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THE ANOMALOUS MAGNETIC MOMENT OF A CHARGED HEAVY LEPTON

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ABSTRACT : We study the mass-dependence of the theoretical contributions to a charged heavy lepton-anomalous magnetic moment $a_\ell = \frac{1}{2}(g_\ell - 2)$. In particular it is shown that the weak interactions effects in the Weinberg-Salam model become very important and are greater than the hadronic effects for a mass of the heavy lepton $\gtrsim$ 5 GeV. In order to illustrate our analysis, we give the theoretical value of the anomalous magnetic moment of the new charged lepton τ (1.9 GeV). We conclude with a discussion of the theoretical implications of an eventual measurement of the anomaly of a charged heavy lepton.

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1. INTRODUCTION

The τ-lepton discovered in e^+e^- colliding beam experiments[1] seems to be the third of the charged lepton-family and probably there will be other charged heavy leptons[2]. The anomalous magnetic moment of the two first types of leptons has been measured with a very high-accuracy[3]. There is at present a very good agreement between the theory[4] and the experiments[3].

Although a measurement of the anomalous magnetic moment of the τ is clearly not for the near future, we believe that an analysis of the various theoretical contributions (QED, hadronic, weak interactions) to the anomaly of heavy leptons may help to stimulate new ideas on the physics of heavy leptons in general.

In the following sections, we analyse the mass-dependence of the different theoretical contributions to a_ℓ . This includes : the purely quantum electrodynamic (QED) perturbation theory effects up to sixth order, the hadronic effects at lowest order and the weak interactions effects.

As a quantitative application, we give in Section V the value of the anomaly of the new lepton τ of mass 1.9 GeV[1].

In all this work, we shall use the same notation as in Ref. [4].

2. THE QED CONTRIBUTION UP TO SIXTH ORDER

The QED contribution to the anomaly of a heavy lepton is obtained from the general identity :

$$a_\ell = \frac{1}{N}\sum_{\ell'}\left\{(a_\ell - a_{\ell'}) + a_{\ell'}\right\} \tag{2.1}$$

where ℓ' denotes a lepton of a mass lighter than the ℓ-mass [5], N is the number of these light-leptons. $(a_\ell - a_{\ell'})$ can be deduced from the theoretical expressions of the muon anomaly [4] by the simple interchange of masses, provided the approximation $(m_{\ell'}/m_\ell) \ll 1$ remains valid. $a_{\ell'}$ is deduced from the electron anomaly and from (2.1) by iteration.

In general for $N \geq 3$, a new class of graphs shown in figure 4 contributes to $(a_\ell - a_{\ell'})$ at sixth order in addition to the known graphs of fourth order (fig. 1) and sixth order (figs. 2 and 3) of the muon case [4]. The contribution of this graph of figure 4 is :

$$a_\ell(\text{figure 4}) = \left(\frac{\alpha}{\pi}\right)^3 \cdot 2\left\{\frac{2}{9}\ln\left(\frac{m_\ell}{m_{\ell'}}\right)\cdot\ln\left(\frac{m_\ell}{m_{\ell''}}\right) - \frac{25}{54}\ln\left(\frac{m_\ell^2}{m_{\ell'}m_{\ell''}}\right) + \frac{\pi^2}{27} + \frac{317}{24} + O\left(\left(\frac{m_{\ell'}}{m_\ell}\right)^2\ln\left(\frac{m_\ell}{m_{\ell'}}\right), \left(\frac{m_{\ell''}}{m_\ell}\right)^2\ln\left(\frac{m_\ell}{m_{\ell''}}\right)\right)\right\} \tag{2.2}$$

3. THE HADRONIC CONTRIBUTION AT LOWEST ORDER

It is the contribution of the diagram shown in figure 5. This contribution can be expressed as a convolution integral over the total e^+e^- into hadrons cross-section $\sigma_H(t)$ [6]:

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$$a_\ell(\text{Hadrons}) = \frac{1}{4\pi^2\alpha}\int_{4m_\pi^2}^{\infty} dt\; \sigma_H(t)\, K\left(\frac{t}{m_\ell^2}\right) \tag{3.1}$$

where [9] :

$$K\left(\frac{t}{m_\ell^2}\right) = \left(\frac{\alpha}{\pi}\right)\int_0^1 dx\; \frac{x^2(1-x)}{x^2+(t/m_\ell^2)(1-x)} \tag{3.2}$$

and m_ℓ is the lepton-mass.

The same analysis as in the muon case [6] leads to the results shown in figure 7 :

i) The contribution of the energy-region : $1.2 \leq \sqrt{t} \leq 7.4$ GeV becomes more and more important. In fact it dominates the other hadronic states contributions for a heavy lepton of mass higher than 2.5 GeV.

ii) The relative importance of the $\psi + \psi'$ contribution increases compared to the low-energy states contribution ($\omega + \varphi$, $n\pi\ (n \geq 4)$).

iii) The asymptotic region contribution ($\sqrt{t} \geq 7.4$ GeV at present)[7] becomes greater than the ρ-contribution for a heavy lepton of mass higher than 17 GeV.

This asymptotic region contribution has been estimated from the asymptotic behavior of σ_H ($\sqrt{t} \geq 7.4$ GeV) given by gauge theories with asymptotic freedom[8]:

$$\sigma_H^\infty(t) = \frac{4}{3}\pi\alpha^2\frac{1}{t}\left(3\sum_i q_i^2\right)\left(1+\frac{\delta}{\ln(t/\mu^2)}\right) \tag{3.3}$$

where $i = u, d, s, c\ldots$; q_i denotes the charge of the relevant quark in units of e ; $\delta = (12/25)$ for $SU(3)_{color} \otimes SU(4)_{flavor}$; μ is an arbitrary mass scale, which we have fixed in such a way that $\sigma_H^\infty(\sqrt{t} = 7.4\,\text{GeV}) = \sigma_H^{exp}(\sqrt{t} = 7.4\,\text{GeV})$. It corresponds to $\mu = (5.4 \pm 0.6)$ GeV.

The above points (i) to (iii) are mainly due to the energy-dependence of the QED function $K\left(\frac{t}{m_\ell^2}\right)$. From Eq.(3.2) it can be readily seen that at fixed t, the size of $K\left(\frac{t}{m_\ell^2}\right)$ increases with m_ℓ^2.

IV. THE WEAK INTERACTIONS-CONTRIBUTION

We study the weak interactions-contribution to the charged lepton-anomaly in the simplest gauge model of Weinberg and Salam [10]. We can apply the muon results [4] provided the masses of the lepton and of its associated neutrino are smaller than the gauge bosons-masses.

The contributions of figures 6a and 6b, denoted by a_ℓ^{J} and a_ℓ^{Z}, are proportional to the squared of the lepton mass [11]. Therefore they are important for heavy leptons and become greater than the hadronic contribution for $m_\ell \gtrsim 7$ GeV, as shown in figure 8. Also, contrary to the muon case, the contribution of the Higgs scalar diagram shown in figure 6c could be important. From the expression of this contribution [11]:

$$a_\ell^{\varphi} = \frac{G m_\ell^2}{8\pi^2\sqrt{2}} \cdot 2\int_0^1 dx \frac{x^2(2-x)}{x^2 + \left(\frac{m_\varphi}{m_\ell}\right)^2(1-x)} \tag{4.1}$$

we deduce that :

$$0 \leq a_\ell^{\varphi} \leq \frac{G m_\ell^2}{8\pi^2\sqrt{2}} \cdot 3 \tag{4.2}$$

where m_ℓ is the charged lepton mass and m_φ the Higgs scalar mass.

It is shown in figure 8, that the total contribution of the weak interactions including the upper value of the Higgs-scalar contribution becomes greater than the hadronic-contribution for $m_\ell \gtrsim 4$ GeV.

5. THE ANOMALY OF THE NEW CHARGED LEPTON τ(1.9 GeV)

In order to illustrate our precedent analysis, we give the different theoretical contributions to the anomaly $a_\tau \equiv \frac{1}{2}(g_\tau - 2)$ of the new lepton with mass 1.9 GeV[1] and we make a comparison with the muon case. The results and the comments are summarized in tables 1 and 2.

6. WHAT COULD BE LEARNED FROM THE MEASUREMENT OF THE ANOMALY OF A CHARGED HEAVY LEPTON ?

A Comparison of the theoretical contributions with an eventual measured value of the anomaly of a charged heavy lepton would provide:

i) a check of the hypothesis of a point-like structure of this new charged particle. Indeed the calculation depends on this assumption because we use the quantum electrodynamics (QED) expansion series.

ii) a good test of the short-distance behavior of quantum electrodynamics. Indeed, it has been shown[12] that the asymptotic part of the lepton anomaly, without light-by-light scattering subgraphs, obeys a Callan-Symanzik type equation. The solution of this equation can be expanded in power series of the effective QED coupling constant $\bar{\alpha}(t = \ln \frac{m_\ell}{m_{\ell'}}, \alpha)$ [12] defined by the Gell-Mann Low equation :

$$\frac{d\bar{\alpha}}{dt} = \bar{\alpha}\beta(\bar{\alpha}) \tag{6.1}$$

with $\bar{\alpha}(t=0) = \alpha$, m_ℓ is the heavy lepton mass and $m_{\ell'} < m_\ell$ is the mass of the light lepton occuring in the vacuum polarization insertions in the photon propagator.

It is clear that for a heavy lepton of mass m_ℓ, we are going to test the quantum electrodynamics expansion series in an energy-region $\ell = \ln\left(\frac{m_\ell}{m_e}\right)$ not reached in the muon-case.

iii) a better estimate of the asymptotic region contribution $(\sqrt{t} \geq 7.4$ GeV at present). Indeed for a heavy lepton this contribution becomes important and is greater than the ρ -contribution for a heavy lepton of mass higher than 17 GeV. A test of the validity of the expression of the $e^+e^- \rightarrow$ Hadrons total cross-section given by gauge theories with asymptotic freedom [8] could therefore be obtained.

iv) An alternative way to test the Weinberg-Salam model. Indeed, the anomaly of a charged heavy lepton is very sensitive to the weak interactions effects ; these contributions are important for a heavy lepton as shown in figure 8.

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TABLE 1

The different theoretical contributions to a_μ and a_τ in units of 10^{-9}. The indexes μ and τ refer to the muon and the τ-lepton.

TYPE	a_μ	a_τ	COMMENTS
QED up to 6th order	1165841.1 ± 1.2	1169662.0 ± 3.0	The 6th order contribution is still dominated by the graphs of light-by-light scattering subgraphs (fig.2) and the error comes mainly from its numerical integration [4].
hadrons (lowest order)	70.2 ± 8.0	3841.4 ± 466.8	i) The difference of a_μ and a_τ is due to the energy-dependence of the QED function: $K\left(\frac{t}{m_\ell^2}\right) = \frac{\alpha}{\pi}\int_0^1 \frac{x^2(1-x)}{x^2+(t/m_\ell^2)(1-x)}\,dx$ (m_ℓ is the lepton-mass). ii) The error comes from the measurement of the e^+e^- into hadrons cross section.
Weak-interaction (Weinberg-Salam)	2.1 ± 0.2	827.2 ± 148.	i) Increase of the importance of this effect because of the m_ℓ^2 - dependence. ii) The Higgs-scalar effect could be significant for the τ since : $0 \leq a_\tau^{\varphi} \leq 148.$ for a scalar mass $m_\varphi \geq 4.5$ GeV.
TOTAL	1165913.4 ± 9.4	1174330.6 ± 617.8	

TABLE 2

The different hadronic states and the weak interaction effect compared to the total lowest order hadronic contribution. The results are per cent and normalized by a_μ(Hadrons) for the muon and by a_τ(Hadrons) for the τ.

TYPE	REFERENCES	μ	τ	COMMENTS
Hadrons states				
ρ	[14]	68.5	43	Increase of the relative importance of the high-energy contribution because of the QED function: $K(\frac{t}{m_\ell^2})$
ω, φ	[15]	13	11	
$\psi + \psi'$	[7]	1	3	
$n\pi\,(n \geq 4)$ $(0.99\ \text{GeV} \leq \sqrt{t} \leq 1.2\ \text{GeV})$	[16]	0.6	3	
$1.2\ \text{GeV} \leq \sqrt{t} \leq 7.4\ \text{GeV}$	[17] & [7]	16	37	
asymptotic region $(\sqrt{t} \geq 7.4\ \text{GeV})$	[8]	0.8	5	
Weak interactions (Weinberg-Salam)	[11]	3	21.5	Increase of the relative importance of this effect because of the m_ℓ^2 dependence.

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- FIGURE CAPTIONS -

FIG.1 : α^2- contribution to the ℓ-anomaly from a lepton ℓ';

FIG.2 : Two of the six graphs of light-by-light scattering subgraphs contributing to $(a_\ell - a_{\ell'})^{(6)}$.

FIG.3 : (a) One of the fourteen graphs with second order ℓ'-vacuum polarization subgraphs contributing to $(a_\ell - a_{\ell'})^{(6)}$.

(b) One of the fourth graphs with fourth order ℓ'-vacuum polarization subgraphs contributing to $(a_\ell - a_{\ell'})^{(6)}$.

FIG.4 : One of the two graphs with a double bubble subgraphs constitued by ℓ' and $\ell'' \neq \ell'$-vacuum polarizations, which contribute to $a_\ell^{(6)}$.

FIG.5 : Hadronic vacuum polarization correction to lowest order contribution to a_ℓ.

FIG.6 : (a) Neutrino exchange contributing to a_ℓ in the Weinberg-Salam model.

(b) Z-boson exchange contributing to a_ℓ in the same model.

(c) Higgs scalar exchange contributing to a_ℓ in the same model.

FIG.7 : Plot of the different hadronic states contribution versus the lepton mass m_ℓ.

FIG.8 : Plot of the total lowest order hadronic contribution and of the weak interactions contribution in the Weinberg-Salam model, versus the lepton-mass m_ℓ.

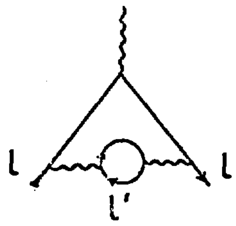

FIG.1

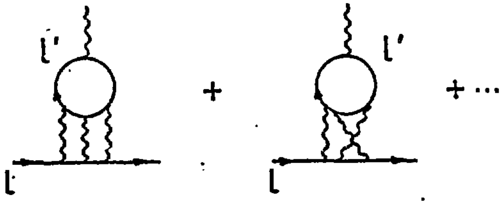

FIG. 2

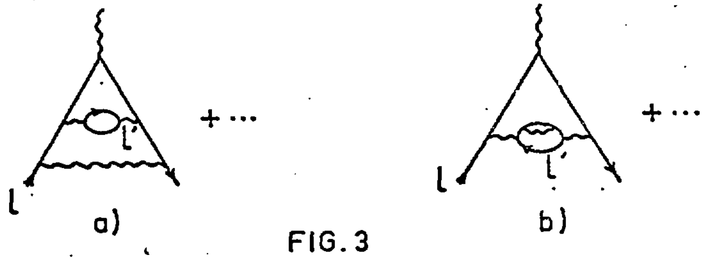

FIG. 3

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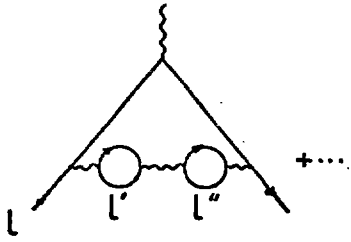

FIG. 4

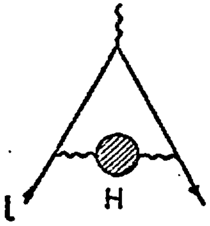

FIG. 5

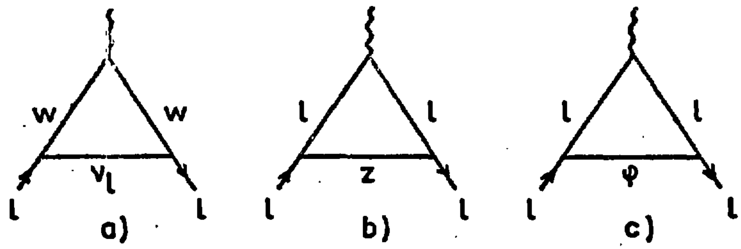

Fig. 6

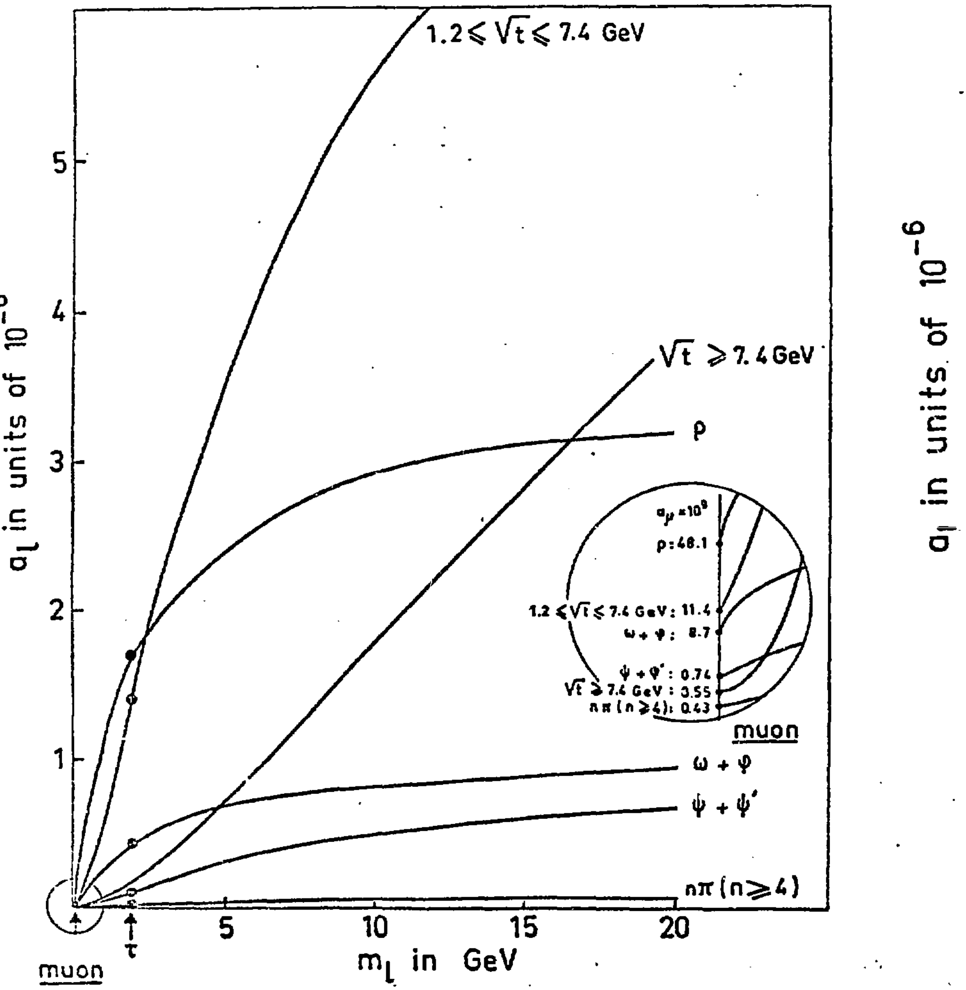
1.2 ≤ √t ≤ 7.4 GeV
√t ≥ 7.4 GeV
ρ
ω + φ
ψ + ψ'
nπ (n ≥ 4)
$a_\mu \times 10^9$
ρ : 48.1
1.2 ≤ √t ≤ 7.4 GeV : 11.4
ω + φ : 8.7
ψ + ψ' : 0.74
√t ≥ 7.4 GeV : 0.55
nπ (n ≥ 4) : 0.43
muon
1
2
3
4
5
5
10
15
20
τ
muon
m_L in GeV
a_L in units of 10^{-6}
a_l in units of 10^{-6}

Fig. 7

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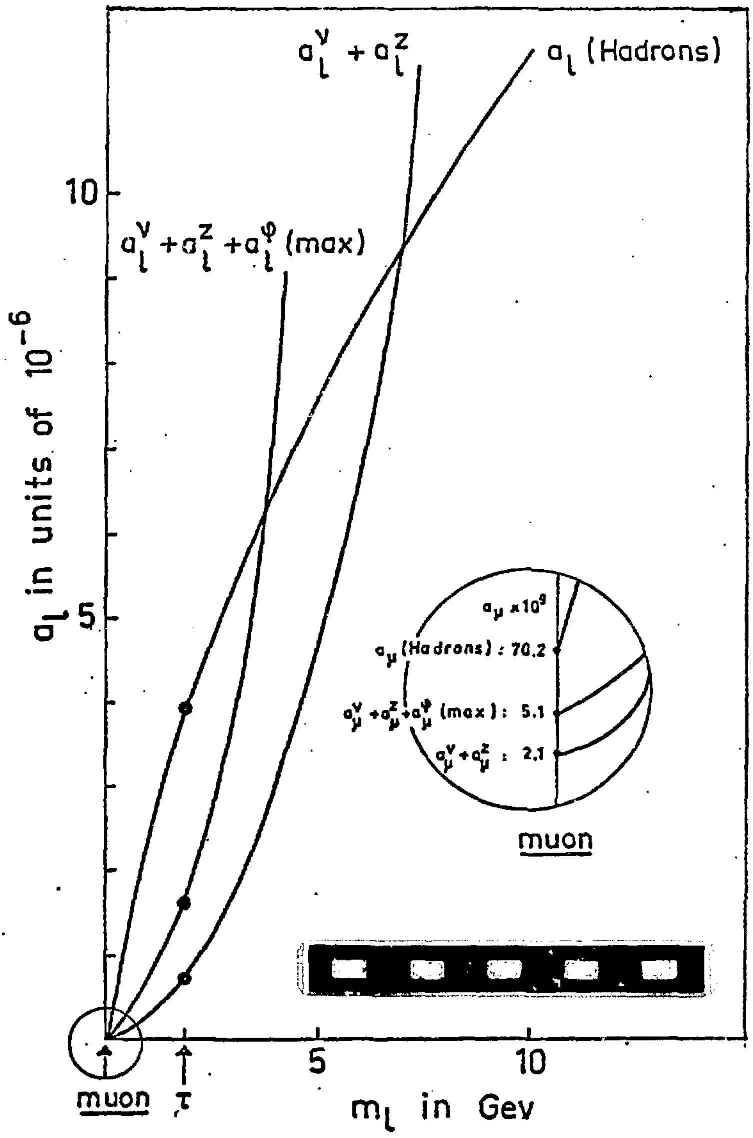

a_l^v + a_l^z
a_l (Hadrons)
10
a_l^v + a_l^z + a_l^φ (max)
a_l in units of 10^-6
5
a_μ x 10^9
a_μ (Hadrons) : 70.2
a_μ^v + a_μ^z + a_μ^φ (max) : 5.1
a_μ^v + a_μ^z : 2.1
muon
5
10
muon
τ
m_l in Gev

Fig. 8