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S.B.Gerasimov

ON THE AXIAL-VECTOR STRUCTURE CONSTANT IN THE RADIATIVE DECAY OF THE CHARGED PION

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Правило сумм для аксиальной структурной константы в радиационном распаде заряженного пиона, вытекающее из алгебры токов и PCAC, вычисляется в рамках кварк-партонного подхода и модели векторной доминантности. Полученное значение близко к нулю и находится в согласии с недавними экспериментальными данными.

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Препринт Объединенного института ядерных исследований. Дубна 1977

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Within the framework of the quark-parton approach and VMD, the sum rule for the axial-vector structure constant, following from the current algebra and PCAC, is calculated. The value close to zero is found in agreement with the recent experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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The angular and energy distribution of photons emitted in the decay $\pi \to \ell \nu \gamma$ was measured in two experiments [1,2], based on the statistics of 143 events [1] and 170±15 events [2], to give the ratio

$$\gamma_{exp} = \frac{a}{b} = \begin{cases} 0{,}4 \text{ or } -2{,}18 \pm .10 \; ^{/1/} & (1a) \\ 0{,}15 \pm 0{,}11 \text{ or } -2{,}07 \pm 0{,}11 \; ^{/2/} . & (1b) \end{cases}$$

where a(b) is the axial (vector) form factor in the "structure-dependent" matrix element

$$M_{SD}(\pi \to \ell \nu \gamma) = \ell_\mu e_\nu [\epsilon^{\mu\nu\rho\sigma} k_\rho p_\sigma b(t) - i(k^\mu p^\nu - g^{\mu\nu}(k \cdot p)) a(t)] , \qquad (2)$$

$k(p)$ is 4-momentum of photon (pion), $t = (k-p)^2$, e_ν is the photon polarization vector, and ℓ_μ is the leptonic current operator. According to the CVC-hypothesis $b(0)$ may be related to the $\pi^0 \to 2\gamma$ decay constant [3]. There is a large number of the theoretical papers devoted to evaluation of $a(t)$ (the references and discussion of the earlier results can be found in the reviews [4,5] and also in some recent articles [6,7]). It seems, that an indication of small value of γ suggested by ref. [2] may present difficulty for many models (see, e.g., refs. [7]). In the present paper the results are presented of calculation of $a(0)$ through the current algebra sum rule of Das et al. [8]

$$\frac{1}{\sqrt{2}} a(0) = \frac{F_\pi}{3} \langle r^2 \rangle_\pi - \frac{1}{F_\pi} \int \frac{dq^2}{q^4} (\rho_1^V(q^2) - \rho_1^A(q^2)) \equiv \frac{F_\pi}{3} \langle r^2 \rangle_\pi - \frac{1}{F_\pi} \rho_{-2}^{V-A} , \qquad (3)$$

where F_π = 93 MeV is the pion decay constant, $\langle r^2 \rangle_\pi$ is the pion e.m. radius, and $\rho_1^{V(A)}(q^2)$ is the spectral density of the two-point vector (axial-vector) current correlation function (our notations and the normalization convention are similar to ref. [6]).

The results obtained in the framework of the quark-parton approach and VMD are as follows.

1. In the one-loop quark diagram approximation for $\langle r^2 \rangle_\pi$ and the integral in Eq. (3) we have for the three-colored quark model:

$$\rho_{-2}^{V-A} = \frac{1}{4\pi^2} \qquad (4)$$

$$\langle r^2 \rangle_\pi = \frac{3}{4\pi^2 F_\pi^2} = 0.34 \text{ fm}^2 \qquad (5)$$

and, further

$$a(0) = 0 \qquad (6)$$

which agrees with Eq. (1b).

The value of $\langle r^2 \rangle_\pi$ in Eq. (5) is in very good agreement with the experiment on a direct determination of the pion radius via the πe-scattering: $\langle r^2 \rangle_\pi^{exp} = 0.33 \pm 0.06$ fm^2 [9]. The numerical value of Eq. (4) is surprisingly close to that given by the pole saturation of the spectral integral ρ_{-2}^{V-A} with the ρ- and A_1-meson contributions

$$\rho_{-2}^{V-A}(\text{VMD}) / \rho_{-2}^{V-A}(\text{Quark loop}) = 4\pi^2 (g_\rho^{-2} - g_A^{-2}) \simeq \\ \simeq \frac{3\pi^2}{g_\rho^2} = 0.99 \pm 0.07 \,, \qquad (7)$$

where the 1st and 2nd Weinberg sum rules are used together with the mass relation $m_{A_1} = \sqrt{2}\, m_\rho$ and the experimental value $g_\rho^2/4\pi = 2.38 \pm 0.18$ [10].

2. In the generalized vector meson dominance model the higher vector mesons with $J^P = 1^\pm$ and $I = 1$ will contribute to ρ_{-2}^{V-A} :

$$\rho_{-2}^{V-A}(\mathrm{GVMD}) = \sum_{n=0}^{\infty}(g_{V_n}^{-2} - g_{A_n}^{-2}) \equiv (g_{\rho}^{-2} - g_{A_1}^{-2})(1 + \sum_{n=1}^{\infty}\Delta_n). \qquad (8)$$

To estimate the correction term in Eq. (8) we assume

$$\Delta_n = \int_{\Delta m_n^2} \frac{dq^2}{q^4}\rho_1^{V-A}(q^2) / \int_{\Delta m_0^2} \frac{dq^2}{q^4}\rho_1^{V-A}(q^2), \qquad (9)$$

$$\rho_1^{V-A}(q^2) = \begin{cases} \mathrm{const}/q^2 & (10) \\ \mathrm{const}/q^4 & (11) \end{cases},$$

where Eq. (10) corresponds to the asymptotical behaviour of $\rho_1^{V-A} = \rho_1^V(q^2) - \rho_1^A(q^2)$ given by the quark-loop diagram with the finite quark masses, while Eq. (11) represents the behaviour of the spontaneously broken chiral symmetry model, generating the q^2 -dependent effective mass of quarks /11/ (for simplicity, we omit the log's terms). The integration in Eq. (9) is carried out in the intervals of $m_{V_n}^2 \le q^2 \le m_{A_n}^2$ with the mass relations for $m_{V_n}^2 (A_n)$ of the form $m_{V_n}^2 = m_\rho^2(1+2n)$ and $m_{A_n}^2 = m_{V_n}^2 + m_\rho^2 = m_A^2(1+n)$, which are characteristic of the dual-resonance models. As a result we have

$$\Delta = \sum_{n=1}^{\infty}\Delta_n = \begin{cases} 0.38 & (12) \\ 0.03 & (13) \end{cases},$$

where Eqs. (12) and (13) correspond to (10) and (11), respectively. Thus, the values of ρ_{-2}^{V-A} obtained in the case of the rapid onset of the asymptotic, chiral-symmetrical regime for $\rho_1^{V(A)}(q^2)$, provide the value of $a(0) \simeq 0$ which agrees favourably with the least of solutions (1b). Note, that the small value of a(0) leads to the corresponding smallness of the pion electromagnetic polarizability, for these quantities are shown in ref. /4/ to be proportional to each other in the soft-pion limit. We stress in conclusion that due to the large compensation of two terms in eq. (3) $a(0)$ is very sensitive even to small variations of each term, so that better experimental accuracy for $\langle r^2 \rangle_\pi$ and $\gamma = a/b$ would be highly desirable.

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