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LOW ENERGY ONE-NEUTRON TRANSFER REACTIONS BETWEEN HEAVY IONS

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ABSTRACT

A constraint is imposed on the possible value of the diffusivity of the imaginary part of the optical model potential used to fit elastic scattering data of light heavy ions at below the barrier energies. This is done by fixing the c.m. energy at which optimal - Q-value one - neutron transfer S-factor crosses the fusion S-factor.

Although it is well accepted that direct reaction contribution to the total reaction cross section of heavy ion collisions at energies near and below the barrier is quite small due to the presence of the Coulomb barrier it has been shown recently[1] that optimal Q-value one-neutron transfer cross section could become comparable and even larger than the fusion cross section at energies well below the barrier. This suggests that the total nuclear S-factor would rise much more steeply with decreasing center of mass energy than the fusion S-factor as was demonstrated in 1), 2). In this note we argue that the one-neutron transfer data could supply an upper limit for the value of the diffussivity, a_w, of the imaginary part of the optical potential used in elastic scattering data analysis. This we demonstrate by estimating the energy E_c at which the one-neutron transfer S-factor crosses the fusion S-factor. We feel that the ambiguity inherent in the values of the parameters of the optical model potential of heavy ions would certainly call for establishing at least upper limits.

The total reaction cross section at very low energy may be expressed in terms of the optical model transmission factor as follows.

$$\sigma_R \simeq \frac{\pi}{k^2} \; T(\epsilon) \tag{1}$$

The nuclear total reaction S-factor is defined as usual by the following

$$S_R = E_{C.M.} \exp(2\pi\eta) \; \sigma_R \tag{2}$$

where η (the Somerfeld parameter) $= \frac{Z_1 Z_2 e^2}{\hbar v}$, v being the asymptotic relative velocity of the two ions. A similar form is defined for the nuclear fusion S-factor

$$S_{FU} = E_{c.m.} \exp(2\pi\eta)\, \sigma_{FU} \tag{3}$$

A way of analysing the relative magnitude of S_R / S_{FU} is through the transmission factor which may be conveniently written as

$$T = c \int_0^\infty |\psi(r)|^2 W(r)\, dr \tag{4}$$

where $c = \frac{8\mu k}{\hbar^2}$, $W(r)$ is the imaginary part of the optical model potential, μ is the reduced mass of the system, k is the asymptotic wave number and $\psi(r)$ is the optical model radial wave function. To simulate the optimum-Q transfer reactions one has to use a rather large value of the diffusivity, a_w, of W(r) since these quasielastic reactions occur mainly at the surface region. This suggest that part of the contribution to T comes from absorption under the barrier[3).]

Accordingly we decompose T into two parts, a volume absorption contribution and a barrier region contribution. This is easily seen to be possible by assuming an exponential from for W(r) $\sim \exp[-r/a_W]$ which is valid at large radii, and using a WKB approximation for $\psi(r)$ i.e. $|\psi(r)|^2 \sim \exp\left[2\int_{a_0}^{r} \mathrm{Re}\, K(r')\, dr'\right]$ where $K(r) = \sqrt{\frac{2\mu}{\hbar^2}(V - iW - E)}$ and a_0 is the outer turning point. Thus the condition of exterma in $|\psi(r)|^2\, W(r)$ yields:

$$2\, \mathrm{Re}\, K(\xi) = \frac{1}{a_W}$$

$$\frac{V'(r_0)}{|W'(r_0)|} \lesseqgtr \frac{1}{\xi}\, \frac{1+\xi^2-\sqrt{1+\xi^2}}{\sqrt{1+\xi^2}} \tag{5}$$

where $\xi \equiv \dfrac{W(r_0)}{V(r_0)-E}$, $V'(r_0) = \dfrac{dV}{dr}(r_0)$ etc.

and $V(r)$ is the real part of the optical model potential.

The first inequality in (5) corresponds to a maximum at r_0^+ . Inspection of eq.(2) reveals that at low energies r_0^+ is larger than the position of the maximum of the barrier R_0 . (Note that $V'(r_0) < 0$ for $r_0 > R_0$ and ξ is always positive). The second inequality in (5) corresponds to a minimum which occurs at $r_0^- < R_0$. Since the integrand has two extrema in the barrier region one may write the two contribution to T(E) as follows

$$
\begin{aligned}
T &= T_1 + T_2 \\
T_1 &\simeq \exp\left\{-2\int_{a_i}^{a_0} \mathrm{Re}\, K(r)\, dr\right\} \\
T_2 &\simeq \beta(E)\, T_1 \\
\beta(E) &\simeq A(E) \exp\left(-\frac{r_0^+}{a_W}\right) \exp\left\{+\int_{a_i}^{r_0^+} \mathrm{Re}\, K(r)\, dr\right\}
\end{aligned}
\tag{6}
$$

where, $a_i(a_0)$ is the inner (outer) turning point, $A(E)$ is a slowly varying function of E, and r_0^+ is the position of maximum in the integrand of T_2 and is given by

$$
2\sqrt{\frac{2\mu}{\hbar^2}}\left(V(r_0) - E\right)^{1/2} = \frac{1}{a_W} \tag{7}
$$

Since fusion is a volume absorption phenomenon one may easily identify T_1 , with fusion. Recent measurement [4] of the fusion has no contribution from absorption under the barrier.

Thus T_2 is the optimal-Q transfer contribution (as well as small contributions from other possible direct reactions). It has been suggested in 5), that the diffusivity a_w that is used in optical model analysis of fusion data should not exceed a critical value given by $a_c = \frac{\hbar}{\sqrt{8\mu E_B}}$ Thus a_w that appears in T_2 must necessarily be larger than a

Clearly T_2 and T_1 must cross at a certain critical value, E_c, of the center of mass energy, $E_{c.m.}$, since below E_c the optimal-Q transfer cross section dominates over fusion even though the former is several order of magnitude smaller than the latter at near the barrier energy. Thus we determone E_c from the condition

$$T_1(E_c) \simeq T_2(E_c) \tag{8}$$

For the above to hold it is clear that $\beta(E_c)$ should be close to unity. This implies that the penetration factor, $\exp\left[+2\int_{a_i}^{r_o^+} \mathrm{Re}\, k(r)\, dr\right]$ must at least be equal to, if not dominate over, the absorption factor, $\exp\left[-r_o^+/a_W\right]$ which would be possible if the difference $r_o^+ - a_i$ attains an optimum value for a given center of mass energy.

Since a_i is basically determined by a_v, the diffusivity of the real part, $V(r)$, of the optical model potential, the above requirement on $r_o^+ - a_i$ implies a rather stringent condition on the value of a_v. However since at low energies, i.e. $E < E_B$ (the height of the Coulomb barrier), r_o^+ is seen to be larger than R_o one expects that the above requirement on a_i, not to be so stringent.

Equation (8) holds when the radius corresponding to the maximum in the integrand in T_1 coincides with the position of its minimum, i.e. at the inflection point. Utilizing this observation we find the following rough estimate

$$E_c \simeq E_B - \frac{\hbar^2}{2\mu} \frac{1}{(2a_w)^2} \tag{9}$$

where E_B is the height of the Coulomb barrier.

It is the clear that the energy E_c differs by much from E_B. In optimal-Q transfer reactions it was found in 1) that the S-factor, at low energies behaves as

$$S_{TR} \sim \exp(-\Lambda E_{C.M.})$$

$$\Lambda = 0.01 \left(\frac{Z_1 Z_2}{E_b^{3/2}}\right)\left(\frac{\mu}{m}\right)^{3/2} \sqrt{\mu c^2} \; [\mathrm{Mev}^{-1}] \tag{10}$$

Where $E_b(m)$ is the binding energy (mass) of the transferred neutron

The above formula clearly indicates that the rate of rise of S'_{TR} with de-creasing center of mass energy depends critically on the binding energy , E_b , of the transferred neutron. The value of $Q_{optimum}$ is given by

$$Q_{optimum} \simeq \frac{m}{\mu} E_b$$

Thus in reactions between heavy ions with closed neutron shell one would expect Q_{opt} to be larger and Λ smaller than the respective values of these quantities in reactions between heavy ions which contain loosely bound neutrons. As has already been stressed above since the value of a_w is closely related with the type of nuclear system being investigated one may conclude that for closed shell nuclear systems $\mathcal{E}_c$ would be quite small making an unambiguous determination of the one-neutron transfer cross section at these very low energies very difficult.

In table 1 we exhibit the values of a_w calculated from eq.(9) for three nuclear systems studied recently[1),2)]. These values are to be considered as supper limits of the a_w's that enter in the analysis of elastic scattering data at low energies as long as other reactions (besides the one neutron transfer) have negligible contribution to the total reaction cross section at these energies which is apparently the case.

We conclude by saying that measurement of the one-neutron optimal Q-value transfer cross section at below the barrier energy would yield information about the value of the diffusivity, a_w, of the imaginary part of the optical model potential used to fit elastic scattering data. This is done by finding $\mathcal{E}_c$, the center of mass energy at which S_{FU} and S_{TR} cross and utilizing the approximate formula (9). This should serve as a constraint on the values of the parameters of the ion- ion optical model potential and it emphasizes once again the point that even at very low energies quasi- elastic reactions must be considered together with elastic data in order to find a rather less ambiguous ion-ion optical model potential.

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Table Caption

Table 1 : The value of a_W estimated from equation (9) for three different systems studied in references 1) and 2).

SYSTEM	E_c(Mev)	E_B(Mev)	a_w(fm)	a_c(fm)
^{14}N + ^{14}N 1)	4.9	8.61	0.45	0.29
^{16}O + ^{9}Be 2)	4.1	5.89	0.71	0.39
^{10}B + ^{18}O 2)	~3.0	7.09	0.45	0.34

TABLE 1