ORNL/TM-6111

# Small Radius Start-up of Tokamak Plasmas with a Moving Limiter



12

T. Uckan



OAK RIDGE NATIONAL LABORATORY OPERATED BY UNION CARBIDE CORPORATION · FOR THE DEPARTMENT OF ENERGY

# **BLANK PAGE**

### Printed in the United States of America. Available from National Technical Information Service U.S. Department of Commerce 5285 Port Royal Road, Springfield, Virginia 22161 Price: Printed Copy \$4.50; Microfiche \$3.00

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, contractors, subcontractors, or their employees, makes any warranty, express or implied, nor assumes any legal liability or responsibility for any third party's use or the results of such use of any information, apparatus, product or process disclosed in this report, nor represents that its use by such third party would not infringe privately owned rights.

## U.S. ENERGY RESEARCH & DEVELOPMENT ADMINISTRATION MAJOR CONTRACTOR'S RECOMMENDATION FOR DISPOSITION OF SCIENTIFIC AND TECHNICAL DOCUMENT

\*See Instructions on Reverse

Form. ERDA 426 (2/75)

ERDAM 3201

1. ERDA Report No.	3. Title Small Radius Start-In of Tokamak Plasman with							
ORNL/TM-6111	a Moving Limiter							
2. Subject Category No.								
	Author: T. Uckan							
4. Type of Document ("X" one)								
🔀 a. Scientific and Technical Report								
b. Conference paper:								
Date of conference								
Exact location of conference	Exact location of conference							
🔲 c. Other (Specify, Thesis, Translatio	n, etc.)*							
5. Copies Transmitted ("X" one or more)								
a. Copies being transmitted for stand	dard distribution by ERDA-TIC.							
b. Copies being transmitted for special distribution per attached complete address list.*								
Twenty-Seven c. The Xan Asian Bills, reproductore copies being transmitted to ERDA-TIC.								
6. Recommended Distribution ("X" one)								
x a. Normal handling (after Patent clearance): no restraints on distribution except as may be required by the security classification.								
b. Make available only to U.S. Government agencies and their contractors.								
c. Make available only within ERDA and to ERDA contractors.								
d. Make available only within ERDA.								
e. Make available only to those liste	e. Make available only to those listed in item 12 below.							
f. Other (Specify)*								
7. Recommended Announcement ("X" one)								
🔀 a. Normal procedure may be follow	ed.*							
b. Recommend following announce	nent limitations:							
8. Reason for Restrictions Recommended in 6	or 7 above.							
a. Preliminary information.								
b. Prepared primarily for internal use.								
c. Other (Explain)								
9. Patent Clearance ("X" one)								
X a. ERDA patent clearance has been granted by responsible ERDA patent group.								
b. Document has been sent to response	onsible ERDA patent group for clearance.							
10. National Security Information (For classif	ied document only, "X" one)							
a. Document does contain national security Information other than restricted data.								
b. Document does not contain nation	b. Document does not contain national security information other than restricted data.							
11. Copy Reproduction and Distribution								
a. Total number of copies reproduced								
b. Number of copies distributed outside originating organization81								
12. Additional Information or Remarks (Cont	inue on separate sheet, if necessary)							

 13. Submitted by (Name and Position) (Please print or type)\*

 P. S. Baker, Classification Officer

 14. Organization

 Oak Ridge National Laboratory

 15. Signature

 16. Date

 December 9, 1977

Who uses this Form: All ERDA contractors except those specifically instructed by their ERDA contract administrator to use the shorter Form ERDA-427.

When to Use: Submit one copy of this Form with each document which is sent to ERDA's Technolal Information Center (TIC) in accordance with the requirements of ERDA Manual Chapter 3201.

Where to send: Forward this Form and the document(s) to:

USERDA- TIC P.O. Box 62 Oak Ridge TN 37830

Item instructions:

Item 1. The first element in the number shall be an LRDAapproved code to be determined as follows: (a) The responsible field office may request TIC approval of a unique code for a contractor, e.g., BNL, BML, HNL, etc.; (b) A program division may request TIC approval of a unique code for a program or series of reports, e.g., I'NE, VUF, etc.; (c) An operations office may instruct a contractor to use the code approved for the operations office, i.e., COO, ORO, IDO, SRO, SAN, ALO, RLO, NVO; and (d) Program divisions shall use the code ERDA for reports which they themselves prepare unless there is reason to use some other approved code.

> The code shall be followed by a sequential number, or by a contract number plus a sequential number, as follows: (a) Contractors or programs with unique codes may complete the report number by adding a sequential number to the code, e.g., HNL-101, HNL-102, etc.; or PNE-1, PNE-2, etc.; or they may add the identifying portion of the contract number and a sequential number, e.g., ABC-2105-1, ABC-2105-2, etc; (b) Contractors using the operations office code shall complete the report number by adding the identifying portion of the contract number and a sequential number, e.g., COO-2200-1, COO-2200-2, etc.; (c) Subcontractor reports shall be identified with the code used by the prime contractor; and (d) Program divisions using the ERDA code shall complete the report number by adding a sequential number which they request from the Library Branch, Division of Administrative Services.

- Item 2. Insert the appropriate subject category from TID-4500 ("Standard Distribution for Uncl., sified Scientific and Technical Reports") or M-3679 ("Standard Distribution for Classified Scientific and Technical Reports") for both classified and unclassified documents, whether or not printed for standard distribution.
- Item 3. Give title exactly as on the document itself unless title is classified. In that case, omit title and state "classified title" in the space for item 3.
- Item 4. If box c is checked, indicate type of item being sent, e.g., thesis, translation, etc.
- Item 5. a. It box a is checked, the number of copies specified for the appropriate category or categories in M-3679 or TID 4500 shall be forwarded to TIC for distribution.

b. If box b is checked, complete address list must be provided TIC.

c. If box c is checked, at least one copy shall be original ribbon or offset and be completely legible. A clear carbon copy is acceptable as a second reproducible copy.

Item 6. If box a is checked for an unclassified document, it may be distributed by TIC (after patent clearance) to addressees listed in TID-4500 for the appropriate subject category, to libraries in the U.S. and abroad, which through purchase of microfiche maintain collections of ERDA reports, and to the National Technical Information Service for sale to the public.

If box a is checked for a classified document, it may be distributed by HC to addressees listed in M-3679 for the appropriate subject category.

If a box other than a is checked, the recommended limitation will be followed unless TIC receives other instructions from the responsible ERDA program division.

Bos f may be checked in order to specify special instructions, such as "Make available only as specifically approved by the program division," etc.

Item 7. a. Announcement procedures are normally determined by the distribution that is to be given a document. If box a in item 6 is checked for an unclassified document, it will normally be listed in the weekly "Accessions of Unlimited Distribution Reports by TIC" (TID-4401) and may be abstracted in "Nuclear Science Abstracts" (NSA).

> A classified document, or an unclassified document for which box b, c, d, e, or f, in item 6 is checked, may be eited with appropriate subject index terms in "Abstracts of Limited Distribution Reports" (ALDR).

b. If the normal announcement procedures described in 7a are not appropriate check 7b and indicate recommended announcement limitations.

- Item 8. If a box other than a is checked in item 6, or if 7b is checked, state reason for the recommended restriction, e.g., "preliminary information," "prepared primarily for internal use," etc.
- Item 9. It is assumed that there is no objection to publication from the standpoint of the originating organization's patent interest. Otherwise explain in item 12.
- Item 10. If box a is checked, document cannot be made available to Access Permit holders (Code of Federal Regulations, 10 CFR, Part 25, subpart 25.6); if box b is checked, TIC will determine whether or not to make it available to them.

Item 11. . ett. c. planatory.

- Item 12. Use this space if necessary to expand on answers given above, e.g., item 6f and item 8.
- Item 13. Enter name of person to whom inquiries concerning the recommendations on this Form may be addressed.

Item 14-16. Self explanatory.

Contract No. W-7405-eng-26

FUSION ENERGY DIVISION

SMALL RADIUS START-UP OF TOKAMAK PLASMAS

WITH A MOVING LIMITER

T. Uckan

Date Published - December 1977

NOTICE This document contains information of a preliminary nature It is subject to revision or correction and therefore does not represent a final report.

> Prepared by the OAK RIDGE NATIONAL LABORATORY Oak Ridge, Tennessee 37830 operated by UNION CARBIDE CORPORATION for the DEPARTMENT OF ENERGY

NOTICE This report was prepared as an account of work sponsor d by the United States Government Neither the Unite' States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assume any legal liability or responsibility for the accuracy, completeness or usefulness of any information, appratus product or process disclosed, or represents that its use would not infrange privately owned rights

.

3.1

DISTRIBUTION OF THIS DECLIMATIVE .

### ABSTRACT

5

The problem of start-up for a large size tokamak plasma is studied with a moving limiter. The plasma "ransport with the presence of the electric field diffusion and heat conduction losses is investigated analytically by the separation of variables during this early phase of the discharge. The results are then applied to a TNS-size plasma. It is shown that a moving limiter may help ameliorate the possible problem of skin effects on the current profile.

### 1. INTRODUCTION

In order to avoid the skin effect on the current profiles in a large, ohmically heated tokamak plasma during the early phase of the discharge, a moving limiter is one of the proposed<sup>1</sup> alternatives to the solution of the problem. Since the plasma column expands by the controlled moving limiter, a desiralle current profile may be obtainable. The plasma transport during this expansion phase is studied analytically by means of separation of variables<sup>2</sup> which are time, t, and  $\rho = r^2h(t)$ , where h(t) is only a function of time. This approach allows one to investigate the plasma discharge parameters in terms of the limiter motion as well as the plasma current rise. Furthermore, the plasma parameters such as poloidal beta, safety factor, flux function, etc., are obtainable by making use of the analytical solutions of the plasma temperature and the current density.

This problem was first studied extensively by Bardotti et al.<sup>2</sup> in the case of pseudoclassical type losses assumed for the electron heat conduction. In this paper, we extend their approach by introducing a model heat conductivity coefficient which in turn provides a general view of the problem of separability. We also observe other possible cases.

In this report, Sect. 2 describes the plasma model and the basic equations, Sect. 3 outlines system equations and Sect. 4 gives the evaluation of the plasma discharge parameters. General discussions and an application of the results are included in Sect. 5.

1

2. PLASMA MODEL AND THE BASIC EQUATIONS

As was done previously,<sup>2,3</sup> a cylindrical plasma column is adopted so that the computational complexities due to toroidal geometry are avoided. Furthermore, we assume:

1) The plasma density is constant and uniform, and the ion density,  $n_i$ , is equal to the electron density,  $n_e$ ;  $n = n_e = n_i$ .

2) The species have the same temperature, T. On the border of the plasma which is defined by the limiter position,  $r_L(t)$ , the temperature is T<sub>o</sub> and constant.

3) Except for the heat conduction losses, all other losses are ignored.

4) The electrical conductivity,  $\sigma$ , is classical and vanishes outside the limiter.

In the model it is assumed that the plasma radius is being increased from zero to  $r_L$  (t =  $\tau$ ) = a during t =  $\tau$  (see Fig. 1). Thus the basic equations for r <  $r_L$ (t) are as follows (mks units are used):

$$\frac{\partial E}{\partial r} = \frac{2B_{p}}{\partial t}, J = \sigma E, \sigma = kT^{3/2}$$

$$\mu J = \frac{1}{r} \frac{\partial}{\partial r} (rB_{p}), 3n \frac{\partial T}{\partial t} = \frac{J^{2}}{\sigma} - \frac{1}{r} \frac{\partial}{\partial r} (rq_{r})$$
(1)

where  $q_r = -n\chi \frac{\partial T}{\partial r}$  is the radial heat conduction flux of the electrons. Here E is the toroidal electric field,  $B_p(r, t) = \mu I(r, t)/2\pi r$  is the induced poloidal magnetic field due to the plasma current  $I(r, t) = 2\pi \int_0^r dr'r' J(r', t), (\mu = 4\pi \times 10^{-7} \text{ H/m})$ , and J(r, t) is the toroidal

2

current density. For the heat conductivity coefficient,  $\chi$ , we introduce the following general form:

$$\chi(r, t) = Cn^{\gamma}r^{2s}/I^{p}r^{q/2}$$

Here C is a dimensional numerical factor that depends on the plasma parameters such as toroidal magnetic field, major and minor radii, etc. The values of s, p, and q are determined later on.

The analytical solutions for the current density and the temperature are sought by introducing two variables, t and  $\rho = r^2h(t)$ . In the following section, the separability condition and the resulting system equations will be discussed.

### 3. SYSTEM EQUATIONS

Let us start with assuming current density profiles of the form

$$J(\mathbf{r}, t) = J_f \left(\frac{t}{\tau}\right)^{\alpha} F(\rho)$$

with an arbitrary constant,  $\alpha$ . Here, at time,  $\tau$ , the limiter reaches the final plasma radius, a; thus the plasma current on the magnetic axis is  $J_f \equiv J(r = 0, t = \tau)$  since  $F(\rho = 0) = 1$ . With this in mind, the system equations are obtained by making use of the variable transformation as well as imposing the separability in t and  $\rho$  on Eq. (1). The resulting equations are then as follows:<sup>3</sup>

$$T(\mathbf{r}, \mathbf{t}) = T_{f} \left(\frac{\mathbf{t}}{\tau}\right)^{2(1+2\alpha)/5} G(\rho)$$
$$I(\mathbf{r}, \mathbf{t}) = I_{f} \left(\frac{\mathbf{t}}{\tau}\right)^{(2-\alpha)/5} \frac{Z(\rho)}{Z_{f}}$$

$$Y(r, t) = \frac{Cn^{\gamma}}{(\pi J_f)^p} \frac{h_f^{(p-s)}}{T_f^{q/2}} \left(\frac{t}{\tau}\right)^{-3(1+2\alpha)/5} P(o)$$
$$h(t) = h_f \left(\frac{t}{\tau}\right)^{-2(1-3\alpha)/5} .$$

The functions F, G, Z, and P must satisfy the following set of nonlinear differential equations: $^{3}$ 

$$\frac{dF}{d\rho} = \frac{3}{2} \operatorname{FSG}^{(q/2-1)} \frac{z^{p}}{\rho^{s+1}} + \frac{G^{3/2}}{2} \left[ \left( \frac{2-\alpha}{10\alpha} \right) \frac{z}{\rho} - \left( \frac{1-z^{\gamma}}{5\alpha} \right) F \right]$$

$$\frac{dG}{d\rho} = \operatorname{SG}^{q/2} \frac{z^{p}}{\rho^{s+1}}$$

$$\frac{dZ}{d\rho} = F$$

$$\frac{dS}{d\rho} = \left[ \operatorname{DG} - \operatorname{F}^{2} \operatorname{G}^{-3/2} - \operatorname{D} \left( \frac{1-3\alpha}{1+2\alpha} \right) \operatorname{SG}^{q/2} \frac{z^{p}}{\rho^{s}} \right] / A ,$$
where
$$S = \rho P \frac{dG}{d\rho} \text{ with } P = \rho^{s} / z^{p} \operatorname{G}^{q/2} .$$
(2)

Here,  $T_f$  denotes the final value of the temperature on the magnetic axis, i.e.,  $T_f \equiv T(r = 0, t = \tau)$ ;  $\rho_f$  is the final value of  $\rho$  at the edge, i.e.,  $\rho_f \equiv \rho(r = a, t = \tau) = a^2 h_f$  and  $Z_t \equiv Z(\rho_f)$ . The constants A, D, and  $\tau$ are given by

$$D = \frac{6}{5} \left(\frac{1+2\alpha}{\alpha}\right) nT_{f}h_{f}/\mu J_{f}^{2}$$

$$A = (4C\sigma_{f}T_{f}n^{\gamma}/J_{f}^{2}) \frac{h_{f}^{(p-s+1)}}{(\pi J_{f})^{p}T_{f}^{q/2}}$$
  
$$\tau = \mu\sigma_{f}a^{2}\alpha/\rho_{f}, \text{ with}$$
  
$$\sigma_{f} = kT_{f}^{3/2}.$$

The separability requires that a, p, q, and s must satisfy

 $\alpha(6 - 6s + p - 2q) + 3 - 2p + 2s - q = 0.$  (3)

This relation gives us some of the acceptable scaling laws for  $\chi(\mathbf{r}, t)$ , which are listed in Table 1. As we see from the table, the pseudoclassical scaling is

$$\chi^{PS} \equiv \chi(s = 1, q = 1, p = 2) = C_{q}^{PS} \rho_{p}^{2} v_{e}$$

where  $\rho_p$  is the poloidal Larmor radius,  $\nu_e$  is the collision frequency of the electrons, and  $C_o^{PS}$  is an enhancement factor. In this case the constant C becomes

$$C^{PS} = C_o^{PS} \left(\frac{4\pi}{\mu}\right)^2 \frac{n}{k}$$
,

with  $C_{0}^{PS} \sim 1 - 10$ . Moreover the system constants D and A have the expressions

$$D = \left(\frac{\epsilon_n}{\mu a^2}\right) \frac{T_f \rho_f}{5J_f^2} \left(\frac{\alpha}{1+2\alpha}\right) \text{ and } A = C_o^{PS} \left(\frac{10}{3} D \frac{\alpha}{1+2\alpha}\right)^2$$

Here, we have made use of the fact that the toroidal conductivity is given by  $\sigma = 2ne^2/m_e v_e$ , where m<sub>e</sub> and e are the mass and the charge of the electron, respectively. The divergent nature of  $\chi^{PS}$  on the magnetic axis can be avoided by introducing

$$\chi^{M} \equiv \left(\frac{r}{a}\right)^{2} \chi^{PS} = \chi(s = 2, q = 1, p = 2)$$
,

•

which is called the Mercier scaling.<sup>4</sup> We are, on the other hand, limited by the value of  $\alpha$ , which is 1/3.<sup>5</sup> The enhancement factor and the system constants in this case become

$$C^{M} \equiv C^{PS}/a^{2}$$
 and  
 $A = C_{o}^{M} \left(\frac{2}{3} D\right)^{2}/\rho_{f}$ 

Here,  $C_o^M$  depends on the inverse aspect ratio,  $\varepsilon = a/R_o$  by the relation

$$C_{0}^{M} = \begin{cases} 2000, \ \epsilon \leq 0.2\\ \\ 2000 \left(\frac{0.2}{\epsilon}\right)^{2}, \ \epsilon > 0.2 \end{cases}$$

where  $R_{o}$  is the major radius of the torus.

Table	1.	The	heat	conductivity	coefficient,
-------	----	-----	------	--------------	--------------

$$\chi = Cn^{\gamma}r^{2s}/I^{p}T^{q/2}.$$

a	S	q	р	χ
Arbitrary	0	3	0	l/T <sup>3/2</sup> , neoclassical
Arbitrary	1	1	2	r <sup>2</sup> /I <sup>2</sup> T <sup>1/2</sup> , pseudoclassical
1/3	2	1	2	r <sup>4</sup> /I <sup>2</sup> T <sup>1/2</sup> , Mercier's like

In the case of neoclassical scaling, we have

$$\chi^{NC} \equiv \chi (s = 0, q = 3, p = 0) = C^{NC}/T^{3/2}$$
.  
Assuming that  $\chi^{NC} = C_0^{NC} a^2 v_e$ , then  $C^{NC} = C_0^{NC} K a^2$  where  $K = v_e T^{3/2}$ . The system constant A then has the form

$$A = \frac{10}{3} C_0^{NC} \mu k KD \frac{\alpha a^2}{(1 + 2\alpha)},$$

where  $kK = 2 ne^2/m_e$ .

We should note that the enhancement factor, i.e.,  $C_0^{NC}$ ,  $C_0^{PS}$ ,  $C_0^M$ , has to be chosen so that the plasma temperature becomes  $T_0$  on the limiter at any time. Furthermore, it is clear that Table 1 may be extended to account for some other possible empirical scaling laws as long as the choice of  $\alpha$ , p, q, and s satisfy the separability condition, Eq. (3).

In order to complete the scheme of obtaining the functions F, G, Z, and S from the set of differential equations given earlier, the boundary and the initial conditions must be stated. For r = 0 we should have F(0) = G(0) = 1 and Z(0) = S(0) = 0.

For a moment let us discuss the time evolution of the limiter. The limiter motion may be found by imposing the condition that there will be no temperature discontinuity on the border of the plasma at any time. This condition is satisfied by the temperature equation as

 $T(r = r_L, t) = constant = T_o$ , which implies that

$$G(\rho_{\rm L}) = \frac{T_{\rm o}}{T_{\rm f}} \left(\frac{t}{\tau}\right)^{2(1+2\alpha)/5}$$

Here  $\rho_L = r_L^2(t) h(t)$ , which defines the evolution of the limiter position,  $r_L(t)$ . This, in turn, gives us the initial time,  $t_o$ , which sets the limiter on the magnetic axis,  $r_L(t_o) = 0$ . Since G(0) = 1,

$$\frac{t_o}{\tau} = \left(\frac{T_o}{T_f}\right)^{5/2(1+2\alpha)}$$

which defines t<sub>o</sub>.

In the coming section, we take up the problem of relating the system parameters, i.e.,  $\alpha$ ,  $\rho_f$ , and D to the plasma discharge parameters.

### 4. PLASMA DISCHARGE PARAMETERS

Knowing the time evolution of the current profile and temperature, the following plasma discharge parameters can be computed in a straightforward manner.

1) The plasma current,  $I_p(t)$ :  $I_p(t)/I_f = \left(\frac{t}{\tau}\right)^{(2-\alpha)/5} \frac{Z(\rho_L)}{Z_f}$ ,

with  $I_f \equiv I(r = a, t = \tau) = \pi J_f Z_f / h_f$ .

2) The normalized safety factor,  $q_s(r, t)/q_s(0, t)$ :

From the definition of the safety factor  $q_s(r, t)$ , we find

$$\frac{q_{s}(r, t)}{q_{s}(0, t)} = \frac{\rho}{Z} \left(\frac{t}{\tau}\right)^{-\alpha}$$

For  $t = \tau$ , this yields

$$q_{b} \equiv \frac{q_{s}(a, \tau)}{q_{s}(0, \tau)} = \frac{\rho_{f}}{Z_{f}}$$

3) The poloidal flux function,  $\psi$ :

Since  $\psi = R_0 \int_0^r dr' B_p(r', t)$ , and from the definition of  $B_p(r, t)$ , we get

$$\psi(\mathbf{r}, \mathbf{t}) = \mu \frac{\frac{R J_f}{2h_f}}{4h_f} \left(\frac{\mathbf{t}}{\tau}\right)^{(2-\alpha)} \int_0^{\mu} d\rho' Z/\rho'.$$

4) The skin time, 
$$\tau_s(r, t)$$
:

We find

$$\frac{\tau}{\tau_{s}} = \frac{2\tau}{5t} (1 - 3\alpha) \left[ \left( \frac{2 - \alpha}{1 - 3\alpha} \right) - \frac{Z(\rho)}{\int_{0}^{\rho} d\rho' Z/\rho'} \right]$$

since  $1/\tau_s \equiv \frac{1}{\psi} \frac{\partial \psi}{\partial t}$ .

5) The averaged poloidal beta,  $\bar{\beta}_{p}$ :  $\bar{\beta}_{p} = \frac{2n \int_{0}^{a} dr 2\pi r T(r, \tau)}{(B_{f}^{2}/2\mu)\pi a^{2}}$  or  $\bar{\beta}_{p}/\beta_{p} = \int_{0}^{\rho_{f}} d\rho G(\rho)/\rho_{f}G_{f}$ ,

where

$$\beta_{p_o} \equiv \frac{2nT_o}{B_f^2/2\mu} = \frac{T_o}{T_B} \text{ and}$$

$$G_f \equiv G(r = a, t = \tau) = T_o/T_f.$$

Here we have used  $B_f \equiv \mu I_f / 2\pi a$  and  $T_B \equiv B_f^2 / 4\mu n$ .

6) The electron energy lifetime, 
$$\tau_E$$
:  
 $\tau_E(\mathbf{r}) = \frac{\frac{3}{2} n \int_0^{\mathbf{r}} d\mathbf{r'r'} T(\mathbf{r', \tau})}{\int_0^{\mathbf{r}} d\mathbf{r'r'} E(\mathbf{r', \tau}) J(\mathbf{r', \tau})}$  or

$$\tau_{\rm E}({\bf r})/\tau_{\rm E_{\rm f}} = \frac{\int_0^{\rm p} {\rm d}\rho' {\rm G}}{\int_0^{\rm p} {\rm d}\rho' {\rm F}^2/{\rm G}^{3/2}} \quad \text{with } \tau_{\rm E_{\rm f}} \equiv \frac{3}{2} {\rm n} {\rm T_{\rm f}}/{\rm J_{\rm f}}^2/\sigma_{\rm f} \ .$$

7) The transformer flux,  $\psi_{tr}$ :

.

Assuming that the transformer coil is located at r = b, the electric field at that point is

$$E(\mathbf{r} = \mathbf{b}, \mathbf{t}) = \frac{1}{2\pi R_0} \frac{\partial \Psi_{\mathbf{tr}}}{\partial \mathbf{t}}$$
.

From this, we compute  $\psi_{tr}$  for t =  $\tau$ , which takes the form

$$\frac{\psi_{\rm tr}}{\mu R_0^{\rm I}_{\rm f}} = \frac{10\alpha}{(2-\alpha)Z_{\rm f}} \left[ 1 - G_{\rm f}^{(1-\alpha/2)/(1+2\alpha)} \right] + \left[ \lambda n(b/a) + \frac{1}{2Z_{\rm f}} \int_0^{\rho_{\rm f}} d\rho Z/\rho \right].$$

### 5. APPLICATION AND DISCUSSION

Before the results are demonstrated on an example, it is convenient to carry out the computation in terms of  $\beta_{p_0}$ ,  $q_b$ , and  $\eta$ , which is defined by

$$n = \Delta t / \tau_B$$
 with  $\tau_B = \tau (T_B / T_f)^{1.5} \rho_f / \alpha$ .

Here  $\Delta t = \tau - t_0$  may be called an opening time of the limiter. For most cases, it is reasonable to assume  $t_0/\tau << 1$ , so that the system parameters become

$$\rho_{f} = \left[ \frac{\alpha}{\eta} \left( \frac{40}{3} \frac{\alpha}{1+2\alpha} D \right)^{1.5} q_{b}^{3} \right]^{0.4},$$

$$G_{f} = \frac{3}{40} \beta_{p_{o}} \left( \frac{1+2\alpha}{\alpha} \frac{\alpha}{Dq_{b}^{2}} \right), \text{ and}$$

$$\beta_{p_o}/G_f = T_f/T_B$$
.

The last relation gives us the value of the final temperature,  $T_{f}$ , on the magnetic axis.

We illustrate the results on a TNS-size plasma with a moving limiter that starts from the magnetic axis and reaches the full-size radius at the end of discharge. The Mercier-like scaling is assumed for  $\chi$  during the computation, since the pseudoclassical scaling case is vastly explored in Ref. 2.

The typical device parameters are:  $R_0 = 6 \text{ m}$ , a = 1.25 m, b = 1.5 m,  $q_s(r = a, t = \tau) = 4$ ,  $n = 6 \times 10^{19} \text{ m}^{-3}$ , and the toroidal magnetic field is 8 T. The system parameters used for the computation are  $\beta_{p_0} = 1.8 \times 10^{-2}$  and  $n = 5 \times 10^{-3}$ , which yield  $T_f = 2.3 \text{ keV}$ ,  $I_f = 2.6 \text{ MA}$ , and  $T_o = 64 \text{ eV}$ . After having solved Eq. (2) numerically, the findings are presented in Figs. 2-9 for  $J(r, t)/J_f$ ,  $T(r, t)/T_f$ ,  $q_s(r, t)/q_s(0, t)$ ,  $\chi(r, t)$ ,  $I_p(t)/I_f$ ,  $\tau/\tau_s(r - 2, t)$ ,  $\psi(r = a, t)$  and  $r_L(t)/a$ . For this particular case the limiter opening time is set to  $\Delta t \approx \tau = 2.3 \text{ sec}$ , and the other plasma parameters found from the computation are  $\bar{\beta}_p = 0.122$ ,  $\bar{\chi} = 16.5 \text{ m}^2/\text{sec}$ ,  $\psi_{tr} = 29.5 \text{ volt-sec}$ , and  $\tau_E(r = a) = 0.284 \text{ sec}$ .

Since we have a current profile peaked off-axis, we can demonstrate the effect of the moving limiter by adjusting the opening time. Fig. 10 (a, b) shows the final forms of the current density profiles for the opening times  $\tau = 2.3$  sec and  $\tau = 4.77$  sec, respectively. We see that the slower motion of the limiter has a tendency to give rise to a profile peaked farther out. That is simply a result of gaining more time for the plasma temperature to grow appreciably in the central part of the discharge. We should also note that the transformer flux for  $\tau = 4.77$  sec is about 36.5 volt-sec compared to 29.5 volt-sec for the previous case. This also indicates that having a moving limiter may prevent the system from the deposition of an excessive amount of magnetic energy and reduce current density skin effects.

### ACKNOWLEDGMENTS

The author appreciates the many helpful discussions with F. B. Marcus, W. A. Houlberg, and N.  $\updownarrow$ . Uckan.



Fig. 1. The geometry of the problem.



Fig. 2. Time evolution of the radial profiles of the current density.



Fig. 3. Time evolution of the temperature profiles.



Fig. 4. Time evolution of the normalized safety factor.



Fig. 5. Evolution of the heat conductivity coefficient.



Fig. 6. The plasma current as a function of limiter position.



Fig. 7. Evolution of the skin time at the edge.



Fig. 8. Evolution of the poloidal flux function at the edge.



Fig. 9. Evolution of the limiter position.



Fig. 10. Final current density profiles for (a)  $\tau$  = 2.3 sec, (b)  $\tau$  = 4.7 sec.

### REFERENCES

- I. N. Golovin et al., Proc. B.N.E.S. Nuclear Fusion Reactor Conf., p. 194 (Culham, 1965).
- 2. G. Bardotti, L. Enriques, and F. Santini, Nucl. Fusion 15, 621 (1975).
- T. Uckan, <u>A Study of Possible Separable Solutions for Plasma Transport</u> <u>in Ohmically Heated Tokamaks</u>, ORNI/TM-6065, Oak Ridge National Laboratory, Oak Ridge, Tennessee (1977).
- 4. C. Mercier, Transport in Tokamak Plasmas, EUR-CEA-FC-812 (1976).
- 5. T. H. Jensen, Phys. Fluids 20, 427 (1977).