(4,1,8_2^*)\oplus(4,1,10^*)\oplus(4,8_1,8_1^*)\oplus(4,8_1,8_2^*)\oplus(4,8_1,10^*)\oplus(4,8_2,1)\oplus$$
$$\oplus(4,8_2,8_1^*)\oplus(4,8_2,8_2^*)\oplus(4,8_2,10^*)\oplus(4,10,1)\oplus(4,10,8_1^*)\oplus(4,10,8_2^*)\oplus$$
$$\oplus(4,10,10^*)$$

The nucleons wave functions are given by:

$$
\begin{aligned}
|P\rangle = 1/\sqrt{108}\,\Big[& 2|p_1\uparrow n_2\downarrow p_3\uparrow\rangle - 2|p_2\uparrow n_1\downarrow p_3\uparrow\rangle + 2|p_2\uparrow n_3\downarrow p_1\uparrow\rangle - 2|p_1\uparrow n_3\downarrow p_2\uparrow\rangle + \\
& + 2|p_3\uparrow n_1\downarrow p_2\uparrow\rangle - 2|p_3\uparrow n_2\downarrow p_1\uparrow\rangle + 2|p_1\uparrow p_2\uparrow n_3\downarrow\rangle - 2|p_2\uparrow p_1\uparrow n_3\downarrow\rangle + 2|p_2\uparrow p_3\uparrow n_1\downarrow\rangle - \\
& - 2|p_1\uparrow p_3\uparrow n_2\downarrow\rangle + 2|p_3\uparrow p_1\uparrow n_2\downarrow\rangle - 2|p_3\uparrow p_2\uparrow n_1\downarrow\rangle + 2|n_1\downarrow p_2\uparrow p_3\uparrow\rangle - 2|n_2\downarrow p_1\uparrow p_3\uparrow\rangle + \\
& + 2|n_2\downarrow p_3\uparrow p_1\uparrow\rangle - 2|n_1\downarrow p_3\uparrow p_2\uparrow\rangle + 2|n_3\downarrow p_1\uparrow p_2\uparrow\rangle - 2|n_3\downarrow p_2\uparrow p_1\uparrow\rangle - |p_1\uparrow p_2\downarrow n_3\uparrow\rangle + \\
& + |p_2\uparrow p_1\downarrow n_3\uparrow\rangle - |p_2\uparrow p_3\downarrow n_1\uparrow\rangle - |p_1\uparrow p_3\downarrow n_2\uparrow\rangle - |p_3\uparrow p_1\downarrow n_2\uparrow\rangle + |p_3\uparrow p_2\downarrow n_1\uparrow\rangle - |p_1\uparrow n_2\uparrow p_3\downarrow\rangle + \\
& + |p_2\uparrow n_1\uparrow p_3\downarrow\rangle - |p_2\uparrow n_3\uparrow p_1\downarrow\rangle + |p_1\uparrow n_3\uparrow p_2\downarrow\rangle - |p_3\uparrow n_1\uparrow p_2\downarrow\rangle + |p_3\uparrow n_2\uparrow p_1\downarrow\rangle - |p_1\downarrow n_2\uparrow p_3\uparrow\rangle + \\
& + |p_3\downarrow n_1\uparrow p_2\uparrow\rangle - |p_2\downarrow n_3\uparrow p_1\uparrow\rangle + |p_1\downarrow n_3\uparrow p_2\uparrow\rangle - |p_3\downarrow n_1\uparrow p_2\uparrow\rangle + |p_3\downarrow n_2\uparrow p_1\uparrow\rangle - |n_1\uparrow p_2\downarrow p_3\uparrow\rangle + \\
& + |n_2\uparrow p_1\downarrow p_3\uparrow\rangle - |n_2\uparrow p_3\downarrow p_1\uparrow\rangle + |n_1\uparrow p_3\downarrow p_2\uparrow\rangle - |n_3\uparrow p_1\downarrow p_2\uparrow\rangle + |n_3\uparrow p_2\downarrow p_1\uparrow\rangle - |n_1\uparrow p_2\uparrow p_3\downarrow\rangle + \\
& + |n_2\uparrow p_1\uparrow p_3\downarrow\rangle - |n_2\uparrow p_3\uparrow p_1\downarrow\rangle + |n_1\uparrow p_3\uparrow p_2\downarrow\rangle - |n_3\uparrow p_1\uparrow p_2\downarrow\rangle + |n_3\uparrow p_2\uparrow p_1\downarrow\rangle - |p_1\downarrow p_2\uparrow n_3\uparrow\rangle + \\
& + |p_2\downarrow p_1\uparrow n_3\uparrow\rangle - |p_2\downarrow p_3\uparrow n_1\uparrow\rangle + |p_1\downarrow p_3\uparrow n_2\uparrow\rangle - |p_3\downarrow p_1\uparrow n_2\uparrow\rangle + |p_3\downarrow p_2\uparrow n_1\uparrow\rangle \Big] ,
\end{aligned}
\tag{A. 10}
$$

$$
\begin{aligned}
|N\rangle = 1/\sqrt{108}\,\Big[& -2|n_1\uparrow p_2\downarrow n_3\uparrow\rangle + 2|n_2\uparrow p_1\downarrow n_3\uparrow\rangle - 2|n_2\uparrow p_3\downarrow n_1\uparrow\rangle + 2|n_1\uparrow p_3\downarrow n_2\uparrow\rangle - 2|n_3\uparrow p_1\downarrow n_2\uparrow\rangle + \\
& + 2|n_3\uparrow p_2\downarrow n_1\uparrow\rangle - 2|n_1\uparrow n_2\uparrow p_3\downarrow\rangle + 2|n_2\uparrow n_1\uparrow p_3\downarrow\rangle - 2|n_2\uparrow n_3\uparrow p_1\downarrow\rangle + 2|n_1\uparrow n_3\uparrow p_2\downarrow\rangle - 2|n_3\uparrow n_1\uparrow p_2\downarrow\rangle + \\
& + 2|n_3\uparrow n_2\uparrow p_1\downarrow\rangle - 2\,[illegible] + 2|p_2\downarrow n_1\uparrow n_3\uparrow\rangle - 2|p_2\downarrow n_3\uparrow n_1\uparrow\rangle + 2|p_1\downarrow n_3\uparrow n_2\uparrow\rangle - 2|p_3\downarrow n_1\uparrow n_2\uparrow\rangle + \\
& + 2|p_3\downarrow n_2\uparrow n_1\uparrow\rangle\ [illegible] - |n_2\uparrow n_1\downarrow p_3\uparrow\rangle + |n_2\uparrow n_3\downarrow p_1\uparrow\rangle - |n_1\uparrow n_3\downarrow p_2\uparrow\rangle + |n_3\uparrow n_1\downarrow p_2\uparrow\rangle - |n_3\uparrow n_2\downarrow p_1\uparrow\rangle + \\
& - |n_1\uparrow p_2\uparrow n_3\downarrow\rangle - [illegible] + |n_1\uparrow p_3\uparrow n_2\downarrow\rangle + |n_3\uparrow p_1\uparrow n_2\downarrow\rangle - |n_3\uparrow p_2\uparrow n_1\downarrow\rangle + |n_1\downarrow p_2\uparrow n_3\uparrow\rangle - \\
& - |n_3\downarrow p_1\uparrow n_2\uparrow\rangle + |n_2\downarrow p_3\uparrow n_1\uparrow\rangle - |n_1\downarrow p_3\uparrow n_2\uparrow\rangle + |n_3\downarrow p_1\uparrow n_2\uparrow\rangle - |n_3\downarrow p_2\uparrow n_1\uparrow\rangle + |p_1\uparrow n_2\downarrow n_3\uparrow\rangle - |p_2\uparrow n_1\downarrow n_3\uparrow\rangle + \\
& + |p_3\uparrow n_2\downarrow n_1\uparrow\rangle - [illegible] + |p_3\uparrow n_1\downarrow n_2\uparrow\rangle - |p_3\uparrow n_2\downarrow n_1\uparrow\rangle + |p_1\uparrow n_2\uparrow n_3\downarrow\rangle - |p_2\uparrow n_1\uparrow n_3\downarrow\rangle + |p_2\uparrow n_3\uparrow n_1\downarrow\rangle - \\
& - |p_1\uparrow n_3\uparrow n_2\downarrow\rangle + |p_3\uparrow n_1\uparrow n_2\downarrow\rangle - |p_3\uparrow n_2\uparrow n_1\downarrow\rangle + |n_1\downarrow n_2\uparrow p_3\uparrow\rangle - |n_2\downarrow n_1\uparrow p_3\uparrow\rangle + |n_2\downarrow n_3\uparrow p_1\uparrow\rangle - |n_1\downarrow n_3\uparrow p_2\uparrow\rangle + \\
& + |n_3\downarrow n_1\uparrow p_2\uparrow\rangle - |n_3\downarrow n_2\uparrow p_1\uparrow\rangle \Big] .
\end{aligned}
$$

QUARK-DIQUARK MODEL

In the "quark-diquark" model mentioned in Sec. (3. 3), in addition to the ordinary Gell-Mann quarks, there are objects called diquarks which belong to the Krönecker product

$$
\begin{aligned}
(2,3) \otimes (2,3) &= (2 \otimes 2,\ 3 \otimes 3) = (1 \oplus 3,\ 3^{*} \oplus 6) = \\
&= (1, 3^{*}) \oplus (1, 6) \oplus (3, 3^{*}) \oplus (3, 6).
\end{aligned}
\tag{A. 11}
$$

Due to the Pauli principle, the possible multiplets of (A.11) compose the 21 - dimensional representation given by:

$$
21 = (1, 3^{*}) \oplus (3, 6) . \tag{A. 12}
$$

Then baryons belong to

$$21 \otimes 6 = 56 \oplus 70 \qquad \text{(A. 13)}$$

and the wave functions of nucleons which belong to 56 - multiplet are given by:

$$|P\uparrow\rangle = \text{sen}\ \Gamma_1\left[\sqrt{\frac{2}{3}}\ |s_1\uparrow n\downarrow\rangle - \sqrt{\frac{1}{3}}\ |s_2\uparrow p\downarrow\rangle\right] + \cos\ \Gamma_1 |t_1\ p\uparrow\rangle$$

$$|N\uparrow\rangle = \text{sen}\ \Gamma_1\left[\sqrt{\frac{1}{3}}\ |s_2\uparrow n\downarrow\rangle - \sqrt{\frac{2}{3}}\ |s_3\uparrow p\downarrow\rangle\right] + \cos\ \Gamma_1 |t_1\ n\uparrow\rangle \qquad \text{(A. 14)}$$

where n, p are the ordinary Gell-Mann quarks, and s_1, s_2, t are the diquarks with charges: 4/3 e, 1/3 e, 4/3 e and spin 1, 1, 0,. Finally Γ_1 is a normalization parameter whose value is $\pi/4$ in absence of symmetry breaking.

It must be noted that in this model mesons are described only in terms of ordinary quarks.

<u>SINGLET-TRIPLET MODEL</u>

Before constructing the wave functions of nucleons, π and K mesons in the singlet-triplet model, we must note that this group symmetry is $U(1) \otimes SU(3)$ and then the nucleons belong to the multiplet $(1) \otimes 8_1$ of the Krönecker product

$$(1) \otimes (3 \otimes 3 \otimes 3) = (1) \otimes 1 \oplus (1) \otimes 8_1 \oplus (1) \otimes 8_2 \oplus (1) \otimes 10. \qquad \text{(A. 15)}$$

The nucleon wave functions are given by:

$$|P\rangle = \frac{1}{\sqrt{6}}\left[2\ |u_0 u_1 u_1 u_2\rangle - |u_0 u_1 u_2 u_1\rangle - |u_0 u_2 u_1 u_1\rangle\right],$$

$$|N\rangle = \frac{1}{\sqrt{6}}\left[-2\ |u_0 u_2 u_2 u_1\rangle + |u_0 u_2 u_1 u_2\rangle + |u_0 u_1 u_2 u_2\rangle\right]. \qquad \text{(A. 16)}$$

Since the mesons are constructed with a triplet and a antitriplet as in SU (3), the wave functions of π and K mesons are given by:

$$|\pi^+\rangle = |u_1 \bar{u}_2\rangle \qquad , \quad |\pi^-\rangle = |u_2 \bar{u}_1\rangle \quad ,$$

$$|K^+\rangle = |u_1 \bar{u}_3\rangle \qquad , \quad |K^-\rangle = |u_3 \bar{u}_2\rangle \quad . \qquad \text{(A. 17)}$$

The charges of the u_0, u_1, u_2, u_3 quarks are respectively: - e , + e , 0, 0.

By including the spin the wave functions of nucleons are expressed in term of the $U(1) \otimes SU(6)$ group symmetry we have:

$$|P\uparrow\rangle = \frac{1}{\sqrt{18}}\Big[2|u_0 u_1\uparrow u_2\downarrow u_1\uparrow\rangle + 2\ |u_0 u_1\uparrow u_1\uparrow u_2\downarrow\rangle + 2|u_0 u_2\downarrow u_1\uparrow u_1\uparrow\rangle -$$

$$- |u_0 u_1\uparrow u_1\downarrow u_2\uparrow\rangle - |u_0 u_1\uparrow u_2\uparrow u_1\downarrow\rangle - |u_0 u_1\downarrow u_2\uparrow u_1\uparrow\rangle - |u_0 u_2\uparrow u_1\downarrow u_1\uparrow\rangle -$$

$$- |u_0 u_2\uparrow u_1\uparrow u_1\downarrow\rangle - |u_0 u_1\downarrow u_2\uparrow u_2\uparrow\rangle\Big],$$

$$|N\uparrow\rangle = \frac{1}{\sqrt{18}}\Big[-2\ |u_0 u_2\uparrow u_1\downarrow u_2\uparrow\rangle - 2|u_0 u_2\uparrow u_2\uparrow u_1\downarrow\rangle - 2\ |u_0 u_1\downarrow u_2\uparrow u_2\uparrow\rangle + \qquad \text{(A. 18)}$$

$$+ |u_0 u_1\uparrow u_2\downarrow u_1\uparrow\rangle + |u_0 u_2\uparrow u_1\uparrow u_2\downarrow\rangle + |u_0 u_2\downarrow u_1\uparrow u_2\uparrow\rangle + |u_0 u_2\uparrow u_2\downarrow u_1\uparrow\rangle +$$

$$+ |u_0 u_1\uparrow u_2\uparrow u_2\downarrow\rangle + |u_0 u_2\downarrow u_2\uparrow u_1\uparrow\rangle\Big].$$

THE FRANKLYN-MODEL

As far as Franklyn model is concerned, we can write the following wave functions for nucleons:

$$|P\uparrow\rangle = \frac{1}{\sqrt{3}} \left[\sqrt{2}\,|p\uparrow p\uparrow n\downarrow\rangle - \frac{1}{\sqrt{2}} \left(|p\uparrow p\downarrow n\uparrow\rangle + |p\downarrow p\uparrow n\uparrow\rangle \right) \right],$$
$$|N\uparrow\rangle = \frac{1}{\sqrt{3}} \left[\sqrt{2}\,|n\uparrow n\uparrow p\downarrow\rangle - \frac{1}{\sqrt{2}} \left(|n\uparrow n\downarrow p\uparrow\rangle + |n\downarrow n\uparrow p\uparrow\rangle \right) \right]. \qquad \text{(A. 19)}$$

Moreover, the wave functions of π and K mesons are identical to those of SU (6).

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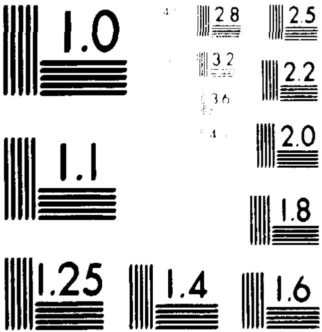
1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
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