QUARK-MIXING WITH SMALL ANGLES)*

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and Gauge Fields",

ABSTRACT :

We try to abstract some general features from symmetry models for the Yukawa-interactions of quarks. We demand that the successful relation $tg^2\vartheta_c = d/s$ (ϑ_c is the Cabibbo-angle, d and s are the masses of the down-quark and the strange-quark) is incorporated into a model for 6 flavours (u,d,c,s, t,b), arranged in three left-handed doublets. If the CP-violation is determined by the generalized GIM-matrix with a "naturally" large phase, we are led to the "prediction" $9 \lesssim t \lesssim 13$ GeV for the mass of the top-quark. The new mixing angles turn out to be very small ($\lesssim \vartheta_c/10$). In anticipation of this result we develop also a simple phenomenology, which at small angles may be more useful than the standard one by Kobayashi and Maskawa.

1. INTRODUCTION

The least artificial explanation of the narrow upsilon resonance [1] suggests the existence of a further quark with new flavour. This new quark could be the "bottom" in an additional doublet (t',b'), besides the usual left-handed ones:

$$\begin{pmatrix} q_i^{(u)} \\ q_i^{(d)} \end{pmatrix}_L = \begin{pmatrix} u' \\ d' \end{pmatrix}_L , \begin{pmatrix} c' \\ s' \end{pmatrix}_L , \begin{pmatrix} t' \\ b' \end{pmatrix}_L , \dots \tag{1.1}$$

In (1.1) the quark states are the "bare" ones. The standard mechanism to create masses for the "dressed" quarks of a unified theory of real and electromagnetic interactions is the introduction of Yukawa interactions $\bar q_L h \phi q_R$, which respect the gauge-invariance. In the most general case $h\phi$ is a matrix allowing for different couplings h to different scalar fields ϕ. Suitable self-interactions of the ϕ - fields trigger spontaneous breaking of the gauge symmetry with vacuum expectation values $\langle\phi\rangle$. As a consequence the Lagrangian contains mass terms $\bar q_L^{(u)} \mathcal{M}^{(u)} q_R^{(u)}$ and $q_L^{(d)} \mathcal{M}^{(d)} q_R^{(d)}$ with matrices $\mathcal{M}^{(u)}$ and $\mathcal{M}^{(d)}$ mixing the up-quarks and down-quarks separately. They are diagonalized by four independent unitary matrices

$$\begin{aligned} U_L^{(u)} \mathcal{M}^{(u)} U_R^{(u)} &= \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & \ddots \end{pmatrix} \\ U_L^{(d)} \mathcal{M}^{(d)} U_R^{(d)} &= \begin{pmatrix} m_1' & & \\ & m_2' & \\ & & \ddots \end{pmatrix} \end{aligned} \tag{1.2}$$

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$$q^{(u)}_{\binom{L}{R}} = \mathcal{U}^{(u)}_{\binom{L}{R}} u_{\binom{L}{R}} \,, \qquad q^{(d)}_{\binom{L}{R}} = \mathcal{U}^{(d)}_{\binom{L}{R}} d_{\binom{L}{R}} \,, \tag{1.3}$$

where the squares of $|m_i| = u, c, t, \ldots$, $|m_i'| = d, s, b, \ldots$ are the eigenvalues of $\mathcal{M}^{(u)}\mathcal{M}^{(u)\dagger}$ and $\mathcal{M}^{(d)}\mathcal{M}^{(d)\dagger}$ respectively. It is clear that all these steps will also depend on the way the q_R are defined (doublets as in (1.1) [2] or singlets [3]). Only the combination

$$\mathcal{U} = \mathcal{U}_L^{(u)\dagger}\, \mathcal{U}_L^{(d)} \,, \tag{1.4}$$

the GIM-matrix [4] , is observable in the charged currents of weak interactions:

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}}\, \bar{u}_{iL}\, \mathcal{U}_{ij} W_{+}\, d_{jL} + h.c. \tag{1.5}$$

For 2N flavours the NxN unitary matrix $\mathcal{U}$ may be parametrized in terms of N(N-1)/2 "angles" ϑ_i (the real parameters of O(N)) and N(N+1)/2 "phases" . The latter may be separated into N "diagonal" matrices in the N-dimensional realization of $\mathcal{U}$(N) and into $\frac{N(N-1)}{2}$ phases δ_i which we define to be the phases of complex "angles" $\Theta_i = \vartheta_i e^{i\delta_i}$. A redefinition of the 2N-1 relative phases of the u_i and d_i reduces the total number of the phases to $\frac{(N-1)(N-2)}{2}$. Therefore an "intrinsic" phase causing CP-violation may occur for N $\geq$ 3 only [5] . E.g. for N = 3 we remain thus with one phase and three (real) angles in $\mathcal{U}$.

It is clear that any relation between the "observable" masses of the quarks [6] and the Cabibbo-type angles ϑ_i , as well as the CP violating phases δ_i - provided such a relation exists at all ! - must have its roots in some symmetry of the original Yukawa couplings. A classical example [7] of such a relation (for N = 2 , $\theta = \vartheta_c$ is the Cabibbo-angle), which is well satisfied numerically, is

$$\mathrm{tg}^2\vartheta_c = d/s \quad . \tag{1.6}$$

Within the present approach (1.6) is seen follow from mass-matrices

$$\begin{aligned} \mathcal{M}^{(u)} &= \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \\ \mathcal{M}^{(d)} &= \begin{pmatrix} \alpha' & \varepsilon' \\ 0 & \alpha' \end{pmatrix} \end{aligned} \tag{1.7}$$

by trivial algebra [6]. We remark in parenthesis that the experimental values of ϑ_c = 0.22 and the standard values for the quark masses [6]

$$\begin{aligned} u &= 4.2\ \mathrm{MeV}, \qquad c = 1.15\ \mathrm{GeV} \\ d &= 75\ \mathrm{MeV}, \qquad s = 150\ \mathrm{MeV} \end{aligned} \tag{1.8}$$

are also in agreement with

$$\mathcal{M}^{(u)} = \begin{pmatrix} \alpha & \varepsilon \\ 0 & \alpha \end{pmatrix} \quad , \qquad \mathcal{M}^{(d)} = \begin{pmatrix} \alpha' & \varepsilon' \\ 0 & \alpha' \end{pmatrix} \quad , \tag{1.7'}$$

because the two equations for the two angles in $\mathcal{U}^{(u)}$ and $\mathcal{U}^{(d)}$

$$\mathrm{tg}^2\,\vartheta^{(u)} = u/c \quad , \qquad \mathrm{tg}^2\,\vartheta^{(d)} = d/s \tag{1.8}$$

lead to

$$\vartheta_c = \left| \vartheta^{(u)} - e^{i\hat{\delta}}\,\vartheta^{(d)} \right| \tag{1.9}$$

where $\hat{\delta}$ is a relative phase determined by the (complex) numbers α, ε, α', ε' . Numerically $\vartheta^{(u)}$ is not negligible as compared to $\vartheta^{(d)}$ [8] , but e.g. a "maximal" $\delta = \frac{\pi}{2}$ yields an acceptable value for ϑ_c . This is a first illustration of the difficulties which face comparisons of "theoretical" predictions of this type with experiment.

In order to arrive at such predictions, the structure (1.7) or (1.7') of the mass-matrices must be the consequence of some symmetry principle. The corresponding group can be a subgroup of the global $\mathcal{U}(N)_L \times \mathcal{U}(N)_R$ symmetry of that part of the Lagrangian, which contains the gauge fields and 2N flavours of quarks.

A continuous subgroup must be ruled out, because it creates - after spontaneous breaking - unacceptable massless Goldstone-bosons [10] . In fact the first models in which (1.7) or (1.7') have been reproduced, were relying on discrete groups. They were also based upon left-right-symmetrc gauge-theories of the type $SU(2)_L \times SU(2)_R \times U(1)$ [6,9] . The models of the last ref. [9] are especially pretty, because they not only lead to the relation (1.6), but also predict independently

the ratio $d/s = (2-\sqrt{3})/(2+\sqrt{3})$.
Actually gauge models of this type are not yet required by the present experimental data: Apart from the somewhat confused situation concerning parity violating effects of neutral currents in atoms, the standard $SU(2)_L \times U(1)$ - model [3], augmented by the additional flavours and a generalized G/M-mechanism (1.4), (1.5) seems to be nowhere in serious disagreement with the experimental data [11].
Therefore symmetry models within the $SU(2)\times U(1)$ - theory should be investigated. Suppose that a general discrete (permutation) symmetry

$$\begin{aligned} q_L &\rightarrow K^L q_L \\ q_R &\rightarrow K^R q_R \\ \phi^k &\rightarrow R^{ks}\phi^s \end{aligned} \qquad (1.10)$$

with unitary matrices K^L, K^R and R leaves

$$-\mathcal{L}_{Yukawa} = \bar{q}_L h^k q_R \phi^k + \bar{q}_L h'^k q_R \tilde{\phi}^k + h.c. \qquad (1.11)$$

invariant [12], i.e.

$$\begin{aligned} K^{L\dagger} h^k K^R R^{ks} &= h^s \\ K^{L\dagger} h'^k K^R R^{*ks} &= h'^s . \end{aligned} \qquad (1.12)$$

For the mass-matrices of u_i and d_i, $\mathcal{M}^{(u)} = h^s \varepsilon_s$, $\mathcal{M}^{(d)} = h'^s \varepsilon_s^*$ this means ($\varepsilon_i = \langle 0|\phi^i|0\rangle$)

$$K^{L\dagger} \mathcal{M}^{(u)} \mathcal{M}^{(u)\dagger} K^L = h^s (h^t)^\dagger R^{*sk} R^{t\ell} \varepsilon_k \varepsilon_\ell^* \qquad (1.13)$$

and an analogous relation for $\mathcal{M}^{(d)}$. Thus if in a certain model R is diagonal, i.e.

$$R^{il}\varepsilon_l = \lambda \varepsilon_i \, , \quad |\lambda| = 1 \, , \tag{1.14}$$

(1.13) implies

$$[K^L, \mathcal{M}^{(u)}\mathcal{M}^{(u)+}] = 0$$
$$[K^L, \mathcal{M}^{(d)}\mathcal{M}^{(d)+}] = 0$$

which means - following the argument of ref. [13] further - that no nontrivial relation for the Cabibbo-angles can occur. (1.14) is trivially true for one ϕ-field or for two ϕ-fields, one of which couples to h and the other to h'. This is the case of natural flavour-conservation in the scalar-couplings. If more scalar fields are present and if (1.14) does not hold (nontrivial permutation of ϕ^i), relations between the angles and the quark masses may follow. Thus it must be concluded that the Yukawa-couplings cannot conserve the flavours in a "natural" [14] way. Explicit models of this type are the ones in ref. [15], of which, however, only the second one contains (1.6). It has N=3 and six complex doublets of Higgs-fields.

A common feature of the known models are the not altogether "simple" assumptions about the representations of the permutation group for the quarks and the numerous scalar fields. Therefore it is the purpose of this lecture to search a common "natural" semi-phenomenological background and investigate possible consequences.

Recent theoretical considerations [16] consistently lead to small angles $\vartheta_i \lesssim \vartheta_c$ for the new ϑ_i in (1.4). We believe that such small angles can be understood naturally [14] only in terms of small angles in $\mathcal{U}_L^{(+)}$ and $\mathcal{U}_L^{(A)}$ (1.3) separately. The consequences of such an assumption for the separate diagonalization of the $\mathcal{M}^{(+)}$ and the $\mathcal{M}^{(A)}$ are discussed in sect. 2. For small angles this is a simple exercise in perturbation theory for matrices as examplified by the special case N = 3. But small values of the ϑ_i imply that ϑ_c and the "new" angles are essentially independent to $\mathcal{O}(\vartheta)$. Any further restriction must rely on phenomena, which are of higher order in ϑ. Hence the first subject of section 3 is a repetition of the argument of the last ref. [16], which uses the success of the prediction by Gaillard and Lee [21] for the charmed quark mass from the K_{O1}-K_{O2}-mass difference as a constraint. Then we check by estimating orders of magnitude, whether a matrix like (1.4) with small angles may "naturally" account for all the CP-violation in the K_o-$\bar{K}_o$ system [17]. The results as exhibited in the Conclusions are not discouraging at all.

2. Small Mixing Angles and Mass-Matrices

2a) The standard GIM matrix (1.4) in the parametrization of Kobayashi and Maskawa [5] shares with the description of $O(3)$ in terms of Euler-angles the disadvantage that the small - θ limit cannot be obtained by systematically neglecting powers of θ up to a certain order: e.g. in linear order two angles remain instead of three. It is obvious that, on the other hand

$$
\begin{aligned}
&U = e^{\Theta} = 1 + \Theta + \dots \\
&\Theta^{\dagger} = -\Theta
\end{aligned}
\qquad (2.1)
$$

does not suffer from this defect. A GIM-matrix for N = 3 may be based upon

$$
\Theta = \begin{pmatrix} 0 & \Theta_3 & -\Theta_2 \\ -\Theta_3^* & 0 & \Theta_1 \\ \Theta_2^* & -\Theta_1^* & 0 \end{pmatrix} \qquad (2.2)
$$

The "diagonal" phases (β_i = real)

$$
i \begin{pmatrix} \beta_1 & & \\ & \beta_2 & \\ & & \beta_3 \end{pmatrix}
$$

in the linear term (2.2) have been dropped already, because these β_i and the phases of Θ_2 and Θ_3 can be eliminated in the linear term of (2.1) by an appropriate redefinition of (2.1) which changes the phases of the u_i and d_j :

$$\tilde{\mathcal{U}} = A'^{\dagger}\,\mathcal{U}\,A \tag{2.3}$$

$$A = \begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & e^{i\gamma} \end{pmatrix}, \quad A' = \begin{pmatrix} e^{i\alpha'} & & \\ & e^{i\beta'} & \\ & & e^{i\gamma'} \end{pmatrix}$$

$$\theta_1 = \vartheta_1 e^{i\delta}, \quad \theta_2 = \vartheta_2, \quad \theta_3 = \vartheta_3 \tag{2.4}$$

Thus (2.1) with (2.2) becomes for N=3

$$\mathcal{U} = 1 + \Theta + \tfrac{1}{2}\mathcal{U}^{(2)} + \mathcal{O}(\theta^3)$$

$$\mathcal{U}^{(2)} = \begin{pmatrix} -|\theta_2|^2 - |\theta_3|^2 & \theta_1^* \theta_2 & \theta_1 \theta_3 \\ \theta_1 \theta_2^* & -|\theta_1|^2 - |\theta_3|^2 & \theta_2 \theta_3^* \\ \theta_1^* \theta_3^* & \theta_2^* \theta_3 & -|\theta_1|^2 - |\theta_2|^2 \end{pmatrix}, \tag{2.5}$$

which is the form to be used for the individual unitary $\mathcal{U}^{(u)}$ and $\mathcal{U}^{(d)}$ transformations of the u_i and d_i , eq. (1.3). The GIM-matrix to $\mathcal{O}(\theta^2)$ reads as (2.5) with $\theta_i \longrightarrow \hat{\theta}_i$ and

$$\begin{aligned} \hat{\theta}_1 &= \theta_1^{(d)} - \theta_1^{(u)} + \tfrac{1}{2}\left(\theta_2^{(u)} \theta_3^{(d)*} - \theta_2^{(d)} \theta_3^{(u)*}\right) \\ \hat{\theta}_2 &= \theta_2^{(d)} - \theta_2^{(u)} + \tfrac{1}{2}\left(\theta_1^{(d)} \theta_3^{(u)} - \theta_1^{(u)} \theta_3^{(d)}\right) \\ \hat{\theta}_3 &= \theta_3^{(d)} - \theta_3^{(u)} + \tfrac{1}{2}\left(\theta_2^{(d)} \theta_1^{(u)*} - \theta_2^{(u)} \theta_1^{(d)*}\right), \end{aligned} \tag{2.6}$$

if the "diagonal" phases $\hat{\beta}_i$ (which are of $\mathcal{O}(\theta^2)$) in $U = U_L^{(u)+} U_L^{(d)}$ are transformed away. The $\hat{\theta}_1$ can be simplified according to (2.4).

The ordinary Cabibbo-angle may be determined independently from a comparison between μ-decay ($\cos\vartheta_c$) and nucleon-β-decay on the one side and the semileptonic decay of strange particles ($\sin\vartheta_c$) on the other side [22] . In our notation of (2.5) with (2.4), writing for simplicity ϑ_1 instead of $\hat{\vartheta}_1$

$$\vartheta_2 \lesssim \vartheta_3 = \vartheta_c \qquad (2.4)$$

follows, because in the first line of U the experimental values

$$1 - \vartheta_2^2 - \vartheta_3^2 = \text{"}\cos^2\vartheta_c\text{"} = 0.94181 \pm 0.004$$

$$\vartheta_3^2 = \text{"}\sin^2\vartheta_c\text{"} = 0.0529 \pm 0.014 \qquad (2.5)$$

"saturate" the relation $\cos^2\vartheta_c + \sin^2\vartheta_c = 1$ within the experimental error. If one could make a similar argument for the second line of U , using sufficiently precise data on c-s-couplings from semileptonic charmed particle decays, it could be checked, whether ϑ_1 is also small [23] . We shall this assume to be the case:

$$\vartheta_1 \lesssim \vartheta_3 = \vartheta_c \qquad (2.6)$$

Then the 2x2 sector of the flavours u,d,c,s is weakly mixed with flavours of very heavy quarks and it is "natural" to assume this to be true for up-quarks and for down-quarks separately ($\mathcal{U}_L = \mathcal{U}_L^{(u)}$ or $\mathcal{U}_L^{(d)}$) as already pointed out in section 1. We are thus lead to consider [24]

$$\mathcal{U}_L = \begin{pmatrix} U_1 & \Delta \\ \Delta' & U_0 \end{pmatrix} , \quad (\Delta, \Delta') \ll U_0, U_1 \tag{2.7}$$

where U_1 is a 2x2-matrix and U_0 an (N-2)x(N-2) matrix, Δ and Δ' are rectangular ((N-2)x2 and 2x(N-2)):

$$U_1^+ U_1 = 1 + O(\Delta'^2)$$
$$U_0^+ U_0 = 1 + O(\Delta^2) \tag{2.8}$$

$$U_1^+ \Delta + \Delta'^+ U_0 = 0 \tag{2.9}$$

$M = \mathcal{M}\mathcal{M}^+$ may be decomposed analogously:

$$M = \begin{pmatrix} M_1 & V \\ V^+ & M_0 \end{pmatrix} \tag{2.10}$$

M is diagonalized by (2.7), if

$$U_1^+ M_1 U_1 + U_1^+ V \Delta' + \Delta'^+ V^+ U_1 + \Delta'^+ M_0 \Delta' = \text{diagonal} \tag{2.11a}$$

$$U_0^+ M_0 U_0 + U_0^+ V^+ \Delta + \Delta^+ V U_0 + \Delta^+ M_1 \Delta = \text{diagonal} \tag{2.11b}$$

$$U_1^+ M_1 \Delta + U_1^+ V U_0 + \Delta'^+ M_0 U_0 + \Delta'^+ V^+ \Delta = 0 \tag{2.11c}$$

$$U_0^+ V^+ U_1 + U_0^+ M_0 \Delta' + \Delta^+ M_1 U_1 + \Delta^+ V \Delta' = 0 \tag{2.11d}$$

in such a way that terms of $\mathcal{O}(\Delta^2, \Delta'^2, \Delta\Delta')$ are neglected. Thus the last terms on the l.h.s. of (2.11) may be dropped right away. From (2.11c) and (2.9) we have

$$V = \Delta u_0^\dagger M_0 - M_0 \Delta u_0^\dagger + \mathcal{O}(\Delta^2) = \mathcal{O}(\Delta) \tag{2.12}$$

This is consistent with the other eqs. (2.11).
On the other hand $M = \mathcal{M}\mathcal{M}^\dagger$ is determined from

$$\mathcal{M}_{ij} = \delta_{i1}\rho^{(1)}_j + \delta_{i2}\rho^{(2)}_j + \sigma^{(i)}_j \quad (\sigma^{(1)} = \sigma^{(2)} = 0) \,, \tag{2.13}$$

where we have distinguished the first two lines $\rho^{(1)}$ and $\rho^{(2)}$ from the others:

$$\begin{aligned} M_{ik} &= (\delta_{i1}\rho^{(1)}_j + \delta_{i2}\rho^{(2)}_j)(\delta_{k1}\rho^{(1)*}_j + \delta_{k2}\rho^{(2)*}_j) \\ (M_0)_{ik} &= \sigma^{(i)}_j \sigma^{(k)*}_j \\ V_{ik} &= (\delta_{i1}\rho^{(1)}_j + \delta_{i2}\rho^{(2)}_j)\,\sigma^{(k)*}_j \,. \end{aligned} \tag{2.14}$$

Baring some accidental smallness of the internal products of the lines of $\mathcal{M}$ it is again "natural" to assume

$$|\rho| \ll |\sigma|$$

in order to be in agreement with (2.12). This implies that M_1 is still smaller than V, i.e.

$$M = \begin{pmatrix} 0 & 0 \\ 0 & M_0 \end{pmatrix} + \tau \begin{pmatrix} 0 & V \\ V^\dagger & 0 \end{pmatrix} + \tau^2 \begin{pmatrix} M_1 & 0 \\ 0 & 0 \end{pmatrix} + \dots \,, \tag{2.15}$$

where the expansion parameter τ has been introduced in order to keep track of different orders of magnitude.

2b) The <u>diagonalization of the general 3x3 matrix</u> is an instructive example for the simple perturbation theory which is required for the solution of an arbitrary model with a mass-matrix

$$\mathcal{M} = \begin{pmatrix} \alpha_1 & \varepsilon_3 & -\varepsilon_2 \\ -\delta_3 & \alpha_2 & \varepsilon_1 \\ \delta_2 & -\delta_1 & \alpha_3 \end{pmatrix} \tag{2.16}$$

referring to either the up- or the down-quarks. According to (2.13) - (2.15) we diagonalize

$$M = \mathcal{M}\mathcal{M}^+ = \begin{pmatrix} \mu_1 & E_3 & -E_2 \\ E_3^* & \mu_2 & E_1^* \\ -E_2^* & E_1 & \mu_3 \end{pmatrix} \tag{2.17}$$

with

$$\mu_i = \mathcal{M}_{ij} \mathcal{M}_{\underline{i}j}^* \qquad \text{(no sum of } \underline{i} \text{ !)} \tag{2.18}$$

and

$$\begin{aligned} E_1 &= -\alpha_2^* \delta_1 + \alpha_3 \varepsilon_1^* - \delta_3^* \delta_2 \\ E_2 &= -\alpha_1 \delta_2^* + \alpha_3^* \varepsilon_2 + \varepsilon_3 \delta_1 \\ E_3 &= -\alpha_1 \delta_3^* + \alpha_2^* \varepsilon_3 - \varepsilon_1^* \varepsilon_2 \end{aligned} \tag{2.19}$$

Thus M in (2.15) to leading order should contain μ_3 only and therefore, at least one quantity among α_3 , δ_1 and δ_2 (cf. (2.18)) must be large as compared to the others in (2.16). We thus have the perturbation problem (2.15) in this special case:

$$M e^{(i)} = \lambda^{(i)} e^{(i)} \qquad (2.20)$$
$$M = M_0 + \tau V_1 + \tau^2 V_2$$

$$M_0 = \begin{pmatrix} & & \\ & & \\ & & \mu_3 \end{pmatrix} \qquad V_1 = \begin{pmatrix} & & -\varepsilon_2 \\ & & \varepsilon_1^* \\ -\varepsilon_2^* & \varepsilon_1 & 0 \end{pmatrix}$$

$$V_2 = \begin{pmatrix} \mu_1 & \varepsilon_3 & \\ \varepsilon_3^* & \mu_2 & \\ & & 0 \end{pmatrix} . \qquad (2.21)$$

With eigenvectors

$$e^{(i)} = e^{(i,0)} + \tau e^{(i,1)} + \tau^2 e^{(i,2)} + \dots \qquad (2.22)$$

and eigenvalues

$$\lambda^{(i)} = \lambda^{(i,0)} + \tau \lambda^{(i,1)} + \tau^2 \lambda^{(i,2)} \qquad (2.23)$$

we obtain to zero order in τ

$$\lambda^{(1,0)} = \lambda^{(2,0)} = 0 \quad , \quad \lambda^{(3,0)} = \mu_3 \qquad (2.24)$$

$$e^{(1,0)} = (\cos\vartheta,\ \sin\vartheta\, e^{i\delta},\ 0)$$
$$e^{(2,0)} = (-\sin\vartheta\, e^{-i\delta},\ \cos\vartheta,\ 0)$$
$$e^{(3,0)} = (\ 0,\ 0,\ 1) \qquad (2.25)$$

and to first order the relations $\lambda^{(i,1)} = 0$ and

$$(e^{(1,0)} e^{(3,1)}) = -(e^{(3,0)} e^{(1,1)})^* = \hat{E}/\mu_3$$
$$(e^{(2,0)} e^{(3,1)}) = -(e^{(3,0)} e^{(2,1)})^* = \hat{E}'/\mu_3 \qquad (2.26)$$
$$\hat{E} = -E_2 \cos\vartheta + E_1^* \sin\vartheta\, e^{-i\delta}$$
$$\hat{E}' = E_2 \sin\vartheta\, e^{i\delta} + E_1^* \cos\vartheta \ .$$

From the orthonormalization of $e^{(i)}$ to $\mathcal{O}(\tau)$ we get $\mathrm{Re}(e^{(i,1)} e^{(i,0)}) = 0$ (no sum over $\underline{i}$!), whereas the imaginary part of the same quantity is undetermined and may be chosen to be zero. We also take $(e^{(1,0)} e^{(i,1)}) = (e^{(2,0)} e^{(i,1)}) = 0$ ($i = 1,2$), which simplifies the evaluation of (2.20) to $\mathcal{O}(\tau^2)$:

$$(e^{(i,0)} e^{(3,1)}) = (e^{(3,0)} e^{(j,2)}) = 0 \qquad (i,j = 1,2) \qquad (2.27)$$
$$\lambda^{(3,2)} = (|E_1|^2 + |E_2|^2)/\mu_3 \qquad (2.28)$$
$$\mathrm{tg}\,\delta = -s/t \qquad (2.29)$$
$$t + is = \mu_3 E_3 + E_1 E_2 \qquad (2.30)$$
$$\mathrm{tg}\,2\vartheta = -2\sqrt{w}/z \qquad (2.31)$$

$$W = s^2 + t^2 , \tag{2.32}$$

$$z = \mu_3 (\mu_2 - \mu_1) + |\varepsilon_2|^2 - |\varepsilon_1|^2 .$$

The eigenvalues $\overset{(1,2)}{\lambda} = m_1^{\;2}$, $\overset{(2,2)}{\lambda} = m_2^{\;2}$ (the masses of the light quarks) are

$$\left. \begin{matrix} \overset{(2,2)}{\lambda} \\ \overset{(1,2)}{\lambda} \end{matrix} \right\} = \left(\xi \pm \sqrt{z^2 + 4W} \right) / 2\mu_3 \tag{2.33}$$

$$\xi = \mu_3 (\mu_1 + \mu_2) - |\varepsilon_1|^2 - |\varepsilon_2|^2 . \tag{2.34}$$

From the eigenvectors to $\mathcal{O}(\tau^2)$ - we put $\tau = 1$ now - the unitary matrix is given in the notation (2.8) by

$$U_0 = 1 - \left(|\varepsilon_1|^2 + |\varepsilon_2|^2 \right) / 2\mu_3^2 \tag{2.35}$$

$$U_1 = \begin{pmatrix} \cos\vartheta + \varepsilon_2^* \varepsilon_3 / 2\mu_3^2 & -\sin\vartheta \, e^{i\delta} + \varepsilon_2^* \hat{\varepsilon}' / 2\mu_3^2 \\ \sin\vartheta \, e^{-i\delta} - \varepsilon_1 \hat{\varepsilon} / 2\mu_3^2 & \cos\vartheta - \varepsilon_1 \hat{\varepsilon}' / 2\mu_3^2 \end{pmatrix} \tag{2.36}$$

$$\Delta = \frac{1}{\mu_3} \begin{pmatrix} -\varepsilon_2^* \\ \varepsilon_1 \end{pmatrix} \tag{2.37}$$

$$\Delta' = - \frac{1}{\mu_3} \left(\vec{\varepsilon} , \hat{\varepsilon}' \right) \tag{2.38}$$

Here we have retained terms $\mathcal{O}(|E|^2/\mu^2)$, which have been neglected consistently in (2.8). Comparison of (2.35 - 38) with (2.5) yields to $\mathcal{O}(\vartheta^2)$

$$\begin{aligned} \Theta_1 &= (E_1 - E_2^* \Theta_3^*/2)/\mu_3 \\ \Theta_2 &= (E_2^* - E_1 \Theta_3/2)/\mu_3 \\ \Theta_3 &= -\vartheta e^{i\delta} \quad . \end{aligned} \tag{2.39}$$

A general feature of (2.39) is that e.g. for $E_1=0$ the corresponding angle Θ_1 is proportional to Θ_3 and of $\mathcal{O}(\vartheta^2)$ only. The model of the second ref. [15] is of this type (with $\Theta_i^{(1)} = \hat{\Theta}_i$, $\vartheta_i^{(1)} = 0$) .

So far all these formulas may be used for an arbitrary model. Let us assume now that the model is such that

$$tg^2\vartheta = \sqrt{\lambda^{(12)}/\lambda^{(21)}} = m_1/m_2 \tag{2.40}$$

i.e. a relation of the type (1.6) holds. With (2.31) and (2.33) this means

$$z^2 - z\xi - 2w = 0 \quad . \tag{2.41}$$

The three eqs. (2.33) and (2.41) are equivalent to

$$\begin{aligned} \langle\rho|\rho\rangle &= m_1 m_2 \\ \langle\sigma|\sigma\rangle &= (m_2 - m_1)^2 \\ |\langle\rho|\sigma\rangle|^2 &= \langle\rho|\rho\rangle\,[\langle\sigma|\sigma\rangle - \langle\rho|\rho\rangle] \end{aligned} \tag{2.42}$$

in terms of complex vectors

$$|\rho\rangle = (\alpha_1, \varepsilon_3) \qquad |\sigma\rangle = (-\delta_3, \alpha_2) \tag{2.43}$$

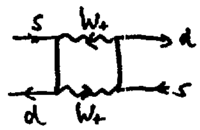
A symmetry model will be characterized by the fact that some (nonvanishing) elements of (2.16) will be equal up to factors of order one. From (2.42) the parameters ε_1 and ε_2, which determine the angles θ_1 and θ_2 (cf. (2.19), (2.37), $\mu_3 \sim |\alpha_3|^2 \sim m_3^2 \gg m_1^2, m_2^2$), are then estimated to be of order $m_1 m_2$ or $m_2 - m_1 \sim m_2$. Θ_1 or Θ_2 will thus be given by four types of relations ($\Theta_3 \sim \sqrt{m_1/m_2}$)

$$
\begin{aligned}
&(\mathrm{I}) \quad |\Theta_{1,2}| \sim m_2/m_3 \\
&(\mathrm{II}) \quad |\theta_{1,2}| \sim \sqrt{m_1 m_2}/m_3 \\
&(\mathrm{III}) \quad |\Theta_{1,2}| \sim |\theta_{2,1}|\,|\theta_3| \sim \begin{cases} \sqrt{m_1/m_2}\; m_2/m_3 & (a) \\ \sqrt{m_1/m_2}\; \sqrt{m_1 m_2}/m_3 = m_1/m_3 & (b) \end{cases} \\
&(\mathrm{IV}) \quad |\theta_{1,2}| \sim 0
\end{aligned} \tag{2.44}
$$

In the case (III) E_1 (or E_2) is zero in the specific model. Numerically III(a) coincides with II.

3. K_o - Mass - Difference and CP - Violation

An orientation about the magnitude of θ_1 and θ_2 can be obtained [25] from the successful prediction of the mass of the charmed quark in a model with N = 2 by Gaillard and Lee [21]. In our notation of the GIM-matrix the graph for a transition of quarks with $\Delta S = 2$, $\Delta c = 0$, appropriate for a calculation of the K_{01}-K_{02}- mass-difference is

$A =$ 

$\propto G_F^2 m_2^2 \hat{\theta}_3 \left[\hat{\theta}_3 - \hat{\theta}_1^{2} \hat{\theta}_2 \, y(m_1^2/m_2^2) \right]$

$$y(x) = 1 + 2 \log x/(1-x) \tag{3.1}$$

where $m_3 = t$, $m_2 = c \gg m_1 = u$ ($m_W \gg t$). Because the first term in the sqare bracket gave the good prediction for $m_2 = c$ we must have ($\hat{\theta}_2 = \vartheta_c = 0.22$)

$$|\hat{\theta}_1||\hat{\theta}_2|/|\hat{\theta}_3| \ll |y(x)| \tag{3.2}$$

$$\hat{\theta}_1 \sim \theta_1^{(d)} - \theta_1^{(u)}$$
$$\hat{\theta}_2 \sim \theta_2^{(d)} - \theta_2^{(u)} \tag{3.3}$$

(3.2) can be combined with any of the situations (I),(II),(III),(IV) in (2.44) for the $\theta_i^{(u)}$ or $\theta_i^{(d)}$ (i=1,2).

First let us consider models in which a $\theta_i^{(u)}$ or a $\theta_i^{(d)}$ vanishes (case IV) in (3.3) so that each is determined in terms of one angle alone. A straightforward discussion of all those cases with the tentative identification $b = 4.7$ GeV [26] shows that (3.2) is not restrictive at all for the mass t of the top quark; typical limits are $t \leq t_{max}$ with $t_{max} \gtrsim 300$ GeV (where (3.2) cannot be used any more !) for $\hat{\theta}_i = \theta_i^{(d)}$, or $t \geq t_{min}$ with $t_{min} \lesssim 0.4$ GeV for $\hat{\theta}_i = \theta_i^{(u)}$ and $\hat{\theta}_1 = \theta_1^{(u)}, \hat{\theta}_2 = \theta_2^{(d)}$. Only in the two cases $\hat{\theta}_i = \theta_i^{(u)}(I)$ ($t \gtrsim 36$ GeV) and $\hat{\theta}_1 = \theta_1^{(u)}(I)$, $\hat{\theta}_2 = \theta_2^{(u)}(II)$ ($t \gtrsim 5$ GeV) some restriction for the t-mass is observed. Therefore it seems pointless to consider more complicated cases for $\hat{\theta}_i$ at all.

Now we turn to the more interesting CP-violation. We have seen in sect. 1 that a nontrivial relation between the angles in the GIM-matrix demands the introduction of more than one doublet of scalar bosons. This opens up the possibility of CP-violating Yukawa interactions [20], but such a mechanism is without much predictive power, even as far as orders of magnitude are concerned [27]. However, if the relevant boson-masses are large enough so as to eliminate this source of CP-violation (together with flavour-changing neutral interactions of those scalar-fields !), it may happen that the CP-violation resides in the GIM-matrix (2.5) with (2.4) only. A phenomenological analysis based upon this assumption in the parametrization of ref. [5] has been carried out by Ellis, Gaillard and Nanopoulos [16]. In our notation of

(3.1) the CP-violating parameter of the K_o-$\overline{K}_o$ system [28] becomes [29]

$$\varepsilon_K \sim \operatorname{Im} A / 2 \operatorname{Re} A \sim \tfrac{1}{2} \operatorname{Im}\tau / (1 - \operatorname{Re}\tau)$$

$$\tau = \frac{|\hat\theta_1||\hat\theta_2|}{|\theta_3|} e^{i\delta} y(c^2/t^2) \tag{3.4}$$

again for $m_W \gg t,c$. For the D_o-$\overline{D}_o$-system ε_D is similar, except $c/t \rightarrow s/b$; for "bottonium" after a similar calculation ($m_2 \ll m_y$)

$$\varepsilon_B \sim \frac{\sin\delta\, |\hat\theta_1||\hat\theta_2|}{2\left[\, |\theta_1| + |\hat\theta_1||\hat\theta_2| \cos\delta \,\right]} \tag{3.5}$$

can be obtained [25] .

It is tempting to assume that $|\varepsilon_K|$ takes its experimental value 10^{-3} in a "natural" way, i.e. for a big phase $\delta \sim \frac{\pi}{2}$ in (3.4) and to try to find a combination of the alternatives (I) - (IV) in (2.44) yielding such a result. We restrict ourselves to an "allowed" range of $b \lesssim t \lesssim 30$ GeV, where the upper limit is determined by the validity of the approximation used in (3.1) [30] . We consider all possibilities (2.44) for $|\hat\theta_1|$ and $|\hat\theta_2|$ and take in those cases, where $|\hat\theta_i|$ obtaines contributions from <u>both</u> $\theta^{(u)}$ and $\theta^{(d)}$ a situation, where one of the two angles is "big" [31] as compared to the other. The result is surprising: The values of t are far outside the "allowed" range, <u>except</u>

(1) $t \sim 8.8\ GeV$ for $|\hat{\theta}_1| \sim |\hat{\theta}_2| \sim \frac{\sqrt{uc}}{t} = 7.9 \cdot 10^{-3} \gg \theta^{(d)}$

(2) $t \sim 12.9\ GeV$ for $|\hat{\theta}_1| \sim |\hat{\theta}_2| \sim \frac{\sqrt{ds}}{b} = 7.1 \cdot 10^{-3} \gg \frac{u}{t}$

In the case (1) $\hat{\theta} \sim \theta^{(u)}$ corresponds to alternative II in (2.44), whereas all alternatives are allowed for $\theta^{(d)}$; in case (2) $\theta^{(u)}$ can be only the one in alternative III(b). It is perhaps more than a coincidence that the formulas for the new Cabibbo-angles must be again of the type encountered for θ_3, $|\theta_3| = \sqrt{m_1/m_2} = \sqrt{m_1 m_2}/m_2$, in order to yield a theory with "natural" explanation of the CP-violation. Note that the small angles $\hat{\theta}_1$ and $\hat{\theta}_2$ essentially drop out in (3.5), so that a large value $\varepsilon_B \sim 0{\cdot}1$ results [25] for a "maximal" $\delta \sim \pi/2$. Our result is also completely consistent with the consequences of (3.2) above.

For completeness we mention another possible situation: In some model those small angles may be produced by a cancellation $\theta^{(u)} \sim \theta^{(d)}$ in (3.3). Again the only "solution" for t in the allowed range is the one for situation II or IIIa) :

$$\frac{\sqrt{uc}}{t} \sim \frac{\sqrt{ds}}{b}$$

with $t \sim 9{\cdot}8$ GeV, i.e. again with a value in the vicinity of 10 GeV.

4. CONCLUSIONS

We have seen by rather model-independent arguments that the top-mass of a sequential doublet $(t,b)_L$ can be expected to lie in the range $9 \lesssim t \lesssim 13$ GeV i.e. the vector meson of the "topponium" may be looked for in the mass-range 18 - 25 GeV. We have only assumed that a (global) symmetry model for the quark doublets and scalar mesons reproduces $\vartheta \sim \sqrt{m_1/m_2}$ in the 4 flavour subspace of "old" quarks and that this symmetry makes new elements of the mass-matrix "equal" (i.e. that relative factors are of order one) to old ones or zero. In addition CP-violation has been assumed to depend on a ("naturally") large phase. Moreover the ovserved CP-parameter ε_K in the K_o-system should be determined by the CP-violation in the GIM-matrix alone. It is clear that the values from current algebra for the quark masses u, d, c, s (1.8) together with b = 4,7 GeV influence this "prediction" as well. We have linked the smallness of ε_K to mixing angles of $\mathcal{O}(10^{-2})$ for the new flavours, which are thus an order of magnitude below the Cabibbo-angle ϑ_C. Replacing in the estimate of the last ref. [16] for the lifetime of a "bottonic" meson the appropriate K.-M.-angle [5] by $|\theta_1|$, the life-time could be as long as 10^{-11} sec, which may produce interesting experimental effects.

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[24] Note that we have actually made a more stringent assumption above in our discussion of $\mathcal{U}$, namely that <u>all</u> angles are small. This is not necessary here.

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[30] The extension to $m_W \lesssim t$ of the result (3.1) and the discussion of consequences for $t \gtrsim 30$ GeV is left as an exercize to the diligent reader. Vector mesons with such masses are still out of the range of present experimental possibilities.

[31] "Big" is defined to be a factor of about 10.

[redacted]