KFKI-1977-91

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Hungarian Academy of Sciences

CENTRAL RESEARCH INSTITUTE FOR PHYSICS

BUDAPEST

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Submitted to Nuclear Physics B.

HU ISSN 0368 5330
ISBN 963 371 336 6

ABSTRACT

SU(3)-rules analogous to the Okubo-Zweig-Iizuka rule for strong decays are given for semileptonic and nonleptonic decays of strange and charmed mesons. In particular, relations among two- and three-body weak decays are derived. In the case of nonleptonic decays the relations depend on the colour structure of currents.

АННОТАЦИЯ

Рассматриваются SU(3)-правила для полулептонных и нелептонных распадов странных и очарованных мезонов,аналогичные правили Окубо-Цвейг-Изука для сильных распадов. В частности, выводятся соотношения между двух- и трехчастичными слабыми распадами. В случае нелептонных распадов эти соотношения зависят от цветовой структуры токов.

KIVONAT

Az erős bomlásokra vonatkozó Okubo-Zweig-Iizuka szabállyal analóg SU(3)-szabályokat vezetünk le a ritka és a bájos mezonok szemileptonos és nemleptonos bomlásaira. A két- és három-test gyenge bomlások között összefüggéseket származtatunk. A nemleptonos bomlások esetében az összefüggések érzékenyek az áramok szin-szerkezetére.

The SU(3)-properties of semileptonic and nonleptonic decays of mesons.

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Abstract: SU(3)-rules analogous to the Okubo-Zweig--Iizuka rule for strong decays are given for semileptonic and nonleptonic decays of strange and charmed mesons. In particular, relations among two- and three--body weak decays are derived. In the case of nonleptonic decays the relations depend on the colour structure of currents.

1. Introduction

After the recent discovery of charmed D- and F-mesons [1,2] an increasing amount of informations is accumulated also on the weak decays of these particles [3-5]. The facts known at present are generally consistent with the Glashow, Iliopoulos, Maiani (GIM) scheme [6] for the weak currents. From the point of view of unified gauge theories it is of great interest, however, to verify the charm interpretation in detail.

Due to their large mass the D- and F-mesons have a rather large number of decay channels. The branching ratios into the different channels can give valuable informations on the decay mechanism. SU(3)-relations following from the SU(3) transformation properties of the currents were derived previously by Kingsley, Treiman, Wilczek and Zee [7]. More restrictive relations for nonleptonic decays follow from 20-plet dominance /analogous to octet dominance in K-meson decays/ [7-11]. In the present paper I shall investigate the consequences of SU(3) symmetry for the weak decays of D- and F-mesons assuming a generalization of Okubo-Zweig--Iizuka (OZI) rule [12] which can be naturally incorporated in a large class of quark models. For definiteness, I shall consider the quark model with phenomenological quark confinement described in detail previously [13,14]. The obtained relations are, however, more generally valid than the model itself. /Examples of similar quark models are given e.g. in Refs. [15-19]./

In Section II. the SU(3)-structure of weak amplitudes is given in general. The Fiertz-transformation properties of the current x current effective Hamiltonian for nonleptonic decays are taken into account emphasizing the role of colour degrees of freedom. Specific relations for the two- and three-body decays are considered in Section III. and IV., respectively. /For details see also the Appendix./ The relations are first tested in both cases for the well known K-meson decays. In Section V. the conclusions are briefly summarized.

II. SU(3)-structure of the weak decay amplitudes of mesons.

The charged hadronic weak current in the GIM scheme [6] can be written as

$$h_\mu(x) = \bar{\psi}(x)\, \mathcal{H}_w\, \gamma_\mu (1-\gamma_5)\, \psi(x) \;, \qquad /1/$$

where $\gamma_{\mu,5}$ denotes Dirac-matrices, $\psi(x) = \{\psi_q(x)\}$ $q=u,d,s,c$ is the quark field operator /colour indices are suppressed for the moment/, and the 4x4 matrix $\mathcal{H}_w$ is given by

$$\mathcal{H}_w = \begin{pmatrix} 0 & \cos\Theta & \sin\Theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\sin\Theta & \cos\Theta & 0 \end{pmatrix} \qquad /2/$$

/Θ is the Cabibbo angle./ The effective current x current weak interaction Hamiltonian density is

$$\mathcal{H}_{eff}(x) = \frac{G}{\sqrt{2}} \left[h_\mu(x) + \ell_\mu(x) \right] \left[h^\mu(x) + \ell^\mu(x) \right]^+ \;, \qquad /3/$$

where G is the Fermi coupling constant and $\ell_\mu(x)$ denotes the charged leptonic weak current containing $e-$, $\mu-$ and $\tau-$ like leptons:

$$\ell_\mu(x) = \sum_{\ell=e,\mu,\tau} \tilde{\psi}_{\nu_\ell}(x)\gamma_\mu(1-\gamma_5)\psi_\ell(x) \quad . \qquad /4/$$

The effective interaction is the low energy limit of some gauge theory coupling mediated by the charged intermediate boson $W^{\pm}$.

In the present paper we shall consider only the leading charm-changing terms in Eq.(3) proportional to $\cos\theta$ in the semileptonic case and to $\cos^2\theta$ in the non-leptonic case.

Semileptonic decays.

The semileptonic interaction comes from the coupling of the hadronic and leptonic currents. Graphically the leading charm-changing piece /proportional to $\cos\theta$ / is represented by Figure 1. In the quark model of Refs. [13,14] the simplest quark graph /the so called "direct term"/ contributing to the semileptonic decays is depicted in Figure 2. /for instance, in the case of D^0-meson decay/ [20]. The Feynman-rules for the calculation of quark graphs [13,14] are briefly summarized in Table 1. In what follows I shall only consider the /flavour/ SU(3) structure coming from the usual meson -matrix part of the wave function

$$M = \sum_{\mathcal{P}} \mathcal{P}\, M(\mathcal{P}) = \begin{pmatrix} \eta_u = \frac{\pi^0}{\sqrt{2}} + \frac{\cos\varphi_P}{\sqrt{2}}\eta + \frac{\sin\varphi_P}{\sqrt{2}}\eta' & \pi^- & K^- & D^0 \\ \pi^+ & \eta_d = -\frac{\pi^0}{\sqrt{2}} + \frac{\cos\varphi_P}{\sqrt{2}}\eta + \frac{\sin\varphi_P}{\sqrt{2}}\eta' & \bar{K}^0 & D^+ \\ K^+ & K^0 & \eta_s = \cos\varphi_P\,\eta' - \sin\varphi_P\,\eta & F^+ \\ \bar{D}^0 & D^- & F^- & \eta_c \end{pmatrix} . \qquad /5/$$

Here, for definiteness, the pseudoscalar mesons are taken, hence $\mathcal{P} = \pi^0, \eta, \eta', \pi^+, \dots, \eta_c$ and the mixing angle φ_P belongs to the pseudoscalar nonet. /The mixing of the charmed quark states is neglected here./ The momentum dependent part $\chi(p,q)$ of the wave function [13,14] will not be specified here, /It will be assumed, however, that the SU(3) breaking effects coming from the quark mass matrix M_q are to a good approximation compensated by the SU(3)-breaking in the meson wave functions like it has been shown for strong meson decays in Ref. [14]./

From Table I. it follows that the SU(3)-structure of the amplitude corresponding to the graphs like in Figure 2. is [20] :

$$\sum_{\pi(1)n} C_n[\pi(\cdot)]\, \mathrm{Tr}\left\{ M(I)\, \bar{M}(\mathcal{P}_{\pi(1)}) \dots \bar{M}(\mathcal{P}_{\pi(n)}) \right\} . \qquad /6/$$

Here $\sum_{\pi(!)n}$ means a summation over the permutations $\{\pi(1), \pi(2), \ldots, \pi(n)\}$ of the numbers $\{1, 2, \ldots, n\}$ / n is the number of hadrons in the final state/. The trace over internal symmetry indices in Eq. (6) is already reduced to /flavour/ SU(3) by the relations following from Eqs. (2,5)

$$\chi_w = \cos\Theta\left[M(\pi^+) + M(F^+)\right] + \sin\Theta\left[M(K^+) - M(D^+)\right];$$

/7/

$$M(D^0)\bar{M}(F^+) = M(K^-),$$

$$M(D^+)\bar{M}(F^+) = M(\bar{K}^0),$$

$$M(F^+)\bar{M}(F^+) = M(\eta_s).$$

/8/

From these relations it follows that the matrix M(I) specifying the "initial" SU(3) quantum numbers of the n-hadron state is given by Eq.(5) if

$$I_{D^0} = K^-, \quad I_{D^+} = \bar{K}^0, \quad I_{F^+} = \eta_s.$$

/9/

The factors $c_n[\pi(\cdot)]$ in the amplitude (6) depend on the momenta and spin indices /only the dependence on the permutation $\pi(\cdot)$ is explicitly indicated/. SU(3) symmetry means that c_n does not depend on internal symmetry indices. The matrices $\overline{M}(\mathcal{P}_j)$ in Eq.(6) stand for the j'th out-going pseudoscalar meson with SU(3) quantum number $\mathcal{P}_j$. In the case of resonances in the final state $\mathcal{P}$ is, of course, replaced by $\mathcal{V}=\rho^+, \rho^0, \dots, \overline{K}^{*0}$ /for vector mesons/, $\mathcal{T}=A_2^+, A_2^0, \dots, \overline{K}^{**0}$ /for tensor mesons / etc.

As far as the SU(3) coupling scheme is concerned Eq.(6) is obviously the generalization of the OZI-rule to semileptonic decays. Graphically it means that it is always possible "to draw a continuous quark line" among the mesons /and the diagram obtained is always connected/. In the quark model the graph in Figure 2. is, however, only the simplest /"direct"/ one, therefore in principle the cther graphs may spoil the behaviour given by Eq.(6). The essential point is that a large class of graphs have the same SU(3)-structure as the "direct" one and, according to the success of the OZI-rule for hadronic couplings, these graphs dominate. Among the more complicated graphs belonging to this class there are the "indirect terms" when one /or more/ ofthe pairs of internal quark lines makes up a /resonating/ internal hadron line. /The pole terms coming from such indirect graphs seem, in fact, to dominate in the form factors for $K_{\ell 3}$ or $\pi_{\ell 3}$ decays/. Examples of such indirect terms are depicted in Figure 3. Other kinds of terms having the same SU(3) structure like Eq.(6) are the ones with internal gluon lines, because the /coloured/ gluons are flavour singlets. In what follows we take Eq.(6) with some unspecified momentum and spin dependent part c_n to be the SU(3)-property of semileptonic decays.

Nonleptonic decays.

The nonleptonic decays are given by the product of two hadronic currents in the effective Hamiltonian in Eq.(3). For later purposes it is convenient to introduce a graphical notation also for the Fiertz-transformed four-quark couplings. In the case of the leading $(i.e.\ \cos^2\theta)$ charm changing nonleptonic interaction the notation is explained by Fig.4.

For the nonleptonic decays the set of /direct/ quark graphs can be devided into two essentially different classes [20]. The"exchange-type" graph is illustrated in the case of D^0 decay in Figure 5. The special case $n_1=0$ is possible only for D^0 decay. It can be called "exchange annihilation" as the quarks $(c\bar{u})$ in D^0 are annihilating by the exchange of a W^+ boson. The "emission-type" graph is shown in the case of F^+ decay in Figure 6. The case $n_1=0$ /"emission annihilation"/ is possible only for F^+ when the quarks $(c\bar{s})$ in F^+ annihilate each other by the emission of a W^+.

From the Feynman-rules in Table 1 it follows that the SU(3)--structure of the amplitude corresponding to the sum of emission and exchange graphs is [20]:

$$\sum_{\pi(!)n} \sum_{n_1+n_2=n} \Big\{ a_{n_1 n_2}[\pi(\cdot)] \mathrm{Tr}\left[M(I)\,\overline{M}(\mathcal{P}_{\pi(1)})\ldots\overline{M}(\mathcal{P}_{\pi(n_1)})\right]$$

$$\cdot \mathrm{Tr}\left[M(\pi^+)\,\overline{M}(\mathcal{P}_{\pi(n_1+1)})\ldots\overline{M}(\mathcal{P}_{\pi(n_1+n_2)})\right] + b_{n_1 n_2}[\pi(\cdot)]$$

$$\cdot \mathrm{Tr}\left[\overline{M}(\mathcal{P}_{\pi(1)})\ldots\overline{M}(\mathcal{P}_{\pi(n_1)})M(I)\,\overline{M}(\mathcal{P}_{\pi(n_1+1)})\ldots\overline{M}(\mathcal{P}_{\pi(n_1+n_2)})M(\pi^+)\right]\Big\} \qquad /10/$$

The SU(3)- matrix M(I) is the same here as in Eq.(6) and it is given by Eqs.(9,5). The amplitudes $a_{n_1 n_2}$ /for the emission graph/ and $b_{n_1 n_2}$ /for the exchange graph/ depend on the momenta and spins.

The generalization of the OZI-rule for nonleptonic decays is given by Eq. (10) . The "indirect" terms and gluon exchange terms have the same SU(3)-structure as Eq. (10) , only the momentum and spin dependent parts are different.

Fiertz transformation.

It can be seen from Figures 4-6. that the exchange and emission graphs for nonleptonic decays are connected by a Fiertz-transformation. After Fiertz-transformation the graph in Figure 5, for instance, goes over into Figure 7.

Let us first forget about the colour of quarks. In this case the Fiertz-transformation of the leading charm-changing piece of the current x current Hamiltonian in Eq. (3) is:

$$\frac{G}{\sqrt{2}}\cos^2\Theta\left[\tilde{\psi}_u(x)\gamma_\mu(1-\gamma_5)\psi_d(x)\right]\left[\tilde{\psi}_s(x)\gamma^\mu(1-\gamma_5)\psi_c(x)\right]=$$

$$=\frac{G}{\sqrt{2}}\cos^2\Theta\left[\tilde{\psi}_u(x)\gamma_\mu(1-\gamma_5)\psi_c(x)\right]\left[\tilde{\psi}_s(x)\gamma^\mu(1-\gamma_5)\psi_d(x)\right]. \qquad /11/$$

It can be seen from here that the structure of the Fiertz--transformed exchange graph is exactly the same as the emission graph the only difference being in the SU(3) quantum numbers of quarks participating in the weak interaction. That is, in the SU(3)-symmetric limit the momentum and spin dependent part of the exchange graph is the same as that of the emission graph:

$$b_{n_1 n_2}\,\mathrm{Tr}\left\{\bar{M}^{n_1} M(I)\,\bar{M}^{n_2} M(\pi^+)\right\}=$$

$$=a_{n_1 n_2}\,\mathrm{Tr}\left\{M(F)\,\bar{M}^{n_1}\right\}\mathrm{Tr}\left\{M(\bar{K}^0)\,\bar{M}^{n_2}\right\}. \qquad /12/$$

Here the SU(3) transformation corresponding to Eq.(11) was performed on the right hand side, hence instead of Eq.(9) we have

$$\mathcal{F}_{D^0} = \eta_u \,, \qquad \mathcal{F}_{D^+} = \pi^+ \,, \qquad \mathcal{F}_{F^+} = K^+ . \tag{13}$$

From the explicit form of the SU(4)-matrices in Eq.(5) and Eqs. (9,13) it follows

$$\mathrm{Tr}\{M(\mathcal{I})\bar{M}^{n_1}\}\,\mathrm{Tr}\{M(\pi^+)\bar{M}^{n_2}\} - (\bar{M}^{n_1})_{si}\,(\bar{M}^{n_2})_{ud}$$

$$\mathrm{Tr}\{\bar{M}^{n_1}M(\mathcal{I})\bar{M}^{n_2}M(\pi^+)\} = \mathrm{Tr}\{M(\mathcal{I})\bar{M}^{n_1}\}\,\mathrm{Tr}\{M(\bar{K}^0)\bar{M}^{n_2}\} =$$

$$= (\bar{M}^{n_1})_{ui}\,(\bar{M}^{n_2})_{sd} \,. \tag{14}$$

The index i is equal to u for D^0, d for D^+ and s for F^+ decay. Comparing the second relation in Eq.(14) with Eq.(12) we obtain:

$$b_{n_1 n_2} = a_{n_1 n_2} \,. \tag{15}$$

If the quarks would obey Bose-statistics instead of Fermi--statistics [15,21-25] there would be an additional negative sign in Eq. [11] ,therefore instead of Eq.(15) it would give

$$b_{n_1 n_2} = -\,a_{n_1 n_2} \,. \tag{16}$$

Up to now the quarks were assuemd to be colourless . /for a recent review on colur see Ref. [26] . In QCD with coloured quarks the Fiertz-transformation in Eq. (11) is replaced by

$$\frac{G}{\sqrt{2}}\cos^2\Theta\sum_{\alpha,\beta}\left[\tilde{\psi}_{u\alpha}(x)\gamma_\mu(1-\gamma_5)\psi_{d\alpha}(x)\right]\left[\tilde{\psi}_{s\beta}(x)\gamma^\mu(1-\gamma_5)\psi_{c\beta}(x)\right]=$$

$$=\frac{G}{\sqrt{2}}\cos^2\Theta\sum_{\alpha,\beta}\Big\{\frac{1}{3}\left[\tilde{\psi}_{u\alpha}(x)\gamma_\mu(1-\gamma_5)\psi_{c\alpha}(x)\right]\left[\tilde{\psi}_{s\beta}(x)\gamma^\mu(1-\gamma_5)\psi_{d\beta}(x)\right]+$$

$$+\frac{1}{2}\sum_{\alpha',\beta'}\sum_{i=1}^{8}\left[\tilde{\psi}_{u\alpha}(x)(\lambda_i)_{\alpha\alpha'}\gamma_\mu(1-\gamma_5)\psi_{c\alpha'}(x)\right]\left[\tilde{\psi}_{s\beta}(x)(\lambda_i)_{\beta\beta'}\gamma^\mu(1-\gamma_5)\psi_{d\beta'}(x)\right]\Big\} \qquad /17/$$

The colour indices are denoted here by greek letters and λ_i stands for the Gell-Mann SU(3)- matrices. EQ. (17) shows that the colour "spoils" the simple relations following from the similarity of the Fiertz-transformed interaction to the original one. It seems hard to imagine that a relation like Eq. (15) can be derived in generality unless some specific dynamical assumptions are made.

An example of dynamical assumptions in the connection with colour is made in the geometrodynamics approach of Preparata [18], namely that colour contributions are unimportant /or simply not there/. In this case the second term of the Fiertz - transformed expression /containing λ_i / cannot contribute as the colour trace over the quark loops gives zero. Therefore, the form in Eq. (11) is essentially restored, only with an extra factor 1/3 . This leads to

$$b_{n_1 n_2}=\frac{1}{3}a_{n_1 n_2} \quad . \qquad /18/$$

In the following Sections we shall investigate also the experimental consequences of Eqs. (15,16,18).

III. Two-body decays.

Symmetry breaking kinematics.

The SU(3) is in general a good symmetry for the amplitudes /e.g. for the coupling constants in the case of strong decays of resonances/ but substantial symmetry breaking effects come from the kinematics of the decay due to the large mass splittings among pseudoscalar mesons. The usual procedure for the verification of SU(3) - relations is therefore to take the symmetry breaking phase space /and centrifugal barrier/ effects into account by the physical values of masses.

In the case of two-body decays of a pseudoscalar meson into two pseudoscalar mesons $(\mathcal{P} \to \mathcal{P}\mathcal{P})$ the decay width is given by the dimensionless coupling constant g_{PP} like

$$\Gamma_{PP} = |g_{PP}|^2 \frac{w}{8\pi} , \qquad /19/$$

where w is the value of the c.m. momentum. If m is the mass of the decaying particle and m_1, m_2 are the masses of the decay products, respectively, then

$$w = \frac{1}{2m}\left\{ m^4 + m_1^4 + m_2^4 - 2m^2m_1^2 - 2m^2m_2^2 - 2m_1^2m_2^2 \right\}^{1/2} . \qquad /20/$$

For the decay of a pseudoscalar into a vector and a pseudoscalar $(\mathcal{P} \to \mathcal{V}\mathcal{P})$ the corresponding expression is:

$$\Gamma_{VP} = |g_{VP}|^2 \frac{w^3}{8\pi m_1^2} . \qquad /21/$$

There is, of course, some ambiguity here in choosing m_1 in the denominator instead of some other mass. This choice corresponds to the simplest form of the transition amplitude.

Finally, for the decay into two vectors $(\mathcal{P} \to VV)$ we have

$$\Gamma_{VV} = |g_{VV}|^2 \frac{w^5}{8\pi m_1^2 m_2^2} . \qquad /22/$$

/Simılar formulae can be written down also for other kinds of decays containing, for instance, tensor mesons etc./ In what follows I shall assume the SU(3) relations for the dimensionless coupling constants like g_{PP}, g_{VP} and g_{VV}.

For two-body /nonleptonic/ decay amplitudes into two pseudoscalars a_{11}, b_{11}, a_{02} and b_{02} in Eq.(10) can not depend on the permutation $\pi(\cdot)$ as the only variables are the masses which are degenerate (i.e. $m_1 = m_2$) in the symmetric case /assumed for the amplitudes/. Below, the notations a_{PP}, b_{PP}, $\bar{a}_{PP}$ and $\bar{b}_{PP}$ will be used for a_{11}, b_{11}, a_{02} and b_{02}, respectively. For quasi two-body decays into two vector mesons the same quantities are a_{VV}, b_{VV}, $\bar{a}_{VV}$ and $\bar{b}_{VV}$. In the case of quasi two-body decays into a pseudoscalar and a vector meson the amplitudes belonging to the two permutations may be different /as $m_1 \neq m_2$ even in the symmetric case/, therefore the corresponding notations will be a_{PV}, a_{VP}, b_{PV}, b_{VP}, $\bar{a}_{PV}$, $\bar{a}_{VP}$, $\bar{b}_{PV}$ and $\bar{b}_{VP}$.

$K \to 2\pi$ decays.

First it is natural to test the symmetry relations for K-meson decays, when the SU(3) - symmetry is, in fact, reduced to SU(2) /isospin/. The derivation of SU(2)-rules for the strangeness changing decays is completely analogous to the charm changing case, therefore, it is not necessary to repeat again the arguments of the previous Section. The SU(2) quark rule is given by Eq. (6) and Eq. (10) for the semileptonic and nonleptonic decays, respectively. Instead of Eq. (9) the initial quantum numbers of the hadronic states are given by

$$I_{K^+} = \eta_u \quad , \qquad I_{K^0} = \pi^- \ . \qquad /23/$$

The relations in Eq. (14) are replaced by

$$Tr\{M(I)\bar{M}^{n_1}\}\, Tr\{M(\pi^+)\bar{M}^{n_2}\} = (\bar{M}^{n_1})_{iu}(\bar{M}^{n_2})_{ud} \ ,$$

$$Tr\{\bar{M}^{n_1}M(I)\bar{M}^{n_2}M(\pi^+)\} = (\bar{M}^{n_1})_{id}(\bar{M}^{n_2})_{uu} \ ; \qquad /24/$$

where $i=u$ for K^+ decay and $i=d$ for K^0 decay.

The $K \to 2\pi$ amplitudes following from Eqs. (10,23,24) are best displayed by the generating functions

$$G(K^+) = \pi^+\pi^0 \frac{1}{\sqrt{2}}(a_{PP} + b_{PP}) \ ,$$

$$G(K_S) = \pi^0\pi^0 \frac{1}{\sqrt{2}}(\bar{b}_{PP} - b_{PP}) + \pi^+\pi^- \sqrt{2}(\bar{b}_{PP} + a_{PP}) \ . \qquad /25/$$

The calculation of the decay widths from here goes via the substitution

$$A\pi^{+n_+}\pi^{-n_-}\pi^{\circ n_\circ} \Rightarrow |A|^2 \frac{n_+!\,n_-!\,n_\circ!}{(n_+ + n_- + n_\circ)!}\,\varrho(K\to\pi^{+n_+}\pi^{-n_-}\pi^{\circ n_\circ}), \qquad /26/$$

where $\varrho(K\to\pi^{+n_+}\pi^{-n_-}\pi^{\circ n_\circ})$ means the two-body phase-space $(w/8\pi$ in Eq. (19)) belonging to a number n_α of π^α mesons in the final state $(\alpha = +, -, \circ)$.

There are altogether 3 $K\to 2\pi$ decays, hence for the 2 ratios we have 3 complex parameters [say: $a_{pp}/\bar{a}_{pp}$, $b_{pp}/\bar{a}_{pp}$, $\bar{b}_{pp}/\bar{a}_{pp}$] therefore in QCD there are no symmetry relations among these decays. In the case of colourless quarks Eq. (15) or Eq. (16) holds for femion and boson quarks, respectively. In the first case the only /complex/ parameter for the ratios of decay widths is $a_{pp}/\bar{a}_{pp}$:

$$\frac{\Gamma(K^+\to\pi^+\pi^\circ)}{\Gamma(K_s\to\pi^+\pi^-)} = \frac{\varrho(K^+\to\pi^+\pi^\circ)}{\varrho(K_s\to\pi^+\pi^-)}\left|\frac{a_{pp}}{a_{pp}+\bar{a}_{pp}}\right|^2 \quad \{0.0021\pm 0.00003\},$$

$$\frac{\Gamma(K_s\to\pi^\circ\pi^\circ)}{\Gamma(K_s\to\pi^+\pi^-)} = \frac{\varrho(K_s\to\pi^\circ\pi^\circ)}{\varrho(K_s\to\pi^+\pi^-)}\,\frac{1}{2}\left|\frac{\bar{a}_{pp}-a_{pp}}{\bar{a}_{pp}+a_{pp}}\right|^2 \quad \{0.456\pm 0.006\}. \qquad /27/$$

The measured values are given in the curly brackets. They are reproduced by $|a_{pp}/\bar{a}_{pp}| \gtrsim 0.05$, therefore the annihilation diagram is dominating.

The second case /Bose-quarks/ was proposed to explain the $\Delta I = \frac{1}{2}$ rule [15,21-26] resulting in the relations

$$\Gamma(K^+ \to \pi^+\pi^\circ) = 0 \quad ,$$

$$\frac{\Gamma(K_s \to \pi^\circ\pi^\circ)}{\Gamma(K_s \to \pi^+\pi^-)} = \frac{\varrho(K_s \to \pi^\circ\pi^\circ)}{\varrho(K_s \to \pi^+\pi^-)}\,\frac{1}{2} = 0.505 \;. \qquad /28/$$

/As it can be seen from Eq. (27) these relations hold also for $a_{pp} = 0$ in the previous case. The smallness of $|a_{pp}/\bar{a}_{pp}|$ shows how good is $\Delta I = \frac{1}{2}$. /

The data can also be fitted by the geometrodynamics value of b in Eq. (18) where $|a_{pp}/\bar{a}_{pp}|$ turns out to be $\cong 0.025$.

Two-body decays of charmed mesons.

The generating function analogous to Eq. (25) for two-body nonleptonic decays of the D- and F-mesons can be obtained from Eq. (10) using the expressions in Eq. (14). The result is:

$$G(D^\circ) = K^-\pi^+ (a_{pp} + \bar{b}_{pp}) + \bar{K}^\circ\pi^\circ \frac{1}{\sqrt{2}}(b_{pp} - \bar{b}_{pp}) + \bar{K}^\circ\eta \frac{1}{\sqrt{6}}(b_{pp} - \bar{b}_{pp}) + $$
$$+ \bar{K}^\circ\eta' \frac{1}{\sqrt{3}}(b_{pp} + 2\bar{b}_{pp}) \quad ,$$
$$G(D^+) = \bar{K}^\circ\pi^+ (a_{pp} + b_{pp}) \quad , \qquad /29/$$
$$G(F^+) = \bar{K}^\circ K^+ (b_{pp} + \bar{a}_{pp}) + \eta\pi^+ \sqrt{\tfrac{2}{3}}(\bar{a}_{pp} - a_{pp}) + \eta'\pi^+ \frac{1}{\sqrt{3}}(a_{pp} + 2\bar{a}_{pp}).$$

For the pseudoscalar nonet the octet-singlet mixing was neglected here, that is $\cos\varphi_p = \frac{1}{\sqrt{3}}$ was taken /the same will be used also in the rest of this paper/. The amplitudes $a_{pp},\dots,\bar{b}_{pp}$ entering in these expressions are related to the ones in Eq. (25) by SU(4)-symmetry /SU(4) may, however, be badly broken/. To obtain the decay widths from here the same recipe has to be applied as in Eq. (26) /extended, of course, from π^+, π^-, π° to the whole set of pseudoscalar mesons/.

The ratios of the decay widths for the 8 different two-body channels in Eq. (29) are given by 3 complex parameters [say: $a_{PP}/\bar{a}_{PP}$, $b_{PP}/\bar{a}_{PP}$, $\bar{b}_{PP}/\bar{a}_{PP}$] therefore there must be some relations. The following ones are independent from the values of the parameters:

$$\Gamma(F^+ \to \pi^+\pi^0) = 0 \; ,$$

$$\frac{\Gamma(D^0 \to \bar{K}^0\pi^0)}{\Gamma(D^0 \to \bar{K}^0\eta)} = \frac{\varrho(D^0 \to \bar{K}^0\pi^0)}{\varrho(D^0 \to \bar{K}^0\eta)}\, 3 = 3.35 \; . \qquad /30/$$

If the b-amplitudes are related to the a-amplitudes by some of the Eqs. (15), (16) or (18), then there are, of course, much more relations. The parameter free ones are collected in the Appendix in the first and last cases. /The rather unconventional case of Bose-quarks in Eq. (16) is left to the interested reader as a simple exercise./

Quasi two-body decays of charmed mesons.

Similar relations can also be derived for quasi two-body nonleptonic decays producing /in general/ two resonances in the final state. In the present paper only vector mesons will be considered but the generalization to other kinds of resonances is straightforward. The generating functions in this case are /with $\cos\varphi_V = 1$, that is ideal mixing for the vector-meson mixing angle/:

$$
\begin{aligned}
G(D^0) = {} & K^- \rho^+ (a_{PV} + \bar{b}_{PV}) + K^{*-} \pi^+ (a_{VP} + \bar{b}_{VP}) + \bar{K}^0 \rho^0 \tfrac{1}{\sqrt{2}} (b_{VP} - \bar{b}_{PV}) + \\
& + \bar{K}^{*0} \pi^0 \tfrac{1}{\sqrt{2}} (b_{PV} - \bar{b}_{VP}) + \bar{K}^0 \omega \tfrac{1}{\sqrt{2}} (b_{VP} + \bar{b}_{PV}) + \bar{K}^{*0} \eta \tfrac{1}{\sqrt{6}} (b_{PV} + \bar{b}_{VP} - 2\bar{b}_{PV}) + \\
& + \bar{K}^0 \phi \bar{b}_{VP} + \bar{K}^{*0} \eta' \tfrac{1}{\sqrt{3}} (b_{PV} + \bar{b}_{PV} + \bar{b}_{VP}) + K^{*-} \rho^+ (a_{VV} + \bar{b}_{VV}) + \\
& + \bar{K}^{*0} \rho^0 \tfrac{1}{\sqrt{2}} (b_{VV} - \bar{b}_{VV}) + \bar{K}^{*0} \omega \tfrac{1}{\sqrt{2}} (b_{VV} + \bar{b}_{VV}) + \bar{K}^{*0} \phi \bar{b}_{VV} ;
\end{aligned}
$$

$$
G(D^+) = \bar{K}^0 \rho^+ (a_{PV} + b_{VP}) + \bar{K}^{*0} \pi^+ (a_{VP} + b_{PV}) + \bar{K}^{*0} \rho^+ (a_{VV} + b_{VV}) ;
$$

$$
\begin{aligned}
G(F^+) = {} & \bar{K}^0 K^{*+} (b_{VP} + \bar{a}_{VP}) + \bar{K}^{*0} K^+ (b_{PV} + \bar{a}_{PV}) + \rho^+ \pi^0 \tfrac{1}{\sqrt{2}} (\bar{a}_{PV} - \bar{a}_{VP}) + \\
& + \pi^+ \rho^0 \tfrac{1}{\sqrt{2}} (\bar{a}_{VP} - \bar{a}_{PV}) + \pi^+ \omega \tfrac{1}{\sqrt{2}} (\bar{a}_{VP} + \bar{a}_{PV}) + \rho^+ \eta \tfrac{1}{\sqrt{6}} (\bar{a}_{VP} + \bar{a}_{PV} - 2 a_{PV}) + \\
& + \pi^+ \phi\, a_{VP} + \rho^+ \eta' \tfrac{1}{\sqrt{3}} (a_{PV} + \bar{a}_{VP} + \bar{a}_{PV}) + \bar{K}^{*0} K^{*+} (b_{VV} + \bar{a}_{VV}) + \\
& + \rho^+ \omega \sqrt{2}\, \bar{a}_{VV} + \rho^+ \phi\, a_{VV} .
\end{aligned}
$$

/31/

For the ratios of the 8 vector-vector decays there are again 3 complex parameters, wheareas for the ratio of 18 pseudoscalar-vector decays there are 8 complex parameters. In the case of independent b- and a-type amplitudes the following parameter-free relations hold:

$$
\Gamma(F^+ \to \rho^+ \rho^0) = 0 ,
$$

$$
\frac{\Gamma(F^+ \to \pi^+ \rho^0)}{\Gamma(F^+ \to \rho^+ \pi^0)} = \frac{\varrho(F^+ \to \pi^+ \rho^0)}{\varrho(F^+ \to \rho^+ \pi^0)} = 1.00 .
$$

/32/

The parameter free relations in the case of Eq. (15) or Eq. (18) are also listed in the Appendix.

IV. Three-body decays.

For three-body decays it would be in principle possible to compare the probability distributions in the Dalitz-plot, but the direct comparison of decay widths is not possible due to the differences in the Dalitz-plot shapes resulting from the SU(3)-breaking mass differences. A possibility for the estimate of decay width ratios is to approximate the amplitudes by constant /average/ values. In this case only the ratios of the Dalitz-plot areas matter. This approximation will be used in the present Section.

$K \to 3\pi$ decays.

In the case of $K \to 3\pi$ decay SU(3) is reduced to SU(2)/isospin /. The relations following from Eqs. (10,23,24) are given by the generating functions like for $K \to 2\pi$ decays in Eq. (25):

$$G(K^+) = (a_{03} + b_{12} + a_{21})\left(\tfrac{1}{2}\pi^+\pi^0\pi^0 + \pi^+\pi^+\pi^-\right) ,$$

$$G(K_L) = (b_{03} - b_{12} + b_{21})\left(\tfrac{1}{2}\pi^0\pi^0\pi^0 + \pi^0\pi^+\pi^-\right) . \qquad /33/$$

To obtain the decay widths from here /for constant amplitudes/ the substitution in Eq. (26) has to be performed. ρ is now the relativistic phase-space integral /equal to the area of the Dalitz-plot/:

$$\rho(K \to \pi^{+n_+}\pi^{-n_-}\pi^{0n_0}) = \int \delta^4\Big[p - \sum_{\alpha=+,-,0}\sum_{i_\alpha=1}^{n_\alpha} p(i_\alpha)\Big] \cdot \prod_{\alpha=+,-,0}\prod_{i_\alpha=1}^{n_\alpha} \frac{d^3p(i_\alpha)}{2p(i_\alpha)_0} . \qquad /34/$$

/p is the four-momentum of the decaying particle./

There are 2 relations following from Eqs. (33,26), namely

$$\frac{\Gamma(K^{+}\to\pi^{+}\pi^{0}\pi^{0})}{\Gamma(K^{+}\to\pi^{+}\pi^{+}\pi^{-})}=\frac{\rho(K^{+}\to\pi^{+}\pi^{0}\pi^{0})}{\rho(K^{+}\to\pi^{+}\pi^{+}\pi^{-})}\,\frac{1}{4}=0.305 \qquad \{0.309\pm0.01\},$$

$$\frac{\Gamma(K_{L}\to\pi^{0}\pi^{0}\pi^{0})}{\Gamma(K_{L}\to\pi^{0}\pi^{+}\pi^{-})}=\frac{\rho(K_{L}\to\pi^{0}\pi^{0}\pi^{0})}{\rho(K_{L}\to\pi^{0}\pi^{+}\pi^{-})}\,\frac{3}{2}=1.83 \qquad \{1.75\pm0.08\}. \qquad /35/$$

These are expressing the absence of I=3 components in the final state. The measured ratios /in curly brackets/ are in excellent agreement with the theoretical values.

Similarly to the $K\to2\pi$ case, the approximately valid

$$\frac{\Gamma(K^{+}\to\pi^{+}\pi^{+}\pi^{-})}{\Gamma(K_{L}\to\pi^{0}\pi^{+}\pi^{-})}=\frac{\rho(K^{+}\to\pi^{+}\pi^{+}\pi^{-})}{\rho(K_{L}\to\pi^{0}\pi^{+}\pi^{-})}\,2=1.62 \qquad \{1.91\pm0.04\} \qquad /36/$$

follows for Bose-quarks from Eq. (16) or, in the case of colourless fermion quarks, if only the annihilation amplitude $b_{03}=a_{03}$ is present.

Three-body decays of the charmed mesons.

The generating functions for three-body nonleptonic decays of the D-and F-mesons can be obtained from Eqs. (10,14) :

$$G(D^0) = \bar K^0 \pi^+ \pi^- (b_{03}+a_{21}+b_{21}) + \bar K^0 \pi^0 \pi^0 \frac{1}{2}(b_{03}+b_{21}-b_{12}) +$$
$$+ K^- \pi^+ \pi^0 \frac{1}{\sqrt{2}}(a_{21}+b_{12}) + \bar K^0 K^+ K^- (b_{03}+b_{21}+a_{12}) + \bar K^0 K^0 \bar K^0 b_{03} +$$
$$+ K^- \pi^+ \eta \frac{1}{\sqrt{6}}(2a_{12}+b_{12}-a_{21}) + K^- \pi^+ \eta' \frac{1}{\sqrt{3}}(2a_{12}+b_{12}+2a_{21}+3b_{03}) +$$
$$+ \bar K^0 \pi^0 \eta \frac{1}{\sqrt{3}}(b_{21}-b_{12}) + \bar K^0 \pi^0 \eta' \frac{1}{\sqrt{6}}(b_{12}+2b_{21}-3b_{03}) + \bar K^0 \eta\eta \frac{1}{6}(b_{21}-b_{12}+3b_{03}) \;;$$

$$G(D^+) = K^- \pi^+ \pi^+ (a_{21}+b_{12}) + \bar K^0 K^+ \bar K^0 (a_{12}+b_{21}) - \bar K^0 \pi^+ \pi^0 \frac{1}{\sqrt{2}}(a_{21}+b_{12}) +$$
$$+ \bar K^0 \pi^+ \eta \frac{1}{\sqrt{6}}(2a_{12}+2b_{21}-a_{21}-b_{12}) + \bar K^0 \pi^+ \eta' \frac{2}{\sqrt{3}}(a_{12}+b_{21}+a_{21}+b_{12}) \;;$$

$$G(F^+) = \pi^+ \pi^+ \pi^- a_{03} + \pi^+ \pi^0 \pi^0 \frac{a_{03}}{2} + \pi^+ K^+ K^- (a_{03}+a_{21}+b_{12}) +$$
$$+ \pi^+ K^0 \bar K^0 (a_{03}+b_{21}+a_{21}) + \bar K^0 K^+ \pi^0 \frac{1}{\sqrt{2}}(b_{21}-b_{12}) +$$
$$+ \pi^+ \eta\eta \frac{1}{6}(3a_{03}+4a_{21}-4a_{12}) + \pi^+ \eta\eta' \frac{\sqrt{2}}{3}(3a_{03}-2a_{21}-a_{12}) -$$
$$- K^+ \bar K^0 \eta \frac{1}{\sqrt{6}}(2a_{12}+b_{12}+b_{21}) + K^+ \bar K^0 \eta' \frac{1}{\sqrt{3}}(a_{12}+2b_{12}+2b_{21}+3a_{03}) \,. \qquad /37/$$

The amplitudes $a_{03}, b_{03}, \dots, b_{21}$ appearing here are related to the ones for K-meson decay in Eq. (33) by SU(4). It is important to note that the channels are understood here as the "direct" three-body ones, without the resonances /quasi two-body channels/.

In the genaral case $(b \neq a)$ the following parameter-free relations follow from Eqs. 37,26 :

$$\frac{\Gamma(D^+ \to \bar{K}^0 \pi^+ \pi^0)}{\Gamma(D^+ \to K^- \pi^+ \pi^+)} = \frac{\varrho(D^+ \to \bar{K}^0 \pi^+ \pi^0)}{\varrho(D^+ \to K^- \pi^+ \pi^+)} \frac{1}{4} = 0.249 \,,$$

$$\frac{\Gamma(F^+ \to \pi^+ \pi^0 \pi^0)}{\Gamma(F^+ \to \pi^+ \pi^+ \pi^-)} = \frac{\varrho(F^+ \to \pi^+ \pi^0 \pi^0)}{\varrho(F^+ \to \pi^+ \pi^+ \pi^-)} \frac{1}{4} = 0.252 \,,$$

$$\frac{\Gamma(D^0 \to K^- \pi^+ \pi^0)}{\Gamma(D^+ \to K^- \pi^+ \pi^+)} = \frac{\varrho(D^0 \to K^- \pi^+ \pi^0)}{\varrho(D^+ \to K^- \pi^+ \pi^+)} \frac{1}{4} = 0.249 \,,$$

$$\frac{\Gamma(F^+ \to K^+ \bar{K}^0 \pi^0)}{\Gamma(D^0 \to \bar{K}^0 \pi^0 \eta)} = \frac{\varrho(F^+ \to K^+ \bar{K}^0 \pi^0)}{\varrho(D^0 \to \bar{K}^0 \pi^0 \eta)} \frac{3}{2} = 2.56 \,. \qquad /38/$$

Calculating the Dalitz-plot area the masses m_{D^0} =1863 MeV, m_{D^+} = 1868 MeV, m_{F^+} =2030 MeV [3-5] were used. These relations are expected to hold somewhat less rigurously than the corresponding ones for $K \to 3\pi$ decays (35). The reasons are that SU(3) can be broken more seriously than SU(2) /the mass differences are more important/, therefore the approximation of the amplitudes by constants /in the larger Dalitz-plot area/ is less good and finally the subtraction of the resonance /quasi two-body/ contributions is by no means unambiguous.

The parameter-free relations following from Eqs. (15) or (18) expressing the b-type amplitudes by the a-type ones are also given in the Appendix.

Semileptonic decays.

The $SU(3)$-properties of the semileptonic decays are given by Eq. (6). In the case of a single hadron /in general a resonance/ we have a three-body decay and the approximation of constant amplitudes /here constant $K_{\ell 3}$, $D_{\ell 3}$ and $F_{\ell 3}$ form-factors/ may be taken in order to get an estimate for the branching ratios.

In the generating functions only 1 or 2 pseudoscalar mesons will be written out. The resonance nonets can, of course, also be included without further ado. The results are:

$$G(K^+) = \ell^+\nu_\ell \left\{ \pi^0 \frac{c_1'}{\sqrt{2}} + \left(\pi^+\pi^- + \pi^0\pi^0 \frac{1}{2}\right) c_2' \right\} ;$$

$$G(K_L) = \left(\ell^+\nu_\ell \pi^- + \ell^- \bar{\nu}_\ell \pi^+\right) \frac{c_1'}{\sqrt{2}} ;$$

$$G(D^0) = \ell^+\nu_\ell \left\{ K^- c_1 + \left[\bar{K}^0 \pi^- + K^- \pi^0 \frac{1}{\sqrt{2}} - K^- \eta \frac{1}{\sqrt{6}} + K^- \eta' \frac{2}{\sqrt{3}} \right] c_2 \right\} ;$$

$$G(D^+) = \ell^+\nu_\ell \left\{ \bar{K}^0 c_1 + \left[K^- \pi^+ - \bar{K}^0 \pi^0 \frac{1}{\sqrt{2}} - \bar{K}^0 \eta \frac{1}{\sqrt{6}} + \bar{K}^0 \eta' \frac{2}{\sqrt{3}} \right] c_2 \right\} ;$$

$$G(F^+) = \ell^+\nu_\ell \left\{ \eta' \frac{c_1}{\sqrt{3}} - \eta c_1 \sqrt{\frac{2}{3}} + \left[\eta'\eta' \frac{1}{3} + \eta\eta \frac{2}{3} - \eta\eta' \frac{2\sqrt{2}}{3} + K^+K^- + K^0\bar{K}^0 \right] c_2 \right\} . \qquad /39/$$

For $K_{\ell 3}$ decay /taking the average of electronic and muonic widths/ it follows:

$$\frac{\Gamma(K_L \to \pi \ell \nu_\ell)}{\Gamma(K^+ \to \pi^0 \ell^+ \nu_\ell)} = 2 \frac{\rho(K_L \to \pi^- e^+ \nu_e) + \rho(K_L \to \pi^- \mu^+ \nu_\mu)}{\rho(K^+ \to \pi^0 e^+ \nu_e) + \rho(K^+ \to \pi^0 \mu^+ \nu_\mu)} = 1.99 \qquad \{1.98 \pm 0.04\} \qquad /40/$$

in excellent agreement with the experimental number /in curly brackets/. The corresponding relations for the charmed mesons are:

$$\frac{\Gamma(D^0\to K^-\ell^+\nu_\ell)}{\Gamma(D^+\to \bar{K}^0\ell^+\nu_\ell)} = \frac{\varrho(D^0\to K^-\ell^+\nu_\ell)}{\varrho(D^+\to \bar{K}^0\ell^+\nu_\ell)} = 0.999 \ ,$$

$$\frac{\Gamma(F^+\to \eta'\ell^+\nu_\ell)}{\Gamma(D^+\to \bar{K}^0\ell^+\nu_\ell)} = \frac{\varrho(F^+\to \eta'\ell^+\nu_\ell)}{3\varrho(D^+\to \bar{K}^0\ell^+\nu_\ell)} = 0.177 \ ,$$

$$\frac{\Gamma(F^+\to \eta\ell^+\nu_\ell)}{\Gamma(D^+\to \bar{K}^0\ell^+\nu_\ell)} = \frac{2\varrho(F^+\to \eta\ell^+\nu_\ell)}{3\varrho(D^+\to \bar{K}^0\ell^+\nu_\ell)} = 0.780 \ . \qquad /41/$$

Note that the hadronic final state in D^+ semileptonic decay is obtained from D^0 semileptonic decay by an isospin transformation, therefore the full semileptonic widths of D^+ and D^0 are equal /by SU(2)-symmetry/.

From Eq. (39) it is straightforward to obtain the relations for resonance and for two pseudoscalar meson production, too.

V. Conclusions.

In the present paper the derivation of the SU(3)-relations for mesonic weak decays is based on the following assumptions:

i/ the validity of the GIM - current for weak interactions [6] /only the dominant parts, i.e. the $\cos\Theta$ term for semileptonic and the $\cos^2\Theta$ term for non-leptonic decays, were considered/;

ii/ SU(3)-symmetry for amplitudes and /usual/ SU(3)-breaking phase space factors; /for three-body decays the estimates of branching ratios were given for constant amplitudes on the Dalitz-plot/;

iii/ the validity of SU(3)-coupling rule analogous to the OZI-rule in strong decays [12], which can be easily incorporated into a quite general class of quark models [13-19].

The actual framework explicitly considered is the quark model with phenomenological quark confinement [13-14] applied previously to calculate form factors and the strong decays of resonances. In this model the simplest quark diagrams /"direct terms"/ for the weak decays [20] have the same SU(3)-structure as a large class of graphs containing also internal hadron lines ["indirect terms" including the pole contributions] and/or internal gluon lines. In the spirit of the OZI-rule this class of diagrams dominates, determining the SU(3) coupling scheme among the hadrons in the final state.

The internal symmetry relations obtained were tested first for the well known case of K-meson decays. The two relations in Eq.(35) for $K \to 3\pi$ are very well satisfied. The amazing accuracy is due to the fact that only isospin symmetry is involved in these relations, and besides, the phase space for $K \to 3\pi$ is rather small, therefore the constant amplitude approximation is good. Although not surprising /as they express the absence of I=3 in the final state/, these relations also shed new lignt on the quark rule generally known as OZI-rule. /In this respect note also the $K_{\ell 4}$ relation given below which can be one of the best places for checking the accuracy of this quark rule./

As far as the charmed meson decays are concerned the experimental verification of the relations in Eqs.(30,32,38) would give strong support to the first one among the above three assumptions /GIM-current/ as the SU(3) symmetry of hadron couplings and the OZI-rule is well established.

A very interesting feature, in my opinion, is the possibility to investigate the effects of quark colour in nonleptonic decays. In general there are two sets of amplitudes /a-and b-type/. For colourless quarks /or if coloured gluon exchange is negligible/ the two sets are connected by some of the relations in Eq. (15,16,18). These produce a number of additional SU(3) -relations among nonleptonic channels/see e.g. in the Appendix/. More generally, the connection of the a-and b-type amplitudes depends on the Fiertz-transformation properties of the effective current x current nonleptonic interaction and this gives a handle to investigate the colour properties of the weak current.

It is interesting to note that the preliminary data known at present seem to indicate that $D^0 \to K^- \pi^+$ and $D^+ \to \bar{K}^0 \pi^+$ occur at roughly the same rates. According to Eq.(29) this means that neither b= -a /boson-quarks/ nore the dominance of the annihilation process $(a_{pp} \cong b_{pp} \cong 0)$ work for D-meson decays. This is in sharp contrast to what we learned from K-meson decays. If more data will be available it will be very informative to know which part of the nonleptonic amplitudes /if any/ is dominating.

In principle it is possible to extend the results of the present paper also to multiparticle channels. The estimates based on the approximation of constant amplitudes may be very useful in this case. As an example, the relation for $K_{\ell 4}$ decays /obtained from Eq.(6) with constant amplitudes/ [27]:

$$\frac{\Gamma(K^+ \to \pi^+ \pi^- e^+ \nu_e)}{\Gamma(K^+ \to \pi^0 \pi^0 e^+ \nu_e)} = 2 \frac{\rho(K^+ \to \pi^+ \pi^- e^+ \nu_e)}{\rho(K^+ \to \pi^0 \pi^0 e^+ \nu_e)} = 1.71 \quad \{2.1 \pm 1.2\}$$

seems to work well within the experimental errors. /ρ denotes here the four-body relativistic phase space integral./ Similar relations hold also for charmed mesons. In general, also the multiparticle production aspects of the weak decays /average multiplicities, inclusive distributions etc. [20] /may be investigated along these lines.

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Appendix.

In this appendix the additional parameter-free relations resulting from the Fiertz-transformation properties of nonleptonic current-current interaction are collected. Two cases are considered:

i/ colourless /fermion/ quarks, when Eq.(15) , i.e. b=a holds;

ii/ coloured quarks in the geometrodynamics approach [18] when Eq. (18), i.e. $b=\frac{1}{3}a$ is valid. Otherwise the same notations and assumptions are made as in the text.

Two-body decays, b=a.

$$\frac{\Gamma(D^0\to K^-\pi^+)}{\Gamma(F^+\to K^+\bar{K}^0)} = \frac{\rho(D^0\to K^-\pi^+)}{\rho(F^+\to K^+\bar{K}^0)} = 0.971$$

$$\frac{\Gamma(D^0\to \bar{K}^0\eta')}{\Gamma(F^+\to \pi^+\eta')} = \frac{\rho(D^0\to \bar{K}^0\eta')}{\rho(F^+\to \pi^+\eta')} = 0.721$$

$$\frac{\Gamma(D^0\to \bar{K}^0\pi^0)}{\Gamma(F^+\to \pi^+\eta)} = \frac{\rho(D^0\to \bar{K}^0\pi^0)}{\rho(F^+\to \pi^+\eta)}\,\frac{3}{4} = 0.689$$

Two-body decays, $b=\frac{1}{3}a$.

$$\frac{\Gamma(D^0\to \bar{K}^0\eta')}{\Gamma(F^+\to \pi^+\eta')} = \frac{\rho(D^0\to \bar{K}^0\eta')}{\rho(F^+\to \pi^+\eta')}\,\frac{1}{9} = 0.0801$$

$$\frac{\Gamma(D^0\to \bar{K}^0\pi^0)}{\Gamma(F^+\to \pi^+\eta)} = \frac{\rho(D^0\to \bar{K}^0\pi^0)}{\rho(F^+\to \pi^+\eta)}\,\frac{1}{12} = 0.0766$$

Quasi two-body decays involving vector mesons, b=a.

$$\frac{\Gamma(D^0\to K^{*-}\rho^+)}{\Gamma(F^+\to \bar{K}^{*0}K^{*+})}=\frac{\varrho(D^0\to K^{*-}\rho^+)}{\varrho(F^+\to \bar{K}^{*0}K^{*+})}=0.632$$

$$\frac{\Gamma(D^+\to \bar{K}^{*0}\pi^+)}{\Gamma(D^+\to \bar{K}^{0}\rho^+)}=\frac{\varrho(D^+\to \bar{K}^{*0}\pi^+)}{\varrho(D^+\to \bar{K}^{0}\rho^+)}=0.875$$

$$\frac{\Gamma(D^0\to K^{*-}\pi^+)}{\Gamma(F^+\to K^{*+}\bar{K}^0)}=\frac{\varrho(D^0\to K^{*-}\pi^+)}{\varrho(F^+\to K^{*+}\bar{K}^0)}=0.935$$

$$\frac{\Gamma(D^0\to K^-\rho^+)}{\Gamma(F^+\to K^+\bar{K}^{*0})}=\frac{\varrho(D^0\to K^-\rho^+)}{\varrho(F^+\to K^+\bar{K}^{*0})}=1.067$$

$$\frac{\Gamma(D^0\to \bar{K}^{*0}\eta')}{\Gamma(F^+\to \rho^+\eta')}=\frac{\varrho(D^0\to \bar{K}^{*0}\eta')}{\varrho(F^+\to \rho^+\eta')}=0.00689$$

$$\frac{\Gamma(D^0\to K^{*-}\rho^+)}{\Gamma(D^0\to \bar{K}^{*0}\omega)}=\frac{\varrho(D^0\to K^{*-}\rho^+)}{\varrho(D^0\to \bar{K}^{*0}\omega)}\,2=2.31$$

$$\frac{\Gamma(D^+\to \bar{K}^{*0}\rho^+)}{\Gamma(F^+\to \rho^+\phi)}=\frac{\varrho(D^+\to \bar{K}^{*0}\rho^+)}{\varrho(F^+\to \rho^+\phi)}\,4=2.98$$

Quasi two-body decays involving vector mesons, $b=\frac{1}{3}a$.

$$\frac{\Gamma(D^0\to \bar{K}^{*0}\eta')}{\Gamma(F^+\to \rho^+\eta')}=\frac{\varrho(D^0\to \bar{K}^{*0}\eta')}{\varrho(F^+\to \rho^+\eta')}\,\frac{1}{9}=0.000766$$

$$\frac{\Gamma(D^+\to \bar{K}^{*0}\rho^+)}{\Gamma(F^+\to \rho^+\phi)}=\frac{\varrho(D^+\to \bar{K}^{*0}\rho^+)}{\varrho(F^+\to \rho^+\phi)}\,\frac{16}{9}=1.32$$

Three-body decays, b=a.

$$\frac{\Gamma(D^+\to \bar{K}^\circ K^+ \bar{K}^\circ)}{\Gamma(D^+\to K^-\pi^+\pi^+)} = \frac{\varrho(D^+\to \bar{K}^\circ K^+ \bar{K}^\circ)}{\varrho(D^+\to K^-\pi^+\pi^+)} = 0.171$$

$$\frac{\Gamma(D^+\to \bar{K}^\circ \pi^+ \eta)}{\Gamma(D^+\to K^-\pi^+\pi^+)} = \frac{\varrho(D^+\to \bar{K}^\circ \pi^+ \eta)}{\varrho(D^+\to K^-\pi^+\pi^+)}\,\frac{1}{12} = 0.0374$$

$$\frac{\Gamma(D^+\to \bar{K}^\circ \pi^+ \eta')}{\Gamma(D^+\to K^-\pi^+\pi^+)} = \frac{\varrho(D^+\to \bar{K}^\circ \pi^+ \eta')}{\varrho(D^+\to K^-\pi^+\pi^+)}\,\frac{8}{3} = 0.181$$

$$\frac{\Gamma(D^\circ\to \bar{K}^\circ K^\circ \bar{K}^\circ)}{\Gamma(F^+\to \pi^+\pi^+\pi^-)} = \frac{\varrho(D^\circ\to \bar{K}^\circ K^\circ \bar{K}^\circ)}{\varrho(F^+\to \pi^+\pi^+\pi^-)} = 0.0855$$

$$\frac{\Gamma(F^+\to \pi^+ K^\circ \bar{K}^\circ)}{\Gamma(D^\circ\to \bar{K}^\circ\pi^+\pi^-)} = \frac{\varrho(F^+\to \pi^+ K^\circ \bar{K}^\circ)}{\varrho(D^\circ\to \bar{K}^\circ\pi^+\pi^-)} = 0.766$$

$$\frac{\Gamma(D^\circ\to \bar{K}^\circ K^+ K^-)}{\Gamma(F^+\to \pi^+ K^+ K^-)} = \frac{\varrho(D^\circ\to \bar{K}^\circ K^+ K^-)}{\varrho(F^+\to \pi^+ K^+ K^-)} = 0.222$$

$$\frac{\Gamma(F^+\to \pi^+ \eta\eta')}{\Gamma(D^\circ\to \bar{K}^\circ\pi^\circ\eta')} = \frac{\varrho(F^+\to \pi^+ \eta\eta')}{\varrho(D^\circ\to \bar{K}^\circ\pi^\circ\eta')}\,\frac{4}{3} = 2.68$$

$$\frac{\Gamma(F^+\to K^+ \bar{K}^\circ \eta')}{\Gamma(D^\circ\to K^-\pi^+\eta')} = \frac{\varrho(F^+\to K^+ \bar{K}^\circ \eta')}{\varrho(D^\circ\to K^-\pi^+\eta')} = 0.117$$

Theree-body decays, $b=\frac{1}{3}a$

$$\frac{\Gamma(D^\circ\to \bar{K}^\circ K^\circ \bar{K}^\circ)}{\Gamma(F^+\to \pi^+\pi^+\pi^-)} = \frac{\varrho(D^\circ\to \bar{K}^\circ K^\circ \bar{K}^\circ)}{\varrho(F^+\to \pi^+\pi^+\pi^-)}\,\frac{1}{9} = 0.00950$$

$$\frac{\Gamma(F^+\to \pi^+ \eta\eta')}{\Gamma(D^\circ\to \bar{K}^\circ\pi^\circ\eta')} = \frac{\varrho(F^+\to \pi^+ \eta\eta')}{\varrho(D^\circ\to \bar{K}^\circ\pi^\circ\eta')}\,12 = 24.1$$

$$\frac{\Gamma(D^+\to \bar{K}^\circ \pi^+ \eta)}{\Gamma(D^+\to \bar{K}^\circ\pi^+\eta')} = \frac{\varrho(D^+\to \bar{K}^\circ \pi^+ \eta)}{\varrho(D^+\to \bar{K}^\circ\pi^+\eta')}\,\frac{25}{128} = 1.29$$

Table 1.

Feynman rules for first order weak interaction quark graphs /single quark loop for semileptonic and two quark loops for nonleptonic decays/.

1/ Label the internal quark line four-momenta taking into account four-momentum conservation.

2/ Write a factor

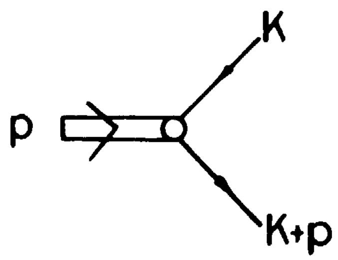
$$(2\pi)^{2(m_1+m_2)-8}\, i^{2+p_1+p_2} \int d^4k_1\, d^4k_2\, Tr\{\dots\}_1\, Tr\{\dots\}_2$$

where

k_j = four-momentum of some of the quark lines
p_j = number of quark lines
m_j = number of BS vertices /open circles/
$Tr\{\dots\}_j$ = trace over SU(4) and Dirac-indices

} in the quark loop with index j= (1,2)

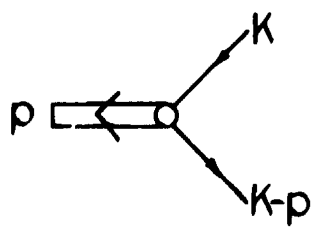
3/ For incoming mesons



$$= M\chi(p, K+\frac{p}{2})$$

where M is the internal symmetry matrix of the meson and χ is the momentum dependent part of its BS wave-function.

4/ For outgoing mesons



$$= \bar{M}\bar{\chi}(p, K-\frac{p}{2})$$

where $\bar{M}=M^T$ and $\bar{\chi}$ are the conjugate wave functions.

5/

$$-(M_q - K\cdot\gamma)$$

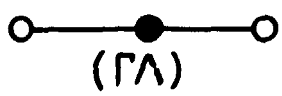
where M_q is the quark /effective/ mass matrix.

6/

$$-\frac{iG}{\sqrt{2}}(2\pi)^4\delta^4(p_{in}-p_{out})$$

This assures overall four-momentum conservation.

7/



$$-\ \Gamma\Lambda$$

for the local current operator ● $(\Gamma\Lambda)$ with Dirac-part Γ and internal symmetry part Λ .

Figures.

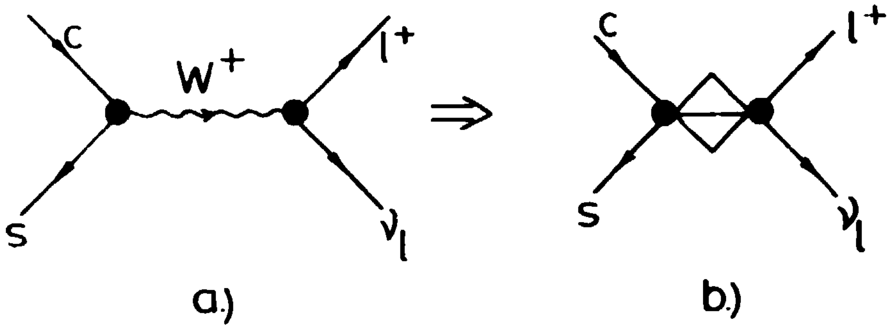


Figure 1. The leading semileptonic coupling for charm decay. The intermediate vector boson coupling (a) is equivalent at low energies to the four-fermion coupling (b).

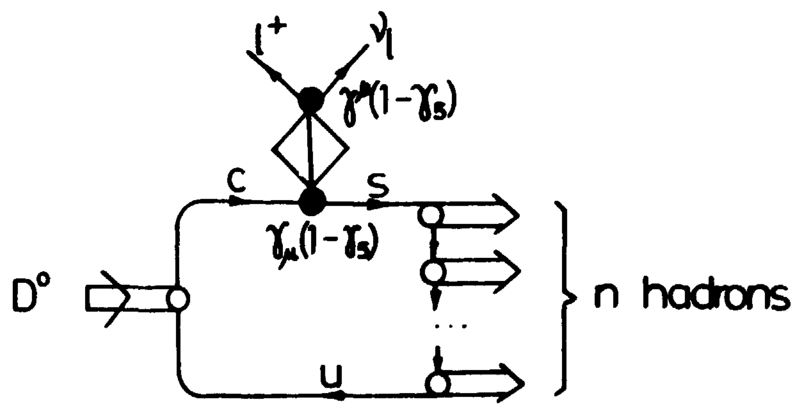


Figure 2. The simplest /"direct"/ quark graph contributing to the semileptonic decay of D^0.

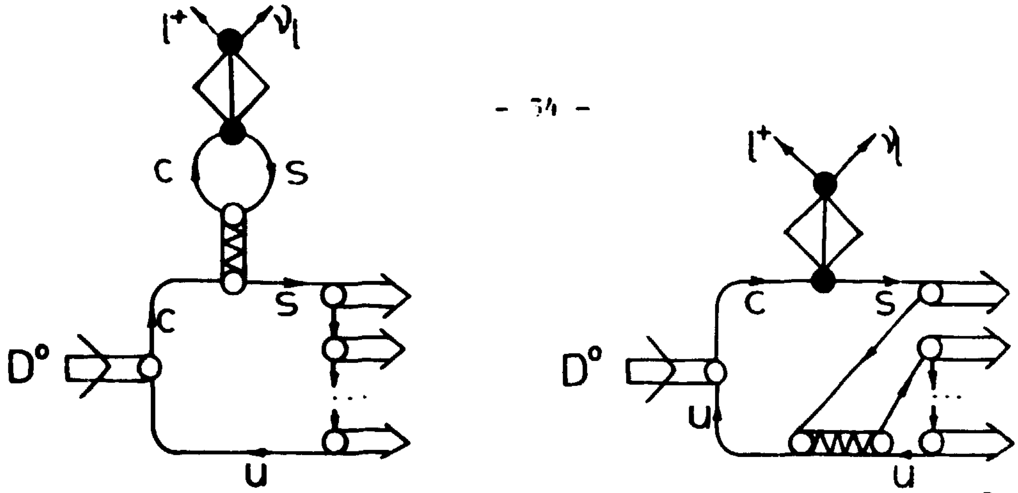


Figure 3. Examples of indirect terms to semileptonic D^0 decay / is an internal hadron line/.

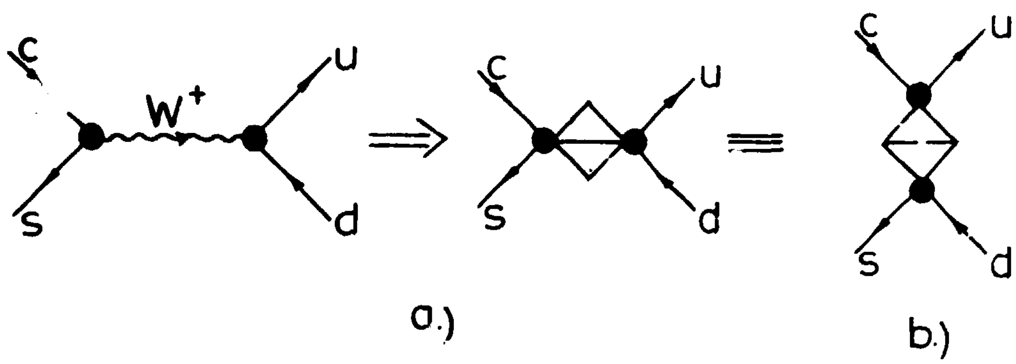


Figure 4. The leading nonleptonic coupling for charm decay. The four-fermion coupling can be given in two equivalent forms /a and b/ connected by a Fiertz-transformation.

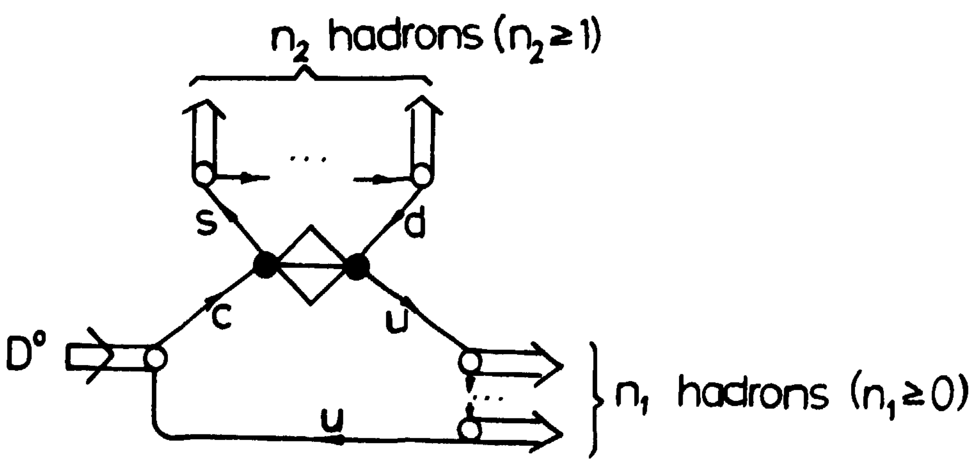


Figure 5. "Exchange" quark graph for D^0 nonleptonic decay.

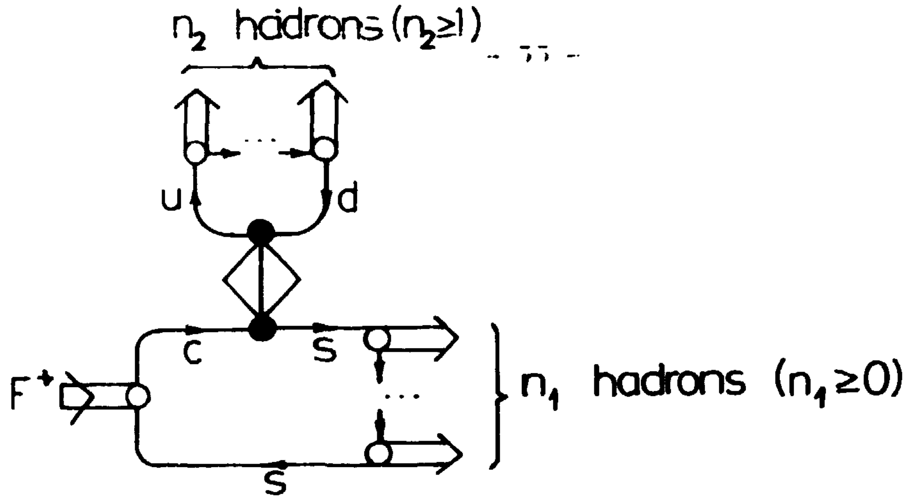


Figure 6. "Emission" quark graph for F^+ nonleptonic decay.

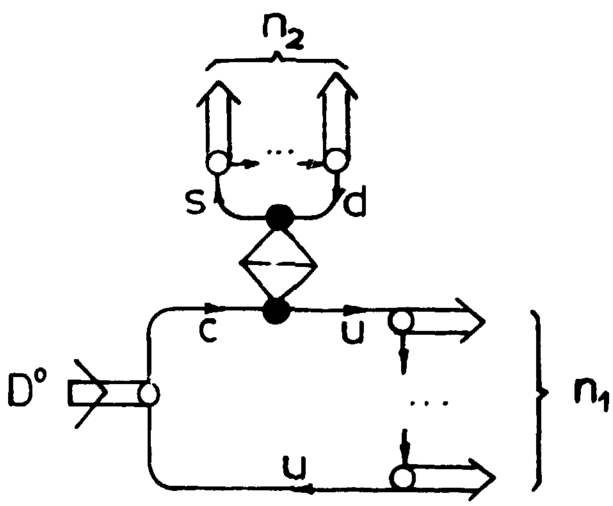


Figure 7. The "exchange" quark graph after Fiertz-transformation.

Kiadja a Központi Fizikai Kutató Intézet
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Budapest, 1977. november hó