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Волошин М.Б.

О СВОЙСТВАХ 1^1D_2 УРОВНЯ ЧАРМОНИЯ

A b s t r a c t

Theoretical estimates of the radiative and annihilative decays of the 1^1D_2 charmonium level are discussed within the framework of the nonrelativistic potential model of charmonium. It is argued that the state $\chi(3.45)$ possibly seen in ψ' radiative decay cannot be identified with the 1^1D_2 charmonium level.

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1. Introduction

It is well known[1] that the interpretation of the state at 3.455 GeV possibly observed in ψ' radiative decays as 2^1S_0 charmonium level (η_c') encounters a lot of troubles because of the experimental bound[2]

$$B(\chi(3.45) \to J/\psi + \gamma) > 25\% , \qquad (1)$$

while in the standard nonrelativistic charmonium model one expects[1]

$$B(\eta_c' \to \psi\gamma) \lesssim 1\% .$$

Harari[3] has made a proposal that the bound (1) can be accomodated if $\chi(3.45)$ is the 1^1D_2-level of charmonium. The width of the annihilative decay of 1^1D_2 into hadrons is determined as well as that of η_c' by the two-gluon annihilation rate[4]. However, in case of a D-state the latter is heavily suppressed because of vanishing of the D-state quark wave-function at the origin. The radiative decay $^1D_2 \to J/\psi+\gamma$ is also strongly suppressed but the hope expressed in[3] is that the latter suppression is less strong than the former one, so that the branching ratio (1) would be of the right order of magnitude.

Below some estimates will be presented concerning the widths of various decay modes of a $^1D_2$ charmonium level, from which it can be inferred that the expectations of Ref. [3] are not substantiated, i.e. that the suppression of the $^1D_2$ decay into $J/\psi+\gamma$ seems even more strong than that of the annihilative decays. Besides that it will be shown that the decay of 1^1D_2 into $1^1P_1+\gamma$ which was not considered in

[3] at all might turn out to be the dominant decay mode of the 1^1D_2-state. In spite of the fact that it seems to be impossible to identify the 1^1D_2-state with either of the charmonium levels observed so far, an investigation of this level as well as others of paracharmonium ($^1P_1$, ^1S) promises to be very instructive for understanding the dynamics of charmonium.

2. $^1D_2$-Level Annihilation into Hadrons

Using the simple technique presented in [5] one can estimate the rate of a $^1D_2$-state annihilation into two gluons. However, it is simplier to calculate first the two-photon annihilation rate of a positronium-like $^1D_2$-state i.e. with the unit charge and with no colour, and then convert the result thus obtained into the charmonium annihilation into gluons [5,6]. Since the positronium D-wave eigenfunction vanishes at the origin as r^2 the amplitude of the $^1D_2$-state annihilation is given by terms of the order $(v/c)^2$ in the expansion of $e^+e^- \to 2\gamma$ amplitude by powers of v/c (v being the electron velocity in the c.m. frame).

Neglecting the terms describing the $(v/c)^2$ corrections to a $^1S_0$-state annihilation one arives at the following expression for the amplitude of $^1D_2 \to 2\gamma$ annihilation

$$A = i\,\frac{4\pi\alpha\sqrt{2}}{m^3}\,\chi_0\,\varepsilon_{ikl}\,a_{1i}\,a_{2k}\,n_l\left((\vec{p}\vec{n})(\vec{p}\vec{n})-\frac{\vec{p}^{\,2}}{3}\right) \qquad (2)$$

where $\vec{p}$ is the electron momentum in the c.m.frame, m is its mass, $\vec{a}_{1,2}$ are the photon polarization vectors, $\vec{n}$ is the unit vector directed towards one of the photons' momentum and χ_0 is the pseudoscalar spinor part of the $^1D_2$

wave function ($|\chi_0|^2 = 1$). The coordinate D-state eigenfunction has the form

$$\psi^{(D)}_{mn}(\vec{r}) = \sqrt{\frac{15}{8\pi}}\left(r_m r_n - \frac{1}{3} r^2 \delta_{mn}\right) \frac{R_D(r)}{r^2}\chi_0$$
$$\left(\int_0^\infty |R_D(r)|^2 r^2 dr = 1\right). \tag{3}$$

From where one gets

$$\Gamma(^1D_2^{(e^+e^-)} \to 2\gamma) = \frac{\alpha^2 |R_D''(0)|^2}{m_e^6} \tag{4}$$

and converting this into charmonium annihilation rates one finds

$$\Gamma(^1D_2^{(c\bar{c})} \to 2\gamma) \simeq 3\alpha^2 Q_c^4 \frac{|R_D''(0)|^2}{m_c^6} \tag{5}$$

$$\Gamma(^1D_2 \to 2\,gluons) \simeq \frac{2}{3}\alpha_s^2 \frac{|R_D''(0)|^2}{m_c^6} \tag{6}$$

where α_s is the quark-gluon coupling constant which as determined from an analysis of J/ψ-meson decays is equal to $\alpha_s \simeq 0.2$. The numerical values of the estimates depend on the form of the potential. The harmonic oscillator potential model [5] on one hand in all other cases gives reasonable results close to those obtained in the linear potential model [6,7,8] and provides with estimates calculable in an analytic form, on the other. In this model one readily finds

$$|R_{1D}''(0)|^2 \simeq \frac{64}{15}\left(\frac{\lambda^7}{\pi}\right)^{1/2} \simeq 6.1\times10^{-2}\ \mathrm{GeV}^7 \tag{7}$$

($\lambda \simeq$ 0.35 GeV^2 [5]).

The same model requires $m_c \simeq 2.3$ GeV, so that the rate of 1^1D_2 annihilation into two gluons can be estimated as

$$\Gamma(1^1D_2 \to \text{hadrons}) \simeq 10 \text{ keV}. \tag{8}$$

On the other hand, if one substitutes in (6) $M_D/2 \simeq 1.73$ GeV instead of m_c (as it is usually done in Ref.[6]) then one arrives at

$$\Gamma(1^1D_2 \to \text{hadrons}) \simeq 70 \text{ keV} \tag{9}$$

(It is worth reminding that in the nonrelativistic model it is impossible to distinguish m_c and $M/2$).

In spite of the roughness of such estimates the numerical values obtained seem to be quite reasonable. It can be also noted that in Ref. [9] from dispersion relations and the asymptotic freedom of Quantum Chromodynamics it was obtained that

$$\Gamma(1^1D_2 \to \text{hadrons}) \simeq 80 \div 120 \text{ keV if } M_D \simeq 3.45\text{-}3.5 \text{ GeV},$$

which is in a good agreement with the estimate (9).

3. Magnetic Radiative Transitions Involving 1^1D_2 - Level

In this section we shall first consider the radiative transition $\psi' \to {}^1D_2\gamma$ and then we shall proceed to a discussion of the radiative decay of the $^1D_2$ level. The Hamiltonian which governs the magnetic transitions between the spin triplet charmonium levels ($^3S_1$, $^3D_1$) and the $^1D_2$-level has the form

$$\mathcal{H} = -\mu(\vec{\sigma}_1 - \vec{\sigma}_2)(\vec{H}(\vec{r}_1) + \vec{H}(\vec{r}_2))/2 \tag{10}$$

where $\vec{H}(\vec{r})$ is the magnetic field of the photon emitted in the transition, $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the c- and $\bar{c}$-spin operators and

$$\mu^2 \simeq \frac{Q_c^2 \alpha}{4 m_c^2}.$$

If ψ' were a pure $^3S_1$ charmonium state, the width of the decay $\psi' \to {}^1D_2 + \gamma$ would have been determined by the following formula

$$\Gamma(2\,^3S_1 \to 1\,^1D_2 + \gamma) = \frac{8\pi\mu^2}{3}\omega^2 |F|^2 \simeq \frac{80}{27}\alpha \frac{\omega^3}{m_c^2}|F|^2 \qquad (11)$$

where ω is the photon energy and

$$F = \int_0^\infty R_{2S}(r) R_{1D}(r) j_2\left(\frac{\omega r}{2}\right) r^2 dr \simeq$$
$$\simeq \frac{\omega^2}{60}\int_0^\infty R_{2S}(r) R_{1D}(r) r^4 dr \qquad (12)$$

$$\left(j_2(x) = 3(\sin x - x\cos x)/x^3 - \sin x / x \right)$$

In the harmonic oscillator model one has

$$F \simeq -\frac{\omega^2}{4\lambda}\frac{1}{3}\sqrt{\frac{2}{5}}$$

Therefore if $m(^1D_2) = 3.45$ GeV ($\omega \simeq 240$ MeV) one obtains

$$\Gamma(2\,^3S_1 \to 1\,^1D_2 + \gamma) \simeq \left(\frac{2}{3}\right)^5 \alpha \frac{\omega^3}{m_c^2}\left(\frac{\omega^2}{4\lambda}\right)^2 \simeq 4\,eV.$$

However it was argued elsewhere (see, e.g. [5]) that ψ' can contain an essential admixture of a $^3D_1$-state. (The mixing angle may reach 0.2-0.25). For a $^3D_1$-state the transition rate into $^1D_2 + \gamma$ is given by

$$\Gamma(^3D_1 \to {}^1D_2 + \gamma) = \frac{16}{3} \mu^2 \omega^3 |M|^2$$

where

$$M \simeq \int R_{^3D_1}(r)\, R_{^1D_2}(r)\, r^2 dr \simeq 1$$

and here it is assumed that the spin-dependent interaction does not spoil essentially the coordinate wave functions. If $\omega \simeq 240$ MeV one finds

$$\Gamma(^3D_1 \to {}^1D_2 + \gamma) \simeq \frac{16}{21} \alpha \frac{\omega^3}{m_c^2} \simeq 10 - 20 \text{ KeV}$$

where the lower numerical value refers to the case $m_c \simeq 2.3$ GeV while the upper one corresponds to $m_c \simeq 1.64$ GeV [8]. Thus in case the admixture of the $^3D_1$ wave function in ψ' does reach 0.2-0.25 the decay rate of ψ' into $^1D_2 + \gamma$ may reach about 1 keV and the decay can be accessible for an experimental study.

For the J/ψ -meson the $^3D_1$-admixture should be much less than that in ψ' because of a larger energy denominator determining the mixing angle. Therefore the transition rate of $^1D_2 \to J/\psi + \gamma$ decay can be obtained from eqs.(11) and (12) where one should only allow for the difference of the statistical weights of the $^1D_2$ and J/ψ. The result is

$$\Gamma(1\,^1D_2 \to J/\psi + \gamma) \simeq 16 \mu^2 \omega^3 |F|^2 .$$

In the harmonic oscillator model for 1D → 1S transition the overlap integral F is given by

$$F \simeq \frac{\omega^2}{8\sqrt{15}\,\lambda}$$

from where we estimate ($\omega \simeq$ 350 MeV)

$$\Gamma(1^1D_2 \to J/\psi + \gamma) \simeq \frac{4}{135}\,\alpha\,\frac{\omega^3}{m_c^2}\left(\frac{\omega^2}{4\lambda}\right)^2 \simeq 13-25\,eV$$

(m_c= 2.3 - 1.6 GeV).

4. The Decay $^1D_2 \to {}^1P_1\gamma$

The decay $^1D_2 \to {}^1P_1\gamma$ is a usual E1 transition and it is not suppressed. The width of this transition is given by

$$\Gamma(1^1D_2 \to 1^1P_1 + \gamma) \simeq \frac{4}{3} Q_c^2 \,\alpha\, \omega^3 \cdot \frac{2}{5}\,|I|^2 \qquad (13)$$

where

$$I = \int R_{1P}(r)\, R_{1D}(r)\, r^3\, dr .$$

In the harmonic oscillator model $I = \sqrt{\frac{5}{2\lambda}}$, hence the decay width can be parametrized as follows

$$\Gamma(1^1D_2 \to 1^1P_1 + \gamma) \simeq \frac{16}{27}\,\alpha\,\omega^3/\lambda \simeq 12\,KeV\left(\frac{\omega}{100\,MeV}\right)^3 . \qquad (14)$$

The numerical value of the rate depends heavily on the mass spacing of $^1D_2$ and $^1P_1$ states. In the oscillator model the spacing is one half of that of ψ' and J/ψ , i.e. $\omega \simeq$ 300 MeV. In this case the decay width is about 300 keV. In the linear potential model the spacing should be less but the matrix element I is expected to be larger, so that in either case the estimate

$$\Gamma(1^1D_2 \to 1^1P_1 + \gamma) \simeq 100\text{–}300\ \text{keV} \qquad (15)$$

seems to be quite moderate and reasonable, since that is the right order of E1 transition probability between modeless levels of a characteristic radius $R \sim (0.5\ \mathrm{GeV})^{-1}$ and with the available decay energy $\omega \sim$ 200-300 MeV. Thus, this decay mode should dominate over those considered above: $^1D_2 \to$ hadrons and $^1D_2 \to J/\psi + \gamma$.

5. Discussion of the Results

Thus, from the estimates of the previous sections it seems plausible that $B(1^1D_2 \to J/\psi + \gamma) \sim 10^{-3}$-$10^{-4}$. Hence, in spite of the roughness of our estimates it can be inferred that there is no way to identify the $\chi(3.45)$ state with the 1^1D_2 charmonium level. And the latter still awaits its discovery.

On the other hand, it can be noted that one would expect that

$$B(1^1D_2 \to 1^1P_1 + \gamma) \gtrsim 50\%.$$

Therefore one can imagine a peculiar cascade of γ-transitions

$$\psi' \to \gamma\, 1^1D_2 \to \gamma\gamma\, 1^1P_1 \to \gamma\gamma\gamma\, \eta_c \qquad (16)$$

where only the first transition is a "narrow path". As to the other two their branching ratios are thought to be $\gtrsim 50\%$ in case the estimates of Refs.[5,6] of the hadronic width of the 1^1P_1 charmonium level

$$\Gamma(^1P_1 \to 3\ \text{gluons} \to \text{hadrons}) \simeq 100\text{-}300\ \mathrm{KeV}$$

are correct. An observation of the cascade (16) would be a beautiful opportunity to study the paracharmonium levels.

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