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NULL-PLANE QUANTIZATION AND QUASIPOTENTIAL EQUATION FOR COMPOSITE PARTICLES

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Methods of investigation of relativistic bound systems are known from the time of creation of quantum field theory. At present, (as always, of course) the problem is to develop more simpler and economical ways for dealing with bound states.

We are inclined to think that the null-plane [1] quantum field theory is to be more adapted to the problems under consideration because, if we eliminate the $p^+ = 0$ modes, null-plane canonical commutation relations have simplest, Fock, representation even in the presence of interaction [2]. Bound state wave functions at the equal $\tau = x^0 + x^3$ "times" for constituents are maximally close to nonrelativistic expressions without the transition to the infinite momentum frame [3,4].

Bound state problem for two spinless particles on the one null-plane was considered in [5] *using Tamm-Danooff approximation and* in [6] – on the basis of DGS spectral representation.

From our point of view, the most successive approach to the bound state problem in relativistic quantum field theory is a quasipotential method [7]. Equal-time quasipotential method had been successfully applied in many investigations [8]. Null-plane quasipotential equation was considered in papers [9] for spinless particles and in [10,11] – for spinorial particles.

In this report we shall discuss the main features of null-plane quasipotential approach and give an application to the asymptotic behaviour of composite particle form factors at large $\vec{P}_\perp$.

Taking into account that the $p^+=0$ modes may be eliminated [12] from the Fock-space with the help of second class constraints [13] it is possible to prove /14/ the formal equivalence of S-matrix in null-plane and ordinary covariant field theories.

For the illustration of quasipotential method let us consider first the sector $a^+ b^+ |0\rangle$ of the Fock-space which consists of one quark and one antiquark. In coordinate space the meson wave function with momentum P and helicity h has the form /15,16/

$$\langle 0 | q_+^{+\beta}(y) q_+^{\alpha}(x) | P, h \rangle = e^{-\frac{i}{2} P(x+y)} \psi_+^{\alpha\beta}(z, P, h)_{,\, z=x-y,\, z^+=0} \quad (1)$$

The module of this wave function is related to the probability for finding quark and antiquark inside the meson. Definning the covariant wave function

$$\langle 0 | \bar{q}^{\beta}(y) q^{\alpha}(x) | P, h \rangle = e^{-\frac{i}{2} P(x+y)} \psi^{\alpha\beta}(z, P, h) \quad (2)$$

we have

$$\psi_+(z, P, h) = \Pi_+ \psi(z, P, h) \gamma^0 \Pi_+, \quad \Pi_+ = \frac{1}{2}(1+\alpha_3) \quad (3)$$

Wave functions $\psi^{\alpha\beta}(z)$ may be connected apparently with very important characteristics such as the bilocal current matrix elements, weak and electromagnetic decay constants, etc. It is evident that in composite quark model these wave functions contain full dynamical information on the hadron structure, hence it is much desirable to have for them dynamical equations.

In the case of null-plane quantization the start point is the null-plane Bethe-Salpeter (BS) amplitude

$$\chi_{P,h}(x, y) = \langle 0 | T_+ (\bar{q}(x) q(y)) | P, h \rangle$$

Since the anticommutators of "good" operators q_+ and q_+^+ are C-numbers the projected BS amplitude, $\Pi_+ \chi_{P,h}(x,y) \gamma_0 \Pi_+ |_{x^+ = y^+}$ coincides with Tamm-Dankoff wave function.

This formalism can be generalized to the arbitrary numbers of constituents and to the full operators /10,11/. We shall give the main results underlining the advantages of general quasipotential approach in null-plane QFT. Let the hadron consists out of the N pointlike spinorial particles with 4-momenta $P_1, P_2, \ldots, P_N$. Let $P = \sum_{i=1}^{N} P_i$, $P_i^+ = x_i P^+$, $\sum_{i=1}^{N} x_i = 1$, $0 < x_i \leq 1$ (as the spectrum of P^+ generator is positive-defined).

We introduce a momentum-space quasipotential wave function $\psi_{P\alpha}$ /9-11/

$$(P^+)^{N-1} \prod_{i=1}^{N} x_i \psi_{P\alpha}(\{x_i, P_{i\perp}\}) = \int_{-\infty}^{\infty} \prod_{i=1}^{N} dP_i^- \, \delta\Big(P^- - \sum_{i=1}^{N} P_i^-\Big) \chi_{P\alpha}(\{P_i\}) \quad (4)$$

This wave function satisfies the equation /10,11/

$$\Big(P^2 - \sum_{i=1}^{N} \frac{(\vec{P}_{i\perp} - x_i \vec{P}_\perp)^2 + m_i^2}{x_i}\Big) \prod_{i=1}^{N} \otimes (\bar{P}_i + m_i) \psi_{P\alpha}(\{x_i, \vec{P}_{i\perp}\})$$
$$= \int \prod_{i=1}^{N} d^2 P'_{i\perp} \, \delta^{(2)}\Big(\vec{P}_\perp - \sum_{i=1}^{N} \vec{P}'_{i\perp}\Big) \int_0^1 \prod_{i=1}^{N} \frac{dx'_i}{x'_i} \delta\Big(1 - \sum_{i=1}^{N} x'_i\Big) \cdot$$
$$\cdot \prod_{i=1}^{N} \otimes (\bar{P}_i + m_i) V_P(\{x_i, P_{i\perp}\}, \{x'_i, P'_{i\perp}\}) \cdot \quad (5)$$
$$\cdot \prod_{i=1}^{N} \otimes (\bar{P}'_i + m_i) \psi_{P\alpha}(\{x'_i, P'_{i\perp}\})$$

Here V_P is a quasipotential matrix defined according to the known recept /10,11/ with the aid of T-matrix, and $\bar{P}^\mu = (P^+, P_\perp, (\vec{P}_\perp^2 + m^2)/P^+)$. We can easily reconstruct an eq. (5) to Faddeev form in the case of $N \geq 3$ /11/.

The main features of eq. (5) are: maximal proximity to nonrelativistic expressions, invariance under the group isomorphic to the Gallilean one, similarity to the quark-parton picture. On the other hand no reference frame is fixed and no approximation is made, i.e., full information of relativistic QFT is preserved. For these reasons our formalism may be successfully applied in any multiparticle problems. Quasipotential equation on the null-plane had been applied in studying of composite particle scattering /17/, inclusive processes /18/ as well as in composite particle form factors /19/ at large transverse momenta.

Composite particle form factors are defined by the matrix elements of local operator (current) [9-11]

$$\langle p\alpha' | J_\mu(0) | p\alpha \rangle = \int \prod_{i=1}^{N} d^2 p'_{i\perp}\, \delta^{(2)}\Big(\vec{p}'_\perp - \sum_{i=1}^{N} \vec{p}'_{i\perp}\Big) \cdot \int_0^1 \prod_{i=1}^{N} \frac{dx'_i}{x'_i}\, \delta\Big(1 - \sum_{i=1}^{N} x'_i\Big) \int \prod_{i=1}^{N} d^2 p_{i\perp}\, \delta^{(2)}\Big(\vec{p}_\perp - \sum_{i=1}^{N} \vec{p}_{i\perp}\Big)$$

$$\int_0^1 \prod_{i=1}^{N} \frac{dx_i}{x_i}\, \delta\Big(1 - \sum_{i=1}^{N} x_i\Big)\, \bar{\Psi}_{p\alpha'}(\{x'_i, \vec{p}'_{i\perp}\})$$

$$\prod_{i=1}^{N} \otimes (\hat{p}'_i + m)\, \tilde{\Gamma}_\mu (p'\{x'_i, \vec{p}'_{i\perp}\}; p\{x_i, \vec{p}_{i\perp}\})$$

$$\prod_{i=1}^{N} \otimes (\hat{p}_i + m)\, \Psi_{p\alpha}(\{x_i, \vec{p}_{i\perp}\}) \qquad (6)$$

Asymptotic behaviour of composite particle form factors at $|\vec{p}_\perp - \vec{p}'_\perp| \to \infty$ was studied in papers [20] on the basis of various dynamical equations. In our equations (5),(6) the variables x_i (x'_i) having the meaning of portions of total momentum p^+ are confined in the region (0,1). Moreover for every spinorial constituent there are corresponding projective operators $(\hat{p}_i + m)$. (Note that the projective properties follow automatically in Kadyshevsky quasipotential approach as well [22]). These projective properties simplify the derivation of the automodel counting rule [22] (up to logarithms) in the frame of perturbation theory for any renormalizable models with dimensionless coupling constants.

In conclusion we may note that it is easy to include internal symmetries (light-cone algebra, $SU_W(6)$ and etc.) in our approach. Then after the lightlike kinematical decompositions [15,16] there remain equations for dynamical structure functions. There are wide classes of dynamical problems (lightlike quark symmetries, $\vec{p}_\perp$ phenomena, nuclear and atomic bound states , etc.) in which the application of null-plane quasipotential equations seems powerful.

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