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TURBULENT DESTABILIZATION AND SATURATION OF THE UNIVERSAL DRIFT MODE IN A SHEARED MAGNETIC FIELD

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ABSTRACT

In a sheared magnetic field, turbulent diffusion of electrons in the vicinity of a mode rational surface can eliminate the stabilizing influence of nonresonant electrons and lead to an absolute instability at small but finite wave amplitudes. As the turbulence grows, the inverse electron Landau resonance is broadened in both velocity and configuration space, and the convective shear damping due to ions is enhanced by turbulent spatial broadening of the mode until saturation occurs.

The original work of Pearlstein and Berk[1] indicated the existence of an absolute universal instability of a confined plasma ($\nabla p \neq 0$) in a sheared magnetic field. Recently, numerical integration of the exact differential equation describing the radial structure of the drift wave eigenmode showed the absence of an absolute instability, regardless of how weak the shear or how large the poloidal wave number.[2,3] The stability of the universal mode in these improved treatments is due to the inclusion of nonresonant, nonadiabatic electrons in the region about the mode rational surface where $k_{\parallel}(r) = [m - nq(r)]/Rq \lesssim \omega/v_{Te}$. Here, m and n are poloidal and toroidal mode numbers, respectively, $q(r) = rB_T/RB_P$ is the safety factor, ω is the mode frequency, and $v_{Te} = (2T_e/m_e)^{1/2}$ is the electron thermal velocity. Thus, instability might be recovered by an effect altering the electron response in the region around the rational surface.

In this paper it is shown that turbulent diffusion of electrons across the rational surface, due to a combination of shear ($\partial k_{\parallel}/\partial r \equiv k_{\parallel}' \neq 0$) and random $\vec{E} \times \vec{B}$ fluctuations and/or stochastic magnetic perturbations, results in a finite amplitude-induced version of the

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absolute universal instability. Physically, the turbulent scattering of electrons across the rational layer leads to an effective finite value for $k_\parallel$ which destroys the stabilizing influence of the nonresonant electrons. At larger amplitudes, the electron growth is reduced and the ion shear damping is enhanced by spatial broadening of the mode, yielding nonlinear stabilization.

The turbulent diffusion process in a sheared magnetic field produces a resonance broadening mechanism for the electrons which is fundamentally different than the process, due to random $\vec{E} \times \vec{B}$ drifts alone, in a shearless field.[4,5] With shear, stochastic radial motion combines with parallel electron streaming to induce random poloidal motion. The decorrelation frequency resulting from this random motion of electrons in a sheared field can exceed the magnitude of the *real* part of the linear eigenfrequency for low levels of turbulence.

The electron distribution function for a turbulent plasma in a sheared magnetic field is written $f_e = \overline{F}_e + \tilde{f}_e$, where $\overline{F}_e$ is the phase averaged part of f_e and $\tilde{f}_e$ is the fluctuating response. The phase averaged distribution satisfies a quasilinear type of equation:

$$\left(\frac{\partial}{\partial t} + v_\parallel \vec{n} \cdot \nabla\right)\overline{F}_e + \vec{n} \cdot \left\langle \nabla\Phi \times \nabla\tilde{f}_e(\overline{F}_e)\right\rangle / B + |e| v_\parallel \left\langle \vec{n} \cdot \nabla\Phi \; \partial\tilde{f}_e/\partial\varepsilon \right\rangle = 0 \; , \tag{1a}$$

where $\vec{E} = -\nabla\Phi$, $\varepsilon = 1/2 \, m_e v^2$, $\vec{n} = \vec{B}/B$, and brackets denote the phase average. The fluctuating part of f_e satisfies the nonlinear drift equation:

$$\left(\frac{\partial}{\partial t} + v_{\parallel}\vec{n}\cdot\nabla - \frac{\nabla\Phi\times\vec{B}}{B^2}\cdot\frac{\partial}{\partial\vec{r}} + |e|v_{\parallel}\vec{n}\cdot\nabla\Phi\frac{\partial}{\partial\varepsilon}\right)\tilde{f}_e$$
$$= (-|e|v_{\parallel}\vec{n}\cdot\nabla\Phi + i\omega_{*e}|e|\Phi)\partial\overline{F}_e/\partial\varepsilon \quad . \tag{1b}$$

Here, $\omega_{*e} \equiv i\,\vec{n}\times\nabla\overline{F}_e/(|e|B\partial\overline{F}_e/\partial\varepsilon)\cdot\nabla\,\ell n\,\Phi = k_\theta\,\partial\overline{F}_e/\partial r(|e|\,B\partial\overline{F}_e/\partial\varepsilon)^{-1}$ is the electron diamagnetic frequency and $k_\theta = m/r$ is the poloidal wave number. The effects of magnetic field fluctuations have been neglected here, but will be mentioned later.

Integrating Eq. (1b) along perturbed electron trajectories yields for the phase coherent part[4] of $\tilde{f}_e = \Sigma\,\tilde{f}_e^{\vec{k}}(r)\exp[-i\omega t + i(m\theta - n\phi)]$:

$$\tilde{f}_e^{\vec{k}} = -|e|\Phi_{\vec{k}}\,\partial\overline{F}_e/\partial\varepsilon - i(\omega - \omega_{*e})|e|(\partial\overline{F}_e/\partial\varepsilon)R_{\vec{k}}\Phi_{\vec{k}} \quad , \tag{2}$$

where $\Phi = \Sigma_{\vec{k}}\,\Phi_{\vec{k}}(r)\exp[-i\omega t + i(m\theta - n\phi)]$ and the resonance operator is:

$$R_{\vec{k}}\Phi_{\vec{k}} \equiv \int_0^\infty \exp(i\omega'\tau)\left\langle \Phi_{\vec{k}}\,[r(\tau)]\exp[-im\delta\theta(\tau) + in\delta\phi(\tau) - ik_{\parallel}'v_{\parallel}\int_0^\tau \delta r(\tau')d\tau']\right\rangle d\tau \quad , \tag{3}$$

where $\omega' = \omega - k_{\parallel}v_{\parallel}$. The quantity in the phase average in Eq. (3) represents the nonlinear response to random $\vec{E}\times\vec{B}$ fluctuations, e.g., $d\delta r(\tau)/d\tau = -\nabla_\theta\Phi/B$. Its role is to broaden the linear wave-particle resonance $\omega - k_{\parallel}v_{\parallel} \approx 0$ over a width τ_c^{-1}, where τ_c is the correlation time arising from the turbulent scattering of the electron orbit. The terms $\propto \delta\theta(\tau)$ and $\delta\phi(\tau)$ have been previously computed for a uniform magnetic field.[4,5] The new term $\propto k_{\parallel}'\int^\tau\delta r$ is an additional change in θ due to nonlinear radial motion, $d\delta\theta/d\tau = -(v_{\parallel}/Rq)(\partial\,\ell n\,q/\partial r)\delta r(\tau)$. It gives the dominant broadening in a sheared field. To show this, the phase average in Eq. (3) is evaluated using a cumulant expansion:

$$R_{\vec{k}}\Phi_{\vec{k}} \approx \int_0^\infty \exp[i\omega'\tau - \frac{1}{2} m^2\langle\delta\theta^2\rangle - \frac{1}{2}(k_\parallel' v_\parallel)^2\left\langle\left(\int_0^\tau \delta r d\tau'\right)^2\right\rangle] G(\Phi)d\tau \quad \text{(4a)}$$

where

$$G(\Phi) = \int_{-\infty}^{\infty} \frac{dr'}{\sqrt{2\pi\langle\delta r^2(\tau)\rangle}} \exp[-(r - r')^2/(2\langle\delta r^2(\tau)\rangle)]\Phi(r') \quad . \quad \text{(4b)}$$

The average displacements appearing in Eq. (4) may be evaluated by substituting $\tilde{f}_e^{\vec{k}}$ from Eq. (2) into Eq. (1a) and taking the $\delta\theta^2$ and δr^2 moments of the resulting diffusion equation for $\overline{F}_e$. This yields $\langle\delta\theta^2\rangle = 2D_{\theta\theta}\tau/r^2$ and $\langle\delta r^2\rangle = 2D_{rr}\tau$, where the diffusion tensor for electrostatic turbulence is:

$$\overleftrightarrow{D} = B^{-2} \sum_{\vec{k}} \left\langle \vec{n} \times \nabla\Phi_{\vec{k}}^* \vec{n} \times \nabla R_{\vec{k}}\Phi_{\vec{k}} \right\rangle \quad . \quad \text{(5)}$$

The inclusion of finite β effects on the drift modes considered here results in an enhanced radial diffusion coefficient due to magnetic fluctuations:[6]

$$D_{rr} = v_\parallel^2 B^{-2} \sum_{\vec{k}} \left\langle b_{r\vec{k}}^* R_{\vec{k}} b_{r\vec{k}} \right\rangle \quad . \quad \text{(6)}$$

Noting $\langle\delta r(t_1)\delta r(t_2)\rangle = \langle\delta r^2(|t_1 - t_2|)\rangle = 2D_{rr}|t_1 - t_2|$, Eq. (4) becomes:

$$R_{\vec{k}}\Phi_{\vec{k}} = \int_0^\infty d\tau \exp[i\omega'\tau - \tau/\tau_{c0} - (\tau/\tau_c)^3]G(\Phi) \quad \text{(7)}$$

where $\tau_{c0}^{-1} = k_\theta^2 D_{\theta\theta}$ is the shearless decorrelation frequency,[4] and $\tau_c^{-1} = [(k_\parallel' v_\parallel)^2 D_{rr}/3]^{1/3}$ is the decorrelation frequency in a sheared magnetic field, which vanishes in the absence of wave-particle energy transfer. Note that $\xi \equiv \tau_c/\tau_{c0} \approx [L_S D_{rr}(k_\theta \Delta r^3 v_{Te})^{-1}]^{2/3}$, where $L_S = Rq^2/(rq')$ and

$\Delta r \gtrsim k_\theta^{-1}$ is the radial mode width. For tokamaks $\xi << 1$, except near the value of D_{rr} required for saturation of short wavelength modes, for which $\xi \lesssim 1$. Henceforth, terms of $\mathcal{O}(\xi)$ are neglected. For $\xi < 1$, $G(\Phi) \approx \Phi(r) + D_{rr}\tau\, \partial^2\Phi/\partial r^2$, representing turbulent broadening of Φ over a correlation length $L_c = \sqrt{D_{rr}\tau_c}$. This contrasts with the shearless case where $\Delta r^2/(L_c^2\xi^{-1}) \sim 1$ and $G(\Phi) = \Phi(r) \exp\,[-k_r^2 D_{rr}\tau]$ contributes to the resonant wave particle energy transfer.

Using Eq. (7) to calculate the electron density perturbation, assuming $\overline{F}_e$ is a Maxwellian, and invoking the linear ion response (for $\xi < 1$) together with quasineutrality yields the eigenmode equation:

$$\frac{\partial^2\Phi}{\partial x^2} - \left[\Lambda - \mu^2 x^2 + \frac{\sigma(x)}{x}\right]\Phi = 0 \quad . \tag{8}$$

In Eq. (8), $x = (r - r_0)/\rho_i$, where $\rho_i = (T_i m_i)^{1/2}/eB$ is the ion Larmor radius, $q(r_0) = m/n$ defines the location r_0 of the rational surface, and $\Lambda = [1 + \tau(1 - \Gamma_0) - \Gamma_0\omega_{*e}/\omega]d^{-1}$ contains the basic drift wave response. Here, $\tau = T_e/T_i$, $\Gamma_n = I_n(b)\exp(-b)$, and $b = (k_\theta\rho_i)^2$. The shear parameter is $\mu = \tau^{-1}(L_n/L_S)(\omega_{*e}/\omega)[\Gamma_0(\tau + \omega_{*e}/\omega)d^{-1}]^{1/2}$, with $L_n^{-1} = -\partial\, \ell n\, n/\partial r$. The destabilizing electron contributions are contained in $\sigma(x) = \sigma_0 Z[(x_e + ix_c)/x]$, where $\sigma_0 = (\omega/\omega_{*e} - 1)\alpha\, d^{-1}$, $x_e = \alpha\omega/\omega_{*e}$, $x_c = \alpha\, \omega_c/\omega_{*e}$, $\alpha = (1/2\, \tau\, m_e/m_i)^{1/2}\, L_S/L_n$, Z is the plasma dispersion function, and $\omega_c = [(k_\parallel' v_{Te})^2 D_{rr}/3]^{1/3}/\Gamma(4/3)$. To perform the velocity space integrals of $\tilde{f}_e$ in terms of the Z-function, a Lorentzian form for the resonance function was chosen. The quantity $d = (\Gamma_0 - \Gamma_1)(\tau + \omega_{*e}/\omega) + 1.2\, i\, (1 - \omega/\omega_{*e})(\omega_{*e}/\omega_c)x_c^2$ includes both ion gyroradius and turbulent broadening effects. Eq. (8)

is valid provided $(\partial \ln \Gamma_0/\partial b)\partial^2/\partial x^2 << 1$. In a sheared magnetic field, the neglect of ion Landau damping in Eq. (8) requires $(k_\parallel' r_{eff} v_{Ti}/\omega)^2 < 1$, where $r_{eff} = \Delta r + \rho_i$ is the mode width $\Delta r = \rho_i/|\sqrt{\mu}|$ broadened by the finite ion gyroradius.

The turbulence enters the electron response function $\sigma(x)$ through the effective collision frequency ω_c. However, turbulence does not affect the electrons in the same way as a local (in real space) number conserving collision operator, which is known to have a stabilizing influence on drift waves in slab geometry.[7] Indeed, the $\vec{E} \times \vec{B}$ (or magnetic) fluctuations scatter the particle orbits in real space, producing a turbulent flux of electrons in the kinetic Eq. (1) for f_e.

For $\omega_c << \omega$, Eq. (8) reduces to the eigenvalue problem solved in Refs. 2, 3. At small turbulence levels (which may be initially present, for example, due to small amplitude tearing activity), it is possible to achieve $\omega_c > \omega$. For $\omega/\omega_{*e} \lesssim 1/3$ and $b \gtrsim 1$, ω_c is already comparable to ω for a turbulent diffusion coefficient nearly as small as the neoclassical value. Thus, the effects of turbulence are well illustrated in the limit $\omega/\omega_c < 1$. Then the Z-function in the electron response becomes purely imaginary and there is no longer any nonresonant electron contribution, which previously led to stabilization of the linear universal mode in a quiescent plasma.[2,3]

The destabilizing electron contribution to Eq. (8) can be treated by perturbation theory[3] (for $\mathrm{Im}\omega/\mathrm{Re}\omega < 1$), using the full electron Z-function. The dispersion relation for the most unstable modes becomes

$$\Lambda + i\mu + \Sigma_e(x_e, x_c) = 0 \tag{9}$$

where $\Sigma_e = \int_0^\infty \phi_0^2(x)[\sigma(x)/x]dx / \int_0^\infty \phi_0^2 dx$ and $\phi_0(x) = \exp(-i\mu x^2/2)$ is the lowest order eigenmode corresponding to the propagation of energy away from the rational surface[1] for $x > |\sqrt{\mu}|$. Treating $\partial\omega_c/\partial x \approx 0$ for $|x| < |\sqrt{\mu}|$ yields $\Sigma_e = 2i\sigma_0\sqrt{i\mu}\, H[-2i(x_e + ix_c)\sqrt{i\mu}]$, where $H(z) = \int_0^\infty e^{-zt}(1 + t^2)^{-1/2}\, dt$. The branch $\mathrm{Re}\sqrt{i\mu} > 0$ for $\mathrm{Im}\mu < 0$ is chosen.

For relatively small values of $\omega_c/\omega_{*e} \gtrsim 0.1$, the figure shows that with $k_\theta\rho_i \gtrsim 1.0$ and moderate shear ($L_S/L_n = 16$), the turbulence destabilizes the drift mode, with maximum growth rates $\mathrm{Im}\omega/\mathrm{Re}\omega \sim 0.2$. There is good agreement between the numerical results using the shooting code described in Ref. 2 (which predicted stability for $\omega_c = 0$) and the analytic dispersion relation in Eq. (9). As the turbulence level increases, the electron growth arising from Σ_e is weakened and finally reduced to a value where shear damping, enhanced by turbulent broadening of the mode, leads to stabilization. There is a narrow range of values for ω_c, $0.1 \lesssim \omega_c/\omega_{*e} \lesssim 2$, corresponding to a variation in D_{rr} over three orders of magnitude, over which the nonlinear instability is excited and finally saturates.

The value of ω_c, and hence the turbulent diffusion coefficient, required for saturation of this instability can be determined by solving Eq. (9) at marginal stability. As the turbulence grows, $x_c|\sqrt{\mu}|$ approaches unity (the mode width is limited to $\Delta x \gtrsim x_c$ by turbulent broadening). In this limit, the approximate stability criterion becomes (for $b \gtrsim 1$, corresponding to the modes most difficult to stabilize):

$$(\omega_{*e}/\omega_c)^2 - \mu_0^2\,(\omega_{*e}/\omega)(8\pi b^3)^{-1/2} - 0.36\,\mu_0^4 x_c^4 = 0\ , \qquad (10)$$

where $\Lambda_0 = \Lambda d$ and $\mu_0 = \mu d^{1/2}$. (Recall that $x_c = \alpha\, \omega_c/\omega_{*e}$, where $\alpha \approx 1$ depends on the shear length.) There is also a nonlinear increase in the frequency determined by $\Lambda_0 = 0.6\, \mu_0^2 x_c^2$. In Eq. (10), the first term represents broadened electron growth, the second is due to linear shear damping and the last is enhanced shear damping resulting from the turbulent spatial broadening of the mode which decreases the effective shear length [the Φ'' term in $G(\Phi)$, cf. Eq. (7)]. A stabilization mechanism similar to this latter one has been computed for a Q-machine in Ref. 8. There, however, $\tau_c/\tau_{c0} \gg 1$, so that mode coupling in the *ion* kinetic equation was the dominant nonlinearity (with adiabatic electrons).

The maximum diffusion coefficient obtained from Eq. (10) occurs for $b \equiv b_0 \simeq (1 + \tau)^3 \tau^{-2} (L_n/L_S)^3 (m_i/m_e)$, where the nonlinear and linear shear damping become comparable. Typically, $b_0^{1/2} \lesssim b_1^{1/2} \equiv \tau(1 + \tau)^{-1} \times (L_S/L_n)$, where $b \lesssim b_1$ is sufficient to neglect ion Landau damping in Eq. (8). Also, the mode width $\Delta x \sim \alpha\, b_1^{1/2} \gtrsim 1$, which justifies the use of the differential Eq. (8). The diffusion coefficient which results from maximizing D_{rr} with respect to b is:

$$D_{rr} = \frac{2 \times 10^6}{\tau(1 + \tau)} \left(\frac{\tau}{1 + \tau}\, \frac{L_S}{L_n}\right)^{7/2} \frac{T_e^{3/2}}{B^2 L_n}\ \mathrm{cm^2/sec}\ , \qquad (11)$$

with B in kg, T_e in keV, and L_n in cm. The associated electron thermal conduction coefficient is $\kappa_e = 3/2\, D_{rr}$. For the ISX-A discharge[9] with plateau regime electrons, where the main electron energy loss channel might correspond to the effects considered here, Eq. (11) yields the correct order of magnitude to account for electron heat transport outside the q = 1 surface.

Equations (5-6) and (11) can be used to estimate the fluctuation level required to stabilize these modes. If magnetic braiding is the dominant stochastic mechanism, Eq. (6) indicates $\tilde{b}^2/B^2 \sim 10^{-8}$ is sufficient for stabilization.[10] If $\vec{E} \times \vec{B}$ turbulence is dominant, Eq. (5) yields the following result in the strong turbulence limit $\omega_c \gtrsim \omega'$:

$$\frac{\tilde{n}}{n} = \frac{1}{4} \frac{\tau^2}{(1 + \tau)^4} \left(\frac{L_s}{L_n}\right)^3 \frac{\rho_e}{L_n} \quad . \tag{12}$$

For typical ISX-A data, $\tilde{n}/n \approx 0.04$ is obtained from Eq. (12).

In conclusion, destabilization and saturation of the drift mode in a sheared field have been shown to result from a resonance broadening mechanism that dominantly affects electrons. This contrasts with previous turbulence theories in a shearless field,[4] where nonlinear ion damping led to saturation and the electron dynamics were linear. Thus, whereas recent theory[5] indicates that for tokamak parameters, ion nonlinearity is not a viable saturation mechanism for electrostatic drift modes, the present theory predicts saturation at modest fluctuation levels.

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FIGURE CAPTION

Figure 1 Growth rate (normalized to real frequency) vs ω_c (normalized to ω_{*e}) for $T_e/T_i = 1$, $L_S/L_n = 16$, and various values of $k_\theta \rho_i$, obtained numerically (solid line) and from analytic dispersion relation (dashed line).

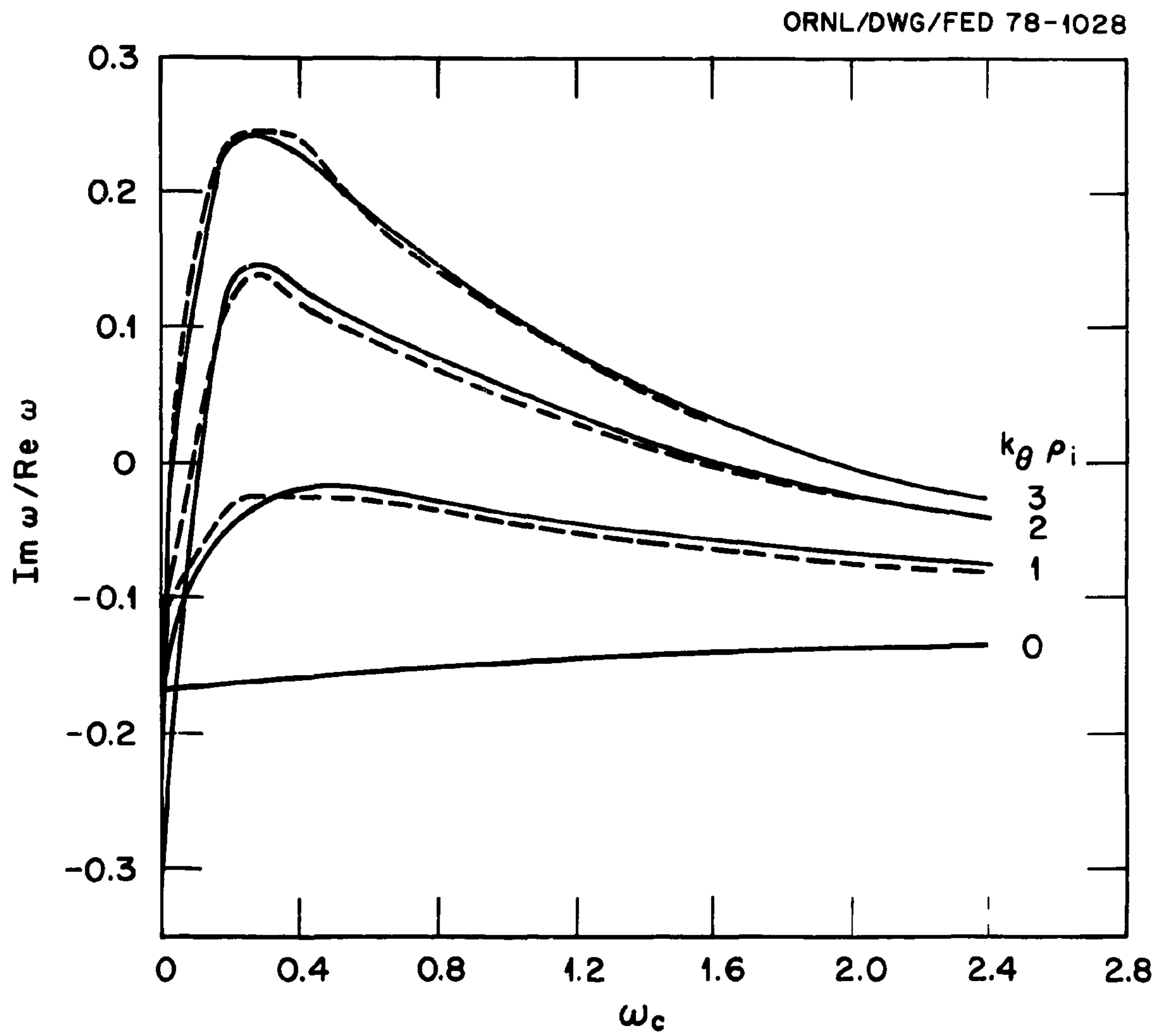
ORNL/DWG/FED 78-1028
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
Im ω/Re ω
$k_\theta \rho_i$
3
2
1
0
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
ω_c

Fig. 1