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ON THE EFFECT OF TRAPPED PARTICLES IN THE REGIME OF CYCLOTRON RESONANCE

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ABSTRACT

A motion of particles in a finite amplitude wave, propagating obliquely to the homogeneous magnetostatic field is discussed. As it follows from simple integral properties, in the neighbourhood of Doppler - shifted cyclotron resonance similar trapping effects appear as in a plasma without magnetostatic field. Consequences of this trapping are discussed, in particular the possibility of a strong absorption of the wave and the origin of stochastic instabilities, caused by the perturbation of an effective trapping potential and leading to the acceleration of particles.

1. INTRODUCTION

An influence of finite amplitude wave on the dynamics of particles and consequently on the interaction wave-plasma is an often discussed problem. Since this interaction is basically strongly nonlinear, the appropriate solution requieres numerical analysis. On the other hand, DAWSON and SHANNY [1] have presented a simple analytic estimation of region of parameters, where strong absorption and even breaking of a wave in a plasma without magnetostatic field appears We have extended their estimation on the case of a magnetoactive plasma. Here, perturbations of the effective trapping potential appear in principle. We have therefore discussed also their effect on the detrapping of particles and on the existence of stochastic instabilities (see, e.g. [2]).

The suitable canonical formalism is used for the description of the motion of particles. Its advantage is - besides simple expressions - the possibility of following of a dynamics of particles in similar way, as has been done in [1]. A short communication on this topic appeared in [3].

2. MOTION OF A PARTICLE IN THE ELECTROSTATIC WAVE WITH $k_{\parallel} \neq 0$, $k_{\perp} \neq 0$ IN THE MAGNETOSTATIC FIELD NEAR RESONANCE $k_{\parallel} v_{\parallel} + \omega_c - \omega \sim 0$.

2.1 The description of the model

We consider homogeneous plasma in homogeneous magnetostatic field and the electrostatic wave, propagating with frequency ω and components of wave vector $k_{\parallel}$, $k_{\perp}$ oblique to the magnetostatic field. Considering wave as a given entity, we are discussing its influence on individual plasma particles (the problem is therefore not-selfconsistent). For concrete estimations, we consider waves generated in beam-plasma experiment [4].

2.2 Canonical formalism and integral properties of the motion of particles

Let us consider electrostatic wave with potential V in the form

$$V = V_0 \cos(k_{\parallel} Q_3 + k_{\perp} x - \omega t) \tag{1}$$

where Q_3 and x are the longitudinal coordinate (along magnetic field lines) and the perpendicular coordinate, respectively. The corresponding Hamiltonian function of a particle in a magnetostatic field with cyclotron frequency ω_c has the form

(2) $$H = \omega_c P_1 + \frac{P_3^2}{2m} + e V_0 \cos\left[k_{\parallel} Q_3 + k_{\perp} \left(\frac{2P_1}{m\omega_c} \right)^{\frac{1}{2}} \sin Q_1 - \omega t \right] .$$

Here Q_3, P_3, x, p_x are longitudinal and perpendicular coordinates and momenta respectively, and Q_1, P_1 are new perpendicular canonical coordinate and momentum, given by the transformation [5]

(3) $$x = \left(\frac{2P_1}{m\omega_c} \right)^{\frac{1}{2}} \sin Q_1 \qquad p_x = \left(\frac{m\omega_c P_1}{2} \right)^{\frac{1}{2}} \cos Q_1 .$$

Let us expand (2) by means of Bessel functions,

(4) $$H = \omega_c P_1 + \frac{P_3^2}{2m} + e V_0 \cos(k_{\parallel} Q_3 - \omega t) \cdot$$

$$\cdot \left[J_0(\alpha) + 2 \sum_{n=1} J_{2n}(\alpha) \cos 2n Q_1 \right] + \left[2 \sum_{n=0} J_{2n+1}(\alpha) \sin(2n+1) Q_1 \right] e V_0 \sin (k_{\parallel}$$

where

(5) $$\alpha = k_{\perp} \left(\frac{2P_1}{m\omega_c} \right)^{\frac{1}{2}} = k_{\perp} \rho$$

and let us rewrite (4) into the form

$$H = \omega_c P_1 + \frac{P_3^2}{2m} + e V_0 J_1(k_{\perp}\rho) \cos(k_{\parallel} Q_3 - \omega t + Q_1) + e V_0 J_0(k_{\perp}\rho) \cos(k_{\parallel} Q_3 - \omega t) +$$

(6) $$+ \sum_{n \neq 0} e V_0 J_{2n}(k_{\perp}\rho) \cos(k_{\parallel} Q_3 - \omega t - 2n Q_1) + \sum_{n \neq 0} e V_0 J_{2n}(k_{\perp}\rho) \cos(k_{\parallel} Q_3 - \omega t + 2n Q_1) +$$

$$+ \sum e V_0 J_{2n+1}(k_{\perp}\rho) \cos\left[k_{\parallel} Q_3 - \omega t - (2n+1) Q_1 \right] +$$

$$+ \sum e V_0 J_{2n+1}(k_{\perp}\rho) \cos\left[k_{\parallel} Q_3 - \omega t + (2n+1) Q_1 \right] .$$

Let us now transform canonically (2), (3) by means of following set of generating functions $F_2^{(1)}$, $F_2^{(2)}$, $F_2^{(3)}$

(7) $$F_2^{(1)} = Q_1 P_1^{(1)} - \omega_c t \cdot P_1^{(1)} + Q_3 P_3^{(1)}$$

$$F_2^{(2)} = (-\omega t + \omega_c t + k_{\parallel} Q_3^{(1)} + Q_1^{(1)}) \frac{P_3^{(2)}}{k_{\parallel}} + Q_1^{(1)} P_1^{(2)} + Q_3^{(1)} m \left(\frac{\omega}{k_{\parallel}} - \frac{\omega_c}{k_{\parallel}} \right)$$

$$F_2^{(3)} = Q_1^{(2)} P_1^{(3)} + Q_3^{(2)} \cdot P_3^{(3)} - \frac{m}{2} \left(\frac{\omega}{k_\parallel} - \frac{\omega_c}{k_\parallel} \right)^2 t \ .$$

In new coordinates we obtain

(8) $$H^{(3)} = \frac{1}{2m} P_3^{(3)^2} + eV_0 J_1 (k_\perp \rho) \cos k_\parallel Q_3^{(3)} + eV_0 J_0 (k_\perp \rho) \cos (k_\parallel Q_3^{(3)} - Q_1) +$$

$$+ \sum_{n=1} eV_0 J_{2n}(k_\perp \rho) \cos \left[k_\parallel Q_3^{(3)} - (2n+1) Q_1^{(3)} \right] + \sum_{n=1} eV_0 J_{2n}(k_\perp \rho) \cos \left[k_\parallel Q_3^{(3)} + (2n-1) Q_1^{(3)} \right] +$$

$$+ \sum_{m=1} eV_0 J_{2n+1}(k_\perp \rho) \cos \left[k_\parallel Q_3^{(3)} - (2n) Q_1^{(3)} \right] + \sum_{m=1} eV_0 J_{2n+1}(k_\perp \rho) \cos \left[k_\parallel Q_3^{(3)} + 2n Q_1^{(3)} \right]$$

with corresponding transformations

$$P_1 = P_1^{(3)} + \frac{1}{k_\parallel} P_3^{(3)} \qquad\qquad P_3 = P_3^{(3)} + m \left(\frac{\omega}{k_\parallel} - \frac{\omega_c}{k_\parallel} \right)$$

(9) $$Q_1^{(3)} = Q_1 - \omega_c t \qquad\qquad k_\parallel Q_3^{(3)} = -\omega t + k_\parallel Q_3 + Q_1$$

$$H^{(3)} = H - \omega_c P_1 - (\omega - \omega_c) \frac{P_3^{(3)}}{k_\parallel} - \frac{1}{2} m (\omega - \omega_c)^2 \frac{1}{k_\parallel^2} \ .$$

Let us express Hamiltonian (8) in the form

(10) $$H^{(3)} = \frac{1}{2m} P_3^{(3)^2} + eV_0 J_1 (k_\perp \rho) \cos k_\parallel Q_3^{(3)} + \Delta H$$

$$\rho^2 = \frac{2}{m\omega_c} \left(P_1^{(3)} + \frac{1}{k_\parallel} P_3^{(3)} \right) .$$

and let us consider the important resonance case

(10a) $$k_\parallel v_\parallel + \omega_c - \omega = 0 \ .$$

Considering (9), (10a) and (3) we can state that the first two terms on the right hand posses in (10) slow dependence on time, whereas ΔH is formed by quick oscillating terms. Since principially (for $k_\perp \rho << 1$ as well for $k_\perp \rho \geq 1$) the term $J_1(k_\perp \rho)$ is not dominant in the total expansion (8), we have first to estimate the effect of ΔH . For this purpose we shall use the method of BOGOLJUBOV (e.g. in [6]).

Let us suppose we have expressed (10) in the action-angle coordinate system J_1, w_1, J_3, w_3

$$H = \omega_c(J_1, J_3) J_3 + \sum\sum \phi_{mnp}(J_1, J_3) \cos(m w_1 + n w_3 + p\omega_c t) = H_0 + \Delta H;$$

(11)

$$H_0 = \omega_c(J_1, J_3) J_3 \quad .$$

Using method of BOGOLJUBOV [6], we shall show that for ϕ_{mnp} of the order of $\varepsilon << 1$ the motion of particles can be described in great part of phase space for time $\Delta t < \frac{1}{\varepsilon}$ by means of Hamiltonian H_0 .

Taking canonical equations for (11) in the form

(12) $$\frac{dx_k}{dt} = f_k(x_i, t, \theta); \qquad \theta = \frac{t}{\varepsilon}, \qquad \varepsilon << 1$$

one can expand x_k [6]

$$x_k = \xi_k + \varepsilon \hat{f}_k + \varepsilon^2 (\qquad) + \varepsilon^3 (\qquad) + \dots$$

where ξ_k is the solution of the equation

(13) $$\frac{d\xi_k}{dt} = \bar{f}_k + \varepsilon \sum \frac{\partial \bar{f}_k}{\partial \xi_i} \hat{f}_i + \varepsilon^2 (\qquad) + \dots$$

and where

$$f_k = \bar{f}_k + \tilde{f}_k$$

$$\bar{f}_k = (2\pi)^{-1} \int_0^{2\pi} f_k \, d\theta$$

$$\hat{f}_k = \int_0^{\theta} \tilde{f}_k \, d\theta \quad .$$

Since canonical equations for Hamiltonian (10) have the form

(14) $$\frac{dx_k}{dt} = f_k(x_i, \omega_c t)$$

we must properly choose the parameter ε . Taking $t = \nu_3^{-1} \tau$, we obtain

$$\frac{dx_k}{d\tau} = \nu_3 f_k \left(x_i, \frac{\omega_c}{\nu_3} \tau\right).$$

Provided $\nu_3 << \omega_c$ (let us choose ν_3 as the frequency of oscillations of particles in effective potential trough), we take $\varepsilon = \nu_3/\omega_c$ and $\theta = \frac{\omega_c}{\nu_3} \tau$.

Considering e.g. J_1 , and neglecting terms of $\mathcal{O}(\varepsilon^2)$, we have

$$\frac{d\xi_{J1}}{d\tau} = \nu_3 \bar{f}_{J1} + \nu_3 \varepsilon \sum_i \frac{\partial f_{J1}}{\partial \xi_i} \hat{f}_i = 0$$

$$\left(\bar{f}_{J1} = 0, \quad \frac{\partial f_{J1}}{\partial \xi_i} \hat{f}_i = 0\right)$$

and therefore

$$J_1 = J_{10} + \varepsilon \hat{f}_{J_1} = J_{10} - \varepsilon \sum \sum \frac{1}{m} \frac{\partial}{\partial J_1} \phi_{mnp} / \sin \alpha_{mnp} \cdot \sin \alpha_{mno} /$$

where

$$\alpha_{mnp} = m w_1 + n w_3 + p\theta$$

$$\alpha_{mno} = m w_1 + n w_3 \quad .$$

Secular changes are of the order $\epsilon(\varepsilon^2)$; one can therefore neglect for times $\Delta t \lesssim \frac{1}{\varepsilon}$ the perturbated part of Hamiltonian ΔH and in majority of occupied phase space regard only the influence of H_0 . Nevertheless, it is well known from the theory of nonlinear systems (see e.g. [2], [8]) that in the occupied phase space there exist trajectories, in neighbourhood of them is the motion strongly unstable;the phase trajectory possesses here erratic motion and consequently "stochastic instability". These trajectories are determined by resonant conditions and in our case by the condition

$$\left(m \frac{dw_1}{dt} + n \frac{dw_3}{dt} \right)_{\varepsilon \to 0} = 0$$

The more exact discussion of region of stochastic instability requiere numerical solution [7] (they are strongly dependent on parameter ε -see e.g. [7]). Approximately, it is possible to say that the most important stochastic region appears in the neighbourhood of separatrix [2], defined by the unperturbed part of Hamiltonian. The broadness of this region is proportional to ε [2]. Considering therefore $t < \varepsilon^{-1}$ $\varepsilon << 1$, we can in the main part of occupied phase space suppose only unperturbed part of Hamiltonian H_0 . This will be done in this and next chapters. Chapt. 4 will be devoted to the also very interesting inverse case of strong nonlinearity.

Returning to our original Hamiltonian (10) and supposing ν_1/ω_c , $\nu_3/\omega_c \sim \varepsilon$, it is obviously impossible to fulfil the simpliest resonant condition

$$\omega_c \pm \nu_1 \pm \nu_3 = 0 . \tag{15}$$

It is therefore necessary to suppose that the secularity will take place only in the order ε^2 . Neglecting approximation of this order, we can for times $\Delta t \sim \frac{1}{\varepsilon}$ suppose only the time independent part of Hamiltonian, namely H_0 ,

$$H_0^{(3)} = \frac{1}{2m} P_3^{(3)^2} + e V_0 J_1 (k_\perp \xi) \cos k_\parallel Q_3^{(3)} \quad . \tag{16}$$

Hamiltonian (16) and dynamics of particle in approximation under discussion has following important properties:

1) $H_0^{(3)}$ is cyclic in $Q_1^{(3)}$; it is possible to express $H_0^{(3)}$ in full cyclic form

$$H_0^{(3)} = H_0^{(3)} (J_1 , J_3)$$

2) $H_0^{(3)}$ is independent on time.

The first property implies

(17) $$P_1 \mp \frac{1}{k_\parallel} P_3 = const .$$

whereas the latter one enables to determine separatrix and consequently to distinguish trapped and untrapped particles.

It follows from (17) that limit in the change of P_3 implies limit in the change of P_1 . Since for trapping there exists only finite change of P_3 (then particle starts to be untrapped and therefore seeing out from resonant region), also resonant change of P_1 can be only limitted.

As well as in one-dimensional case, also here we can define trapping in a following way. Particles are considered to be trapped if Hamiltonian (16) (therefore in the coordinate frame, moving with velocity $v_{dopp} = \frac{\omega}{k_\parallel} - \frac{\omega_c}{k_\parallel}$) possesses libration. Particles will be therefore trapped, if the following condition is valid

(18) $$H_0^{(3)} \leq e V_0 J_1 (k_\perp \rho) .$$

The separatrix is defined by the condition

(19) $$H_0^{(3)} = e V_0 J_1 (k_\perp \rho) .$$

(The analogy with one-dimensional case is not complete. Separatrix (19) is unsymetrical in P_3 . It is namely

$$\frac{P_{3\,max\pm}^{(3)\,2}}{2m} = e V_0 J_1 \left\{ k_\perp \left[\frac{2}{m\,\omega_c} \left(P_1^{(3)} + \frac{1}{k_\parallel} P_{3\,max\pm}^{(3)} \right) \right]^{\frac{1}{2}} \right\} .$$

Since $H_0^{(3)}$ has the meaning of longitudinal energy of a particle in the coordinate system of wave (see (9)), the trapping condition is obvious. Particles are trapped, if the magnitude of the effective potential trough is greater then the kinetic energy of particle in the wave system. From the integral from (17) follows $\Delta P_{1max} = \frac{1}{k_\parallel} \Delta P_{3max}$. (During preparation of the paper to the publication, similar canonical formalism and consequently also invariant (17) appeared in [8]. Since cited paper deals with strong stochastic instability regime, and because this effect has been roughly estimated also by us, we shall discuss its relation to our results in the subsequent Chapt. 4).

3. EFFECT OF TRAPPING ON THE ABSORPTION OF THE WAVE ENERGY

Let us now try to estimate the effect of the trapping mechanism of the rate of absorption of the wave energy. The exact analytic solution consists in the determination of **nonlinear** dispersion relation, respecting the trapping effect. Analytic solution requieres only weak nonlinearities. Strong nonlinearity is solvable only numerically. Analytic procedure, which is discussed here, can therefore give only first estimation of the complex complicated problem.

The estimation will be established analogously to the onedimensional model of DAWSON and SHANNY [1]. Simultaneously we shall use integral properties, described in foregoing Chapt. 2 .

The mechanism of model [1] is following. Given potential of electrostatic wave ϕ determines region of velocities Δv , characterizing trapped particles. For trapped particles the strongiest interaction is to be expected. The amplitude of ϕ at the same time determines the amount of kinetic energy, which particle can obtain during its motion in potential trough. From the form of distribution function (Maxwell distribution is supposed) and from the value of Δv the total amount of kinetic energy ΔW_{ab} is established, which is approximately lost by particles during the first half oscillation perioed. If ΔW_{ab} is comparable with the field energy in the nonlinear region, strong nonlinear effect is to be expected. For particle oscillations near the bottom of the potential trough, absorption corresponds to linear damping. For greater amplitude of oscillations, the absorption is strongly enhanced due to the fact that number of particles which enlarge their energy also increases.

Let us now discuss the similar situation in magnetostatic field. As follows from foregoing Chapt. 2., in the case of cyclotron resonance the trapping mechanism also exists - with effective potential strongh with amplitude $\sim V_0 J_1(k_\perp \rho)$. From integral properties follows $\Delta P_1 = \frac{1}{k_{\parallel}} \Delta P_3$; the maximal change of longitudinal and perpendicular energy is also **limitted** and is given by the potential trough. Nonlinearity of motion therefore (as well as in one-dimensional problem) brings into the consideration finite region of velocities for trapping - and therefore for strongest interaction - and finite value of possible change of energies.

The amount of absorbed energy ΔW_{ab} can be formally expressed as

$$(20) \qquad \Delta W_{ab} = \iiint dt\, dP_1\, dP_3 \left(\frac{dW_\perp}{dt} + \frac{dW_\parallel}{dt} \right) f_1$$

where $\frac{dW_\perp}{dt}$, $\frac{dW_\parallel}{dt}$ is time change of perpendicular and longitudinal energy, respectively, and f_1 is the perturbation of stationary distribution function, caused (as well as $\frac{dW_\perp}{dt}$ and $\frac{dW_\parallel}{dt}$) by means of discussed wave. For approximative calculation we shall choose following procedure. According [1], the integration in time will be substitute by means of maximal change of perpendicular and longitudinal kinetic energy of trapped particles (according (17), both energies change simultaneously). We express ΔW_{ab} therefore in the form

$$(21) \qquad \Delta W_{ab} \sim \delta N\, \delta \left(\Delta W_{\perp\, max} + \Delta W_{\parallel\, max} \right)$$

where δN is that part of trapped particles, which causes the total absorption. For one-dimensional case and for small amplitudes of rf field δN has been determined as [1]

$$(22) \qquad \delta N \doteq \frac{\partial f}{\partial v} \, \frac{\Delta v_{trapp}^2}{2} \Big/ v_\phi = \frac{\omega}{k}$$

and exactly

$$2\delta N = \int_{v_\phi}^{v_\phi - \Delta v} f(v)\, dv - \int_{v_\phi + \Delta v}^{v_\phi} f(v)\, dv$$

where $f(v)$ is distribution function and Δv region of trapping for longitudinal velocities. In [1] the following parameter R is defined

$$(23) \qquad R = \frac{1}{\frac{1}{2} \varepsilon_0 E^2} \, \Delta W_{ab}$$

where $\frac{1}{2} \varepsilon_0 E^2$ is the energy of incident field. Consequently, R is given by equation

$$R = \frac{1}{\frac{1}{2} \varepsilon_0 E^2} \, \delta N \cdot \delta \left(\Delta W_{\parallel\, max} \right)$$

or, according [1] more exactly ($\epsilon^2 = \frac{E^2}{4 n m v_T^2}$, cgse)

$$R = \frac{1}{2\epsilon^2}\left[\frac{1}{2}\left(\frac{v_\phi}{v_T}\right)^2 + \frac{v_\phi}{v_T}\frac{\epsilon}{3} - \frac{1}{16}\right]\left[erf\,\frac{1}{\sqrt{2}}\left(\frac{v_\phi}{v_T} + \sqrt{\frac{2v_\phi}{v_T}\epsilon}\right) - \right.$$

(23a) $$\left. - erf\,\frac{1}{\sqrt{2}}\left(\frac{v_\phi}{v_T} - \sqrt{\frac{2v_\phi}{v_T}\epsilon}\right)\right] - \frac{1}{16\sqrt{2}\epsilon^2}\left\{\left(\frac{v_\phi}{v_T} + \sqrt{\frac{2v_\phi}{v_T}\epsilon}\right)\times\right.$$

$$\left.\times\exp\left[-\frac{1}{2}\left(\frac{v_\phi}{v_T} + \sqrt{\frac{2v_\phi}{v_T}\epsilon}\right)^2\right] - \left(\frac{v_\phi}{v_T} - \sqrt{\frac{2v_\phi}{v_T}\epsilon}\right)\exp\left[-\frac{1}{2}\left(\frac{v_\phi}{v_T} - \sqrt{\frac{2v_\phi}{v_T}\epsilon}\right)^2\right]\right\}.$$

Supposing distribution function $f = f(P_1, P_3)$, two-dimensional analogy of (22) will be obviously

(24) $$\delta N \doteq \frac{\partial F}{\partial P_3}\frac{\Delta P_3^2}{2}\ ; \qquad F = \int_0^{P_{10}} f(P_1, P_3)\, dP_1$$

and more exactly

(25) $$\delta N = \int_0^{P_{10}-\Delta P_1;\ P_{30}-\Delta P_3} f(P_1, P_3)\, dP_1 dP_3 - \int_0^{P_{10};\ P_{30}} f(P_1, P_3)\, dP_1 dP_3\ .$$

Let us choose distribution function in Maxwell form both in longitudinal and perpendicular velocities

(26) $$f(P_1, P_3) = \frac{2}{\sqrt{\pi}}\ \frac{1}{\sqrt{2mkT}}\cdot\frac{\omega_c}{kT}\, n_c\, e^{-\frac{\omega_c P_1}{kT} - \frac{P_3^2}{2mkT}}$$

supposing the same longitudinal and perpendicular temperature $T_\perp = T_\parallel$. Substituting into (25), we obtain

(27) $$\delta N \sim \frac{n_0 e}{2}^{\alpha t_0 + \frac{\alpha_0}{4}}\left\{2\,erf\,(t_0) - erf\,(t_1) - erf\,(t_2)\right\}$$

where $erf\,(t)$ are error functions of the type

(28) $$erf\,(t) = \frac{2}{\sqrt{\pi}}\int_0^t e^{-\xi^2}\, d\xi$$

and t_i are defined

$$t_0 = \frac{P_{30}}{\sqrt{2mkT}} \qquad t_1 = \frac{P_{30}}{\sqrt{2mkT}}\left(1 + \frac{\Delta P_{3\,max}}{P_{30}}\right)$$

$$t_2 = \frac{P_{30}}{\sqrt{2mkT}}\left(1 - \frac{\Delta P_{3\,max}}{P_{30}}\right)$$

P_{30} is here defined as

(29) $$P_{30} = m v_{30} = m v_{dopp}$$

where v_{dopp} is Doppler-shifted velocity

(30) $$v_{dopp} = \frac{\omega - \omega_c}{k_{\parallel}} .$$

According (16), ΔP_3 is defined as

(31) $$\frac{1}{2m} \Delta P_{3max}^{\,2} = e V_0 J_1 (k_\perp \rho) .$$

Defining coefficient of absorption

$$R = \frac{1}{\frac{1}{2} \varepsilon_0 E^2} \Delta W_{abs} \qquad (MSI)$$

where E is intensity of incident wave (supposing with constant amplitude) and using expressions (17), (21), (27), (29), (30), (31) we obtain for R (supposing $k_\perp \rho \ll 1$)

(32) $$R = \frac{1}{\varepsilon_0 E^2} n_0 e^{\alpha t_0 + \frac{\alpha^2}{4}} \left\{ 2\,\mathrm{erf}(t_0) - \mathrm{erf}(t_1) - \mathrm{erf}(t_2) \right\} \times$$
$$\times kT(\alpha + 2t_0) \cdot \frac{\Delta P_3}{\sqrt{2mkT}}$$

More exact expression (analogously to (23a)) is given by the formula

(32a) $$R = \frac{1}{\varepsilon_0 E^2} \left\{ \left(\frac{1}{2} m v_{dopp}^2 + \frac{\Delta P_3^2}{2m} - \frac{kT}{2} \right) \times \right.$$
$$\left. \times n_0 \left[\mathrm{erf}(t_1) - \mathrm{erf}(t_2) \right] + kT n_0 \frac{1}{\sqrt{\pi}} \left[t_1 e^{-t_1^2} - t_2 e^{-t_2^2} \right] \right\} .$$

Expressing now (32) for concrete parameters, chosen according to the experiment [4]

(33) $\omega_c = 6{,}7.10^9 sec^{-1}$; $\omega = 1{,}5\omega_c$; $\lambda \sim 10^{-2} m$; $n_0 = 10^{16} m^{-3}$

and normalizing E and kT to $E_o = 10^4\, Vm^{-1}$ and $kT_o = 100\, eV$ by means of expressions

(34) $kT = \tau kT_o$; $E = \varepsilon E_o$

we can rewrite (32) into the compact form

(35) $$R = 1{,}1 . 10^2 \frac{\tau^{\frac{3}{4}}}{\varepsilon^{\frac{3}{2}}} e^{\frac{5{,}6}{\tau}} \left\{ 2\phi(t_o) - \phi(t_1) - \phi(t_2) \right\}$$

where $t_o = \frac{3{,}9}{\tau^{\frac{1}{2}}}$

$$t_1 = \frac{3{,}9}{\tau^{\frac{1}{2}}} \left(1 + 3{,}3 . 10^{-2} \varepsilon^{\frac{1}{2}} \tau^{\frac{1}{4}}\right)$$

$$t_2 = \frac{3{,}9}{\tau^{\frac{1}{2}}} \left(1 - 3{,}3 . 10^{-2} \varepsilon^{\frac{1}{2}} \tau^{\frac{1}{4}}\right)$$

Fig. 1 presents parametrical curves $R = R(\tau, \varepsilon)$ of (35) for τ = const and for parameters $\varepsilon\,(10^{-2} \div 10^2)$ and $\tau\,(0{,}5 \div 10)$, representing $E\,(1 \div 10^4\, V/m)$ and $T_e\,(50 \div 10^3 eV)$. For small ε and small τ, the coefficient R follows curve $\sim \varepsilon^{-\frac{1}{2}}$, which corresponds to linear Landau damping (similar as in one-dimensional model [1]). For higher temperatures τ (in our region for $T_e \geq 100\, eV$) and for higher intensities ε (for $E \sim 10^4 \div 10^5\, V/m$) a nonlinear effect appears and the curve starts to increase very sharp. For yet higher value of τ (in the region of $T_e \sim keV$) this increasing again ceases (due to fact that working point v_ϕ / v_T nears to the maximum of Maxwell distribution). Comparing with parameters of experiment [4] we can state that absorption is deeply in linear regime (and, of course **extremely weak** due to the high value of v_ϕ / v_T). Strong nonlinear regime can be expected for $T_e \geq 10^2 eV$, $E \sim 10^4 \div 10^5\, V/m$; the breaking of waves requieres time $\Delta t \sim \left[\frac{ek_\parallel}{m} E_\parallel J_1(k_\perp \rho)\right]^{-\frac{1}{2}}$, therefore for our range $T_e = 10^2 eV$, $E = 10^5 Vm^{-1}$, $\Delta t \sim 10^{-9}$. Breaking of waves with lower intensities can be expected for the regime with ω nearer to ω_c (here, we have supposed $\omega/\omega_c = 1{,}5$). Such situation could appear in the case of externally driven field.

4. INFLUENCE OF STOCHASTIC INSTABILITIES ON THE TRAPPING MECHANISM

Till now the change of energy of a particle under the influence of only one wave and only one component of Fouriere expansion of (4) has been supposed. We have neglected the effect of higher harmonics of the expansion, as well as a splitting of an incident single wave in a narrow spectrum, often detected. We shall now try to estimate these effects from the point of view of stochastic instabilities ([2], [7], [8]).

It is well known that under special conditions, the development of nonlinear system can be (at least qualitatively) described by means of diffusion equation. Parameters of such systems fulfil conditions of existence of stochastic instabilities; particles move in similar way as under influence of stochastic rf fields [2]. Since the analytic description of stochastic instabilities follows from perturbatinn analysis, the region of validity for this description is too narrow. Nevertheless, the intensive analytical and numerical work (see, e.g. [7], [8]) show that in strong nonlinear cases (therefore not only for small perturbations) similar "erratic" motion of particles in e.g. two waves is to be expected. These cases contradict to the validity of expressions (16) and to neglections of terms $\sim \varepsilon^2$ in the BOGOLJUBOV (6) chain. Supposing ΔH in (10), this term could then lead to strong detrapping effects and has to be seriously discussed.

Let us first discuss the effect of diffusion in so called stochastic layer [2]. ZASLAVSKIJ [2] has shown that for Hamiltonian

$$H = \omega(J)\,J + \varepsilon V(J, w, t)\,; \qquad \varepsilon \ll 1 \tag{36}$$

where $\omega(J)\,J$ represents Hamiltonian of a particle in potential trough of a wave with potential φ_0 and εV a small perturbation due to second interacting wave of a small amplitude φ_1 , $\varphi_1/\varphi_0 \ll 1$, phase trajectories of particles near separatrix possess erratic motion. This region (so called stochastic layer) has thickness $\Delta J \sim \varepsilon$. We wee that our Hamiltonian (10) is formally the same as (36). Consequently, the similar ergodic layer appears also in the neighbourhood of the separatrix (19). Particles can therefore penetrate outside of the separatrix, but, as well as in [2], they can change their energy only by small parameter $\Delta J \sim \omega_t^{-1} \cdot eV_0 J_2(k_\perp \varrho)$ (supposing $J_1 \gg J_2$). Supposing strong convergence of the Fouriere expansion ($k_\perp \varrho \ll 1$), the effect of higher harmonics appears only in the existence of a thin ergodic layer around separatrix (19). The same effect appears, if we (beside harmonic expansion (10)) suppose further small amplitude wave, interacting with particles together with effective potential of corresponding Hamiltonian (16). The existence of ergodic layer suggests that also for $\varepsilon \sim 1$ in (36) the similar behaviour as for $\varepsilon \ll 1$ can be expected. Nevertheless, such problem can be solved only numerically. SMITH and KAUFMAN [8] used the same Hamiltonian (2), as we have supposed in Chapt. 3. They discussed conditions, under which the detrapping of particles from the region of separatrix (19) into the region of further harmonics of expansion (10) can be expected. This condition is therefore just disjunctive to the conditions, under which our model in Chapt. 3 is working. Since the possibility of detrapping by means of the harmonics has appeared to be important also for us, also we have done some estimations (only in a rough way, nevertheless).

We have solved the similar problem as in [8] for concrete region of our parameters in Chapt. 3. For simplicity, we have not used numerical procedure as in [8], but only following analytic estimation. We have supposed that the detrapping sure takes place (at least in some measure) if separatrixes of two neighbouring components of Fourier expansion (e.g. these, which correspond to $J_1(k_\perp \varrho)$ and $J_2(k_\perp \varrho)$) just touch. It gives amplitudes of intensity $E_t = eV_0 k_\parallel$ inside interval $E_{th} < E_t < E_{gr}$, where E_{th} and E_{gr} are intensities, at which the threshold of detrapping appears and intensity, by which particles, occupying gross of phase space inside trapping region are subjected to exodus outside separatrix, respectively.

E_{th} is in the case of touching of two separatrixes given by equation

$$\omega_c = \sqrt{2}\,\omega_t \left(1+\sqrt{\frac{E_{t2}}{E_{th}}}\right) - \sqrt{2}\left(\frac{ek_\parallel E_{th}}{m}\right)^{\frac{1}{2}} \cdot J_1^{\frac{1}{2}}(k_\perp \varrho)\left\{1+\left[\frac{J_2(k_\perp \varrho)}{J_1(k_\perp \varrho)}\right]\right\}. \tag{37}$$

Therefore, $E_{th} < E_{gr}$, $E_{th} \sim (2 \div 8)\, E_{t2}$, if we take results of SMITH and KAUFMAN [8], where E_{t_2} is the amplitude of the nearest harmonic and where

$$\omega_{th} = \left(\frac{ek_\parallel E_{th}}{m}\right)^{\frac{1}{2}} \sim \frac{\omega_c}{4} \qquad ; \qquad \omega_{gr} \sim 0{,}86\,\omega_{ce} \,. \tag{38}$$

intensity E_{th}, defined by equation (37) and using parameters of Chapt. 3 (i.e. $\omega_c \sim 6{,}7 \cdot 10^{9}\,sec^{-1}$, $\lambda \sim 10^{-2}m$, $k_\perp/k_\parallel \sim 0{,}37$) as a function of $kT_{e\perp}$ is presented on Fig. 2. Due to commarision with results (38) from [8], E_{t_1} is near to values for gross detrapping. Desired intensities are large, larger than in current experiments, wehre usually $E \sim (10^4 \div 10^5)\,Vm^{-1}$ and $kT_e \lesssim 1\,keV$. One can therefore expect for temperatures $\sim 10^2 \div 10^3\,eV$ and for $E \lesssim 10^5\,Vm^{-1}$ only weak effect of detrapping. More interesting and perhaps more efficient is the ion region, mentioned in [8], or effects for experiments with relativistic beams, where larger intensities are expected.

Basic reason for such large intensities, desired for detrapping consists in great frequency difference $\Delta\omega \sim \omega_c$ between subsequent harmonics. If one could apply mechanism, which brings further oscillations of particles with characteristic frequency $\Omega < \omega_c$, the large interval $\Delta\omega \sim \omega_c$ could be decomposed into a train of smaller ones. Here probably smaller intensities for detrapping between subsequent Ω will be desired. The important modulation of velocity of particles takes place in mirror systems. Consequently, it will be very interesting to discuss both trapping effects (trapping in mirrors and trapping in wave) simultaneously. Using results of SMITH and KAUFMAN [8], the model of stochastic instabilities of particles in mirror system under influence of rf field [9] can be generalized.

In some experiments, transient change of phase velocity is observed (e.g. in nonlinear stage of beam-plasma interaction). If after transient decrease of phase velocity an increase follows, a greater amount of trapped particles can be accelerated (energetically on the expense of wave energy).

5. CONCLUSION

The paper discusses two different mechanisms of the interaction of finite amplitude wave with particles in magnetostatic field in one-particle approximation. First we have generalized the effect, discussed by DAWSON and SHANNY [1] in one-dimensional model. We have established that also in two dimensional case similar effect of enhanced absorption due to finite trapping region could exist. Concerning cited experiment [4], its interaction is deeply in linear region. Sharp increase of R can be expected generally for larger intensities and temperatures and cases with $\omega/\omega_c \sim 1$. Further we have discussed the possibility of detrapping of particles trapped in effective potential trough proportional to the $J_1(k_\perp \rho)$ component of Fouriere expansion by means of effective potential of neighbouring $J_2(k_\perp \rho)$ component (this problem already appeared as a letter of SMITH and KAUFMAN [8]. We have estimated this possibility for parameters, near to experiment [4] and we have established that desired amplitude is too large (even if we take less severe requierements of [8]); it seems that this is valid also for current experiments in wave-plasma or beam-plasma interaction. More acceptable conditions appear in ion-region (as has been mentioned in [8]). Some further possibilities could appear if further velocity modulation of particles can be inserted. This takes place e.g. in magnetic mirror systems, where the simultaneous trapping effect due to mirror system and due to finite amplitude wave can bring some new interesting results. Shortly we have mentioned the effect of trapping in wave with transient decrease and increase of its phase velocity.

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6. REFERENCES

[1] Dawson J.M., Shanny R.: Phys. Fluids 11 (1968), 1506.

[2] Zaslavskij G.M.: Statistical Irreversibility in Nonlinear Systems (in Russian), Nauka, Moscow 1970.

[3] Progress Report of Institute of Plasma Physics 1974; IPPCZ-185

[4] Piffl V., Šunka P., Ullschmied J., Jungwirth K., Krlín L.: Proceedings of the Fourth Conference of Plasma Physics, 1971, Madison, USA, No 11.

[5] Krlín L.: Czech J. Phys. 17B (1967), 112.

[6] Voprosy teorii plasmy (in Russian), Gosatomizdat 1963, Vol. II, p. 242.

[7] Walker G.H., Ford J.: Phys. Rev. 188 (1969), 416.

[8] Smith G.R., Kaufman N.: Phys. Rev. Letters 34 (1975), 1615.

[9] Lichtenberg A.J., Jaeger F.: Phys. Fluids 13 (1970), 392.

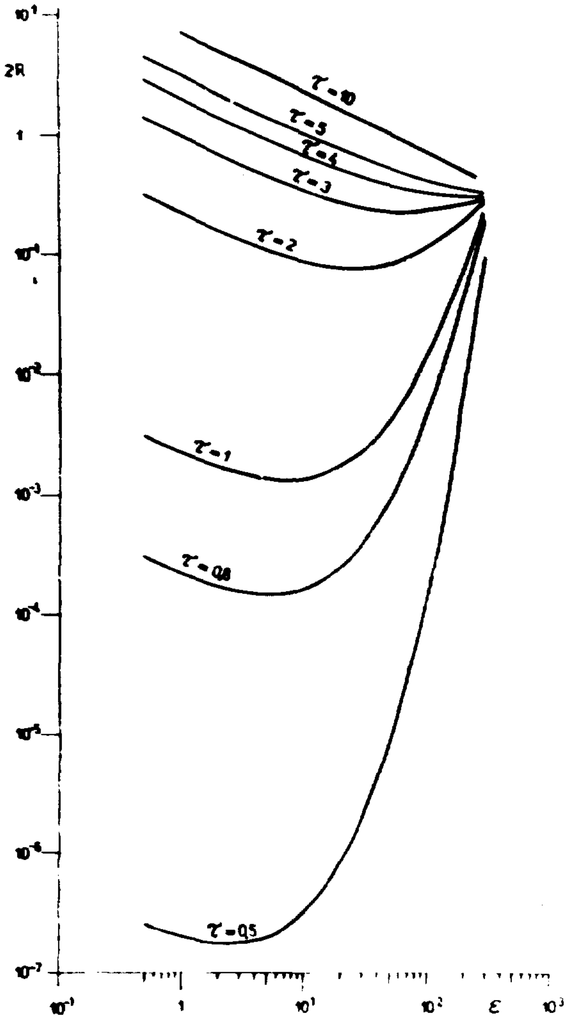

Fig. 1 Coefficient of absorption P as a function of normalized intensity ε (normalized temperature τ is parameter).

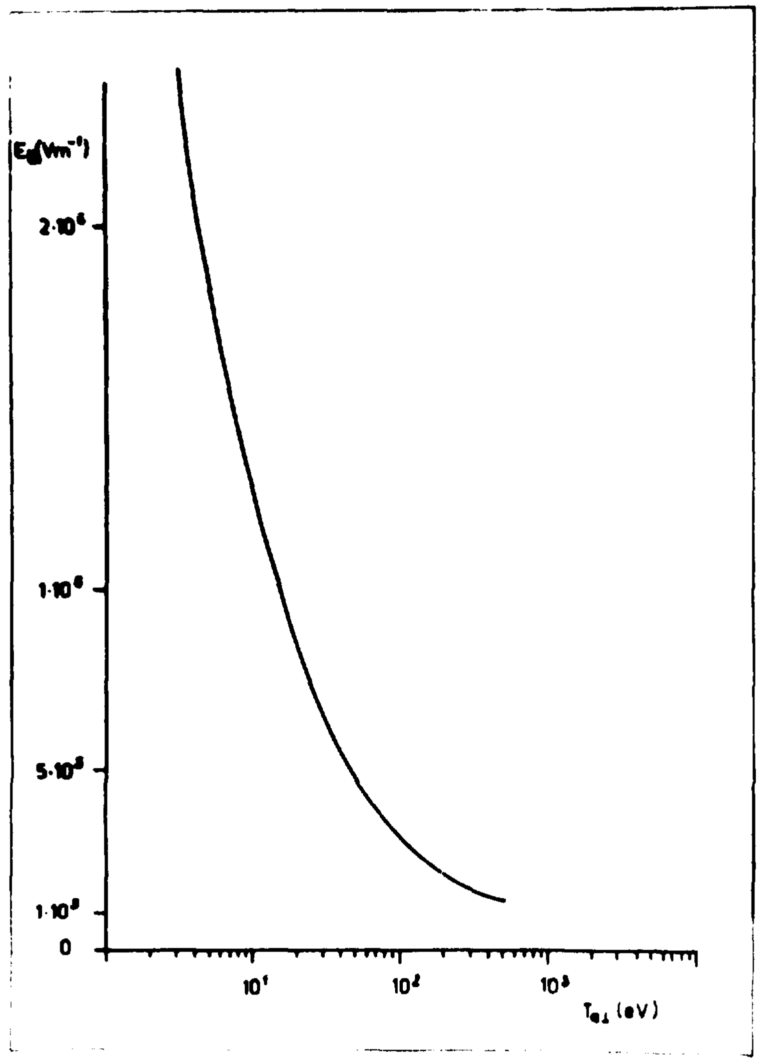

Fig. 2 Threshold intensity E_{th} as a function of perpendicular temperature $T_{e\perp}$.