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FRACTURE MECHANICS APPROACHES FOR ASSESSMENT OF
HTGR STEAM GENERATOR TUBE WELD INTEGRITY

T. TANAKA*, P. BEZLER, AND M. SUBUDHI

* ON LEAVE FROM JAPAN ATOMIC ENERGY RESEARCH INSTITUTE.

DATE PUBLISHED - AUGUST, 1977
DEPARTMENT OF APPLIED SCIENCE BROOKHAVEN NATIONAL LABORATORY
UPTON, NEW YORK 11973



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by

T. Tanaka*, P. Bezler, and M. Subudhi

Structures Analysis Group

Department of Applied Science
Brookhaven National Laboratory
Upton, NY 11973

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July, 1977

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* On leave from Japan Atomic Energy Research Institute.

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Abstract

HTGR steam generator tube welds with circumferential cracks in their highest stress region were assessed by linear elastic fracture mechanics methods. The computed stress intensity factors were below the threshold level for propagation and crack propagation will not occur for the pipe geometry (radius and thickness of tube wall) and the stress levels considered. In addition crack arrest may be expected if the predominant stress field is a thermally induced bending stress field.

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1. Introduction

The justification for applying fracture mechanics approaches to the evaluation of tube weldments derives from the hypothesis that cracks, flaws, and defects may exist in the steam generator tube welds. The approach is then directed toward supplying quantitative information on questions such as:

1. What are the critical crack sizes (i.e., sizes required to cause failure) in the various portions of steam generator tube welds at the expected test and/or operational stress levels?
2. Will cracks initially present, but below critical size, grow to critical size and cause failure during the expected service life of the steam generator?
3. If a critical crack size does exist, will the resulting failure be relatively small in extent, causing a major break in the steam generator?

It may be noted that these three questions involve the three aspects of fracture mechanics analyses:

1. Crack-growth initiation.
2. Crack propagation.
3. Crack propagation arrest.

Small flaws or cracks may grow when subjected to cyclic loads (fatigue), aggressive environments (corrosion fatigue and stress corrosion), or a combination of all these factors. A crack in a tube wall will propagate perpendicularly to the main stress axis. Thus, a crack will advance through the wall thickness at a rate faster than it elongates in the longitudinal or circumferential directions. Having extended through the wall, the crack produces either a leak or a rupture, depending on whether the crack has reached a critical length. A subcritical crack producing leakage may grow longitudinally causing a bulging failure.

To assess the integrity of the HTGR steam generator tube welds, linear elastic fracture mechanics methods were used to evaluate the characteristics of circumferential cracks in tubes with representative dimensions supporting operating loads. In these analyses, the materials were assumed to retain their virgin material mechanical properties. As a refinement of this procedure, the change in the mechanical properties of these materials due to the long-term exposure to elevated temperatures must be made.

Several analysis techniques may be used to solve problems of stress and strain singularities at crack tips. With the finite element method, the singularities may be characterized in the following ways. (1)

1. Direct Method - Conventional elements are used defining the region near the singularity with an extremely fine grid. This is obviously a very costly and inefficient procedure.

2. Energy Release Method - Conventional elements are used defining the singular region with a relatively fine grid. The energy of the system is then monitored as the crack length grows (i.e., several static solutions with increasing crack length are run). This method proves to be more accurate than that shown in Ref. (1) and requires significantly fewer elements, but it remains awkward and time consuming and requires the solution to several similar problems. One drawback of this method is the inability to distinguish between Mode I and Mode II intensities.

3. Superposition Method - Conventional elements with coarse grids are used in conjunction with a classical solution. The method is somewhat cumbersome.

4. Singularity Function Formulations - A special element is formulated that contains the proper singularity. This procedure requires the formulation

of a complicated element stiffness matrix but can be coupled easily to conventional elements. Coarse meshing gives very accurate results.

The present analyses were conducted with the APES⁽²⁾ finite element computer program. This code uses a singularity function formulation to model elastic bodies with singular points. The latest version of APES contains subroutines that calculate the state of strain at the tip of a crack in either plane stress, plane-strain, or axisymmetric geometries. Linear isotropic stress-strain material properties are used and small strain theory is assumed.

2. Weld Geometry

The structural evaluations were performed using the standard weld joint geometry shown in Figure 1a. Various combinations of base metal and weld filler metal were considered. The material combinations and the tube dimensions treated are summarized in Table 1.

Table 1

	<u>Tube Dimensions</u>		
	<u>Material</u>	<u>Inner Radius</u>	<u>Thickness</u>
Superheater Tube	Incoloy 800 (Inconel 82)	0.3625"	0.2"
Superheater Tube	Incoloy 800 + 2 1/4 Cr-Mo (Inconel 82)	0.3625"	0.2"
Reheater Tube	Incoloy 800 (Inconel 82)	0.6725"	0.14"

3. Material Data

The material properties employed in the study for both base and filler metals are shown in Table 2. (3) No attempt was made to account for base metal dilution or its effect upon the properties.

Table 2

Material Data

	<u>Temp.</u>	<u>Incoloy 800</u>	<u>Inconel 82</u>	<u>2 1/4 Cr.Mo</u>
Modulus of Elasticity (psi)	900°F	23.9 x 10 ⁶	27.85 x 10 ⁶	26.05 x 10 ⁶
	1030°F	23.4 x 10 ⁶	27.1 x 10 ⁶	---
Poisson's Ratio	900°F	0.365	0.3075	0.289
	1030°F	0.367	0.314	---
Thermal Coefficient of Expansion (°F ⁻¹)	900°F	9.3 x 10 ⁻⁶	8.25 x 10 ⁻⁶	7.65 x 10 ⁻⁶
	1030°F	9.4 x 10 ⁻⁶	8.4 x 10 ⁻⁶	---

4. Loading Conditions

The tubes were evaluated for the expected normal conditions of operation, both thermal and pressure. In addition, as regard to thermal stresses, the initial stress free state was chosen to correspond to both 1350°F and 70°F. The lower temperature corresponds to analyses wherein welding operation induced residual stress effects are ignored while the higher temperature allows some accounting of these effects.

Table 3

	<u>Loading Conditions</u>			<u>Radial Gradient</u>
	<u>He Side Pressure</u>	<u>Steam Side Pressure</u>	<u>Tube Temp.</u>	
Superheater Tube	710 psi	2,600 psi	900°F	50°F
Superheater Tube (Bimetallic)	710 psi	2,600 psi	860°F	45°F
Reheater Tube	725 psi	640 psi	955°F	85°F

5. Computer Program "APES" (2)

APES is an acronym for "Axisymmetric/Planar Elastic Structures". It is a finite element computer program which incorporates a 12-node quadrilateral isoparametric element having a bicubic displacement assumption and adapted to plane strain, plane stress and axisymmetric conditions of structural behavior. For linear elastic fracture mechanics applications, two different special elements are employed for predicting stress intensity factors: (1) a small circular "core" element which surrounds the crack tip and which reproduces the singular nature of the stresses there, and (2) "enriched" 12-node isoparametric elements which have the elastic singular solution superimposed so that a corner node corresponds to a crack tip. The formulations of these elements are outlined briefly here to indicate the finite element approximations involved in the use of the APES computer program.

1. Conventional Quad-12 Element

The Quad-12 element and its local, nonorthogonal (in general) coordinate system (s,t) are shown in Figure 2. The edges of the element correspond to values of s or t of ± 1 , and the midside nodes correspond to values of s or t of $\pm 1/3$.

The displacement assumptions for the QUAD-12 element are given by

$$\begin{aligned}
 U &= \sum_{i=1}^{12} N_i (s,t) U_i \\
 V &= \sum_{i=1}^{12} N_i (s,t) V_i
 \end{aligned}
 \tag{1}$$

where U and $V = x$ and y components of displacement U_i and $V_i =$ the displacement components at node i .

$N_i (s,t)$ = polynomials which interpolate the displacement over the element.

The element is made to be isoparametric (same parameters) by letting the geometry vary in a fashion as the displacements

$$\begin{aligned}
 x &= \sum_{i=1}^{12} N_i (s,t) x_i \\
 y &= \sum_{i=1}^{12} N_i (s,t) y_i
 \end{aligned}
 \tag{2}$$

where x and y define positions within the element and x_i and y_i are the coordinates of Node i .

The specific N_i for the QUAD-12 element are given by

$$N_1 = \frac{1}{32} (1-t) (1-s) [-10 + 9 (s^2 + t^2)]$$

$$N_2 = \frac{9}{32} (1-t) (1-s^2) (1-3s)$$

$$N_3 = \frac{9}{32} (1-t) (1-s^2) (1+3s)$$

$$N_4 = \frac{1}{32} (1-t) (1+s) [-10 + 9 (s^2 + t^2)]$$

$$\begin{aligned}
N_5 &= \frac{9}{32} (1+s) (1-t^2) (1-3t) \\
N_6 &= \frac{9}{32} (1+s) (1-t^2) (1+3t) \\
N_7 &= \frac{1}{32} (1+s) (1+t) [-10 + 9 (s^2+t^2)] \\
N_8 &= \frac{9}{32} (1+t) (1-s^2) (1+3s) \\
N_9 &= \frac{9}{32} (1+t) (1-s^2) (1-3s) \\
N_{10} &= \frac{1}{32} (1+t) (1-s) [-10 + 9 (s^2+t^2)] \\
N_{11} &= \frac{9}{32} (1-s) (1-t^2) (1+3t) \\
N_{12} &= \frac{9}{32} (1-s) (1-t^2) (1-3t)
\end{aligned} \tag{3}$$

It may be seen that the displacements within the element vary cubically as opposed to a linear variation in the constant-stress element. These shape functions are equivalent to a displacement assumption for a "conventional" quadrilateral finite element with sides parallel to the x- and y- axes given by

$$\begin{aligned}
U &= \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xt + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 x^2 y + \alpha_9 xy^2 \\
&\quad + \alpha_{10} y^3 + \alpha_{11} x^3 y + \alpha_{12} xy^3 \\
V &= \alpha_{13} + \alpha_{14} x + \alpha_{15} y + \alpha_{16} x^2 + \alpha_{17} xy + \alpha_{18} y^2 + \alpha_{19} x^3 + \alpha_{20} x^2 y \\
&\quad + \alpha_{21} xy^2 + \alpha_{22} y^3 + \alpha_{23} x^3 y + \alpha_{24} xy^3
\end{aligned} \tag{4}$$

where the α 's are undetermined parameters (generalized coordinates). The

first three terms in each of Equations (4) correspond to the constant-stress triangular element; the remaining terms demonstrate the "higher order" of the QUAD-12 element. Note that the displacements vary cubically over the element and that the geometry of the element edges may also vary cubically in space, so that strains and stresses vary quadratically over the element.

2. "Core" Crack Tip Element

The special circular crack tip or "core element" is shown in Figure 3. The element is a half-disk for symmetric Mode I problems, and becomes a full circle for combined mode problems. The edges of the QUAD-12 elements that join the core element are curved, providing geometric continuity between the two element types.

The displacement assumption taken from the singular core element corresponds to the first (singular) terms of the elasticity series expansion for the displacements in the immediate vicinity of a crack tip. It has the form:

$$\begin{aligned} \begin{Bmatrix} U \\ V \end{Bmatrix} = \begin{Bmatrix} U_0 \\ V_0 \end{Bmatrix} + \frac{K_I}{4G} \sqrt{\frac{Y}{2\pi}} \begin{Bmatrix} \cos\alpha, -\sin\alpha \\ \sin\alpha, \cos\alpha \end{Bmatrix} \begin{bmatrix} (2K-1) \cos \frac{\sigma}{2} & - \cos \frac{3\sigma}{2} \\ (2K+1) \sin \frac{\sigma}{2} & - \sin \frac{3\sigma}{2} \end{bmatrix} \\ + \frac{K_{II}}{4G} \sqrt{\frac{Y}{2\pi}} \begin{Bmatrix} \cos\alpha, -\sin\alpha \\ \sin\alpha, \cos\alpha \end{Bmatrix} \begin{bmatrix} (2K+3) \sin \frac{\sigma}{2} & + \sin \frac{3\sigma}{2} \\ (2K-3) \cos \frac{\sigma}{2} & - \cos \frac{3\sigma}{2} \end{bmatrix} \end{aligned} \quad (5)$$

where

K_I, K_{II} = Mode I and Mode II stress intensity factors

U, V = displacements

G = elastic shear modulus

K = $(3-4\nu)$ for plane strain or axisymmetric
 $(3-\nu/1+\nu)$ for plane stress

ν = Poisson's ratio

γ, σ = polar coordinates centered at the crack tip as shown in Figure 2. The corresponding stress field is $\sigma_{ij} = \sigma_{ij}(\gamma^{-1/2}, \sigma)$ which tends to infinity as γ tends to zero.

The unknowns associated with the core element are the two crack tip displacements U_0 and V_0 and the stress intensity factors K_I and K_{II} . The singular core element is joined to the conventional QUAD-12 elements by requiring that the displacements of nodes that join the core element match the singular solution evaluated at γ_0 (the outer radius of the core element) and at the appropriate angle σ . The fracture mechanics stress intensity factors are calculated at the same time as the nodal displacement. The incompatibility between the core and the standard finite elements has been found to be of negligible effect provided (1) a sufficient number of elements surround the core element (10 nodes have proven adequate for the Mode I half-disk element, implying that 19 nodes are adequate for the combined mode problem); and (2) that the radius of the core element γ_0 is taken as 2 or 3 percent of the crack length, with a ratio of k (the basic dimension of the QUAD-12 element) to γ_0 in the range of about 6 to 10. Note that thermal loading is not programmed for this element at the present time.

3. "Enriched" QUAD-12 Element

The effects of the singularity are included in this element by "enriching" a bicubic element displacement assumption with terms that give the proper singularity at node (s, t) ,

$$U(s, t) = \alpha_1 + \alpha_2 s + \alpha_3 t + \alpha_4 s^2 + \alpha_5 st + \alpha_6 t^2 + \alpha_7 s^3 + \alpha_8 s^2 t + \alpha_9 st^2 + \alpha_{10} t^3 + \alpha_{11} st^3 + \alpha_{12} ts^3 + K_I f_1(s, t) + K_{II} g_1(s, t) \quad (6)$$

where the α 's are undetermined constants (as are K_I and K_{II}) and $f_1(s,t)$ and $g_1(s,t)$ are the second and third terms of the first of Equation (5) evaluated in terms of the local element coordinates s and t . A similar expression exists for the V component of displacement. In matrix form, Equation (6) may be written

$$U(s,t) = [P(s,t)] \{\alpha\} + K_I f_1(s,t) + K_{II} g_1(s,t) . \quad (7)$$

Evaluating Equation (7) at each of the nodes, the following matrix equation may be written

$$\{U\} = [C] \{\alpha\} + K_I \{f_1\} + K_{II} \{g_1\} \quad (8)$$

in which all matrices are known except $\{U\}$ and $\{\alpha\}$.

Solving Equation (8) for the unknown coefficients $\{\alpha\}$ in terms of the nodal displacements, U_i where $i = 1, 2$, the displacement assumption may be written as

$$U(s,t) = \sum N_i \bar{U}_i + K_I [f_1(s,t) - \sum N_i f_{1i}] + K_{II} [g_1(s,t) - \sum N_i g_{1i}] \quad (9)$$

where the subscripts on f_1 and g_1 indicate "evaluated at Node i ". The analogous expression for the V -component of displacement is

$$V(s,t) = \sum N_i \bar{V}_i + K_I [f_2(s,t) - \sum N_i f_{2i}] + K_{II} [g_2(s,t) - \sum N_i g_{2i}] . \quad (10)$$

The terms enclosed in the parentheses in Equation (9) and (10) account for the singularity. An enriched QUAD-12 element connected to a standard QUAD-12 element would produce a slight incompatibility between the adjoining elements. This incompatibility can be removed by altering the displacement assumption of Equation (9) and (10) to

$$\begin{Bmatrix} U(s,t) \\ V(s,t) \end{Bmatrix} = \begin{Bmatrix} \sum N_i \bar{U}_i \\ \sum N_i \bar{V}_i \end{Bmatrix} + R(s,t) \begin{Bmatrix} K_I [f_1(s,t) - \sum N_i f_{1i}] + K_{II} [g_1(s,t) - \sum N_i g_{1i}] \\ K_I [f_2(s,t) - \sum N_i f_{2i}] + K_{II} [g_2(s,t) - \sum N_i g_{2i}] \end{Bmatrix} \quad (11)$$

where $R(s,t)$ is chosen such that it equals 1 on boundaries adjacent to "enriched" elements and equals 0 on boundaries adjacent to standard elements.

Using the assumption given by Equation (11) in a standard finite element formulation, the left-hand side of the equilibrium equations for an element become

$$\begin{bmatrix} k^{11} & k^{12} \\ k^{21} & 0 \end{bmatrix} \begin{Bmatrix} \bar{U} \\ K_I \\ K_{II} \end{Bmatrix}$$

where

\bar{U} = element nodal point displacements

k^{11} = "regular" stiffness matrix

k^{12} = coupled stiffness matrix from regular and enrichment terms

k^{21} = transpose (k^{12}).

Integration of k^{12} must be done very accurately because they contain singular terms. For this work a high order (8x8) gaussian quadrature is required for the enriched QUAD-12 elements.

6. Results of the Analyses

The stress analyses were conducted in two phases:

1. elastic calculations to determine the stress profiles in defect free tubes under normal conditions of loading;
2. fracture mechanics calculations to evaluate the effects of cracks in the most highly stressed regions.

The elastic analyses were performed with the APES code using only the conventional finite elements. For similar base metal weldments, symmetry about the weld center line was taken advantage of and the finite element grid (Figure 1b) had 514 nodes and 90 elements. For the general weldment, the grid was symmetric about the weld center line and contained 1,009 nodes and 180 elements. For each of the analyses nodes 496-514 were constrained in the longitudinal direction. Pressure loads were applied as radial forces to the inner and outermost nodes and, in addition, thermal loads were applied to each node with node-wise temperature distribution.

The pertinent results of the elastic analyses are depicted graphically in Figures 4-9. Figures 4, 5 and 6 correspond to the 70°F stress-free temperature state assumption while Figures 7, 8 and 9 to the 1350°F stress free state assumption. The results shown are traces of the radial, (σ_r), longitudinal (σ_z) and hoop (σ_θ) stresses along the inner and outer tube surfaces, node lines 1-496 and 19-514, respectively. Figures 4 and 7 correspond to the I800/I82/I800 superheater weldment, Figures 5 and 8 to the I800/I82/I800 reheater weldment, and Figures 6 and 9 to the I800/I82/2 1/4Cr-Mo weldment. For the cases corresponding to the 1350°F assumption, Figures 7-9, the stress states at room temperature and operating temperature are shown. In addition, in Figures 10, 11 and 12 the through thickness stress variations are shown for various node lines and each of the cases.

Referring to Figures 4-9 it is evident that for all cases strong stress gradients exist near the fusion zone, large through thickness (bending) variations exist, and the longitudinal stresses are greatest on the inner surface. For most cases the longitudinal stress component attains the peak stress levels. Referring to Figures 10-12 it is evident that strong through thickness bending stress gradients exist for the longitudinal and hoop stress components. Lastly, for all cases the longitudinal stress component attains its peak value on the inner surface near the fusion line.

For the fracture mechanics analyses, only circumferential cracks were considered. These result from either stress corrosion cracking or sustained load cracking and are commonly observed in tubes. For such cracks the longitudinal load or stress field is the dominant load parameter affecting both crack severity and growth characteristics. For a given weldment the most detrimental circumferential crack will be located at the point where the longitudinal stress component attains a maximum. From the elastic analyses this point is the fusion line at the inner surface for all the cases. This was taken as the crack location for all the fracture analyses.

For the fracture runs, enriched element crack tip models were employed. The gross finite element grid was similar to that used in the elastic analyses with adjustments in the vicinity of the crack. Figure 13 depicts exploded views of the grid surrounding the crack for crack lengths of 12.5, 25, 50, 75, and 87.5 percent of the wall thickness. Each weldment was analyzed for the 12.5% crack while the I800/I82/I800 superheater weldment was analyzed for all the crack lengths.

The computed stress intensity factors are summarized in Table 4 and shown graphically in Figures 14 and 15. Figure 14 shows the results

corresponding to the operating temperature stress fields while Figure 15 shows the results for the room temperature stress fields. In both cases the results are for the 1350^oF stress free state assumption.

The computed stress intensity factors range from 1 to 4 Ksi√in. The highest computed factor is still below K_{th} (6 Ksi√in @ 1000^oF) for the material and consequently, there is no possibility of crack growth.⁽⁴⁾ For the deep cracks, the presence of the crack itself markedly reduces the through wall thickness stress gradients and peak stresses which results in low values the stress intensity factors. This implies that if a small crack propagates in a tube wall having a steep stress gradient, crack arrest will occur after some propagation.

Table 4

Computed Stress Intensity Factors [psi√in Units]

	$\frac{a}{b}$	Superheater Tube I800/I82/I800	Superheater Tube I800/I82/2 1/4Cr-Mo	Reheater Tube I800/I82/I800
Operating Temperature	0.125	3834.4	3046.0	4040.2
	0.125	3172.1 (temp. alone)		3931.8 (temp. alone)
Stress Free at 1350 ^o F	0.125	662.3 (press. alone)		
	0.25	3799.6		
	0.50	2589.2		
	0.75	2460.8		
	0.875	3918.2		
	0.125	2690.0	966.3	1781.5
Room Temperature Stress Free at 1350 ^o F	0.25	1149.3		
	0.50	-1027.9		
	0.75	- 594.6		
	0.875	1519.0		

7. Conclusions

An assessment of HTGR steam generator tube weld integrity has been completed. Circumferential cracks were evaluated using linear elastic fracture mechanics

theory. It was concluded that:

- (a) Circumferential cracks do not propagate for the present pipe geometry (radius and thickness of tube wall) and the stress level.
- (b) For thermally induced stress gradients, crack arrest may be expected if initial crack propagation does occur.

Acknowledgments

The authors wish to thank L. N. Gifford, Jr. for recommending the use of the APES computer code.

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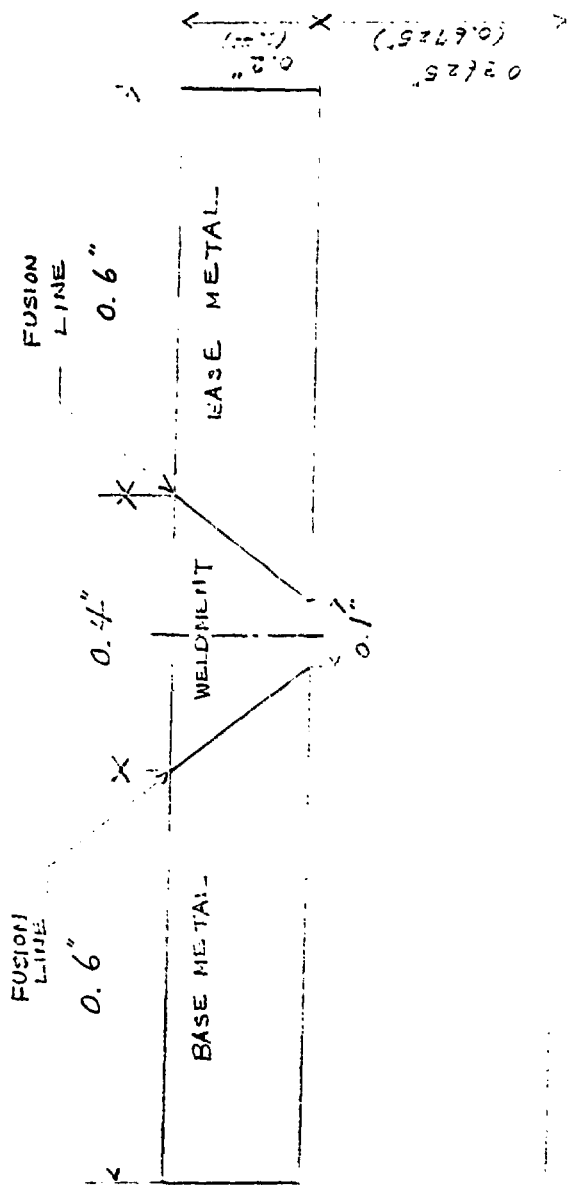


Figure 10. HTGN Low pressure Tub Weldment

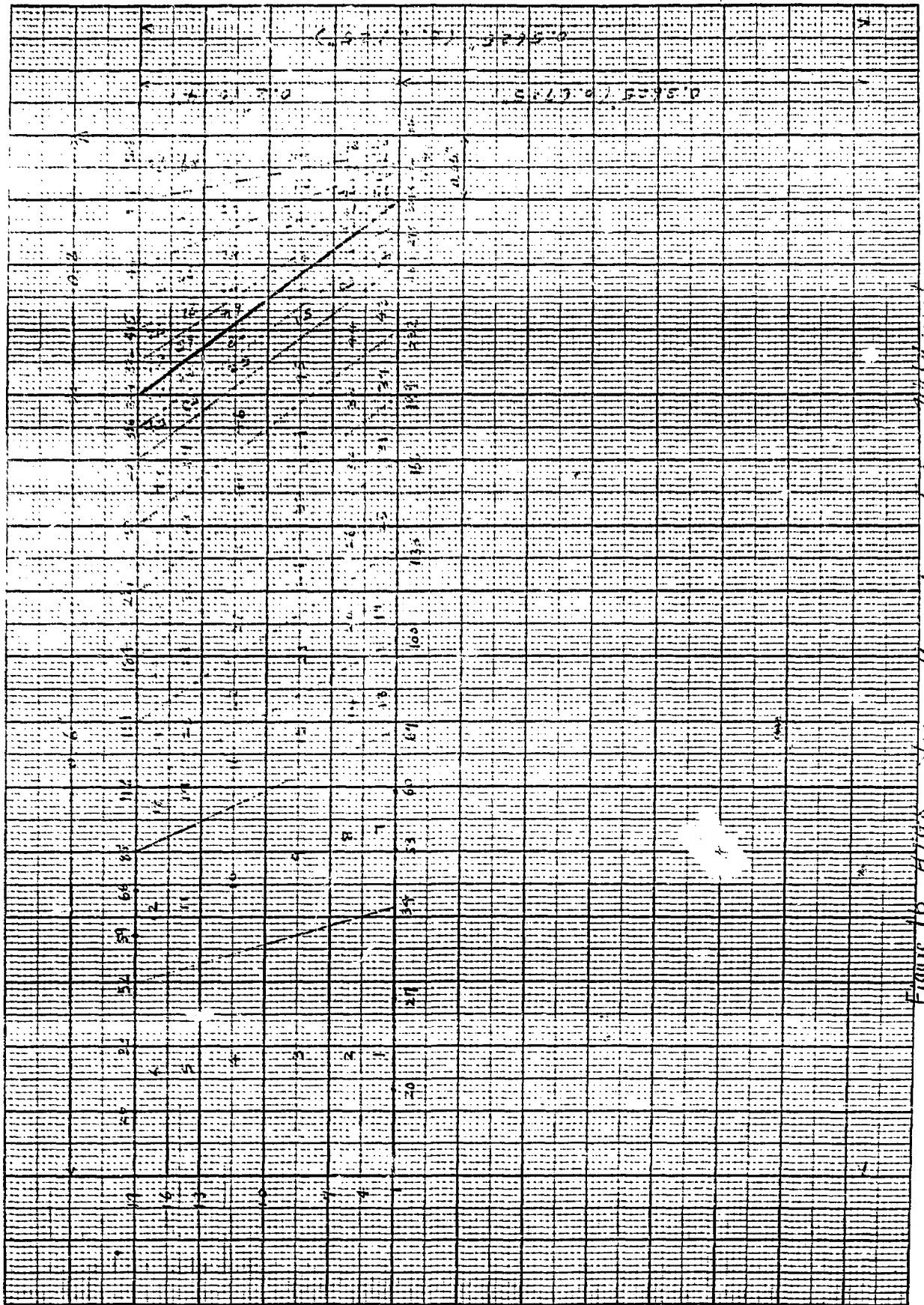


Figure 1b. High...

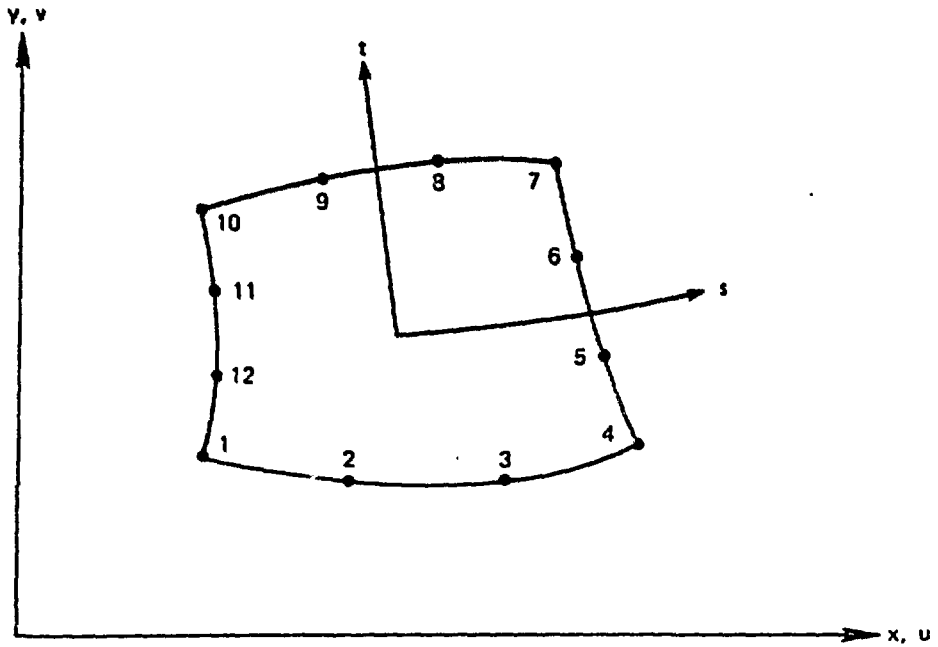


Figure 2 -- QUAD-12 Finite Element and Local Element Node Numbers

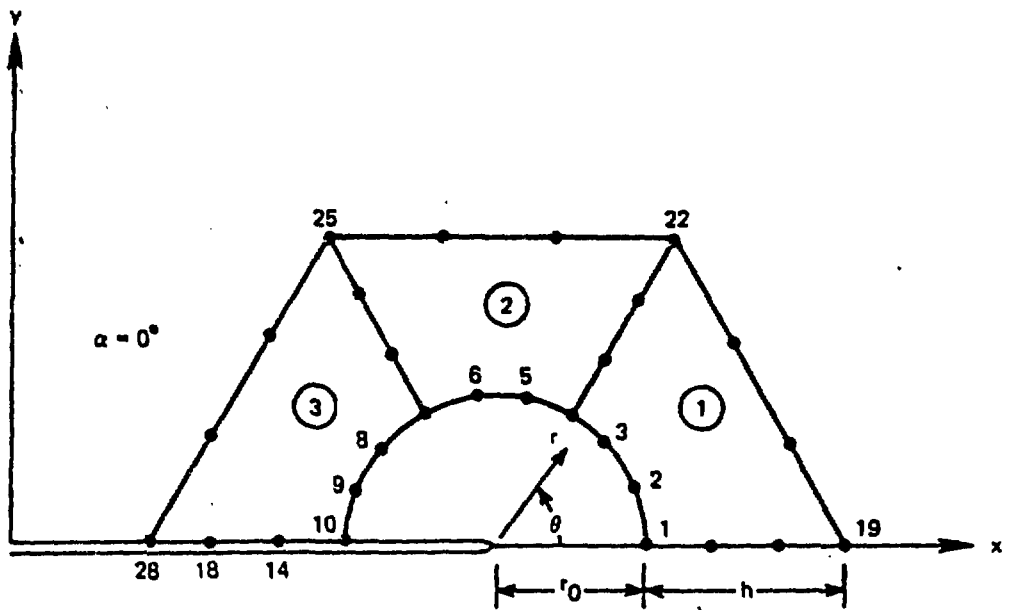


Figure 3a -- Mode I (Symmetric) Crack Tip Model

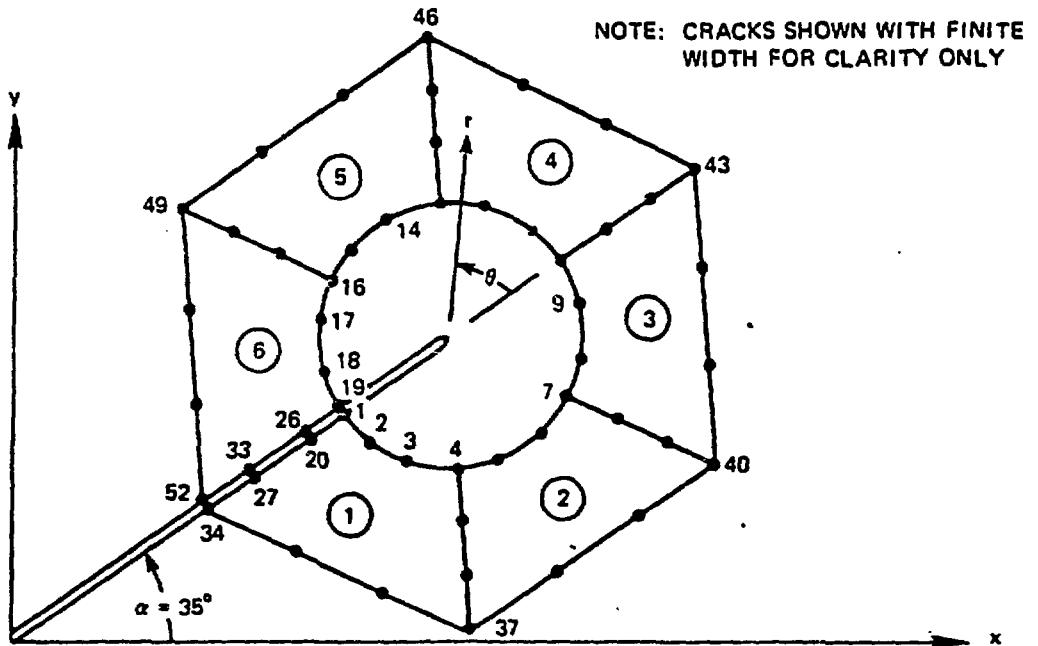


Figure 3b -- Combined Mode (Unsymmetric) Crack Tip Model

Figure 3 -- Special Core Element Crack Tip Models

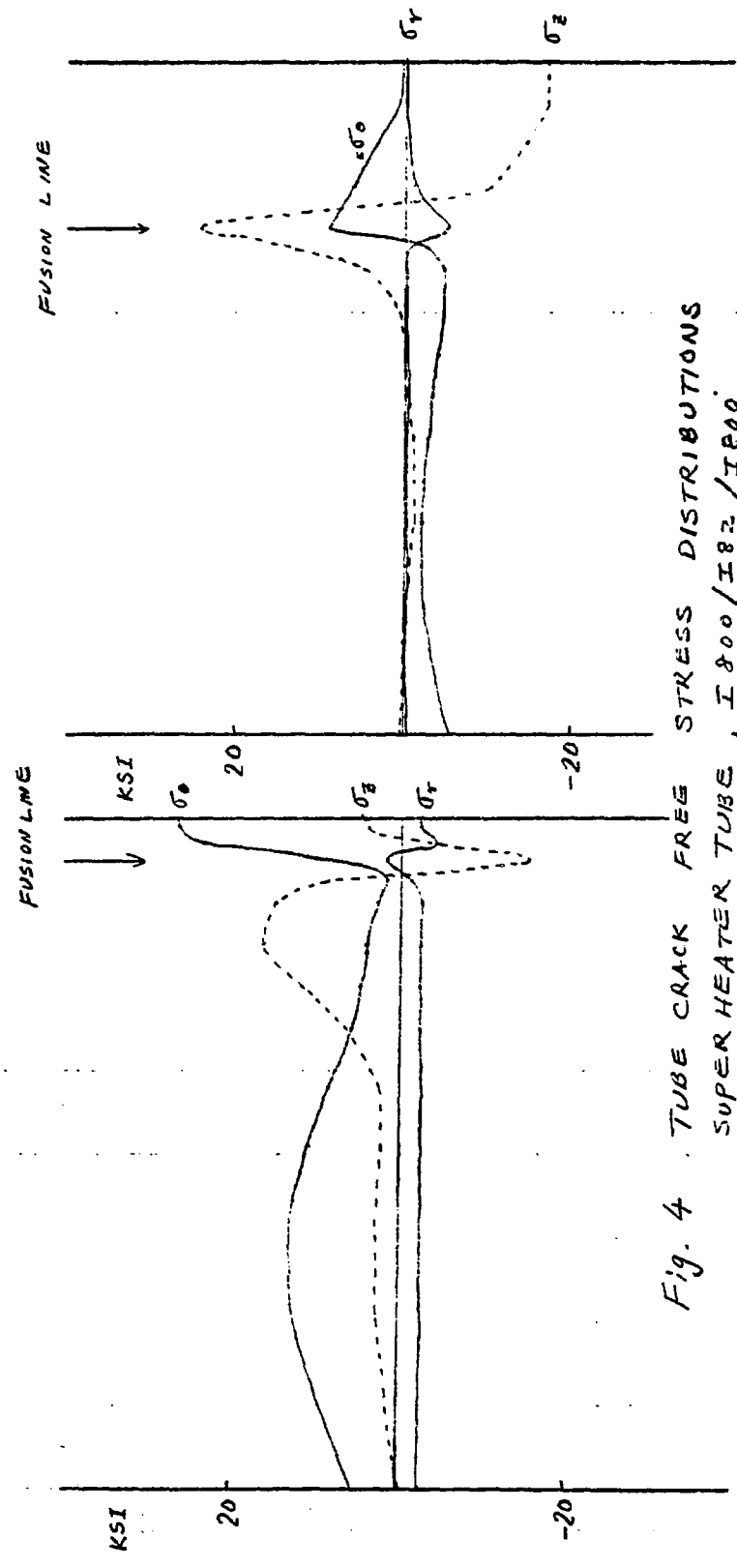
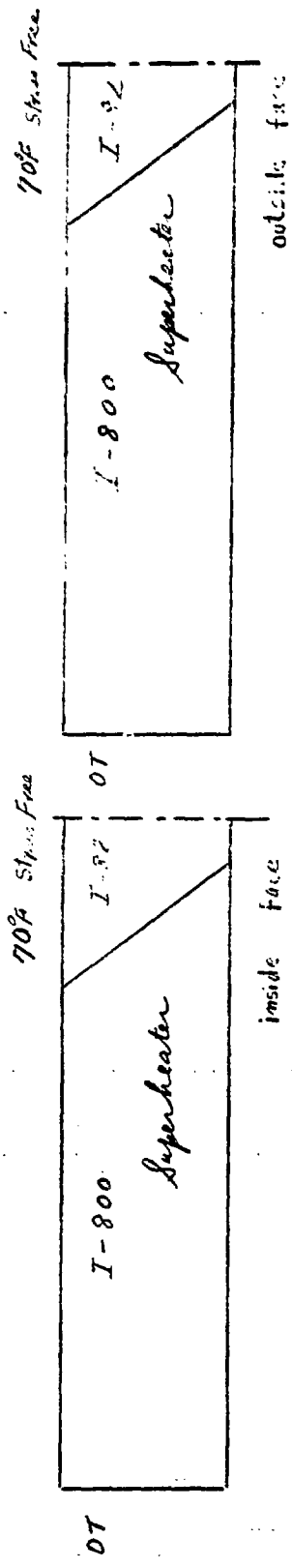


Fig. 4 TUBE CRACK FREE STRESS DISTRIBUTIONS
 SUPER HEATER TUBE, I 800 / I 882 / I 800.
 STRESS FREE AT 1100°F

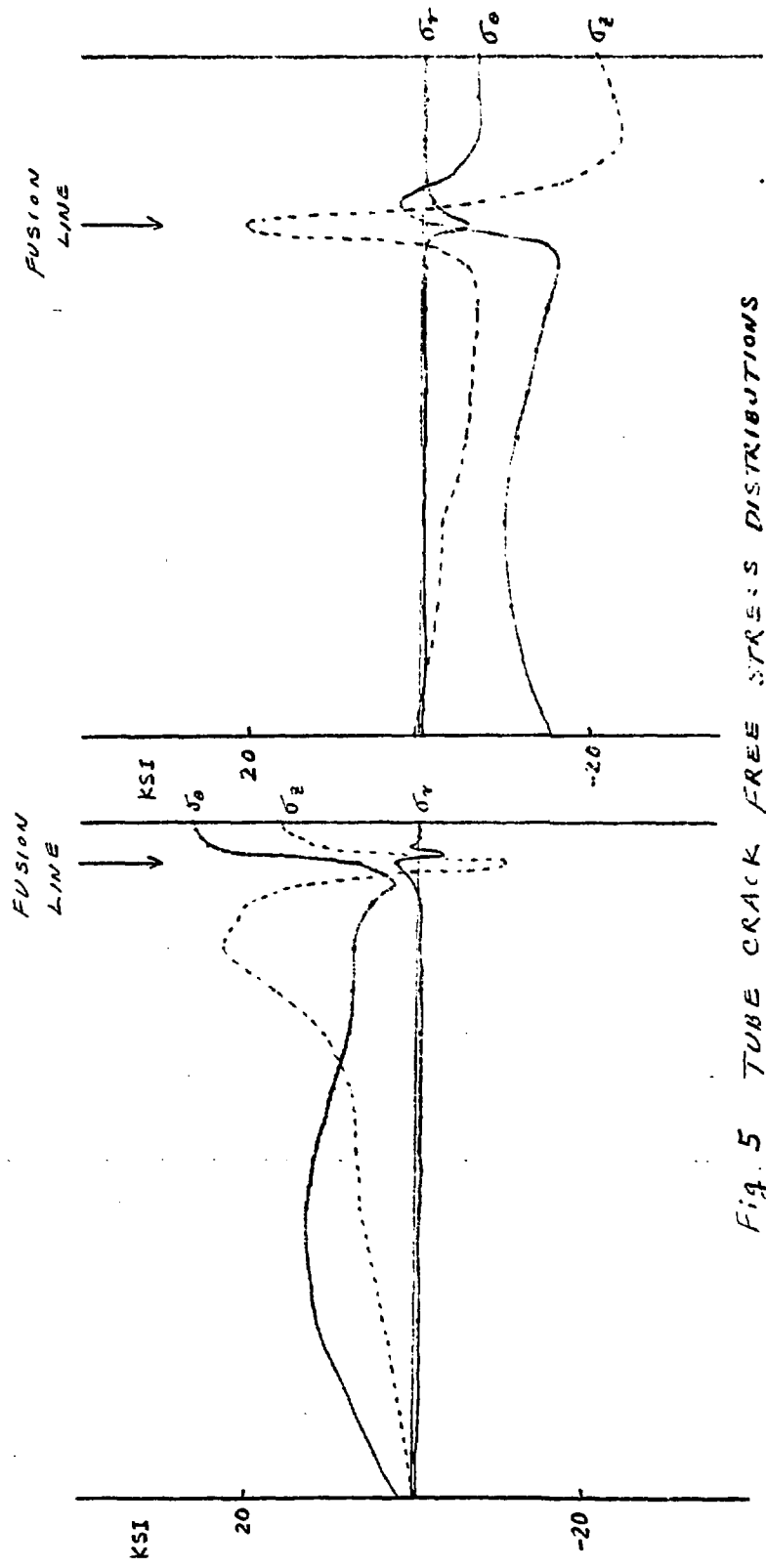
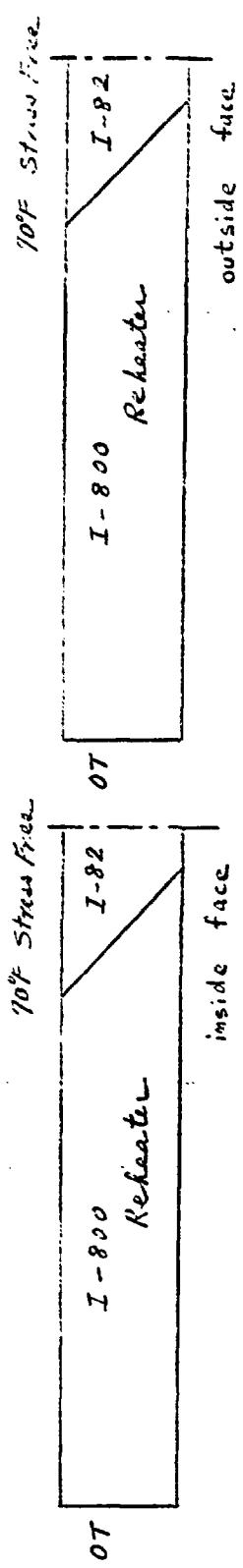


Fig. 5 TUBE CRACK FREE STRESS DISTRIBUTIONS
 REHEATED TUBE, I 800/I 82/I 800
 STRESS FREE AT 70°F

70°F Stress Free

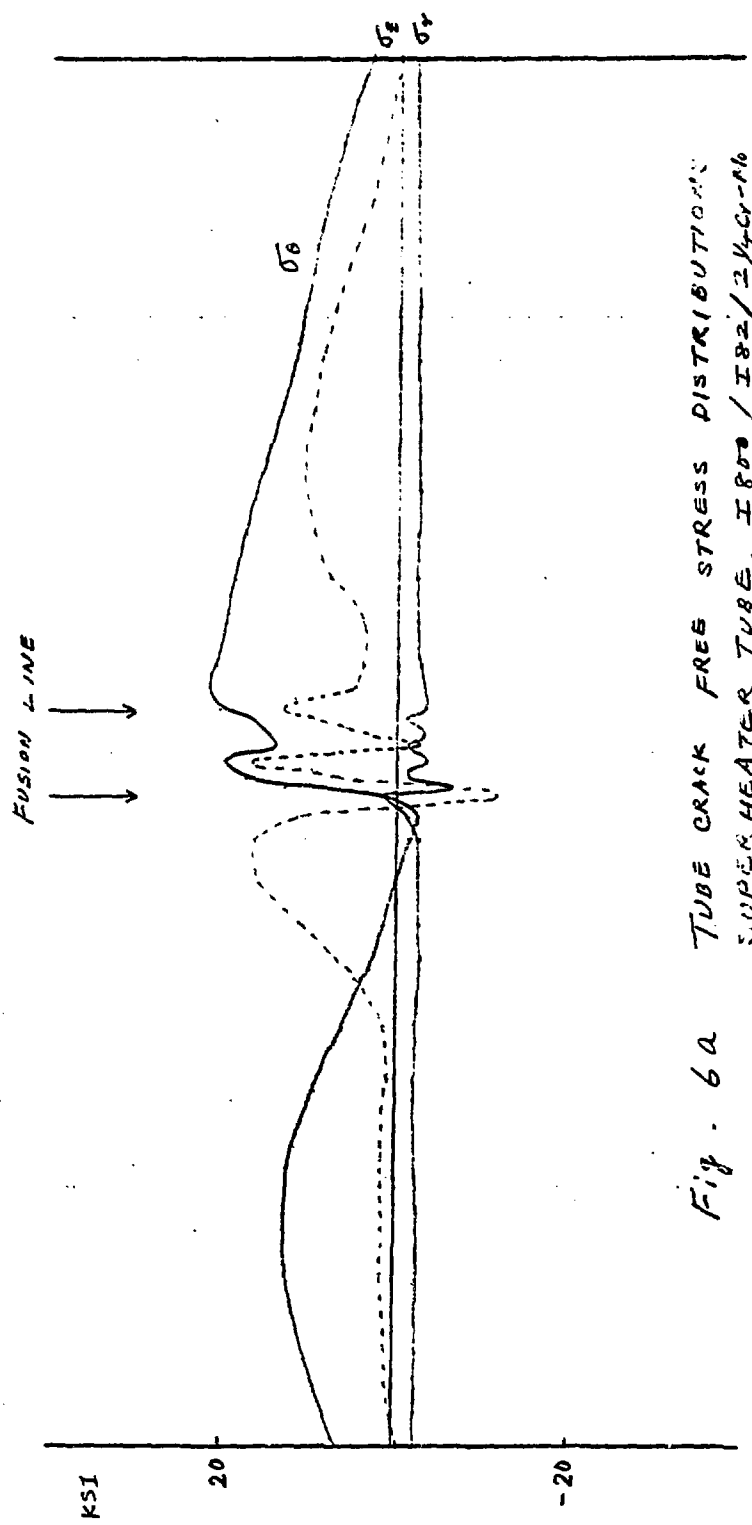
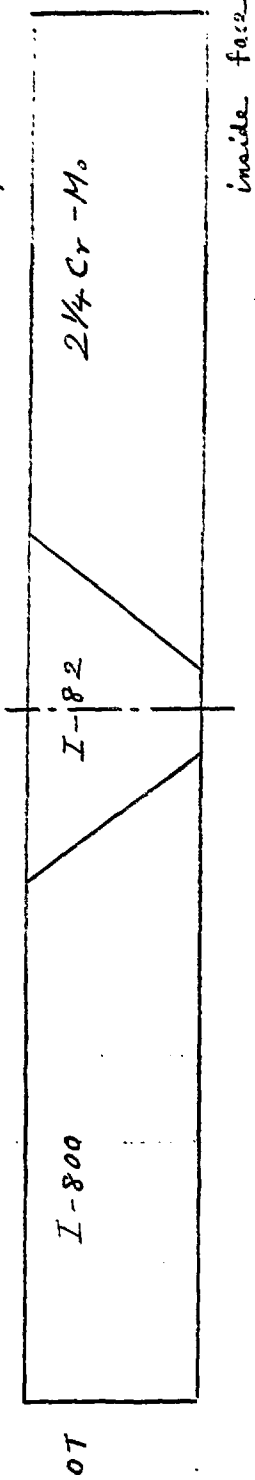


Fig. 6a TUBE CRACK FREE STRESS DISTRIBUTION SUPERHEATER TUBE, I800 / I82 / 2 1/4 Cr-Mn STRESS FREE AT 70°F

10°F Stress Free

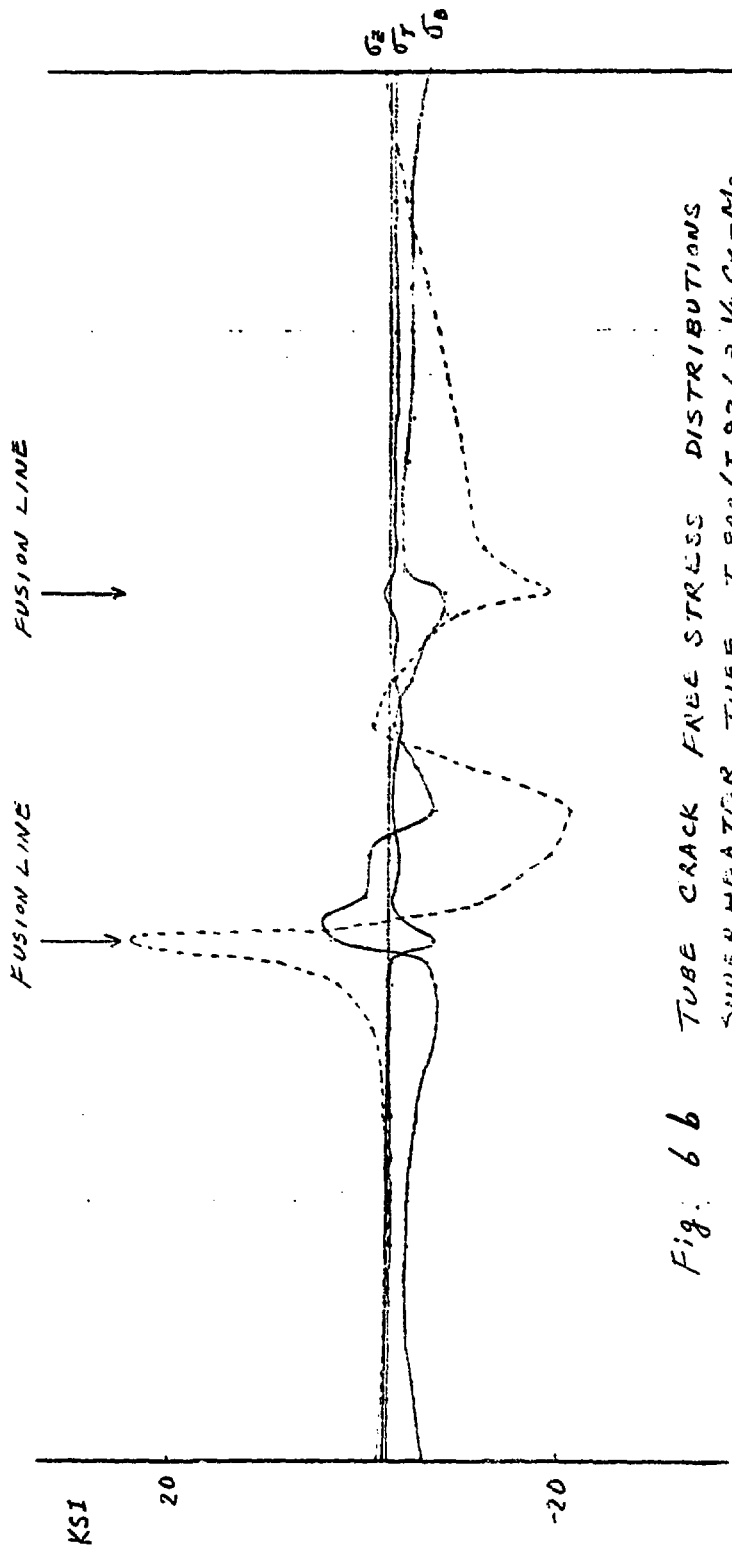
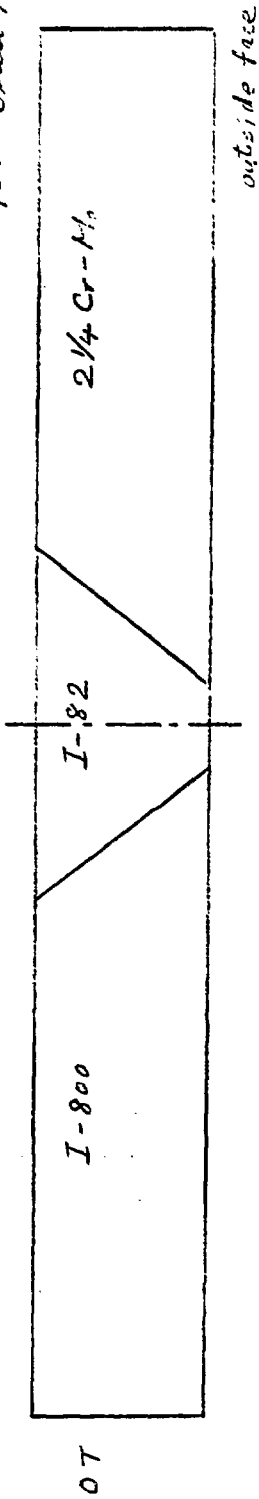


Fig. 66 TUBE CRACK FREE STRESS DISTRIBUTIONS
SUPER HEATER TUBE, I 800/I 82/2 1/4 Cr-Mo
STRESS FREE AT 10°F

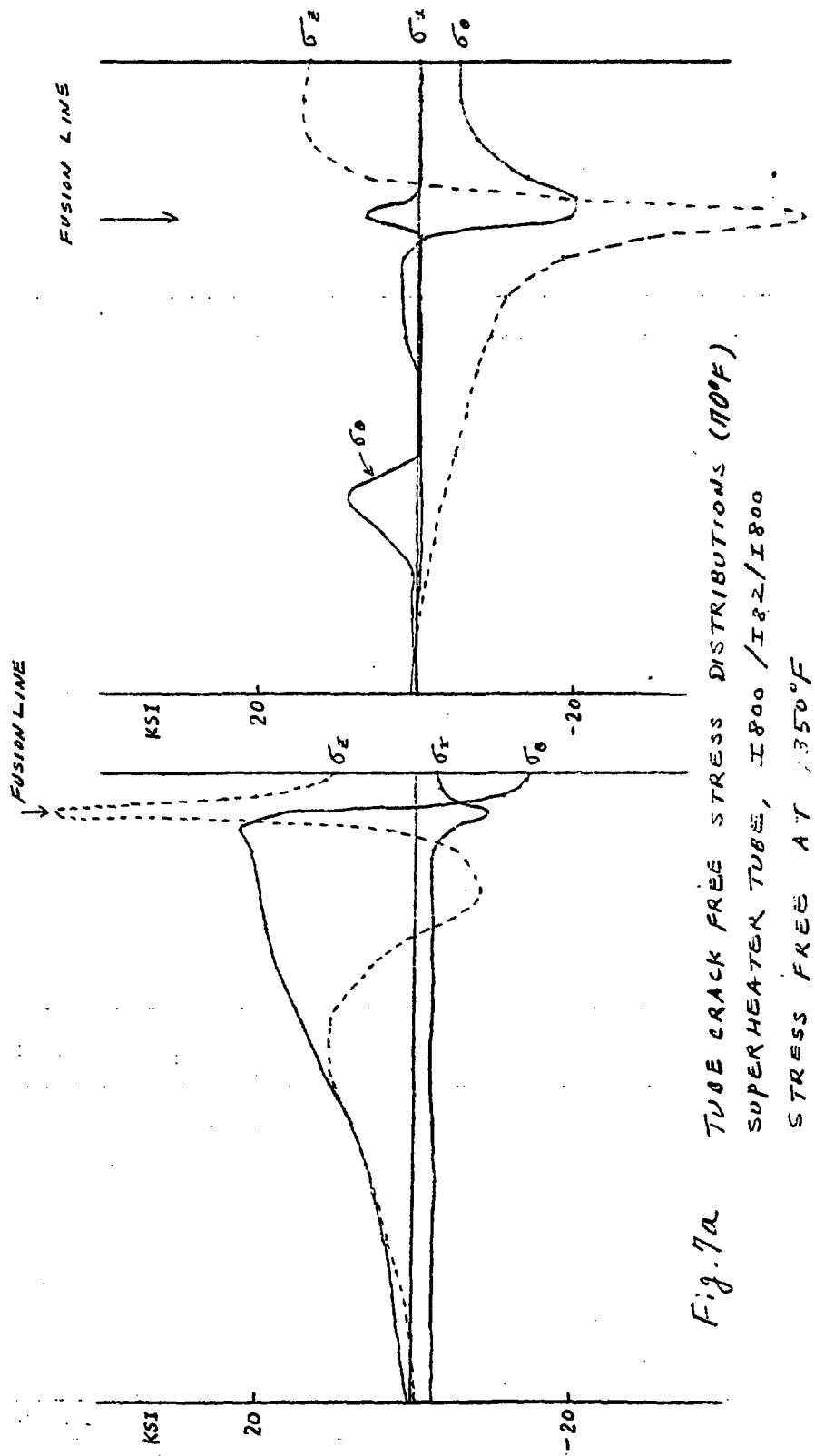
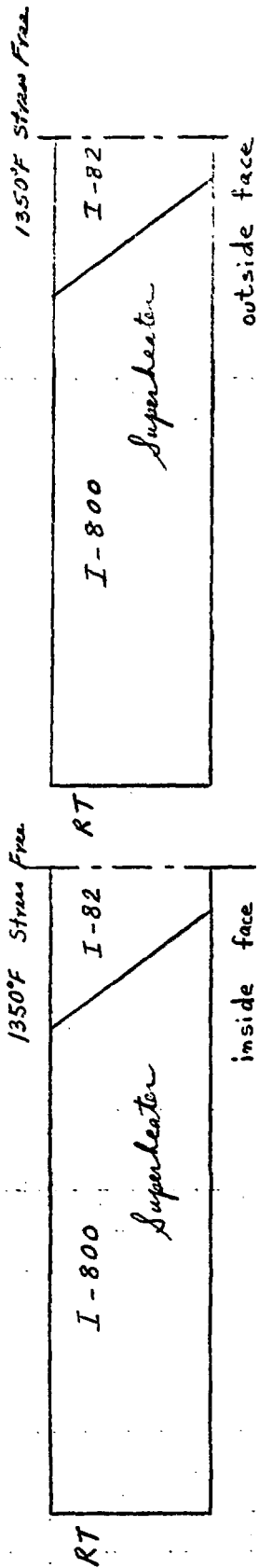


Fig. 7a TUBE CRACK FREE STRESS DISTRIBUTIONS (100°F)
 SUPERHEATED TUBE, I800 / I82 / I800
 STRESS FREE AT 1350°F

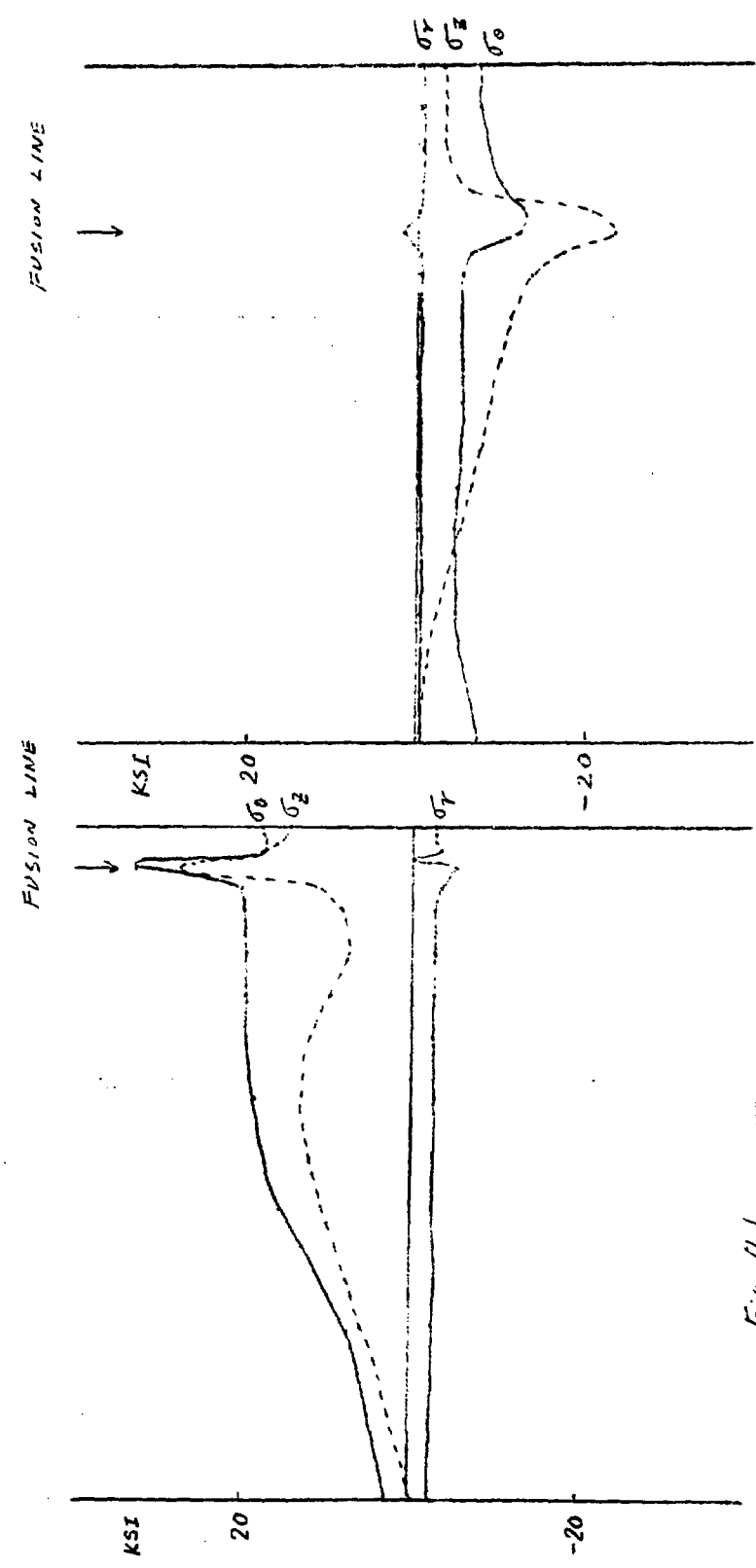
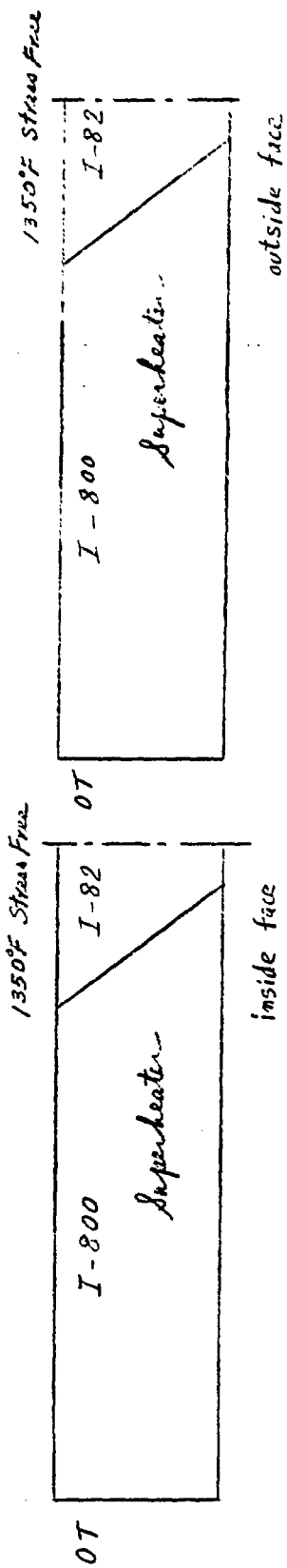


Fig. 16 TUBE CRACK FREE STRESS DISTRIBUTIONS
 SUPERHEATER TUBE, I800/I82/I800
 STRESS FREE AT 1350°F

1/2 in.

1350°F Stress Free

1350°F Stress Free

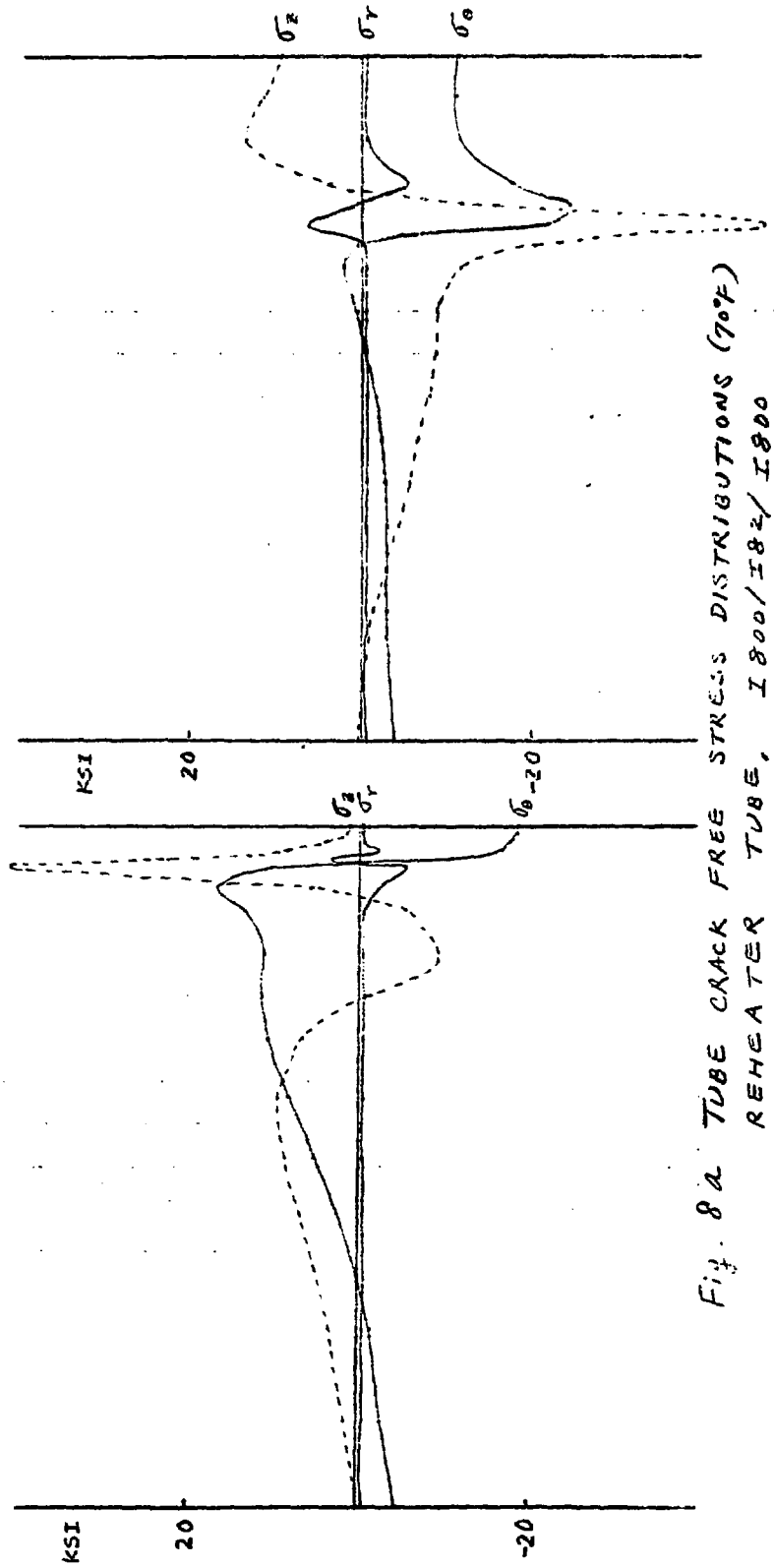
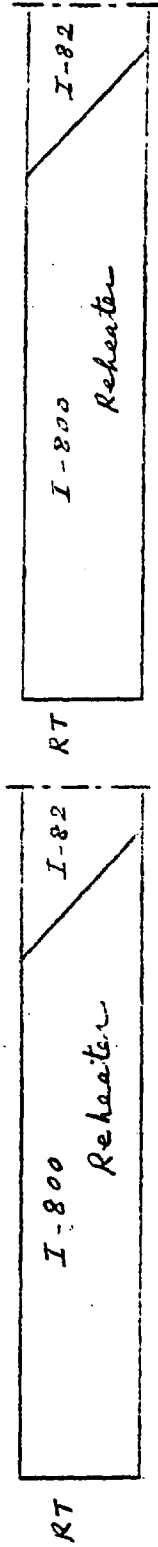
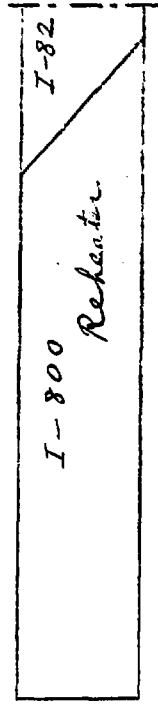


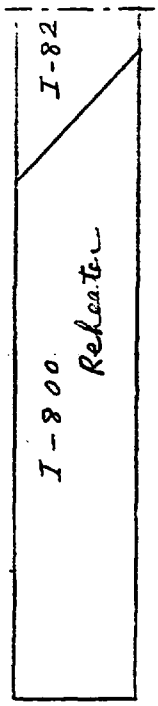
Fig. 8a TUBE CRACK FREE STRESS DISTRIBUTIONS (70°F)
REHEATER TUBE, I800/I82/I800
STRESS FREE AT 1350°F

1350°F Stress Free



outside face

1350°F Stress Free



inside face

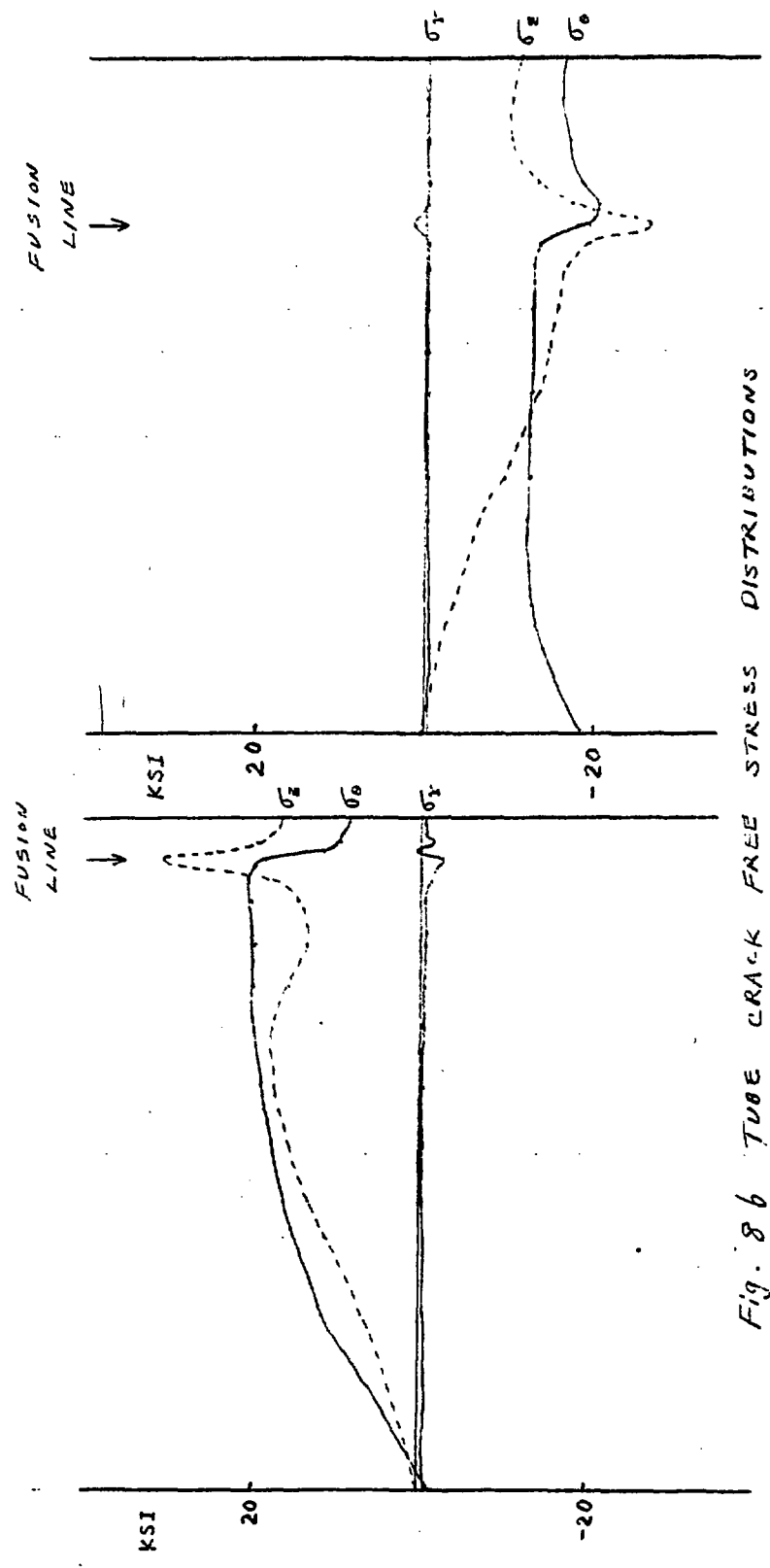
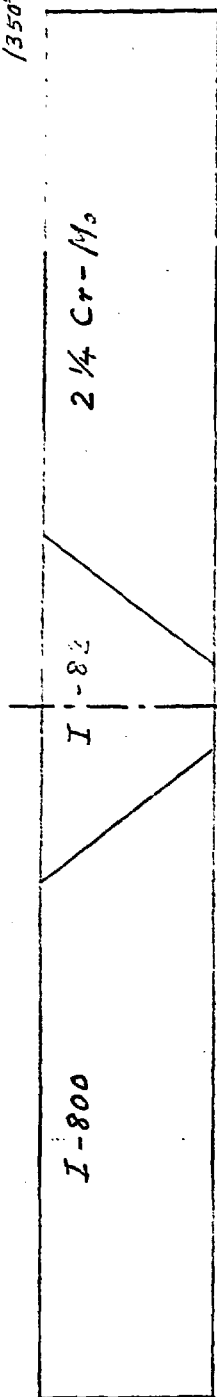


Fig. 86 TUBE CRACK FREE STRESS DISTRIBUTIONS
 REHEATER TUBE, I800/I82/I800
 STRESS FREE AT 1350°F

1350°F Stress Free



inside face

FUSION LINE

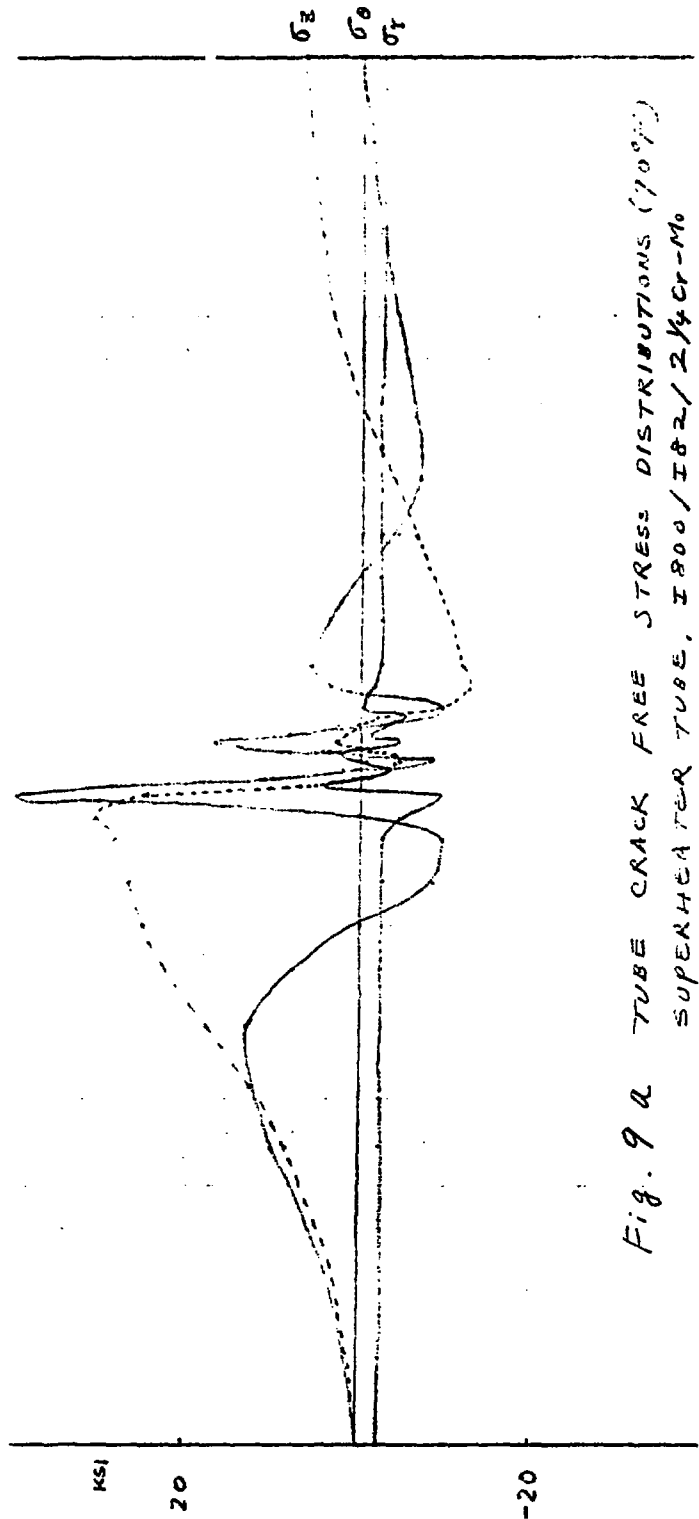
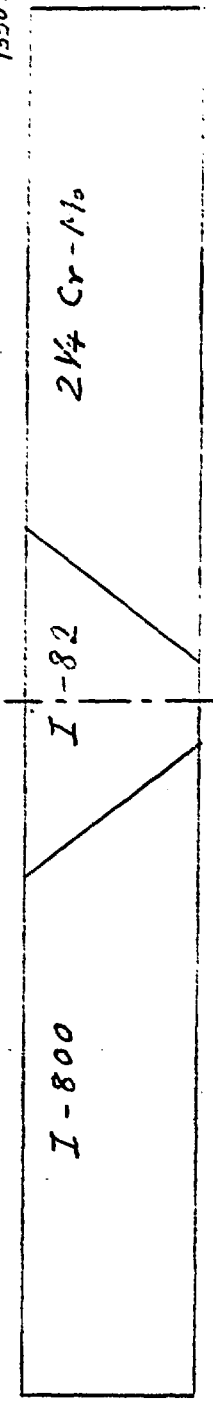


Fig. 9 a TUBE CRACK FREE STRESS DISTRIBUTIONS (70°F)
SUPERHEATED TUBE, I800/I82/2 1/4 Cr-Mo
STRESS FREE AT 1350°F

1350°F Stress Free



outside face

FUSION LINE ↓

FUSION LINE ↓

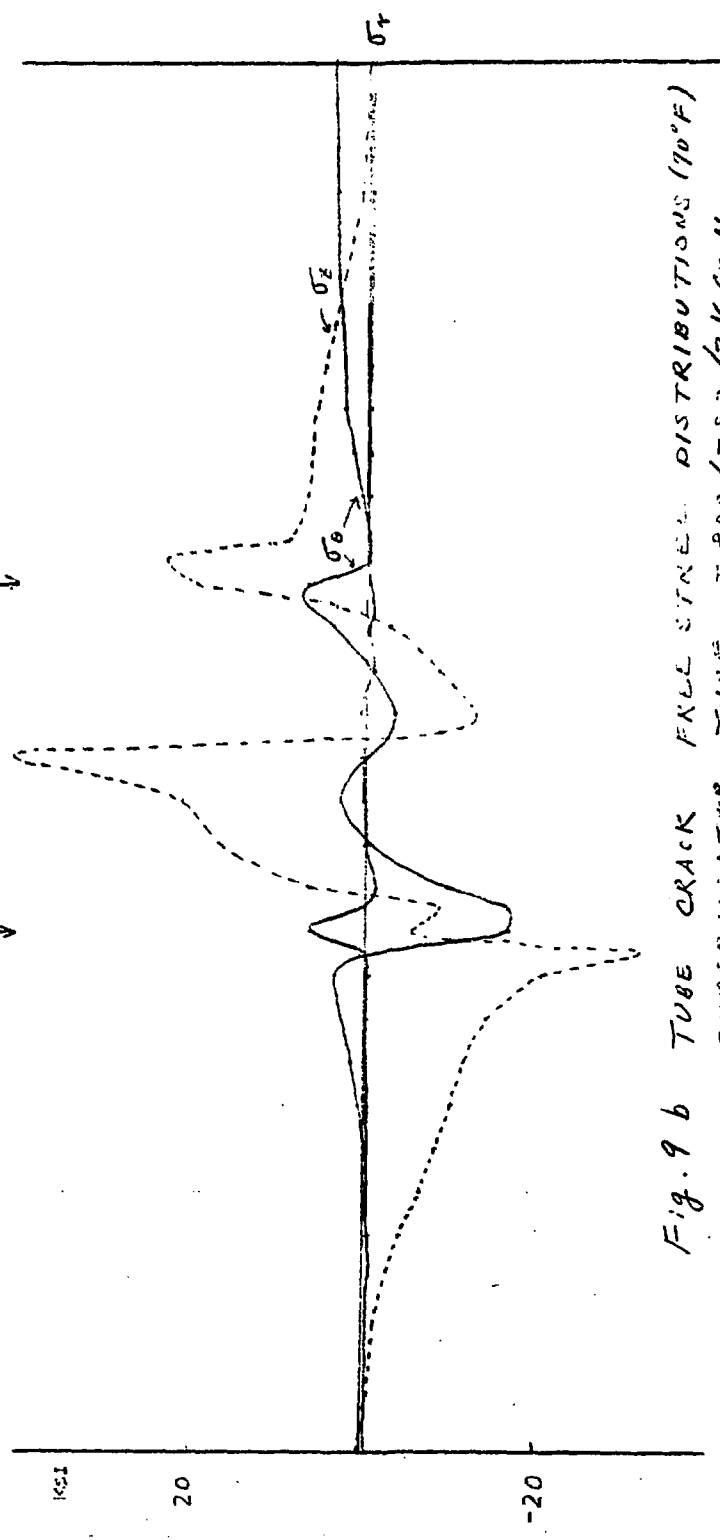
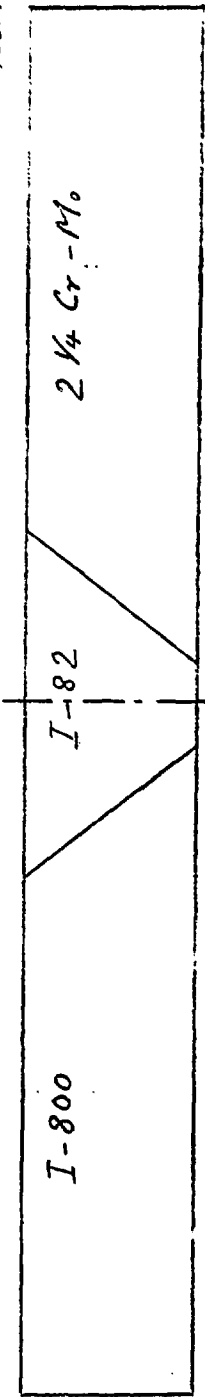


Fig. 9 b TUBE CRACK FILL STRESS DISTRIBUTIONS (70°F)
SUPER HEATER TUBE, I 800 / I 82 / 2 1/4 CR-NI
STRESS FREE AT 1350°F

1350°F Stress Free



inside face

FUSION LINE
FUSION LINE

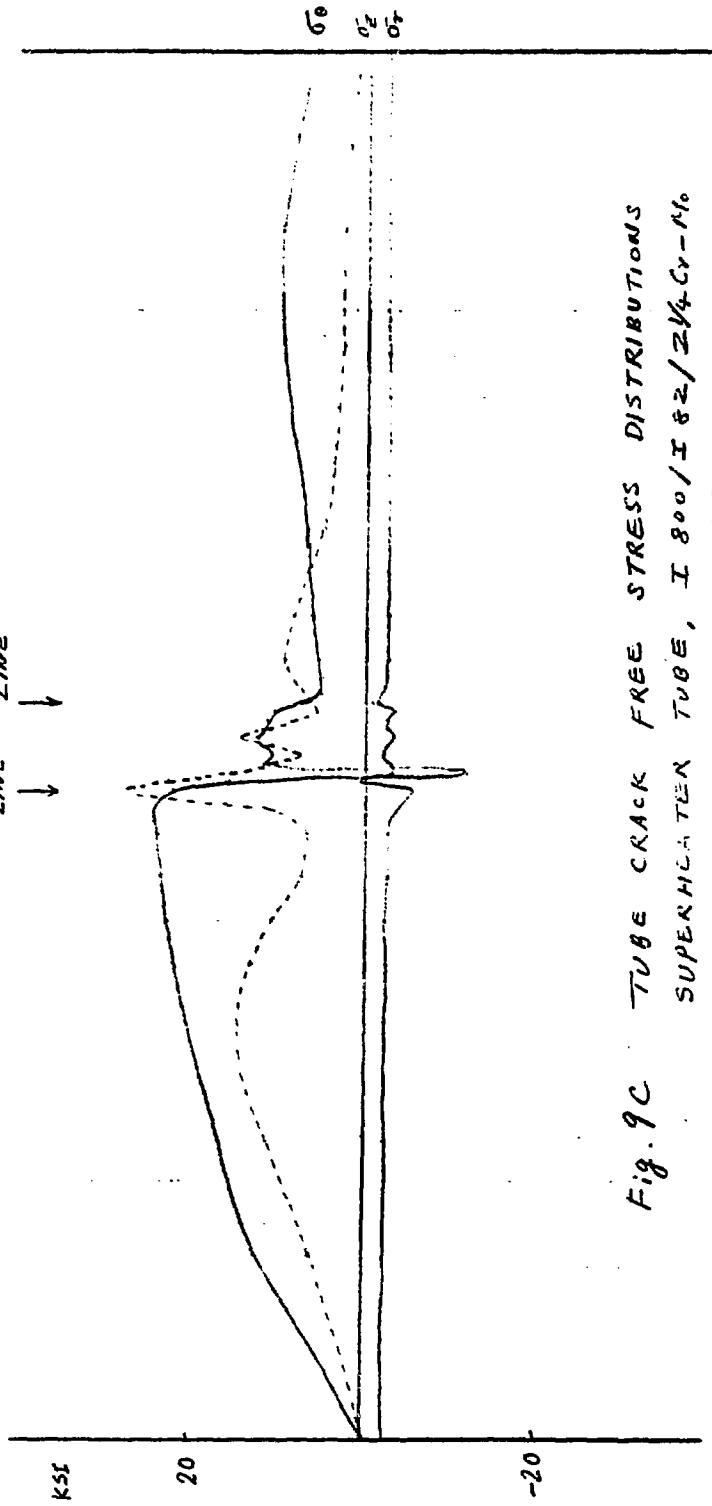


Fig. 9C TUBE CRACK FREE STRESS DISTRIBUTIONS
SUPERHEATED TUBE, I 800/I 82/2 1/4 Cr - Mo
STRESS FREE AT 1350°F

1350°F Stress Free

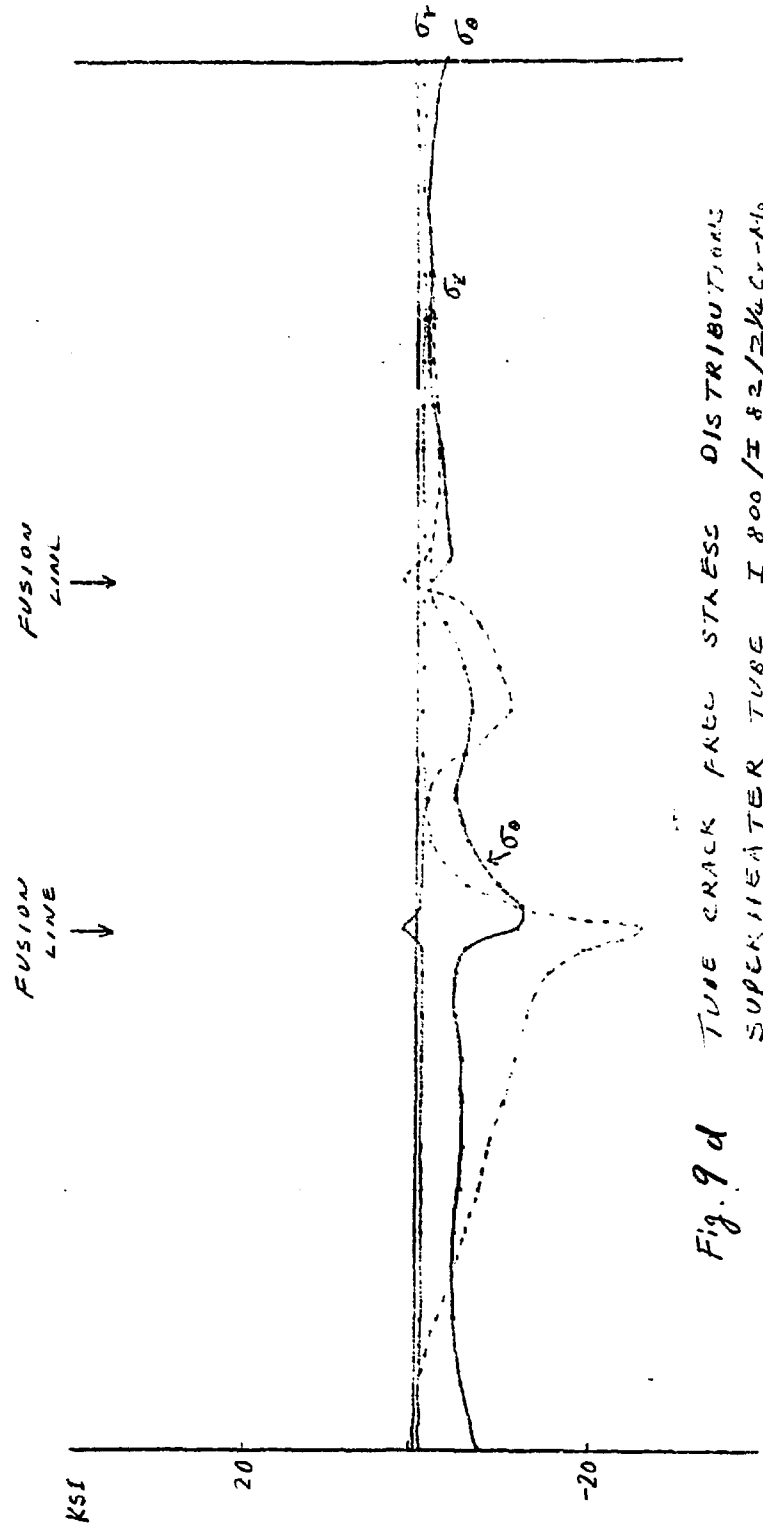
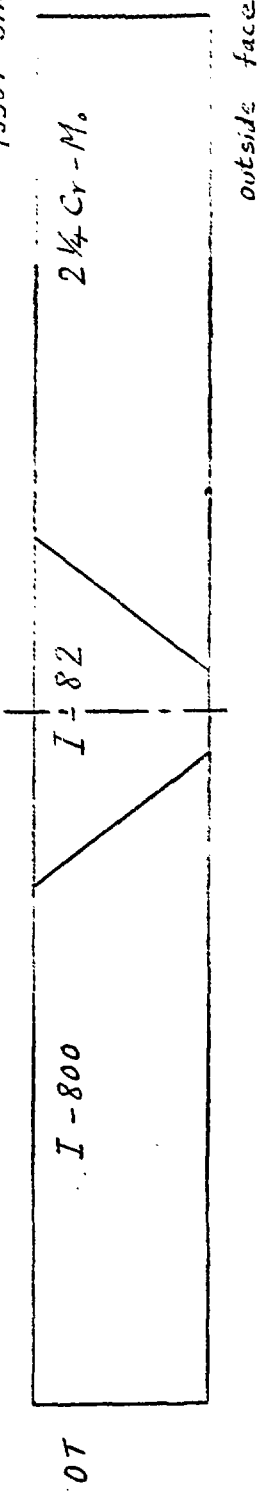


Fig. 9 d TUBE CRACK FACE STRESS DISTRIBUTIONS
SUPERHEATER TUBE, I 800/I 82/2 1/4 Cr-M.
STRESS FREE AT 1350°F

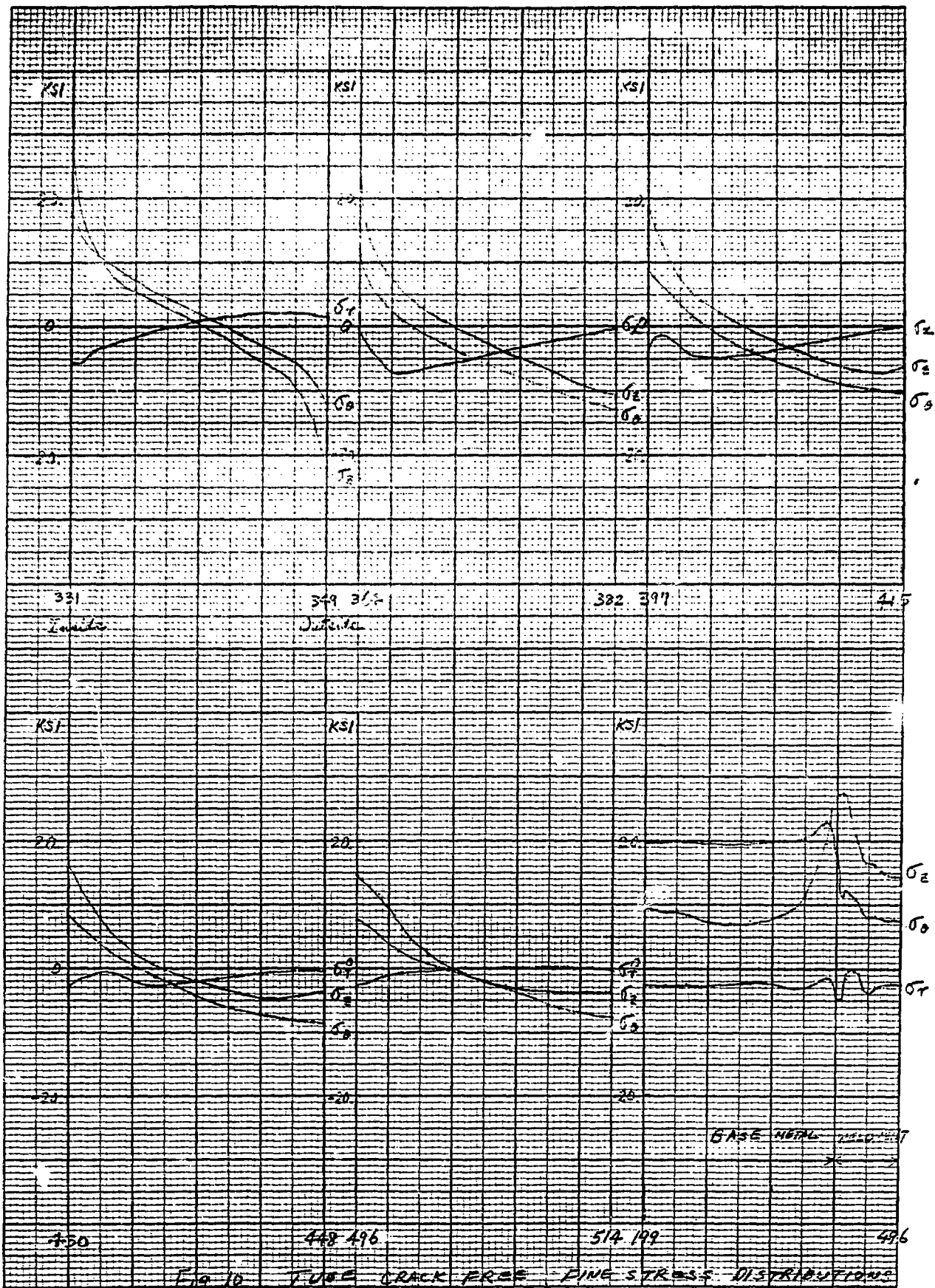


Fig 10 TUBE CRACK FREE PIPE STRESS DISTRIBUTIONS
SUPERHEATER TUBE, I800/I82/I800
STRESS FREE AT 1350°F

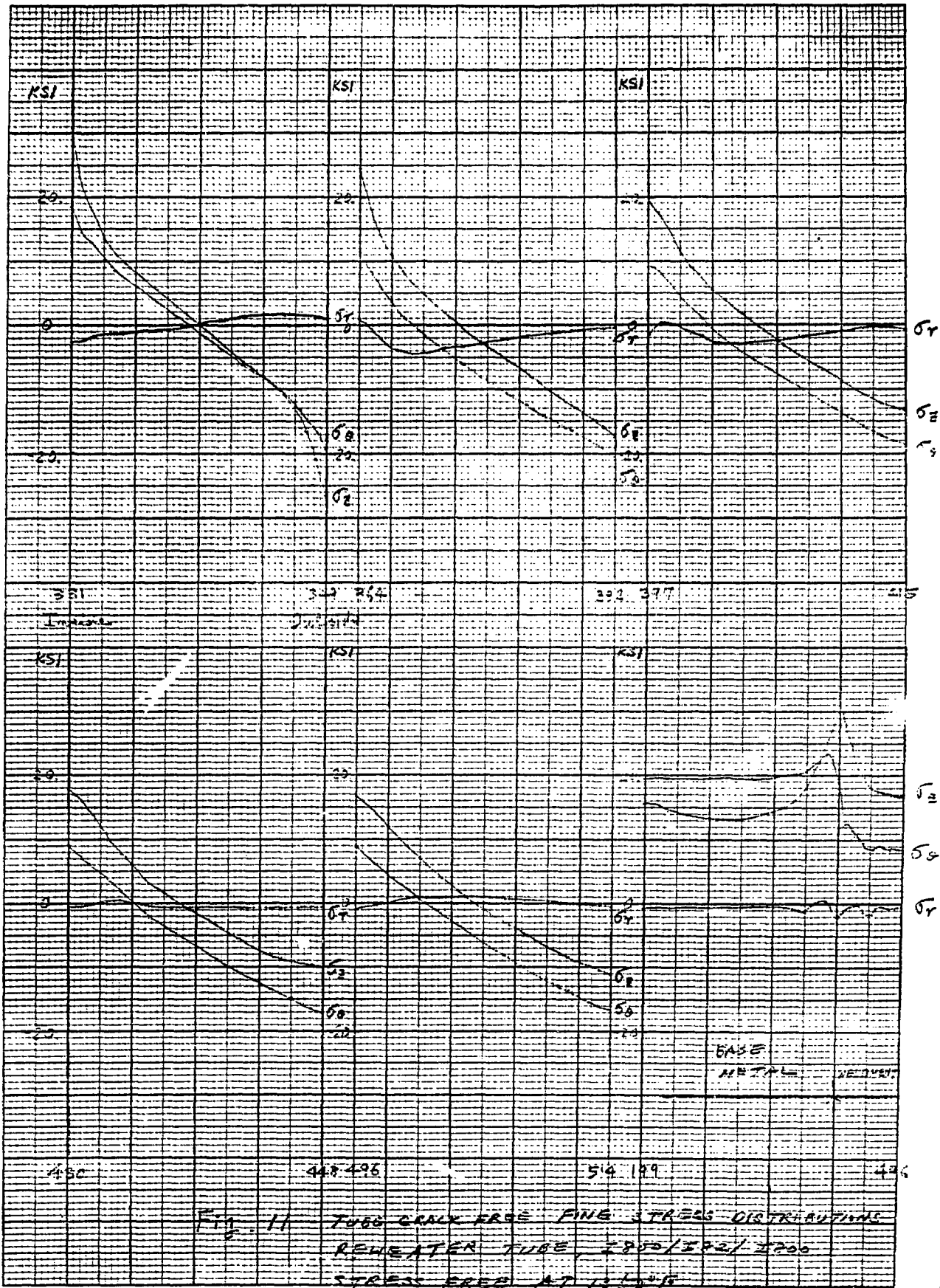


Fig. 11 1000 GRAIN FREE FINE STRESS DISTRIBUTIONS
REHEATED TUBE 2200/2202/2206
STRESS FREE AT 1050°F

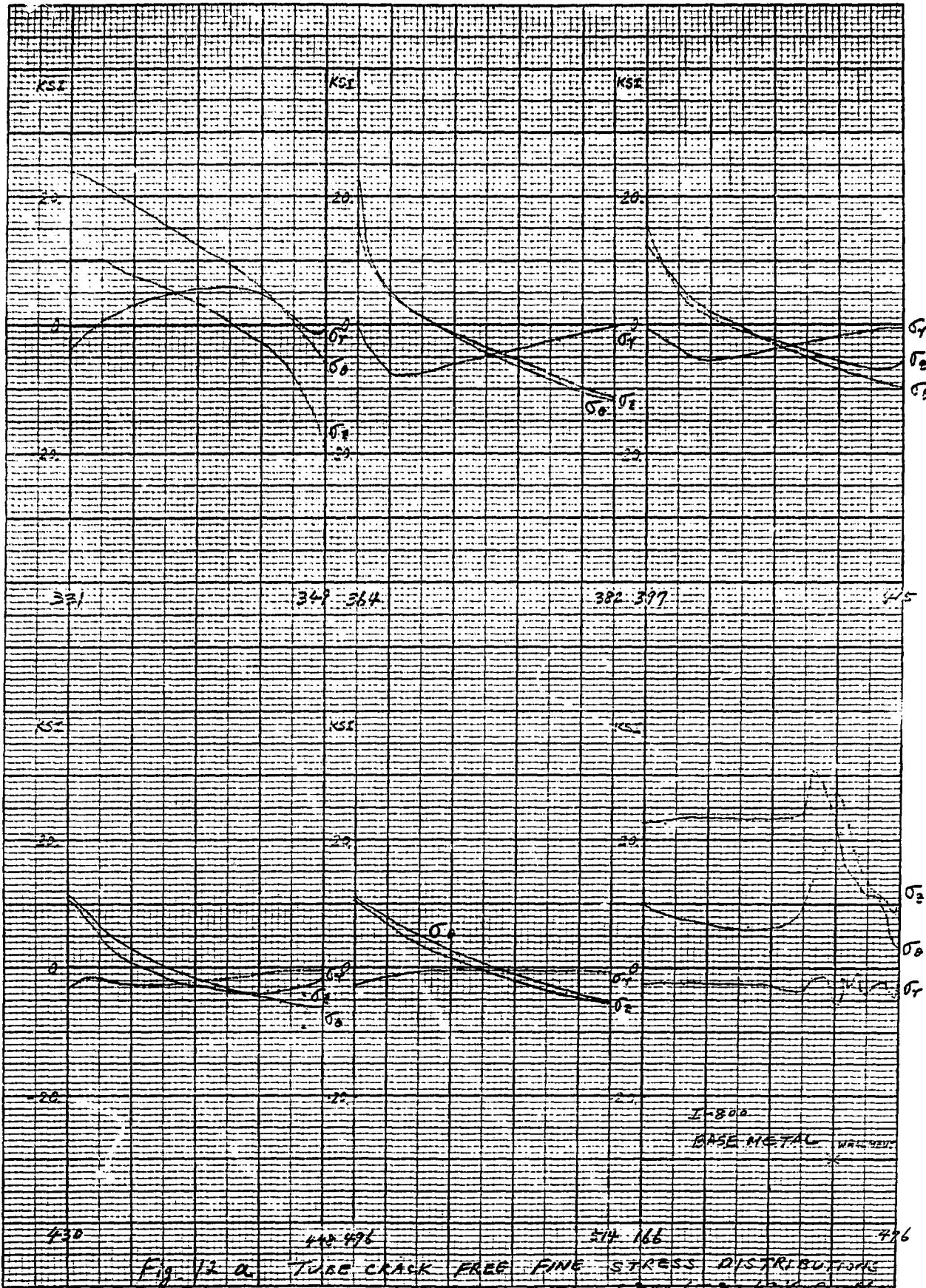


Fig. 12 a
 TUBE CRACK FREE FINE STRESS DISTRIBUTIONS
 SUPER HEATER TUBE, I-800 / 2 3/4" / 2 1/2" CT
 STRESS FREE AT 1350°F

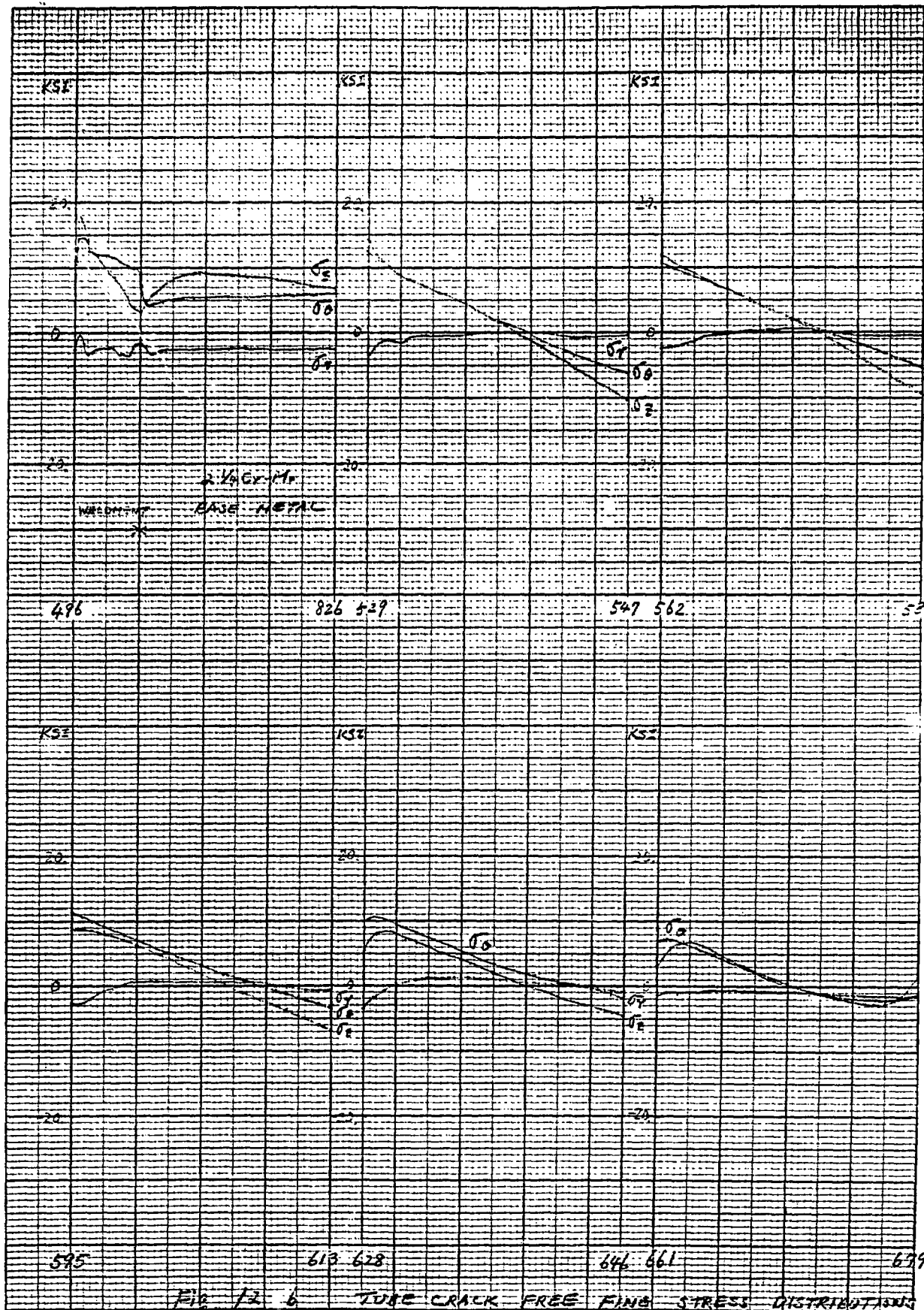
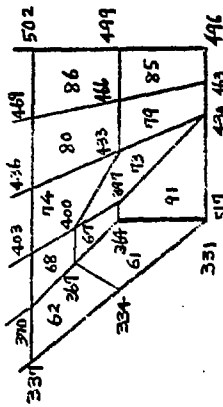
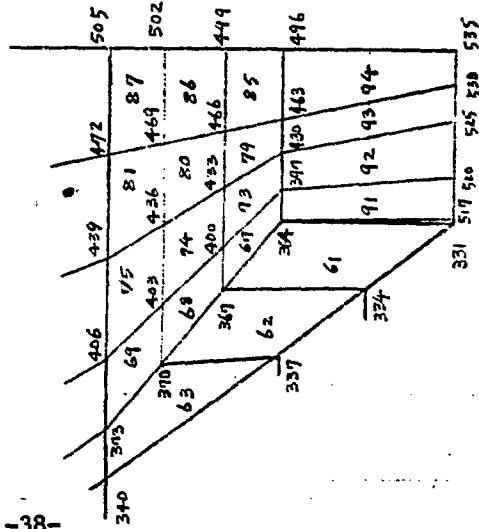


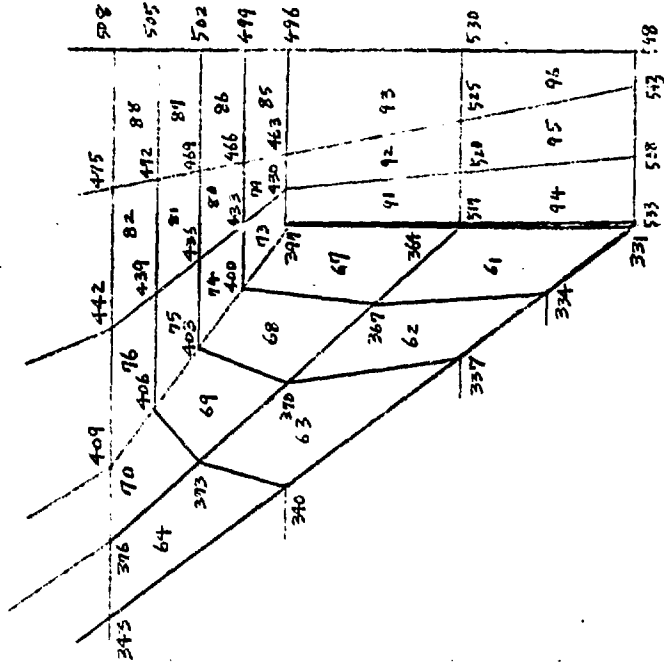
Fig. 12-6 TUBE CRACK FREE FINE STRESS DISTRIBUTION
 SUPERHEATER TUBE, I800/I82/2 1/4 Cr-Mn
 -37- STRESS FREE AT 1350°F



(a) $a/c = 0.0/2.5$
 where a = crack length
 t = wall thickness



(b) $a/c = 0.025$



(c) $a/c = 0.5$

Figure 13. Idealizations for Fracture Mechanics Calculations

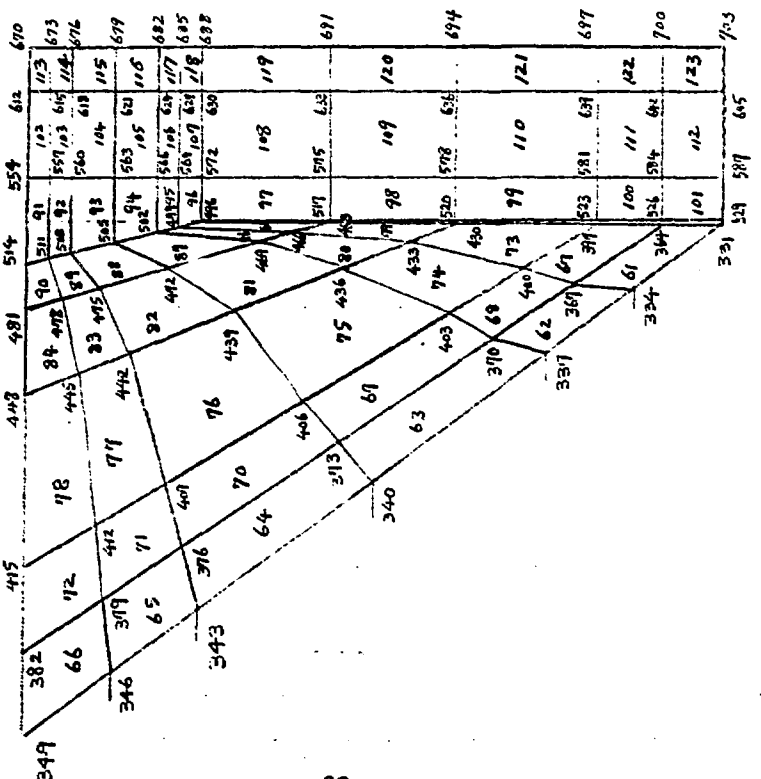
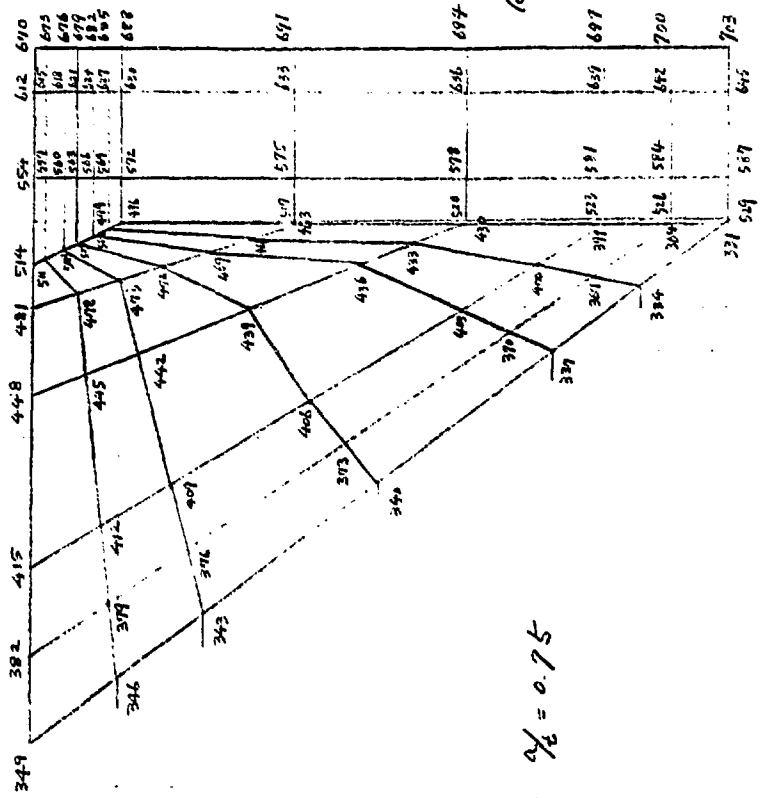


Figure 13. Idealization for Fracture Mechanics Calculation

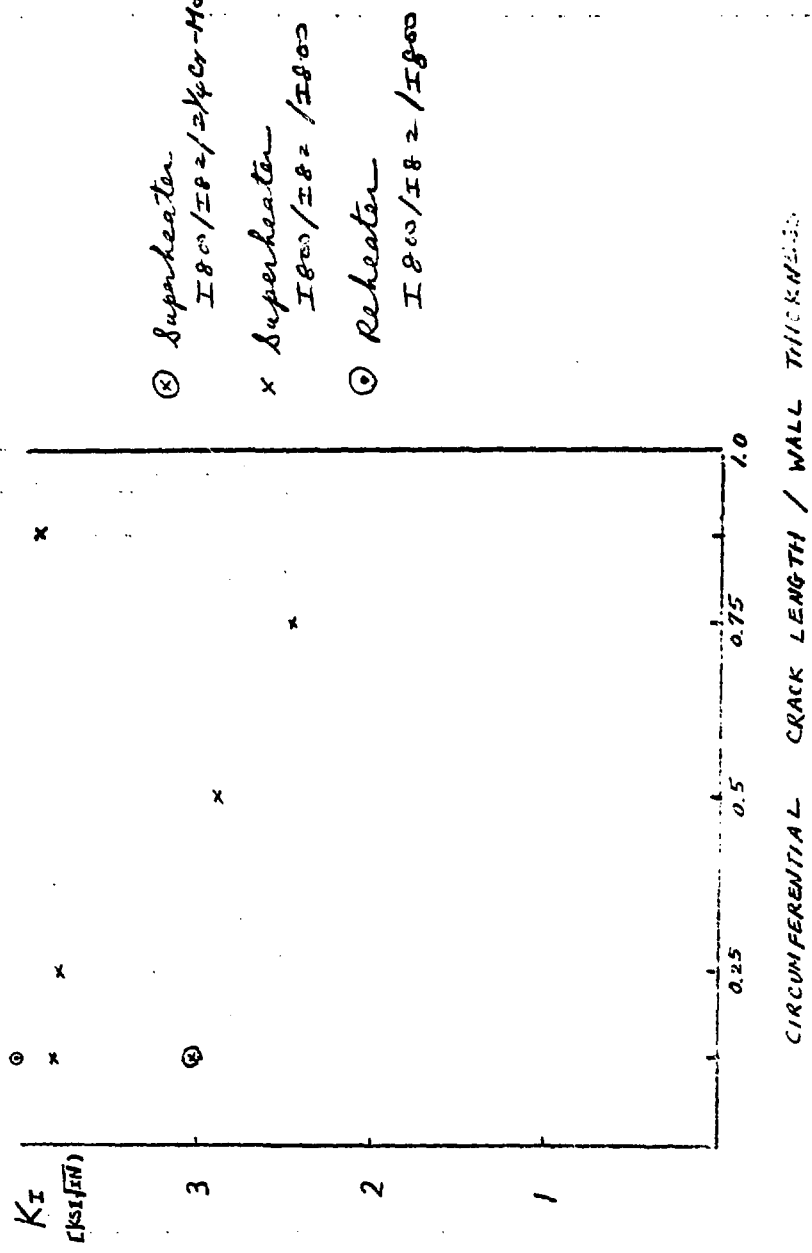


Fig. 14 Computed Stress Intensity Factors at Operating Temperature, Stress Free at 1350°F

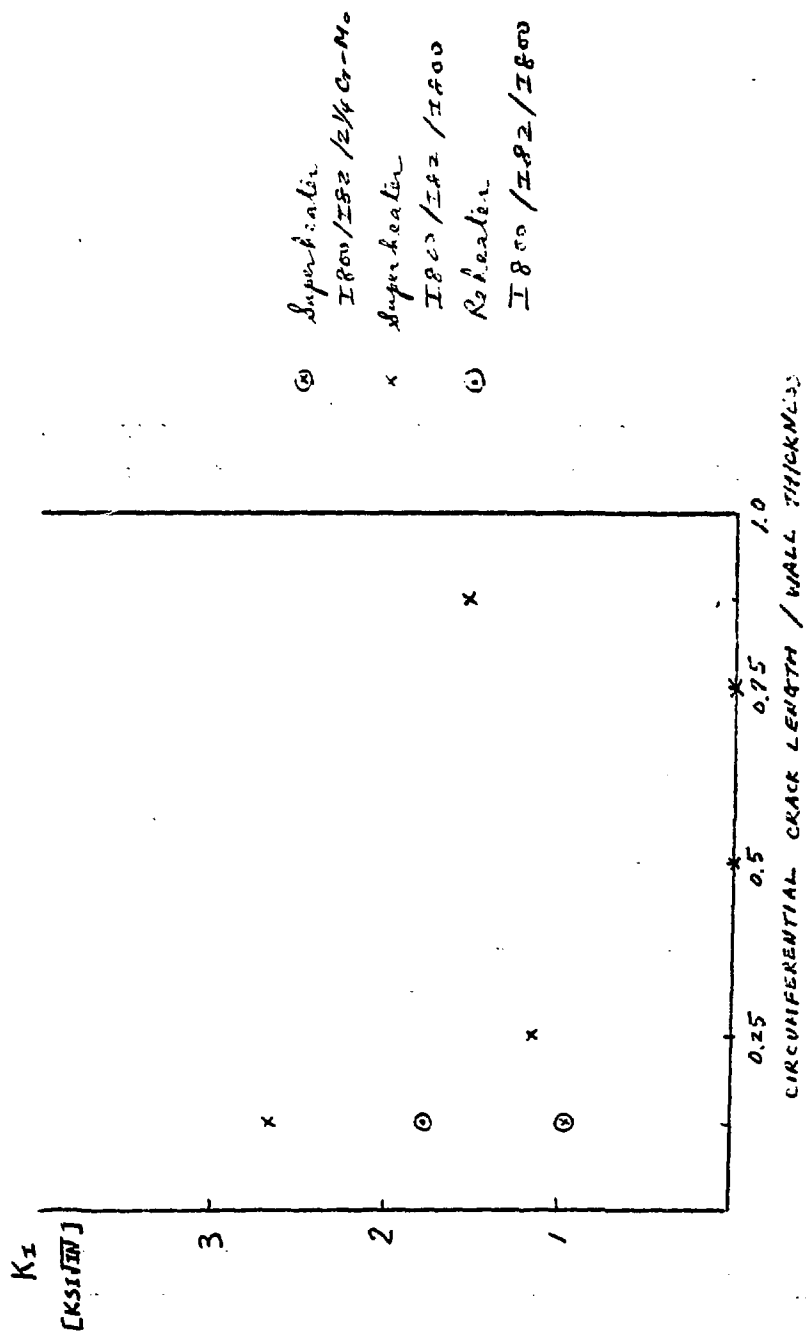


Fig. 15 Computed Stress Intensity Factors at Room Temperature, Stress Free at 1350°F

APPENDIX

Validation of Apes Fracture Mechanics Calculation

The APES computer code was modified to increase the number of nodes (400 → 1500), elements (50 → 200) and nodes across the front (40 → 80), and to include the capability of analyzing thermal loading conditions. The code was checked out with the supplied, as well as newly developed test problems. The validation of the APES fracture mechanics calculations is described below.

1. Single-Edge Crack in a Compact Tension Specimen

The problem of a plate 5 units square and containing a central edge crack of length $a=1.5$ units was considered. The problem is symmetric about the crack line and treated as a plane-strain problem. The reference value for K for this specimen geometry is 4.016. The APES results for K (4.096 and 4.080) shown in Figure A-1 and A-2 were 2.0 percent higher than the reference value.

2. 45-Degree Slant Crack in a Tension Specimen

This example is a combined node plane strain problem. The geometry is shown in Figure A-3 and A-4 along with the finite element idealizations used. The computed results for K_I and K_{II} are also shown in the Figures. The reference values of K_I and K_{II} are 1.86 and 0.88. The results shown in Figure A-3 indicate that the value of K_I varied from 4.0 percent below to 2.4 percent above the reference value and that the value of K_{II} varied from 2.9 percent below to 1.4 percent above the reference value. For an isothermal loading case the values of K_I and K_{II} are zero as expected.

The results using a core crack tip element are shown in Figure A-4. They indicate that the value of K_I was 24 percent below the reference value and K_{II} was 30 percent below the reference value.

3. Cracked Bar

This problem was a bar with a circumferential crack with a depth of half the bar radius. The results shown in Figure A-5 indicate that the value of K was 7.6% below the reference value.

4. Cylinder with Internal Circumferential or Internal Axial Cracks

For the analysis cylinders with internal circumferential or internal axial cracks were evaluated. Loading conditions similar to those used for the Incoloy-800 superheater tube were used. The results shown in Figure A-6 and A-7 were compared with those developed below using the weight function concept.

The crack opening stress $\sigma(x)$ can be approximated by a polynomial.

$$\sigma(x) = \sigma_0 + \left\{\frac{x}{B}\right\} \sigma_1 + \left\{\frac{x}{B}\right\}^2 \sigma_2 + \dots + \left\{\frac{x}{B}\right\}^m \sigma_m \quad (1)$$

For a crack subjected to a node I pressure $\sigma(x)$, the stress intensity factor can be calculated when the weight function $m\left(\frac{x}{a}, \frac{a}{B}\right)$ is known for the crack geometry.

$$K = \sqrt{\frac{2}{\pi}} \int_0^a \frac{m\left(\frac{x}{a}, \frac{a}{B}\right)}{\sqrt{a-x}} \sigma(x) dx \quad (2)$$

Substituting Eq. (1) into Eq. (2) yields

$$K = \sqrt{\pi a} \left[\sigma_0 i_0 \left(\frac{a}{B}\right) + \left\{\frac{a}{B}\right\} \sigma_1 i_1 \left(\frac{a}{B}\right) + \dots + \left\{\frac{a}{B}\right\}^m \sigma_m i_m \left(\frac{a}{B}\right) \right] \quad (3)$$

and

$$i_j \left(\frac{a}{B}\right) = \frac{1}{\pi} \sqrt{\frac{2}{a}} \int_0^a \left\{\frac{x}{a}\right\}^j \frac{m\left(\frac{a}{a}, \frac{a}{B}\right)}{\sqrt{a-x}} dx \quad (4)$$

The value of i_j can be calculated exactly or by using an appropriate numerical method. The limit values of i_j ($\frac{a}{B}$) when $a/B \rightarrow 0$ are the same for all cracks in all structures, that of a crack in a semi-infinite half plane.

These limit values can be calculated using Eq. (4) and the following Equation (5).

$$m \left\{ \frac{x}{a}, 0 \right\} = 1 + 0.6147 \left\{ 1 - \frac{x}{a} \right\} + 0.2502 \left\{ 1 - \frac{x}{a} \right\}^2 \quad (5)$$

Limit values of i_j when $a/B \rightarrow 0$ were taken from J. Heliot and J. Vagner.*

j	0	1	2	3	4
$i_j(0)$	1.12	0.687	0.528	0.446	0.389

For the present analysis ($a/B = 0.125$ and $R/B = 1.825$) i_j (a/B) can be approximated to be $i_j(0)$. Thus Eq. (3) can be given by

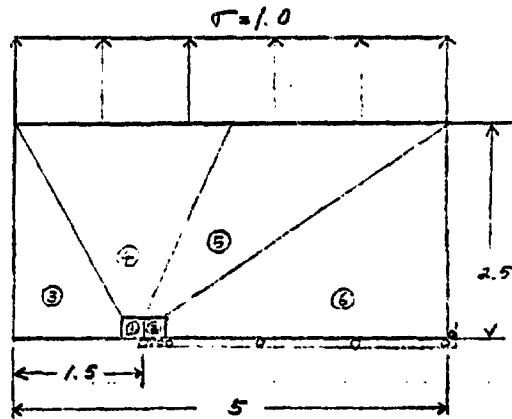
$$K = \sqrt{\pi a} \left\{ 1.12 \sigma_0 + 0.687 \left\{ \frac{a}{B} \right\} \sigma_1 + 0.528 \left\{ \frac{a}{B} \right\}^2 \sigma_2 + 0.446 \left\{ \frac{a}{B} \right\}^3 \sigma_3 + 0.389 \left\{ \frac{a}{B} \right\}^4 \sigma_4 \right\} \quad (6)$$

The stress distribution predicted with the present model is shown in Figure A-8 for an approximate stress function.

The stress intensity factor can then be calculated using Eq. (6). The results are shown below as well as the values predicted with APES.

* Heliot, J. and Vagner, J., "Use of the weight function concept and the crack closing method for calculating stress intensity factors in plane or axisymmetric problems," MS 241, ICF 4, Waterloo, 1977.

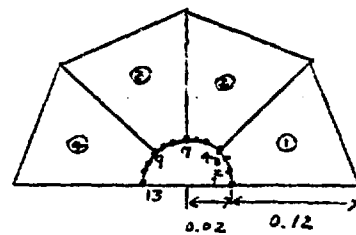
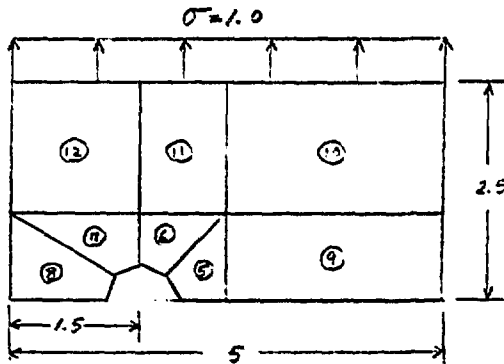
		Reference Value	APES Value
Circumferential Crack	Plane Strain	100,000	109468.5
	Axisymmetric	32,266	32544.8
Axial Crack	Plane Strain	3,355	3589.7



$$K_{Ref} = 4.016^*$$

$$K = 4.09653$$

Figure A-1. Idealization of Single-Edge Crack by Enriched QUAD-12 Elements

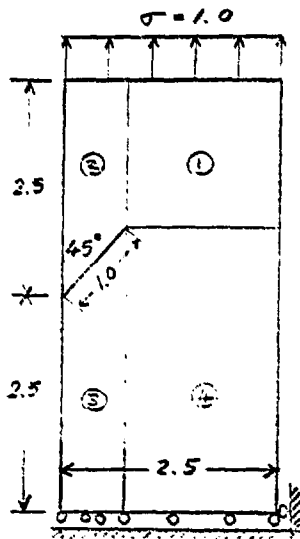


$$K_{Ref} = 4.016$$

$$K = 4.0793$$

Figure A-2. Idealization of Single-Edge Crack by Core Element with 13 nodes

* Bowie, O.L. and Neal, D.M., "Single Edge Crack in a Rectangular Tensile Sheet," J. Appl. Mech., Vol. 32 (1965)



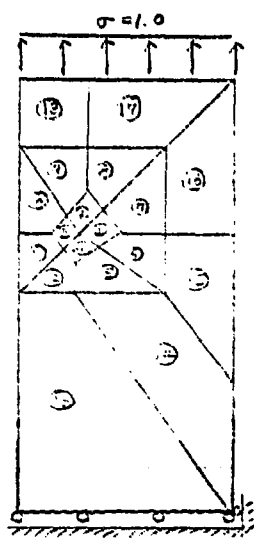
4 Elements

$$K_1 = 1.78636$$

$$K_2 = 0.85406$$

Ref: $K_1 = 1.86$

$$K_2 = 0.83$$

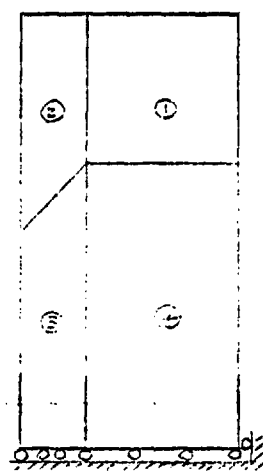


18 Elements

$$K_1 = 1.20533$$

$$K_2 = 0.89196$$

Isothermal Temperature

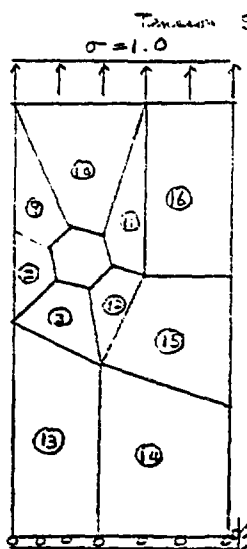


4 Elements

$$K_1 = 0.00001$$

$$K_2 = -0.00000$$

Figure A-3 Idealization of 45-Degree Slant Crack in



$$K_1 = 1.4161$$

$$K_2 = 0.6182$$

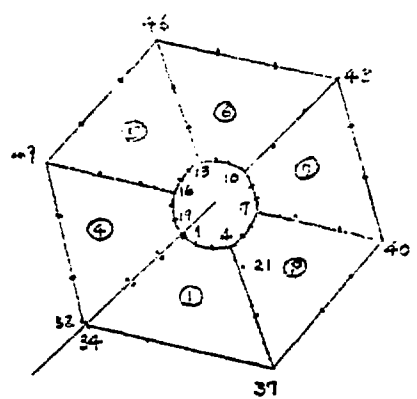


Figure A-4 Idealization of Single-Edge Crack by Core Element with 19 Nodes

Bowie, O.L., "Solution of Plane Crack Problems by Mapping Techniques," "Mechanics of Fracture," Vol. 1, pp. 1-55, Noordhoff, Leyden, Netherlands (1973).

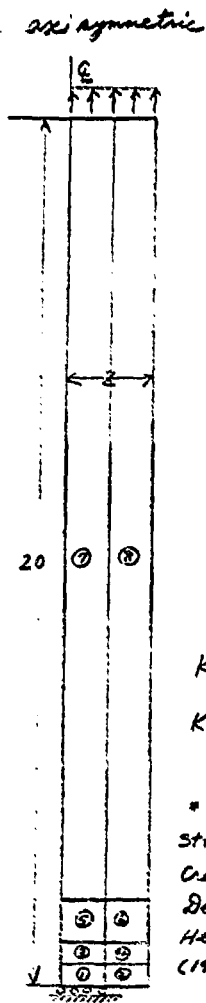


Figure A-5 Idealization of Cracked Bar

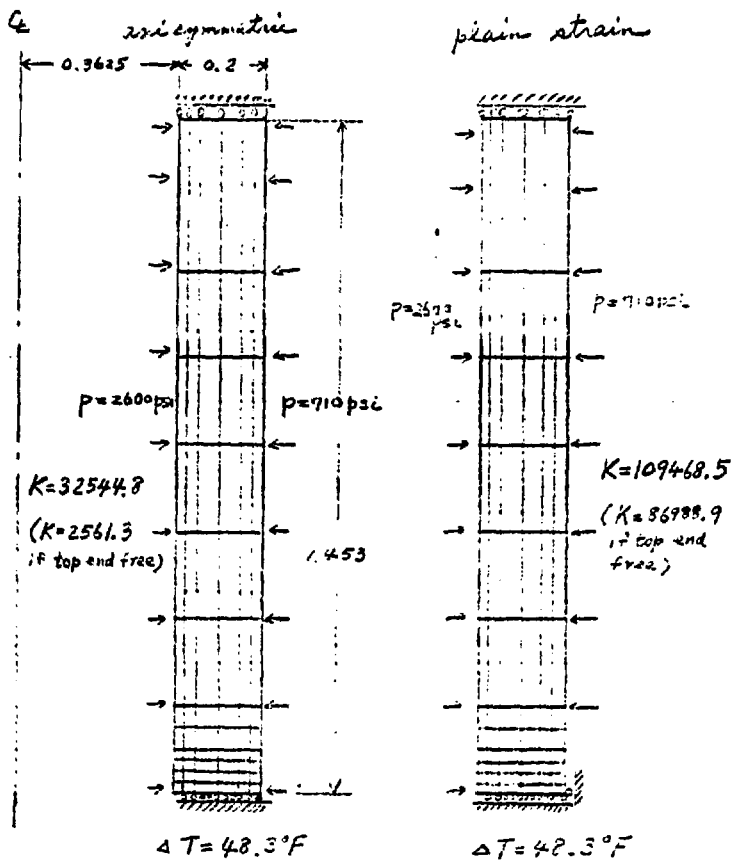


Figure A-6 Idealization of Cylinder with Internal Circumferential Crack

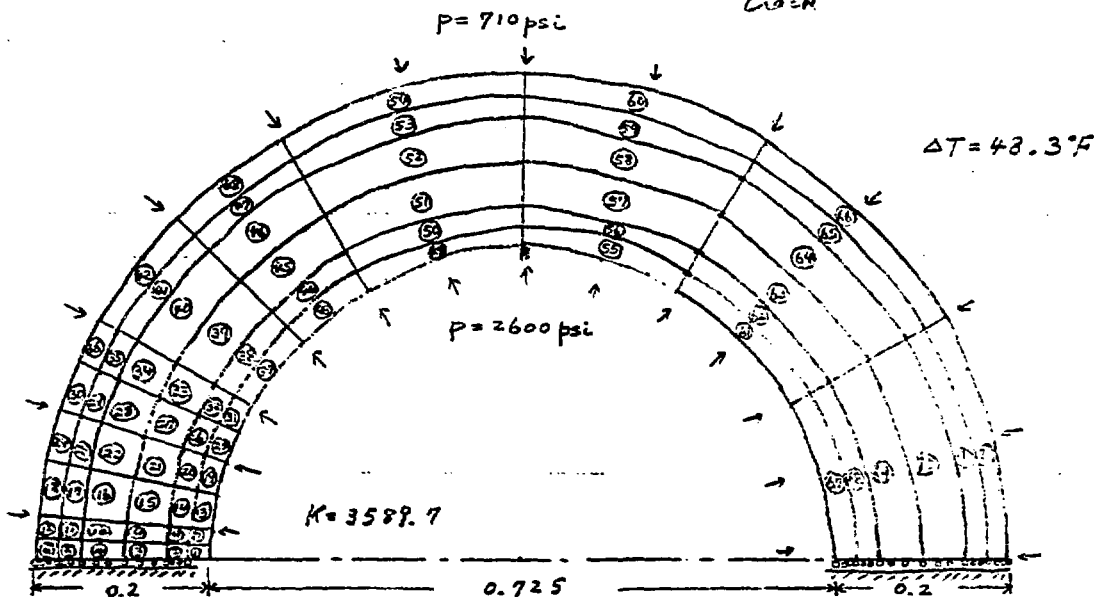


Figure A-7 Idealization of Cylinder with Internal Axial Crack

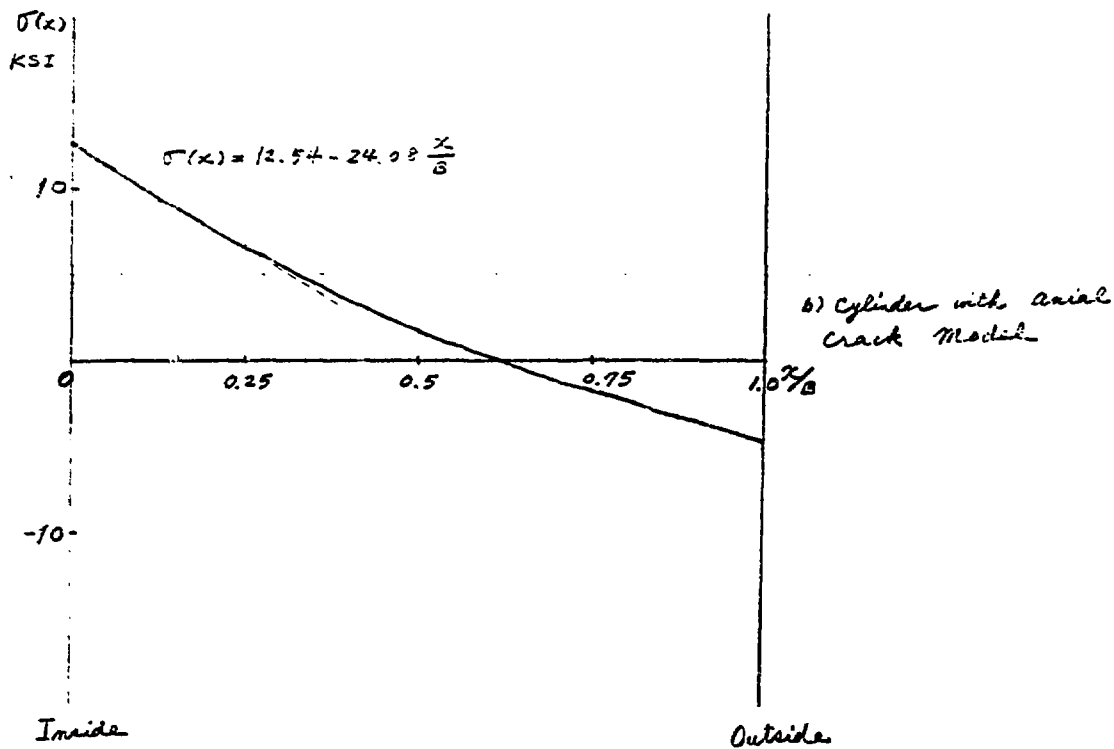
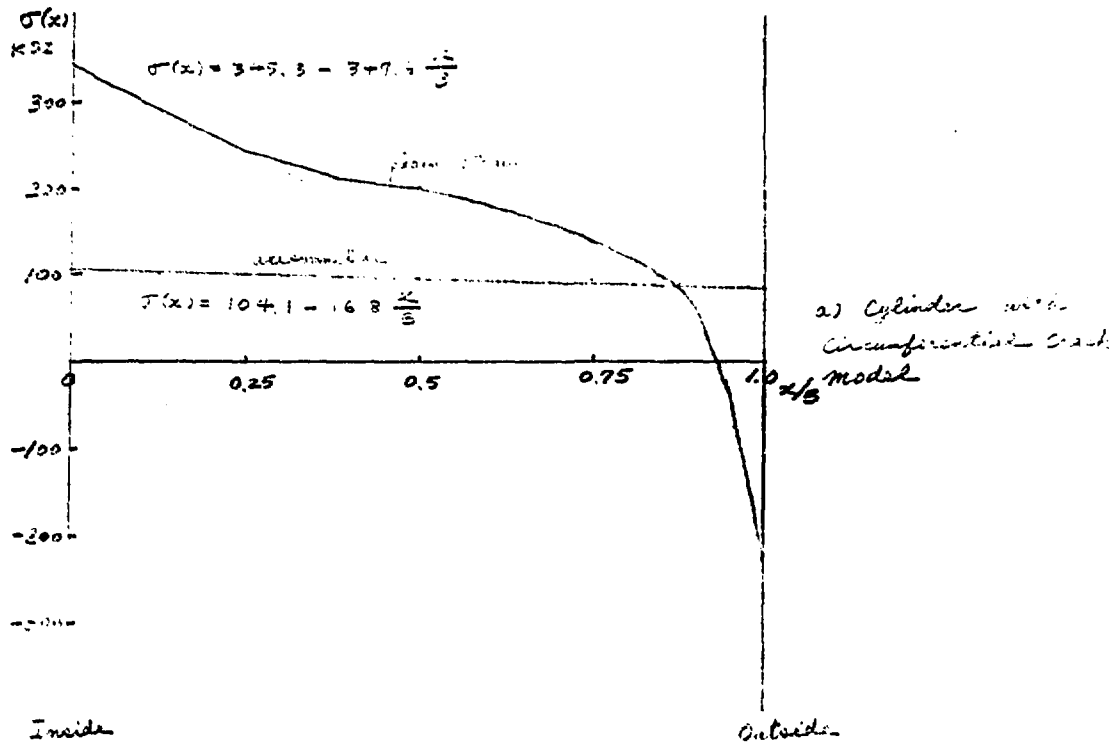


Figure A-8 Crack Opening Stress $\sigma(x)$ Distribution