A NEW PARTICLE WITH THE MASS M=1110 MeV AND VECTOR MESON SPECTRUM IN THE RELATIVISTIC MODEL OF QUARK CONFINEMENT.

N.B.Skachkov

Joint Institute for Nuclear Research, Dubna

These notes concern the proposed several months ago a theoretical model[1] that predicts the existence, in the spectrum of the vector mesons, of a particle with mass equal to that of the particle discovered experimentally in July 1976 at DESY (the communication about this experiment is presented by DESY group at the section "New particle production in hadron collisions"*).

The main feature of the model (I) is the use of the new type of the potential that binds two quarks inside a meson (M-quark mass):

$$V(\rho) = \frac{I}{4\pi\rho} ctg\pi\rho M; \quad 0 < \rho < \frac{I}{M}. \qquad (I)$$

The parameter ρ in (I) is connected with the squared relative distance X^2 between two quarks by formula $X^2 = \frac{I}{M^2} - \rho^2$ and varies in the interval $0 < \rho < I/M$. The potential(I) confines quarks inside a shere of the radius equal to the Compton wave length (see Fig.I)

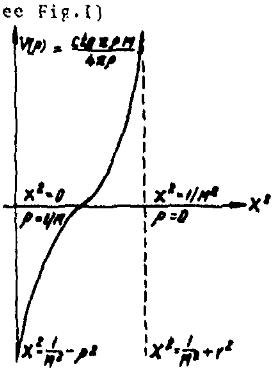

Fig.I

We shall consider the problem of the quark-antiquark bound states in the framework of the relativistic three- dimensional two-body equation of the quasipotential type[3], namely, the Kadyshevsky equation for the wave function of a two-particle relative motion. In c.m.s. this equation is

$$(E_p - E_q)\ \psi_q(\vec{p}) = \int \frac{d\vec{k}}{E_k} V(\vec{p},\vec{k},E_q)\ \psi_q(\vec{k}) \qquad (2)$$

$$E_p = p = \sqrt{M^2+\vec{p}^2}; \qquad E_p \neq E_k$$

(Here, $\vec{p}$ is the quark momentum in the c.m.s. $\vec{p}_1 = -\vec{p}_2 = \vec{p}$ and the quasipotential $V(\vec{p},\vec{k};Eq)$ is constructed from the Feynman diagrams). In the quasipotential equations, in contrast to the Bethe-Salpeter equation, the momenta of all the particles are on the mass shell $k_0^2 - \vec{k}^2 = M^2$. Therefore it is natural to use here the expansion on the Lorentz group, i.e. the system of functions[5]

*See also Denisov's rapporteur talk.

$$\xi(\vec{k},\vec{r}) = \left(\frac{k^0 - \vec{k}\,\vec{n}}{M}\right)^{-1+irM};\quad \vec{r} = r\,\vec{n};\ \vec{n}^2 = I \qquad (3)$$

complete with the volume element $d\Omega_k = \frac{d\vec{k}}{k^0}$ on the hyperboloid $k_0^2 - \vec{k}^2 = M^2$.

In[3] the transform with the functions

$$\psi(\vec{r}) = \frac{I}{2\pi^3}\int \xi(\vec{p},\vec{r})\ \psi(\vec{p})\ \frac{d\vec{p}}{p^0} \qquad (4)$$

was considered as a transform to a new relativistic configurational representation (PCR). It was shown that after this transformation eq.(2) takes the form

$$(\hat{H}_0 + V(\vec{r},Eq))\ \psi_q(\vec{r}) = 2\,Eq\ \psi_q(\vec{r}) \quad , \qquad (5)$$

where the free Hamiltonian $\hat{H}_0$ $(H_0\xi(\vec{p},\vec{r}) = 2E_p\xi(\vec{p},\vec{r}))$ is the differential-difference operator

$$H_0 = 2M\,ch\,\frac{i}{M}\frac{\partial}{\partial r} + \frac{2}{r}\,sh\,\frac{i}{M}\frac{\partial}{\partial r} - \frac{\Delta_{\theta\varphi}}{M\,r^2}e^{\frac{i}{M}\frac{\partial}{\partial r}} \qquad (6)$$

with the step proportional to the Compton wave length I/M. Eq.(5) can be solved exactly for a class of potential, e.g. for the relativistic Coulomb potential[6]

$$V(r) = -\frac{I}{4\pi r}\ \coth\pi\, r\, M \quad ,$$

$V(r)$ is the transform of the quasipotential $V(\vec{p},\vec{k}) = \frac{-I}{(p-k)^2}$ corresponding to the massless gluon exchange propagator (the sign "minus" corresponds to an abstractive potential)**.

The transformation (4) has the group-theoretical meaning of the expansion in principal series (PS) of the unitary representations of SO (3.I). In the non-relativistic limit $\xi(\vec{p},\vec{r}) \to e^{i\vec{p}\vec{r}}$ and (4) is reduced to the ordinary Fourier transformation. Owing to (4) the group parameter r stands for a relativistic generalization of the relative distance between two particles. It is connected to the eigenvalue $\mathcal{X}^2$ of the invariant Casimir operator of SO(3.I) $\hat{C}_L = \frac{1}{2} M_{\mu\nu} M^{\mu\nu}$ (here, $M_{\mu\nu}$ are the generators of SO(3.I) derived by formula

$$\hat{C}_L\ \xi(\vec{p},\vec{r}) = \mathcal{X}^2(\vec{p},\vec{r})\ ;\ \mathcal{X}^2 = \frac{I}{M^2} + r^2,\ 0 < r < \infty \quad (7)$$

Thus, the modulus of the relativistic relative coordinate r is invariant. It was shown in[8] that the invariant mean-square radius definition $\langle r_0^2 \rangle = \frac{\partial F(t)}{\partial t}/_{t=0}$ has also the group-theoretical meaning of the eigenvalue of the Lorentz group Casimir operator for the invariant form factor of the system F(t):

$$\langle r_0^2 \rangle = \frac{\partial F(t)}{\partial t}/_{t=0} = (\hat{C}_L F(t))/_{t=0} \quad (8)$$

So, if we describe the particle strusture in terms of the relativistic coordinate r, i.e. if we perform

** The solution of this equation and the energy spectrum in case of the confinement potential $V(r) = \lambda r$ was obtained in [7]. See also mini-report by Mir-Kasimov.

the transformation (4) of the form factor F(t), then due to (8) and (7) it follows that the distance from the center of the system is measured in terms of eigenvalue χ^2 of the Casimir operator $\hat{C}_L$. In case when the transform of the form factor F(r) in RCR is the function of the constant sign the new coordinate r specifies only the distances beyond the Compton wave lenght $^I/_M$ as is shown in[8] (see also mini-report by Skachkov at section A7).

Following [8] the transition to the distances smaller than the Compton wave lenght $\chi^2 < \frac{I}{M^2}$ may be achieved by including the supplementary series (SS) of SO(3.I) into the wave function expansion (4). The supplementary series are realized by functions

$$\zeta(\vec{p},\vec{\rho}) = \left(\frac{p_0 - \vec{p}\vec{n}}{M}\right)^{-1-\rho M} \qquad (9)$$

that can be formally obtained from the "plane waves" for PS $\xi(\vec{p},\vec{r})$ by changing $r \to i\rho$. The eigenvalue of the Casimir operator for the supplementary series, i.e. the square of the distance from the center, take the following values:

$$C_L\zeta(\vec{p},\vec{\rho}) = \chi^2(\vec{p},\vec{\rho}); \quad \chi^2 = \frac{I}{M^2} - \rho^2, \; 0 < \rho < \frac{I}{M} \qquad (9)$$

The group parameter ρ may be then interpreted as a coordinate reckoned from the boundary of the sphere $R^2 = \chi^2 = \frac{I}{M^2}$ to its center, so that $\rho = {}^I/_M$ corresponds to the origin of the coordinates $\chi^2 = 0$.

In[I] the quasipotential equation was written for the wave function $\psi(\vec{\rho})$ describing the relative motion of the quark-antiquark in the range of the relative distances $0 < \chi^2 < \frac{I}{M^2}$ (no nonrelativistic analogue!):

$$(H_0 + V(\rho))\, \psi_q(\vec{\rho}) = 2E_q \psi_q(\vec{\rho}) \quad (I0)$$

where the free Hamiltonian $H_0 \left(H_0 \zeta(\vec{p},\vec{\rho}) = 2E_p \zeta(\vec{p},\vec{\rho})\right)$ is the differential-difference operator(as in case of r-space)

$$H_0 = 2M \cosh \frac{I}{M}\frac{\partial}{\partial\rho} + \frac{2M}{\rho} \sinh \frac{I}{M}\frac{\partial}{\partial\rho} + \frac{\Delta_{\theta\varphi}}{M\rho^2} e^{\frac{I}{M}\frac{\partial}{\partial\rho}} \quad (II)$$

Now let us consider the analogue of the relativistic Coulomb potential (6) for the distances smaller than $\frac{I}{M}$.(We choose this potential because the Coulomb potential contains the maximal symmetry, and therefore the equation can be exactly solved).Since the transition to the distances $\chi^2 < \frac{I}{M^2}$ may be realized by passing to the supplementary series, we shall make use of the group-theoretical meaning of the modulus of the relativistic coordinate r and we shall obtain the potential $V(\rho)$ for $\chi^2 < \frac{I}{M}$ by changing $r \to i\rho$ in (6). Thus we arrive to the potential (I) which confines quarks inside the sphere $R^2 = \chi^2 = {}^I/_{M^2}$.

In [I] an exact solution for eq.(I0) has been obtained in case of the potential (I).It was found that there may exist only three energy levels in the quark-antiquark system which moves with $\ell = 0$ in the field of the potential(I). The latter may be identified with masses of excited states of one particle:

$$M^I_{bound} \equiv 2Eq^I = I.39798\ M_q; \; M^{II}_{bound} \equiv 2Eq^{II} = 2M_q$$
$$M^{III}_{bound} \equiv 2Eq^{III} = 2.96750\ M_q \qquad (I2)$$

(M_q is the quark mass).

Let us consider the ρ-meson excited state spectrum. Taking $M^I = M_\rho = 773 \pm 3$MeV we obtain from (I2) the value of the quark mass $M_q = 553 \pm 2$MeV.It gives the following masses of the second and third radial excitations $M_{\rho'} = M^{II}_\rho = II06 \pm 4$MeV; $M_{\rho''} = M^{III}_\rho = I645 \pm 6$MeV.

At the time when the spectrum was obtained in [I] only the second excited state of ρ with $M^{III}_\rho = I645 \pm 6$MeV could be identified with the previously observed ρ'' (I650)meson($\Gamma_{\rho''} > 200$ MeV), and there was no experimentally found candidate for our first radial excitation of the ρ-meson with $M^{II}_\rho = II06 \pm 4$MeV. The discovery at DESY of a new vector meson with the mass M=III0 MeV($\Gamma \sim 20$-30 MeV) shows that all three particles appearing in the mass spectrum of our model exist in nature provided that the quantum numbers (isospin,parity, etc) of this vector meson coincide with the quantum numbers of the rho-meson as our model predicts.

Now let us apply this model to the $^J/_\psi$ particle mass spectrum. Equating $M^I = M_{\psi(3095)}$ we obtain the heavy quark Q with the mass $M_Q = 22I6$ MeV. For the radial excitations $\psi(3095)$ it gives the masses $M^{II}_\psi = 4432$ MeV, $M^{III}_\psi = 6576$ MeV with the help of (I2). It is seen that in case of our confinement potential (I) the first radial excitation of the $\psi(3095)$ is not $\psi'(3685)$ but is close to $\psi(44I4)$. Then the $\psi'(3685)$ should be interpreted as a bound state of other two heavy quarks Q' with mass $M_{Q'} = 2635$ MeV. With the help of (I2) the values $M^{II}_\psi = 5270$ MeV and $M^{III}_\psi = 7820$ Mev may be given for the masses of the $\psi'(3585)$ possible excitations. Thus our model requires more than one(charmed)heavy quark in order to describe the $^J/_\psi$ particle spectrum. This is consistent with other schemes including five or six quarks. Note that the mass of the heavy quark $M_Q = 22I6$ appearing in our spectroscopy is in agreement with that obtained from the e^+e^- data analysis in the framework of 6 quark model with the account of heavy leptons.[I0]

I express my sincere gratitude to Profs.Schopper and Höhler for the information about the DESY experiment at the beginning of the Conference.

R e f e r e n c e s:

1. N.B.Skachkov. JETP Letters,23,No I2,713(1976); Preprint JINR E2-9748,Dubna,1976.
2. S.Bartalucci et al. Contributed paper 1258/N 1, Preprint DESY 76/43,1976.
3. A.A.Logunov,A.N.Tavkhelidze,Nuovo Cimento,29, 380(1963); V.G.Kadyshevsky,Nucl.Phys.B6,I25(1968)
4. V.G.Kadyshevsky,R.M.Mir-Kasimov,N.B.Skachkov, Nuovo Cimento,55A,233(1968); Journ.of Particles and Nuclei,2,No 3,69(1972) Translation from Russian.
5. I.S.Shapiro. Doklady Akad.Nauk USSR,I06,647, (1965); Soviet Phys.Doklady,I,91(1956).
6. M.D.Mateev,R.M.Mir-Kasimov,M.Freeman,Nucl.Phys. B12, 107 (1969).
7. K.S.Jhung,K.H.Chung,R.S.Willey,Phys.Rev.DI2, I999(I975).
8. N.B.Skachkov.Teor.i Mat.Fizika,23,313(1975); Soviet Jour.Theot.and Math.Phys.23,No 3(1975); preprint JINR E2-8857,Dubna,1975.
9. S.Ström. Arkiv f.Physic,Bd.38,No 22,373(1968).
10. A.De Rujula,H.Georgi,Phys.Rev.DI3,I296(1976).

SCATTERING PROCESSES IN RELATIVISTIC COMPOSITE MODELS

A.N.Kvinikhidze

Institute of Mathematics,Georgian Academy of Sciences,Tbilisi,USSR

The successes in study of particle structure make us to believe in compositness of hadrons. Therefore the investigation even of two hadron system represents in fact the many-body relativistic problem.

In this talk we present the three dimensional approach to many-body problem in quantum field theory [1]. This approach allows one in particular to construct exact expressions for relativistic scattering amplitudes corresponding to any process of composite system collision. Some simplest approximations repreduce the well-known expressions obtained in naive quark parton model.

We perform our consideration in the null plane quantum field theory, taking into account the fact that it is very useful to avoid some specific many-body problem difficulties existing in usual quantum field theory.

Following to [2,3,4] let us start with "two-time" Green function of n-particles

$$G^n(x^+,y^+;x,y)=\langle 0|T_+\psi_1(x_1)\cdots\psi_n(x_n)\bar\psi_1(y_1)\cdots\bar\psi_n(y_n)|0\rangle\Big|_{\substack{x_i^+=x^+\\ y_i^+=y^+}} \quad (1)$$

where $x^\pm=\frac{1}{\sqrt{2}}(x^0\pm x^3)$; $\underline{x}=(x^-,x_\perp)$

T_+ is x^+ "time" ordering operator $\psi_i(x)$, $\bar\psi_i(x)$ are conjugate to each other Heisenberg field operators of i-th particle.

It is convenient to introduce in the momentum space the following notations

$p^\pm=p^0\pm p^3$; $\underline{p}=(p^+,p_\perp)$ then

$$px=p^-x^++p^+x^--p_\perp x_\perp=p^-x^++\underline{p}\,\underline{x}$$

Determine the Fourier transform of the "two-time" Green function

$$G^n(x^+;x,y)=\frac{i}{(2\pi)^{3n+1}}\int e^{-ip^-x^+-i\sum_1^n(\underline{p}_j\underline{x}_j-\underline{q}_j\underline{y}_j)}$$

$$dp^-\prod_1^n d^3\underline{p}_j\,d^3\underline{q}_j\,\delta(\underline{p}-\underline{q})(p^+)^{\alpha_n}\,g^n(p^-;\underline{p},\underline{q}). \quad (2)$$