a set of three-dimensional equations for the transition operators are obtained. Taking into account the decomposition of quasipotential $\mathcal{V}^{n}$ on the sum of pair interaction, three,- four,- ... - forces terms (following from spectral representation (3)) these equations can be reconstructed to Faddeev form. Therefore the general representations for some distributions obtained in our approach depend crucially an the properties of constituent wave function and their scattering amplitudes /5,6/ .

Note that there are also the equations and normalization conditions for the wave functions entering in (10) /7/ .

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INVESTIGATION OF THE SOLUTIONS OF QUASIPOTENTIAL EQUATIONS

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Quasipotential equations (QE) /1/ in quantum field theory are a convenient method for investigating relativistic bound state problems /2,3/ . In connection with possible applications of QE to the quark models it is necessary to note that there are many forms of QE and therefore it is not clear a priori which form is preferable on general grounds. On the other hand this fact is an advantage when it comes to possible applications /2,3,4/. Different physical problems may require different forms of QE.

Let us consider QE for the scattering amplitude of the equal mass (m) scalar particles (quarks):

$$T(\bar p,\bar p')=V[(\bar p-\bar p')^2]+\int\frac{d^3q\,(K^2+m^2)^{\frac{\nu-1}{2}}}{(q^2+m^2)^{1/2}}\cdot\frac{V[(\bar p-\bar q)^2]\,T(\bar q,\bar p')}{K^2-q^2}, \quad (1)$$

where $\nu=1$ corresponds to the ordinary Logunov-Tavkhelidze quasipotential equation (LTQE), $\nu=2$ - to the modified LTQE (MLTQE). Let $V_0(p,p')$ be the S-wave quasipotential, corresponding in the coordinate representation to the quasipotential of the form $V(r)=-gr^{-1}$. Then eq. (1) can be reduced to the differential boundary value problem of the second order in momentum representation /2/ :

$$\frac{d^2f(x)}{dx^2}+\lambda^2V^{(\nu)}_{(x,E)}f(x)=0,\quad V^{(\nu)}_{(x,E)}=(1+x^2)^{-\nu/2}(x^2+E^2)^{-1} \quad (2)$$

$$f(x)\underset{x\to 0}{\sim}x;\quad f(x)\underset{x\to\infty}{\sim}const, \quad (3)$$

where $x=pm^{-1}$, $\lambda^2=gm^{-1}(1-E^2)^{\frac{\nu-1}{2}}$, $E^2=-K^2m^{-2}$, $x\in X\equiv[0,\infty)$, $E\in\mathcal{E}\equiv[0,1]$.

We have investigated in detail the case $\nu=1,2$. Let us recall now one of the Sturm-Liouville theorems:

THEOREM: If the non-negative function

$V(x,E)$ is incremented in its domain $x \in X$ $E \in \mathcal{E}$ then positive eigenvalues λ^2 will always decrease and negative ones increase.

To apply this theorem to our boundary value problem (2)-(3) it is necessary to find such functions $V_<^{(\nu)}(x,E)$ and $V_>^{(\nu)}(x,E)$, which satisfy the inequality

$$V_>^{(\nu)}(x,E) \geq V^{(\nu)}(x,E) = (1+x^2)^{-\nu/2}(x^2+E^2)^{-1} \geq V_<^{(\nu)}(x,E) \quad (4)$$

and which, when inserted instead of $V^{(\nu)}(x,E)$ make eq. (2) solvable in terms of known special functions. Then, using the mentioned theorem, we can derive exact upper and lower spectral bounds for the original problem (2)-(3):

$$|\lambda_<^2| \geq |\lambda_{exact}^2| \geq |\lambda_>^2|. \quad (5)$$

For the LTQE we use the following approximations:

$$V_<^{(1)}(x,E) = (1+x)^{-1}(x+E)^{-2} \quad (6)$$

$$V_>^{(1)}(x,E) = \begin{cases} (x^2+E^2)^{-1}, & x<1 \quad (7a) \\ x^{-3}, & x \geq 1 \quad (7b) \end{cases}$$

The eigenvalue problem (2)-(3) with potential (6) has the following solution (upper spectral condition):

$$(-1)^{1-2\alpha}\frac{\Gamma(\alpha^*)\Gamma(\alpha^*+1)\Gamma(2\alpha)}{\Gamma(\alpha)\Gamma(\alpha+1)\Gamma(2\alpha^*)} = \frac{F(\alpha,-\alpha^*;2\alpha;\frac{E}{E-1})}{F(\alpha^*,-\alpha;2\alpha^*;\frac{E}{E-1})}\left(\frac{E}{E-1}\right)^{2\alpha-1} \quad (8)$$

where $\alpha = 1/2 + i\,[\lambda_<^2(1-E)^{-1} - 1/4]^{1/2}$.

For the function (7) the corresponding (lower) spectral condition is

$$\frac{1}{2} + \lambda_> \frac{\mathcal{J}_1'(2\lambda_>)}{\mathcal{J}_1(2\lambda_>)} = \frac{\lambda_>^2}{3}\cdot\frac{A(a)E^{2a}F(1+a,-a^*;2a+\frac{1}{2};-E^2)+c.c.}{B(a)E^{2a}F(a,-a^*;2a+\frac{1}{2};-E^2)+c.c.} \quad (9)$$

where $a = 1/4 + i/2\,(\lambda_>^2 - 1/4)^{1/2}$,

$$A(a) = \frac{\Gamma(5/2)\Gamma(1/2-2a)}{\Gamma^2(3/2-a)},$$

$$B(a) = \frac{\Gamma(3/2)\Gamma(1/2-2a)}{\Gamma(1/2-a)\Gamma(3/2-a)}.$$

For the MLTQE we choose the following approximations:

$$V_<^{(2)}(x,E) = (1+x)^{-2}(x+E)^{-2} \quad (10)$$

$$V_>^{(2)}(x,E) = \begin{cases} (x^2+E^2)^{-1}, & x<1 \quad (11a) \\ x^{-4}, & x \geq 1 \quad (11b) \end{cases}$$

These as well as (6)-(7) accomodate properly the analytic and asymptotic properties of the original potentials. The eigenvalue problem (2)-(3) with potential (10) has the solution:

$$\lambda_{n,<}^2(E) = \frac{1}{4}(1-E)^2 + \left[n\pi\frac{1-E}{\ln E}\right]^2, \quad n = 1,2,3,\ldots \quad (12)$$

Taking into account, that $V_>^{(2)}(x,E) \equiv V_>^{(1)}(x,E)$ for $x \in [0,1)$, we obtain the solution of eigenvalue problem (2)-(3) with function (11) from the spectral condition (9) with the help of the substitution:

$$\frac{1}{2} + \lambda_> \frac{\mathcal{J}_1'(2\lambda_>)}{\mathcal{J}_1(2\lambda_>)} \Longrightarrow \lambda_> \, ctg\, \lambda_> \quad (13)$$

Consider now the limit of strongly bounded states ($E = 1$). The analytic results for eigenvalues $\lambda_{n,\mu}^{2(\nu)}(1)$ ($\nu = 1,2$; $\mu = >, <,$ exact) are:

$$\lambda_{n,<}^{2(1)}(1) = \frac{1}{4}\,\varkappa_n^2\,; \quad \lambda_{n,<}^{2(2)}(1) = \pi^2 n^2\,; \quad \lambda_{ex.,n}^{2(2)}(1) = 4n^2-1\,, \quad (14)$$

where $n = 1,2,3,\ldots$ and $\varkappa_n$ is the n-th root of the Bessel function $\mathcal{J}_1(z)$.

Numerically: $\lambda_{1,>}^{2(1)} = 1.155$, $\lambda_{2,>}^{2(1)} = 5.759$, $\lambda_{1,>}^{2(2)} = 1.632$, $\lambda_{2,>}^{2(2)} = 11.162$. These results satisfy the main restriction (5).

In the weak-binding limit ($E \to 0$) we obtained from (8), (9), (13) the following behaviour of the energy eigenvalues:

$$E_{n,\mu}^{(\nu)}(\lambda) = \exp\left\{-\frac{n\pi}{\sqrt{\lambda^2-1/4}} + K_\mu^{(\nu)} + O(\sqrt{\lambda^2-1/4})\right\}, \quad (15)$$

where $K_>^{(1)} = K_>^{(2)} = 1.65$, $K_<^{(1)} = 0.77$, $K_<^{(2)} = 0$ and $\mu = >, <$, $n = 1,2,3,\ldots$

Therefore, using the main inequality (5), we proved, that the behaviour of the exact energy eigenvalues in this limit must be of the type (15)

$$E_{n,ex.}^{(\nu)}(\lambda) = \exp\left\{-\frac{n\pi}{\sqrt{\lambda^2 - 1/4}} + K_{ex.}^{(\nu)} + O(\sqrt{\lambda^2 - 1/4})\right\} \quad (16)$$

with $K_<^{(\nu)} \leq K_{ex.}^{(\nu)} \leq K_>^{(\nu)}$. In connection with this formula we point out that the original eigenvalue problem (2)-(3) cannot be solved in terms of known special functions even if the E is small, and it is the Sturm-Liouville upper and lower spectral bounds technique which renders the problem solvable. The spectral bounds considered make it possible to prove, that:
1) there exists the limit point of the spectrum $E_n(\lambda)$ for $E \to 0$; 2) the dependence $E_{n,ex.}^{(\nu)}(\lambda)$ is nonanalytic, namely of the type (16);
3) there are no energy eigenvalues in the interval $0 \leq \lambda^2 \leq 1/4$. Note, here that boundary value problem (2)-(3) for $\nu = 1$ has been solved formerly /2/ by the (asymptotic) comparison equation method (CEM) /5/ with the following result for eigenvalues ($0 \leq E^2 \leq 1$):

$$\sqrt{\lambda_n^2(E) - 1/4} = \frac{2\,\varkappa_n}{B(1/4, 1/2)\; F(1/2, 1/4; 3/4; 1-E^2)} \quad (17)$$

which for $E \to 0$ reads

$$E_n(\lambda) = \exp\left\{-\frac{\varkappa_n}{\sqrt{\lambda^2 - 1/4}} + \Psi(1) - \tfrac{1}{2}\Psi(\tfrac{1}{2}) - \tfrac{1}{2}\Psi(\tfrac{1}{4}) + O(\sqrt{\lambda^2 - 1/4})\right\}. \quad (18)$$

Then, the Sturm-Liouville upper and lower spectral bounds method as well as CEM may be of great use for solving the broad class of linear differential boundary value problems often met with in different branches of theoretical physics.

In conclusion we note that in the nonrelativistic limit the quasipotential $V(r) = -g r^{-1}$ corresponds to the potential $V(r) = -g' r^{-2}$ in the Schrödinger radial equation, but in our case there is no problem of collapse into scattering centre.

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