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CAN THE EFFECTIVE INTERACTION BE LOCAL IN INCLUSI'VE PROCESS?

Serpukhov 1974

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CAN THE EFFECTIVE INTERACTION BE LOCAL IN INCLUSIVE PROCESS?

Аннотация

Логунов А.А., Мествиришвили М.А., Петров В.А.

Может ли быть локальным эффективное взаимодействие адронов в инклюзивном процессе. Серпухов, 1974.

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В работе показано, что инклюзивные процессы в асимптотической области могут быть описаны структурными функциями, зависящими лишь от переменных ν и $q^2 = (p_a - p_c)^2$. Поскольку эти динамические характеристики не зависят от суммарного импульса $(p_a + p\ )$, эффективное взаимодействие адронов "a" и "b" является как бы локальным.

Abstract

Logunov A.A., Mestvirishvily M.A., Petrov V.A.

Can the Effective Interaction be Local in Inclusive Process? Serpukhov, 1974.

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Ref. 3.

The behaviour of the inclusive spectrum $f_{ab\to c}$ in the asymptotic region ~~$2p_b(p_a - p_c) = \nu \gg m_a^2, m_b^2, m_c^2$; $\frac{q^2}{\nu} = \frac{(p_a-p_c)^2}{\nu}$ fixed,~~ $\frac{2p_b(p_a+p_c)}{\nu} = y \gg 1$ is discussed.

On the basis of the Jost-Lehmann-Dyson representation it is shown that inclusive processes are described by some structure functions, depending only on ν, q^2, under certain restrictions on the J-L-D spectral functions. As these dynamical characteristics (structure functions) do not depend on the sum$(p_a + p_c)$, the effective interaction of hadrons "a" and "c" is as if local.

$(\nu = 2p_b(p_a - p_c),\ q^2 = (p_a - p_c)^2)$

Introduction

Deep inelastic processes

$$e + p \to e + X, \qquad \nu + p \to \mu + X, \qquad (I)$$

where X contains hadrons only, are described by structure functions $F(\nu, q^2)$ dependent on leptonic energy loss and momentum transfer. The structure functions of these processes do not depend on leptonic summed momentum as leptonic interaction lagrangian is taken to be local and the interaction is considered in the lowest order in weak (or electromagnetic) coupling. This circumstance leads to the fact that in the diagrams illustrating processes (1) leptonic lines enter and emerge from one point. It reflects the locality of the interaction. When hadrons interact, even if original strong interaction lagrangian is of local character, locality vanishes and distribution function that describes an inclusive process

$$a + b \to c + X, \qquad (II)$$

depends now not only on energy ν and momentum transfer $(p_a-p_c)^2$, but on the sum (p_a-p_c) as well due to higher approximations in coupling constant that we cannot neglect.

Thus effective interaction of "a" and "c" hadrons is not local any longer due to particle structure. The distribution function of inclusive process (II) $f(\nu,(p_a-p_c)^2, p_b(p_a-p_c))$ depends on three variables. Therefore description of deep inelastic (I) and hadronic (II) inclusive processes seems to differ greatly.

In the present work we will show that in the asymptotic domain when imposing certain requirements on the spectral functions we will gain a possibility to describe hadronic inclusive processes with structure functions dependent on the variables $\nu = 2p_b(p_a-p_c)$ and $q^2 = (p_a-p_c)^2$ only. Distribution function of inclusive process (II) may be expressed via these structure functions. As the structure functions of inclusive process do not depend on the summed momentum (p_a+p_c) then effective interaction of "a" and "c" hadrons is as if it were local. We come to an important physical conclusion that hadronic interaction in inclusive process may be described (in certain asymptotic domain) with a structure function like as in deep inelastic processes. Thus the structure functions seem to be universal characteristics, that describe processes of different kind. We cannot say in what way the structure functions of processes (I) and (II) differ. It is quite possible that structure functions of processes (I) and (II) have some common properties. In any case this statement needs experimental study.

To obtain the aforementioned results we will use Jost-Lehmann-Dyson representation. In the case of inclusive processes this representation was used in /1/ for the first time. An important step in studying deep inelastic processes on the basis of general principles of the quantum field theory was made in /2/ (see also /3/).

I. Representation for Distribution Function of Inclusive Process.

A distribution function for inclusive process (II) is introduced in the following way:

$$f_{ab\to c}(p_a, p_b; p_c) = \sum_{n_c} n_c \frac{d\sigma_{ab\to c}}{d^3 p_c}(s; p_c; n_c). \tag{1}$$

Here

$$\int \frac{d\sigma_{ab\to c}}{d^3 p_c} d^3 p_c = \sigma_{ab\to c}(s, n_c), \quad d^3 p_c = \frac{d\vec{p}_c}{(2\pi)^3 2E_c}, \tag{2}$$

where $\sigma_{ab\to c}(s, n_c)$ is the total cross section for n_c yield of "c" particles.

The distribution function satisfies the normalization condition:

$$\int f_{ab\to c}(p_a, p_b; p_c)\, d^3p_c = \langle n_c\rangle(s)\,\sigma_{ab\to c}(s), \qquad (3)$$

$\sigma_{ab\to c}(s)$ is the total cross section for the yeild of "c" particle:

$$\sigma_{ab\to c}(s) = \sum_{n_c \geq 1} \sigma_{ab\to c}(s, n_c). \qquad (4)$$

If we introduce the local current

$$\Lambda_c(x) = i\,\frac{\delta S^+}{\delta \phi_c(x)}\, S = -\,i\,S^+ \frac{\delta S}{\delta \phi_c(x)}, \qquad (5)$$

then the distribution function of an inclusive process can be presented in the form

$$f_{ab\to c}(p_a, p_b; p_c) =$$

$$= \frac{2\,E_a E_b}{J} \int dx\, e^{-ip_c x} \langle p_a, p_b | \Lambda_c(x)\Lambda_c(0) | p_a p_b\rangle. \qquad (6)$$

Here

$$J = \sqrt{(p_a p_b)^2 - m_a^2 m_b^2}\ .$$

It is easy to see that

$$\int dx e^{-ip_c x} \langle p_a, p_b | \Lambda_c(x) | n\rangle = \qquad (7)$$

$$= \frac{(2\pi)^3\,\delta(p_a + p_b - p_c - \sum_{i=1}^{n} p_i)}{\sqrt{(2\pi)^3\, 2E_a}} \int dt e^{-i\frac{p_i + p_c}{2} t} \langle p_b | \frac{\delta \Lambda_c(\frac{t}{2})}{\delta \phi_a(-\frac{t}{2})} | n\rangle.$$

Taking into account (7) and making expansion in the complete set of physical states, we will write down the distribution function in the form

$$f_{ab\to c}(p_a, p_b, p_c) = \frac{E_b}{J} \sum_n (2\pi)\delta(p_a + p_b - p_c - \sum_1^n p_i) \times$$

$$\times \int dt d\tau\, e^{-iQ(t-\tau)} \langle p_b | \frac{\delta \Lambda_c(\frac{t}{2})}{\delta\phi_a(-\frac{t}{2})} | p_1 \dots p_n \rangle \langle p_1 \dots p_n | \frac{\delta \Lambda_c(\frac{\tau}{2})}{\delta \phi_a(-\frac{\tau}{2})} | p_b \rangle, \quad (8)$$

$$Q = \frac{p_a + p_c}{2} \quad .$$

Hereafter summing in n means both summing in particle quantum numbers and integration in momenta $p_1, \dots, p_n$.

Using the J-L-D representation we will have:

$$T_n(Q, p_b, p_1, \dots, p_n) = \int dt\, e^{-iQt} \langle p_b | \frac{\delta \Lambda_c(\frac{t}{2})}{\delta \phi_a(-\frac{t}{2})} | p_1 \dots p_n \rangle = \quad (9)$$

$$= \frac{1}{\pi} \int du \int d\varkappa^2 \frac{\Phi_n(u, \varkappa^2, p_b; p_1 \dots p_n)}{(Q-u)^2 - \varkappa^2 + i\varepsilon} + \sum_{r=0}^{m-1} Q_{\mu_1} \dots Q_{\mu_r} C^{\mu_1 \dots \mu_r}(p_b, p_1 \dots p_n).$$

The spectral function Φ_n is concentrated in the domain

$$(E_b + \frac{q_0}{2} \pm u_0) \geqslant 0, \qquad (p_b + \frac{q}{2} \pm u)^2 \geqslant 0$$

$$\varkappa \geqslant \max \{ 0, M_1 - \sqrt{(p_b + \frac{q}{2} + u)^2}, \; M_2 - \sqrt{(p_b + \frac{q}{2} - u)^2} \}, \quad (10)$$

$q = p_a - p_c$, $M_{1,2}$ are some masses.

Choosing the coordinate system

$$\vec{p}_b + \frac{1}{2}\vec{q} = 0, \tag{12}$$

and having directed the vector $\vec{p}_b$ along z-axis, after change of the variables

$$u = (E_b + \frac{1}{2} q_o) v,$$

$$\varkappa = (E_b + \frac{1}{2} q_o) \lambda, \tag{13}$$

representation (9) with necessary number of subtractions can be presented in the form:

$$T_n(Q, p_b, p_1, \dots, p_n) = \frac{1}{\pi} \int dv \int d\lambda^2 \frac{\Psi_n(v, \lambda^2, p_b, p_1, \dots, p_n)}{\left(\frac{Q}{E_b + \frac{1}{2} q_o} - v\right)^2 + \lambda^2 + i\epsilon} \times$$

$$\times \left[\theta(\Lambda^2 - \lambda^2) + \theta(\lambda^2 - \Lambda^2) \frac{\left(\frac{Q}{E_b + 1/2\, q_o} - v\right)^{2m}}{\lambda^{2m}} \right] + \tag{14}$$

$$+ \sum_{r=0}^{m-1} Q_{\mu_1} \dots Q_{\mu_r} (E_b + \frac{1}{2} q_o)^{-r} k_r^{\mu_1 \dots \mu_r}(p_b; p_1, \dots, p_n).$$

It should be noted that Q does not go into the spectral function Ψ_n. The spectral function Ψ_n is concentrated in the domain

$$(1 \pm v_o) \geq 0, \quad (1 \pm v_o^2) - \vec{v}^2 \geq 0$$

$$\lambda \geq \max\{0, \frac{M_1}{E_b + \frac{1}{2} q_o} - \sqrt{(1 + v_o)^2 - \vec{v}^2}, \frac{M_2}{E_b + \frac{1}{2} q_o} - \sqrt{(1 - v_o)^2 - \vec{v}^2}\}. \tag{15}$$

As is obvious from (15) the integration domain is limited in variable v.

Let us introduce invariant variables

$$\nu = 2 p_b (p_a - p_c), \qquad \xi = \frac{(p_a - p_c)^2}{\nu}, \qquad \gamma = \frac{2 p_b (p_a + p_c)}{\nu}. \qquad (16)$$

Then (14) will take the form

$$T_n(\gamma, \nu, \xi; \ell_n) = \frac{1}{\pi} \int dv \int d\lambda^2 \frac{\Psi_n(v, \lambda^2, \nu, \xi; \ell_n)}{R(\rho, v, \xi) - \lambda^2 + i\epsilon} \times$$
$$\times [\theta(\Lambda^2 - \lambda^2) + \theta(\lambda^2 - \Lambda^2)\frac{R^m}{\lambda^{2m}}] + \sum_{r=0}^{m-1} \gamma^r g_r(\nu, \xi; \ell_n). \qquad (17)$$

Here ℓ_n denotes the set of the remaining variables,

$$R(\rho, v, \xi) = -2\rho v_0 + v_0^2 - \vec{v}^2 + 2|\vec{v}|\sqrt{\rho^2 - \omega}\,\phi + \omega. \qquad (18)$$

In the region where $\nu \gg m_a^2, m_b^2, m_c^2, \gamma \gg 1$,

$$\rho = \frac{\gamma}{2+\xi}, \qquad \omega = -\frac{\xi}{2+\xi},$$
$$\phi = -(1+\xi)\cos\alpha + \sqrt{-\xi(\xi+2)}\sin\alpha\cos\beta. \qquad (19)$$

Here α and β are polar angles of $\vec{v}$.

Attention should be paid to the fact that the variable ξ is bounded with inequality

$$-1 \leqslant \xi \leqslant 0. \qquad (20)$$

On the basis of (8) the distribution function of inclusive process is expressed via the function $T_n(\gamma, \nu, \xi, \ell_n)$ as

$$f_{ab\to c}(\gamma,\nu,\xi)=$$

$$=\frac{2\pi}{\sqrt{2\gamma\nu}}\sum_n \delta(p_b+q-\sum_{i=1}^{n}p_i)\,|\,T_n(\gamma,\nu,\xi;\ell_n)|^2\,, \tag{21}$$

or in detail

$$f_{ab\to c}(\gamma,\nu,\xi)=\frac{2\pi}{\sqrt{2\gamma\nu}}\,|\,\frac{1}{\pi^2}\int dv_1\,dv_2\int d\lambda_1^2\,d\lambda_2^2\,\times$$

$$\times\,\frac{\Psi(v_1,v_2,\lambda_1^2,\lambda_2^2;\nu,\xi)\prod_{i=1}^{2}[\theta(\Lambda_i^2-\lambda_i^2)+\theta(\lambda_i^2-\Lambda_i^2)\frac{R_i^m}{\lambda_i^{2m}}]}{(R_1-\lambda_1^2+i\epsilon)(R_2-\lambda_2^2-i\epsilon)}\,+$$

$$+\sum_{r=0}^{m-1}\gamma^r G_r(\gamma,\nu,\xi)+\sum_{r=0}^{2(m-1)}\gamma^r D_r(\nu,\xi)\,. \tag{22}$$

The following designations have been introduced:

$$\Psi(v_1,v_2,\lambda_1^2,\lambda_2^2;\nu,\xi)=$$

$$=\sum_n \Psi_n(v_1,\lambda_1^2,\nu,\xi;\ell_n)\,\Psi_n^*(v_2,\lambda_2^2,\nu,\xi;\ell_n)\delta(p_b+q-\sum_{i=1}^{n}p_i),$$

$$G_r(\gamma,\nu,\xi)=\frac{2}{\pi}\,\mathrm{Re}\sum_n\delta(p_b+q-\sum_{i=1}^{n}p_i)\,g_r^*(\nu,\xi;\ell_n)\,\times$$

$$\times\int dv\int d\lambda^2\,\frac{\Psi_n(v,\lambda^2,\nu,\xi;\ell_n)}{R-\lambda^2+i\epsilon}[\,\theta(\Lambda^2-\lambda^2)+\theta(\lambda^2-\Lambda^2)\frac{R^m}{\lambda^{2m}}\,], \tag{23}$$

$$D_r(\nu,\xi)=\sum_{j=0}^{r}\sum_n\delta(p_b+q-\sum_{i=1}^{n}p_i)\,g_{r-j}(\nu,\xi;\ell_n)\,g_j^*(\nu,\xi;\ell_n)\,.$$

Integral representation (22) for the distribution function arises from causality and completness of the set of physical states. It especially should be underlined that one of the independent variables γ (or ρ) is found in the integrand in its explicit form, expressed through the function R. Thus, the unknown spectral functions in (22) are independent of the variable γ. This fact is of importance for the following study of the distribution function behaviour in the region of large values for γ.

2. Structure Functions for Inclusive Process.

Integral representation (22) may be used to study asymptotic expansion in γ if certain conditions for the functions Ψ and G_r be formulated. For simplicity we will study asymptotic expansion of the function $T(\gamma, \nu, \xi; \ell_n)$ and then we will insert the obtained expansion into expression (21) for distribution function $f_{ab\to c}(\gamma, \nu, \xi)$.

Let us consider asymptotic behaviour in the case of large γ for the integral

$$J(\gamma) = \int dv \int_{\Lambda^2}^{\infty} d\lambda^2 \; \frac{\Psi_n(v, \lambda^2, \nu, \xi; \ell_n)}{R(\rho, v, \xi) - \lambda^2 + i\epsilon} \cdot \frac{R^m(\rho, v, \xi)}{\lambda^{2m}} . \qquad (24)$$

It should be noted that in the domain of large γ the function $R(\rho, v, \xi)$ has the form:

$$R(\rho, v, \xi) = -2\rho(v_o - |\vec{v}|\phi) + v_o^2 - \vec{v}^2 + \omega(\xi). \qquad (25)$$

Following ref. /2,3/, we will assume, that spectral functions are polynomially bounded in λ^2 and have the following properties

$$\lim_{\lambda^2 \to \infty} \frac{\Psi_n(v, \lambda^2, \nu, \xi; \ell_n)}{\lambda^{2\ell}} = \Psi_n^o(v, \nu, \xi; \ell_n). \qquad (26)$$

Let the function $\Psi_n^o(v, \nu, \xi; \ell_n)$ have no singularities in v on the surface defined by the equation

$$y(v, \xi) = v_o - |\vec{v}|\,\phi(v, \xi) = 0. \qquad (27)$$

Then it is easy to see that in the asymptotic region in γ the integral $J(\gamma)$ has the form

$$J(\gamma) = -\frac{\pi}{\sin \pi k}\,\gamma^k\,\Phi_n(\nu, \xi; \ell_n), \quad m-1 < k < m, \qquad (28)$$

where

$$\Phi_n(\nu, \xi; \ell_n) = -\left(\frac{2}{2+\xi}\right)^k \int dv\,\Psi_n^0(v, \nu, \xi; \ell_n) \times$$

$$\times [v_o - |\vec{v}|\,\phi(v, \xi) - i\epsilon]^k. \qquad (29)$$

When k is an integer number equal to $m-1$, the asymptotic expansion of the integral $J(\gamma)$ has some different form:

$$J(\gamma) = (-\gamma)^k \ln\gamma\,\Phi_n(\nu, \xi, \ell_n). \qquad (30)$$

If the spectral function Ψ_n has no singularities on the surface $y(\nu,\xi) = 0$ then the integral in the finite domain of integration in the variable λ^2 will be a decreasing function of γ.

Thus, the asymptotic behaviour of the function $T_n(\gamma,\nu,\xi;\ell_n)$ in γ is determined by the asymptotics of integral $J(\gamma)$. Having inserted asymptotic expansion for T_n into (21) we will obtain

$$f_{ab\to c}(\gamma,\nu,\xi) \sim \gamma^{2k-\frac{1}{2}} W(\nu,\xi), \quad m-1<k<m, \tag{31}$$

where

$$W(\nu,\xi) = \frac{\pi^2}{\sin^2\pi_k} \sum_n \delta(p_b + q - \sum_{i=1}^{n} p_i)\,|\Phi_n(\nu,\xi;\ell_n)|^2 \frac{1}{\sqrt{\nu}}, \tag{32}$$

and for integer k

$$f_{ab\to c}(\gamma,\nu,\xi) \sim \gamma^{2k-\frac{1}{2}} \ln^2\gamma \; W(\nu,\xi). \tag{33}$$

From expressions (31) and (33) we see that for large values of γ for inclusive process (II) we can introduce its own structure functions dependent on variables ν, ξ only.It means that description of hadronic processes in the asymptotic region in γ reminds that of deep inelastic processes.Thus the structure functions may be universal characteristics.As for the problem on the relatioship of structure functions for different processes, it remains open.

In the lab system $\vec{p}_b = 0$ the variables γ, ν, ξ have the form

$$\gamma = \frac{E_a + E_c}{E_a - E_c}, \quad \nu = 2m_b(E_a - E_c), \quad \xi = \frac{q^2}{\nu}. \tag{34}$$

If we consider process (II) when energy of "c" particle is equal to $E_c = E_a - E_a^{\alpha}$, where $0<\alpha<1$, in this system at large energies E_a of the incident particle, then we will find ourselves in the region, that is asymptotic both in the variables γ and ν.

Attention should be paid to the fact that if on the surface $y_i(\nu, \xi) = 0$ spectral function in (22) had singularities of the form

$$y_i^{-\frac{1}{2}\sigma(\nu,\xi)}, \tag{35}$$

here σ, generally speaking, may be dependent on ν and ξ then these singularities would lead to arising of additional asymptotic terms of the kind

$$\gamma^{\sigma(\nu,\xi)} F(\nu, \xi). \tag{36}$$

In the case these terms were main in the asymptotic region in γ, we should have

$$f_{ab\to c}(\gamma, \nu, \xi) \sim \gamma^{\sigma(\nu,\xi)-\frac{1}{2}} F(\nu, \xi), \tag{37}$$

i.e. at large γ inclusive processes are still described with structure functions dependent on ν and ξ only.

It worth noticing that if for the distribution function of inclusive process (II) at large values of ν, factorization in ν took place as it seems to be observed for structure functions of deep inelastic processes, then distribution function in the c.m.s. would be of the form

$$f_{ab\to c}(m_{c\perp}, x, z) = m_{c\perp}^{a}\Phi(x, z), \qquad (38)$$

where $x = \dfrac{2\,p_{c\parallel}}{\sqrt{s}}$, $z = \dfrac{2\,m_{c\perp}}{\sqrt{s}}$, $m^2_{c\perp} = p^2_{c\perp} + m_c^2$.

p_C is the momentum of "c" particle in the c.m.s.

It should be underlined that here as well as in/2,3/ we assumed the spectral function should satisfy (26). However it is unknown how much this condition holds in theory. Therefore experimental study of possibility to introduce structure functions dependent on ν and q^2 for inclusive processes (II) seems to be extremely important.

The results obtaind here may be found probably if one considers the light cone singularities.

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