CONF-780157--1

To be presented at the Workshop on
Physics and Engineering in Computerized
Tomography, UC, Irvine, January 17-19, 1978

LBL-8252

3-D OBJECT RECONSTRUCTION EMISSION AND TRANSMISSION TOMOGRAPHY WITH LIMITED ANGULAR INPUT

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October 1978

Prepared for the U. S. Department of Energy
under Contract W-7405-ENG-48

LBL-8[illegible]

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[illegible] of the angular range of data [illegible] in reconstruction [illegible] using the [illegible] method and the [illegible] It is found that in the absence of any [illegible] components in the reconstruction [illegible] However, [illegible] in the case in which the transverse [illegible] concluded that the two [illegible] are mathematically equivalent. The results obtained [illegible] transmission and emission imaging devices.

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SUMMARY

Most emission and transmission imaging methods involve taking data in a continuous range of angles or series of discrete angles. In this paper we investigate the relationship between the angular range of data taking and the quality of the reconstructions by studying the typical problem of imaging in planar positron cameras (Fig. 1). Different algorithms for reconstructing the the object distribution $\rho(\underline{r})$ from the data solve the integral equation

$$\phi(\underline{r}) = \int \rho(\underline{r}')\phi_o(\underline{r} - \underline{r}')d^3\underline{r}' \qquad (1)$$

in different ways (ϕ is the scalar field constructed from the data, ϕ_o is the point response function). One such algorithm Fourier transforms equation (1) to the frequency space (k-space) and solves for $R(\underline{k})$, the Fourier components of $\rho(\underline{r})$. Another algorithm Fourier transforms equation (1) in the x and y dimensions only and solves the resulting integral equation in the z dimension for every spatial frequency (k_x, k_y). We shall refer to these two methods as the deconvolution method [1, 2] and the matrix method [3] respectively.

In the case where the range of integration in equation (1) covers all space, there would be a region in frequency space where $\Phi_o(\underline{k})$ (the Fourier transform of $\phi_o(\underline{r})$) is zero if ϕ_o does not cover the full angular range, as shown in Fig. 2. The consequences are that the corresponding components of $R(\underline{k})$ cannot be determined, and that the integral equation in the z coordinates

in the matrix method does not admit unique solutions. We demonstrated the effect of the size of the camera angle by reconstructing the isotope distribution from the data generated by the same phantom in a two-sided, a four-sided and a six-sided camera whose detectors correspond to one, two and three pairs of opposite faces of a cube with the object at its center. Table I shows the optimum value of X^2, a measure of the deviation of the reconstruction from the phantom, for each configuration.

The undetermined Fourier components can be recovered partly or completely if a priori constraints are utilized. If we assume that the distribution of $\rho(\underline{r})$ in the z-dimension is discrete, most of the zero components of $\Phi_o(\underline{k})$ will become non-zero through sampling. Thus, the corresponding components of $R(\underline{k})$ will be recovered in the deconvolution method, and the degree of undeterminacy will be correspondingly reduced in the integral equation in the z coordinate of the matrix method.

If we make use of the fact that the object is finite in extent, all the undeterminacy is removed. Thus, in the deconvolution method, the facts that (1) the Fourier transform of a finite object is an entire function, and (2) an entire function can be continued throughout the whole complex plane from a knowledge of the function on any finite continuous line segment, make it possible to recover the undetermined components of $R(\underline{k})$. In the case of the matrix method, it can be shown that the integral operator in the z coordinate becomes positive-definite when the range of integration is finite, with the consequence that unique solutions exist for each $k_x \neq 0$; the undetermined components at $k_x = 0$ can be filled in by analytic continuation from $k_x \neq 0$. In our work, an iterative scheme was employed to extend $R(\underline{k})$ obtained from the deconvolution method beyond the deconvolution region. The value of X^2 is significantly improved (Table II).

Table I

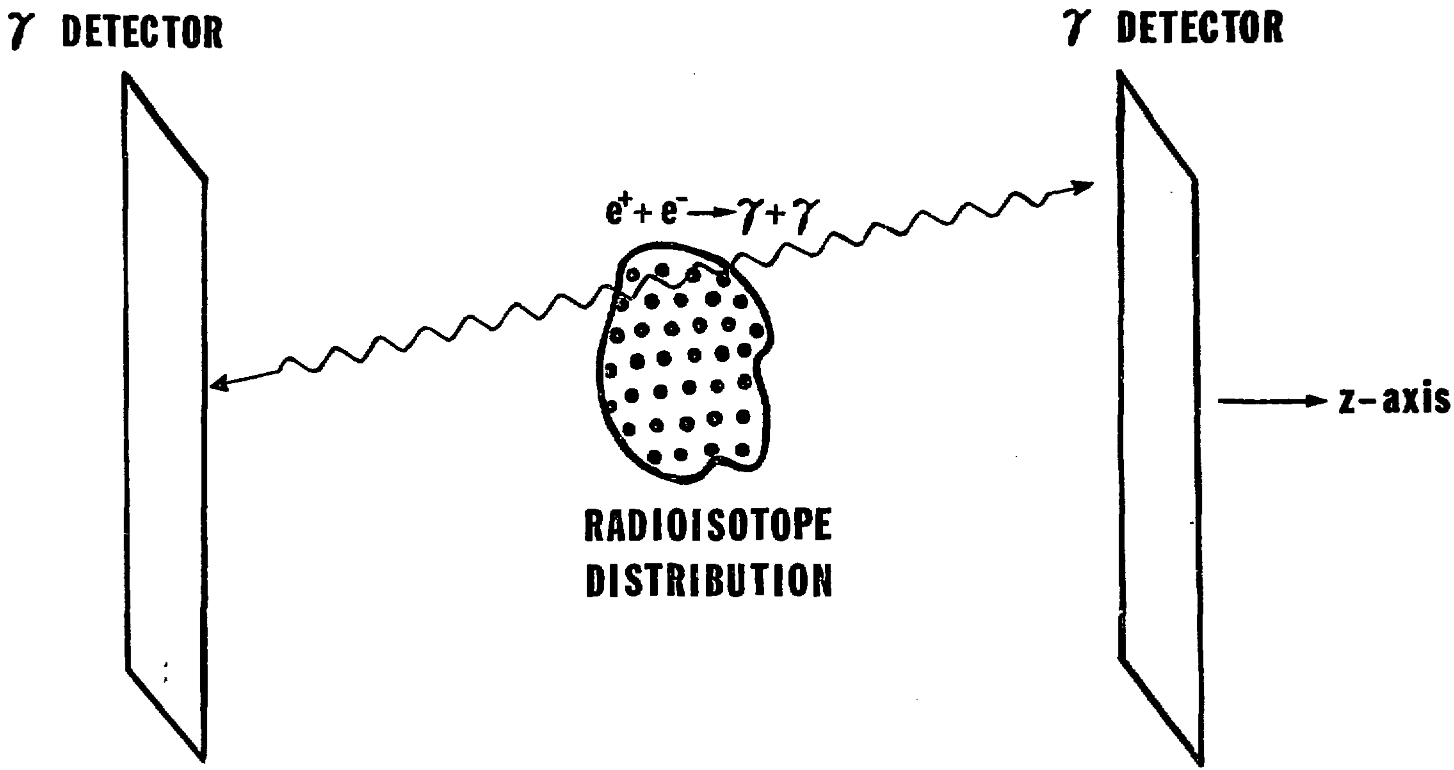
Deconvolution Without Iterations

Number of sides	2	4	6
Optimum x^2	1.000	0.442	0.435

Table II

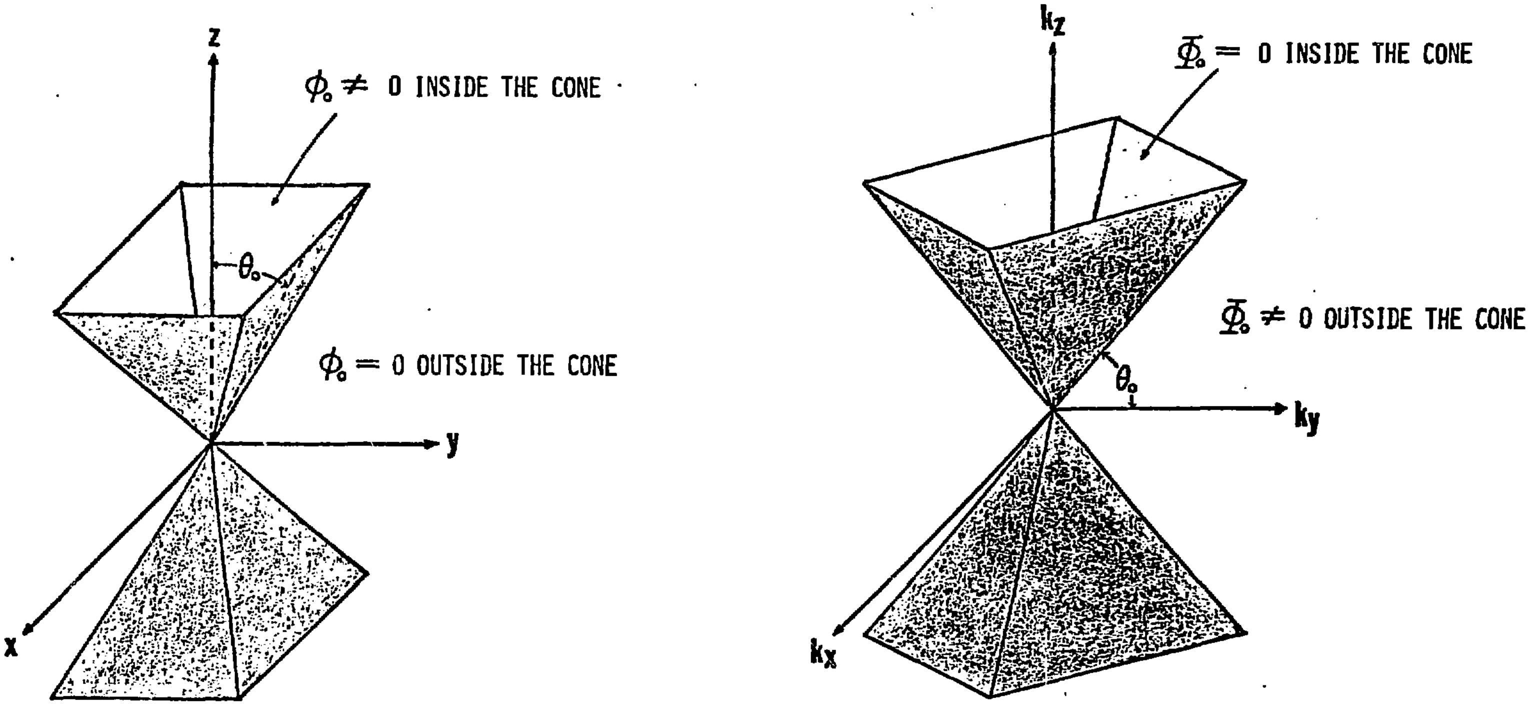
Deconvolution With Iterations

Number of Iterations	0	5	10	15
x^2	1.000	0.431	0.389	0.381



XBL-7810-11996

Figure 1. Schematic figure of planar positron camera.



POINT RESPONSE FUNCTION OF A POSITRON CAMERA WITH RECTANGULAR CONE OF DETECTION.

THE CORRESPONDING OPTICAL TRANSFER FUNCTION ($\Phi_o(\mathbf{k})$).

XBL-7810-11997

Figure 2. The shape of the positron camera point response function and its Fourier transform.