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EARLY STAGES OF THE UNIVERSE

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~~My concern here will be to review~~ In a very simplified, descriptive way, the main trends of Phenomenological Cosmology are reviewed. ~~The word "phenomenological" stands here to distinguish the approach to be followed here from that more severe presented in Novello's Lecture.~~ ~~I will begin with~~ A sketchy view of the fashionable Standard Model[1,2] [illegible] and ~~then proceed to introduce~~ its most pretentious variant, the Symmetric Model, is then discussed.

I) The Standard Model

On a large scale (which could be characterized by lengths of a few hundreds of Megaparsec) the Universe appears to be isotropic and homogeneous. Its content may be described as matter (galaxies, intergalactic gas) immersed in an electromagnetic bath. Matter dominates the energy density nowadays, but we shall see that this was not always the case. Relativistic Cosmology applies Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad (1)$$

to the Universe as a whole.

The energy-momentum tensor is, for an isotropic ideal fluid,

$$T_{\mu\nu} = (p + \rho c^2) u_\mu u_\nu + p\, g_{\mu\nu} . \qquad (2)$$

This should give a fair approximation for times not enormously remote but interactions between the constituents have to be accounted for in stages where high densities appear. Equations (1) and (2) are a system of ten coupled partial differential equations for the components of the metric tensor, eight of which can be solved without great trouble to give the Robertson-Walker metric,

$$ds^2 = c^2\,dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right)\right] \quad (3)$$

(herein integration constants and units have been chosen as to make the expression simpler). The parameter k is restricted to the values 0 and ± 1, and R(t) is a scale function, related to the mass density ρ and pressure p by the two remaining equations:

$$\frac{d}{dt}\left(\rho c^2 R^3\right) + p\,\frac{dR^3}{dt} = 0 \quad (4)$$

$$H(t) \equiv \frac{1}{R}\frac{dR}{dt} = \sqrt{\frac{8\pi G\rho}{3} - \frac{kc^2}{R^2}} \quad (5)$$

Eq. (4) says that the total entropy is constant: it may be written as dE= - pdV and the thermodynamical identity dE= TdS - pdV implies dS=0.

The present value of the function H(t) is Hubble's constant, H_0, $H_0 \approx 50$ km Mpc^{-1} sec^{-1}. It is convenient to define the "critical density"

$$\rho_c = \frac{3H_0^2}{8\pi G} = 4.8\times 10^{-30}\left(\frac{H_0}{50}\right)^2 \, g\, cm^{-3} \quad (6)$$

From eq.(5) we may obtain the expression for the present density:

$$\rho_0 = \frac{3c^2}{8\pi G}\frac{k}{R_0^2} + \rho_c \quad . \tag{7}$$

Notice that the three-dimensional curvature scalar k/R_0^2 of the space section of the four-space defined by eq.(3) has its sign determined by the parameter

$$\Omega = \frac{\rho_0}{\rho_c} \quad . \tag{8}$$

If $\Omega > 1$, the curvature is positive and space is closed. If $\Omega \leq 1$ we have an open Universe. It is of course difficult to know the value of ρ_0 , because of the possible existence of up-to-now undetected material. A recent estimate[3], taking into account galaxy counts, outside limits on the densities of gas and dust, constraints on the number of black-holes and the age of the Universe, besides a few other indirect criteria, gives for Ω a value 0.06±0.02. This means that, to our present knowledge, the Universe is open.

An analysis of eqs.(4) and (5) shows that R(t) is a monotonous function of t and that it has necessarily a zero (in the past, because of the present day expansion). The instant for which R(t)=0 is usually taken as t=0. The history of the Universe is fixed by the functions ρ and p , for which the Standard Model takes

$$\rho = \rho_m + \rho_r \quad ; \quad p = p_m + p_r \tag{9}$$

, the sums of contributions from matter and radiation:

$$\left.\begin{aligned} \rho_r &= \frac{4\sigma}{c^3} T^4 \; ; \; \rho_m = n\left(m_p + \frac{3}{2} k T\right) \\ p_r &= \frac{1}{3}\rho_r c^2 \; ; \; p_m = \frac{3}{2} n k T \end{aligned}\right\} \quad (10)$$

Once these expressions are put into eqs.(4) and (5), one finds that when R(t) decreases ρ_r increases as R^{-4} and ρ_m as R^{-3}. So, for a time small enough ($t \lesssim 10^4$ years), radiation dominates. It is found that

$$RT = \text{constant} \quad (11)$$

and that, when radiation dominates,

$$kT = \left[\frac{3c^3}{128\pi G\sigma}\right]^{1/4} t^{-1/2} \quad (12)$$

The temperature increases indefinitely when $t \to 0$. Here something should be said on the composition of blackbody radiation at very high temperatures. Particles and their respective antiparticles are present if an appreciable amount of photons have energies above the pair creation threshold and their number is easily estimated(4). So, as temperature grows up, the energy density of the radiation is successively shared among the photons, electrons, muons, mesons and baryons. It turns out that the density is dominated by a different kind of particle for each temperature and this leads to a division of the early history of the Universe into eras. Each era is characterized by the prevailing type of particle and by the physical processes which they can experience.

1. <u>Initial period</u> ($t \lesssim 10^{-6}$ sec): density ($\gtrsim 10$[illegible]) and temperature ($kT \gtrsim 1$ GeV) are too large for any known physics to be applicable. There has been of course much speculation about this period, mainly related to the possible formation of primaeval inhomogeneities: Archeons, Planckions and other entities would be haunting the Universe at that time. We shall not discuss this subject here, however great its interest may be.[5]

2. <u>hadron era</u> ($t \lesssim 10^{-4}$ sec, $kT \gtrsim 100$ MeV): strongly interacting particles dominate the composition of universal radiation. The lack of a general theory for the strong interactions is a nasty handicap and the attempts to study this period are based on models. The preferred model has been[6,7,8] Hagedorn's, a statistical description of multiple production which met great fenomenological success[6]. It gives an equation of state for hadron matter, which is a great achievement. It predicts the density of hadron states to be of the form

$$\rho(E) = a\, E^{b}\, e^{E/kT_c} \quad , \qquad (1[illegible])$$

where all the parameters (a, b, T_c) are fitted to experimental results.

The partition function

$$Z(T) = \int_0^{\infty} dE\, e^{-\frac{E}{kT}} \rho(E)$$

diverges as T approaches the critical temperature $T_c \approx 2 \times 10^{12}\,{}^{0}K$, which so appears as an upper limit for all temperatures. If we accept this, while sticking to a

Friedmann universe even at such times, the system would initially be in a metastable state. An alternative would be to abandon Friedmann model or even classical General Relativity, which would delight some of the people who dislike the primaeval singularity.

The main interest in the study of hadron era is the possible existence of condensations which would be at the origin of galaxies and clusters.

Primaeval inhomogeneities could develop in this model(7) but the hadron era would last for larger than in other cases (maybe up to one year). This has the great inconvenient of making cosmic production of Helium quite improbable. Hagedorn's model is of course subject to criticisms(8). Some of its recent reformulations preserve its successful experimental predictions while dropping the existence of a highest temperature(9). To take a definite position about the subject is certainly premature.

3. <u>lepton era</u> ($t \lesssim 1$ sec, $kT \gtrsim 1$ MeV): electrons and muons are dominant. The large amount of energetic electrons make neutrons and protons to be in numerical equilibrium and the synthesis of light elements is possible. The original motivation for Gamow's proposal of what became the Standard Model was precisely Helium synthesis. He obtained a good number for ratio, $\frac{M_{He}}{M_{He}+M_H} \approx 0.25$, and this was its first and lone success up to the discovery of the background radiation in 1965. It should be noticed that at $kT \approx 10$ MeV the mass of the pairs contained in the radiation is of the same order as the present day mass in the Universe. Before that, the amount of the matter which is not in the radiation becomes more and more negligible and

the Universe is practically symmetric.

4. <u>radiative era</u> ($t \lesssim 10^6$ years, $kT \gtrsim 0.3$ eV): photons finally dominate the composition of the blackbody radiation. At $t \approx 10^4$ years matter becomes the main contributor to the total density. This period ends when recombination takes place. The mean free path of the photons becomes very large as matter neutralizes and they decouple to constitute the blackbody background. This solves a great mystery: why should the universal radiation to have a Planckian distribution

$$\rho(\nu)\,d\nu = \frac{8\pi h\nu^3\,d\nu}{c^3}\left[e^{h\nu/kT} - 1\right]^{-1} \tag{14}$$

which is valid for photons in thermal equilibrium with matter, when there is no such equilibrium today ? The answer lays on the existence of equilibrium in the past and on the fact that the distribution (14) is invariant under expansion, because of eq.(11). It is true that only the Rayleigh-Jeans sector of the spectrum has been firmly stablished.

For frequencies in the pure Planckian region, emission in the atmosphere make measurements difficult but several limits on the flux indicate clearly a deviation from Jeans' law in the good direction.

The prediction of the existence of the background radiation is the greatest achievement of the model. Moreover, it can also explain the formation of those light elements which stars seem unable to produce. The difficulties shown by Pacheco in his lecture are small as compared with the positive results. Nevertheless, despite its successes and above all its capacity to coherently arrange all the present-day observational results, the Standard Model is not quite above criticism. For instance, it gives no satisfactory interpretation for the origin of galaxies and clusters. Gravitation instability cannot explain them unless the initial fluctuations are supposed to have an ad hoc amplitude and to have appeared in a convenient epoch. Also the model has an extra free parameter, the entropy per baryon. This entropy is related to the ratio between the photon number density to the baryon number density, which is at present

$$\eta = \frac{n_B}{n_\gamma} = 7.0 \times 10^{-9}\ \Omega \left(\frac{H_0}{50}\right)^2 \quad , \tag{15}$$

through

$$\sigma = 3.6/\eta \quad . \tag{16}$$

Attempts to explain this high value for σ in the usual picture of the model have failed(10) and it remains as something like a universal constant.

It is also a source of uneasiness the fact that initially the baryon over antibaryon excess is negligible for all dynamical purposes and the Universe looked at first like a symmetric one. The excess which is seen today, if we suppose no galaxies to be formed by antimatter, is to be interpreted as coming from a rather arbitrary initial condition.

A certain disconfort, apparently felt only by some physicists, comes from the idea that the strict matter-antimatter symmetry of the equations of fundamental physics finds no correspondence in the large. It is true that no argument exist to suppose this global symmetry, which would mean that all additive quantum numbers (charge, baryon number, lepton numbers) vanish. Only the electric charge can be shown to be zero, by integrating $\nabla \cdot \vec{E} = \rho$ over the universe. For the other numbers, no analogous to Maxwell's equation are known and nothing can be said. All these difficulties, objective or not, are in principle removed by Omnès' model.

II. The Symmetric Model

Symmetric models have been proposed(11) and analysed since many years. I shall here concentrate in Omnès model(8), which seems nowadays to be the most reliable one. It preserves the advantages of the Standard model while trying to remedy the above mentioned drawbacks. It has one fundamental hypothesis there was only the blackbody radiation in the beginning, with all the particles and antiparticles which are its components at very high temperatures. Omnès proposed two independent mechanisms, one for creating primaeval condensations during the hadron era, another to make them grow into protogalaxies or protoclusters during the radiative period. The first mechanism is a phase transition, the second is a coalescence process.

Many arguments were advanced(12,13) for the existence of a phase transition in blackbody radiation at a few hundreds of MeV, by which it becomes an inhomogeneous fluid with a large numbers of regions of matter and antimatter separated by a contact layer where annihilation takes place. A model estimate of the critical temperature(14) is $kT \approx 300$ MeV. Below that, matter and antimatter tend to mix and would annihilate completely were they not kept apart by the pressure exerted by the annihilation products coming from the intermediate layer. This pressure generates important motions in the fluid. Every time two distinct regions of, say, antimatter meet they fuse and so the average size of the regions increase with time. Once this process of coalescence sets up, annihilation is so much reduced that the ratio η becomes nearly constant. Near the critical temperature η was of the order of unity and its decrease as a function of time can be approximately estimated. Coalescence is at work till the epoch of recombination, when the average mass of a region can be of the order of a galaxy mass. A detailed analysis leads to a new division of the early history of Universe into periods, which superpose to the above eras:

1) a separation period (10^{-6}sec$\lesssim t \lesssim 10^{-5}$, 1GeV$\gtrsim$kT$\gtrsim$ 300 MeV), at the end of which a typical condensation mass is $\approx 10^{6\pm1}$g;

2) an annhilation period ($t \lesssim 1.600$ sec, 300MeV$\gtrsim$ kT$\gtrsim$ 30 KeV), during which η decreases to its present day order of magnitude; at its end, a typical mass is $\approx 10^{15\pm2}$g;

3) a coalescence period (1.600 sec$\lesssim t \lesssim 10^{6}$ years, 30 KeV$\gtrsim$ kT$\gtrsim$0.3 e V); recombination is retarded in the symmetric model, due to the ionizing effect of X's rays coming from the thermalization of annihilation γ's ; there is an uncertainty about the events near the end of the period, but it seems clear that the typical mass can be a galaxy's or a cluster's.

Let us make a few comments on each of these periods.

1) The separation period:

Amongst the many attempts(2,7,15) made to find mechanisms for generating seed condensations which could be at the origin of the largest inhomogeneities in the Universe, we shall here only consider Omnès conjecture. Maybe some intuition of the phase transition would come from the following picture: the blackbody radiation, whose temperature is controlled by the metric through eq.(12), contains an enormous amount of baryons and antibaryons. These annihilate (mainly into pions) every time they meet each other, through an exotermic reaction. This would tend to increase the temperature, which is impossible. Then, the particles shall show an inclination to stand away from their antiparticles.

From a more elaborate point of view, this effect is a consequence of an statistical repulsion between nucleons and antinucleons which exists if the mesons are considered to be their bound states, as in old Fermi-Yang model. A feeling of it can be obtained by writting down the free energy of a gas containing mesons, nucleons and anti-nucleons up to the second order in the virial series. The virial coefficient related to N $\bar{N}$ scattering is given by Beth-Uhlenbeck formula[4]. For each partial wave, the contribution is

$$B_{IJ} \propto -\int e^{-p^2/kT} \frac{d\delta_{IJ}}{dp}\, dp \quad . \tag{17}$$

If the partial wave has a bound state, Levinson theorem implies a decreasing phase-shift and so a positive contribution to the virial coefficient. This is characteristic of a repulsive interaction. Of course, the second order is surely inadequate, as the densities are of the order of the nuclear matter. It is even possible that the whole series have no meaning, but we can forget this for a moment and study in detail what happens in this order. Usually, if a phase transition is found in this approximation, the higher orders are able to change drastically the critical point but not to erase the very existence of the phenomenon. By using phase-shifts obtained from a bootstrap model for N $\bar{N}$ interaction, the existence of the separation is stablished at a critical temperature $kT_c \cong 300$ MeV[14]. To have this result, it is essential to suppose that the mesons are themselves part of the thermal radiation, besides being N $\bar{N}$ bound states. Such a procedure is suggested by the S-matrix formulation of Statistical Mechanics[16], but it should be emphasized that no classical analogue to this phenomenon exists. It has been recently analysed from different points of view[17,18] and the issue is not clear. The correct behaviour of the phase-shifts could be found experimentally[19], but this could only [illegible] the [illegible] ner to confirm it.

However, suppose the complete equation of state to present the repulsive character exhibited above. In this case, the system [illegible] kind for which the separation has been rigorously proved[20]: a mixture of two species of particles with repulsion between different particles. This proof allows also a rough estimate of the critical temperature[18] and gives $kT_c \approx 400$ MeV Above this value, the system has the general aspect of an emulsion, with a great number of distinct regions of matter and antimatter.

These is of course no claim to any proof of the existence of the phase transition to have been given. For the time being, it remains a plausible conjecture. Any other mechanism to generate primaeval condensations at the hadron era should be complemented afterwards: the whole matter contained inside one particle's horizon at $t \approx 10^{-4}$ sec would have too small a mass ($\approx 10^{32}$ g). We shall only suppose that at this time the Universe is constituded by a fluid resembling an emulcion[21], with matter and antimatter standing apart and divided in a large number of "bubbles". The considerations which follow do not depend on how this situation has been arrived at. Notice that a numerical uncertainty remains. Even accepting the existence of the phase transition above, the size of the regions could be much modified (as critical parameters usually do) by the higher virial terms. This size will be taken as initial value in the subsequent enlarging processes and uncertainties will result for the final value of η .

2) The annihilation period

Below the critical temperature, baryons and antibaryons tend to mix by diffusion. An important point is that annihilation is concentrated in the intermediate layer, whose width may be shown [illegible] dom walk reasoning) to be $\simeq \sqrt{\lambda_a \lambda}$, where λ and λ_a [illegible] tal and the annihilation proton mean free path. This value [illegible] much smaller than the typical size of a region (roughly [illegible] diffusion length $L = \sqrt{Dt}$), so that the fluid keeps [illegible] like features. Neutrons dominate the diffusion up to $kT \simeq 1$ MeV, when they disappear. After that, the proton diffusion is less important than the simple grouth by expansion to the end of the period. The effects of the annihilation products have been examined in detail[22]. These are, ultimately, neutrinos, γ -rays and electrons. The former decouple very early and the later produce, by acting on the thermal electrons and photons, a large amount of X-rays. These transfer to the medium the energy and momentum. Below $kT \simeq 30$ KeV, the momentum release creates a pressure discontinuity in the layer, which pushes nucleons and antinucleons apartand whose dynamical consequence will be coalescence. Once this becomes effective, annihilation causes no more appre ciable changes in the ratio η , so that at $t \gtrsim 10^3$ sec it has already its present day value.

The real calculation of η is a difficult mathematical problem, involving the solution of the diffusion equation with moving boundaries of arbitrary shape. Some simplified models can be worked down and give encouraging results. The most simple-minded of them supposes matter to be initially concentrated in a great number of delta-like distributions equally spaced at intervals of the order of the typical size. This gives $\eta \approx 10^{-11}$. The situation is complicated by the presence of two effects which oppose each other: the large-scale conservation of the baryon number implies strong correlations (a point raised by Zeldovich) and these are partially broken by turbulence induced by

[illegible] implies strong correlations [illegible]

thermal-neutrino viscosity. The analysis of all these points[23] gives values for η ranging from 5×10^{-12} (no turbulence) to 5×10^{-8} (with maximal turbulence effect).

The knowledge of η as a function of t is of fundamental importance to the study of the nucleosynthesis of light elements, for which some difficulties have been recently[24] reported. At the onset of coalescence, uncertainties in the mass of a typical condensation($\approx10^{15\pm2}$g) reflect the doubts on its value at separation time, besides η's own uncertainties. In any case a strong multiplying mechanism is needed.

3) <u>The coalescence period</u>

Some progress has been made in the knowledge of what goes on during the late radiative era[25,26,22]. The kinetics of annihilation products was worked out in detail and the processes inside the intermediate layer are fairly understood.

It was said above that the system beared some resemblance to an emulsion. Well, emulsions characteristically exhibit the phenomenon of coalescence: regions of the emulsified substance tend to fuse so that their average size increases with time. This is a mere consequence of the surface free energy being proportional to the surface, which so tends to be minimized at equilibrium. However, thermodynamics has to handled with great case in our case, as temperature gradients are present. Moreover, we have equal concentrations of solvent and solute and this is a situation for which Emulsion Science is not well developed[27].

A most significant result from the study of the kinetics is that the momentum release creates a pressure discontinuity through the annihilation layer, and that this discontinuity is, at a point of the "surface" of average radium R, equal to

$$[p] = 2\alpha/R \quad , \tag{18}$$

where $\alpha = (2/3) J \lambda_0$. (J is the momentum flux and λ_0 is the mean free path of the thermal photons). This equation is well known in Surface Science: it is the Laplace-Kelvin formula with a surface tension coefficient α (here perfectly well known), and is valid in any real emulsion[4]. So, our system is much more akin to an emulsion than it could be guessed at first sight. From the kinetics, the hydrodynamic equations for the matter, as well as those for the matter plus radiation system, are obtained. The equations for the matter allow to calculate the number of annihilation per unit of surface and to prove that this rate is small enough to maintain η practically constant. From the equations for the global system, the most interesting is the modified Navier-Stokes equation,

$$\rho \frac{d\vec{v}}{dt} = -\nabla\left(\frac{E}{3}\right) + \frac{\vec{J}}{\lambda_0} + \eta_r \left[\Delta\vec{v} + \frac{1}{3}\nabla(\nabla\cdot\vec{v})\right] . \tag{19}$$

Consider a large volume V of the fluid, inside which the total contact surface is S. The typical size L of a region will be characterized by

$$L = \frac{V}{S} \quad . \tag{20}$$

If we multiply eq.(19) by $\vec{v}$ and take its average over V, the last term can be shown to be negligible during most of the radiative period. When this is the case, the remaining terms lead to an ordinary differential equation for L(t), with the solutions

$$\left.\begin{aligned} L^3(t) &= 5.7\times 10^5\, t^{17/4} && (\text{when } \rho_x \gg \rho_m); \\ L^3(t) &= 4.\times 10^3 \eta^{-1}\, t^{15/4} && (\text{when } \rho_m \gg \rho_x). \end{aligned}\right\} \quad (21)$$

A difficulty comes from the viscosity term in eq.(19): near the end of the period it becomes important, so that the solutions (21) are no more valid (to my knowledge nobody has been able to solve the complete equation by now). Anyhow, they show a very rapid growth of the regions with time. Another difficulty is the recombination epoch, which is longuer and comes later in the symmetric model. Only to check the capacity of coalescence to produce large masses, we may take the usual recombination time ($t \approx 10^6$ years) and extrapolate the second of eqs. (21). We find $M \approx \rho_m L^3 \approx 10^{50}$ g for $\Omega = 0.06$. This has no real significance as a number, but shows that this mechanism is able to develop masses as large as a cluster's.

Reynold's number can be calculated and shown to be small, so that no turbulence is present. At the end of the period this no more valid. It is interesting to recall here two results from Emulsion Science: first, at high concentration of the "solute" there is a high probability of having infinite (that is, with one dimension comparable to the size of the container) droplets; second, that a very efficient means to break them is viscosity[27]. So, at the end of the period the system would consist of a certain number of "infinite" regions of matter (and antimatter), which are progressively broken as the viscosity term becomes important. Quite independently of any consideration of Emulsion Science, Stecker and Puget have shown that turbulence could be at work at the recombination epoch and could be at the origin of the galaxies[28]. Good numbers were found both for the masses and angular momenta.

III. A few remarks on the observational aspects

Despite all calculational difficulties, the Symmetric Model seems to be able to improve on the Standard Model by explaining the high value of the entropy per baryon and the origin of the largest inhomogeneities. Clearly, much remains to be done to lessen the incertainties in the calculations. In particular, the difficulties with Helium synthesis[24] will not be clarified before $\eta(t)$ is reasonably well known and this would be a very good test for the Model. Another test could come from the study of the recombination epoch which lasts for a long time due to the presence of X-rays. This could produce distortions in the black-body radiation spectrum. Also annihilation would cause heating of the medium before recombination and upper limits to this heating have been recently stablished[29]. These are specific tests for Omnès model, the only one for which detailed calculations seem by now feasible. The only observational evidence at present comes from the atribution by Stecker and collaborators[30] of a bump in the diffuse gamma-ray spectrum above 1 MeV to annihilation. The fit is excellent but it is still difficult to exclude the possibility of any other non-trivial mechanism.

More general tests concern the prospection of antimatter in the Universe. Our Galaxy is surely a matter agglomerate and we should look over ways of finding antimatter outside it. It is not excluded that a fraction of the cosmic rays have an extragalactic origin. A theoretical upper limit for antiprotons being produced in our neighbourhood has been proposed[31] recently. If a larger rate were found, we would be gathering cosmic antiprotons. This possibility is by now remote, for they would have fantastic energies and would be very difficult to stop and recognize.

Still another possibility would be the discovery of point γ -sources. This has been analysed by Steigman[32] for a large variety of X - sources, with negative results. Still the emission of γ-rays from other objects is not excluded[33].

Alternative tests, as the det tion of high-intensity positronium rays, are under study.

Summing up: the prospection of antimatter as an widely open field. Any new idea would hardly be too welcome.

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