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# Quadratic Form for Resistive Drift Modes in a Slab with Magnetic Shear

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By deriving a quadratic form it is shown that resistive drift modes in a plasma slab with magnetic shear, with or without finite-$\beta$ effects, are never unstable.

Recently it has been established by analytical and numerical work that drift modes in a resistive plasma slab with magnetic shear, with or without finite-$\beta$ effects, are always stable.[1-4] Similar results have been obtained for a collisionless plasma.[5-7] For the electrostatic collisionless problem, Antonsen[8] has shown how to prove a lack of instability by generating a quadratic form and using an ingenious transformation in complex-x space to establish the sign and the magnitude of various terms. Lee and Chen[9] have extended Antonsen's treatment to prove stability of the collisionless eigenmodes even with finite-$\beta$ effects. In this note we develop a quadratic form for the resistive drift-wave problem with finite-$\beta$ effects and using a trick similar to that of Antonsen[8] we give a proof of the absolute stability of these eigenmodes.

Consider an inhomogeneous (density gradient along $e_x$) plasma in a sheared magnetic field $\underline{B} = B[\hat{e}_z + \hat{e}_y(x/L_s)]$. We describe the electrons by a drift-kinetic equation with a Krook-type number conserving collision operator. Ions are treated as a cold fluid ($T_i = 0$). Since $\beta \ll 1$ we may ignore compressional Alfvén waves ($b_{\parallel} = 0$) and, hence, use $\phi$ and $\underline{A} = A_{\parallel}\hat{e}_{\parallel}$ as perturbed potentials. That is we have $\underline{E} = -\nabla\phi + (1/c) \times \partial(A_{\parallel}\hat{e}_{\parallel})/\partial t$ and $\underline{b}_{\perp} = -\nabla_{\perp} \times (A_{\parallel}\hat{e}_{\parallel})$. Linearizing the basic equations and assuming $\exp(ik_y y - i\omega t)$ dependence, we can readily solve for the density and current perturbations. Substituting these into the quasineutrality condition and the Ampere's law, we get the following set of two coupled equations

$$\rho_s^2 \nabla_{\perp}^2 \phi = F(x) \left( \phi - \frac{\omega}{c} \frac{A_{\parallel}}{k_{\parallel}(x)} \right) , \tag{1}$$

and

$$\rho_s^2 \nabla_\perp^2 A = \frac{\omega c}{k_\parallel(x) V_A^2} F(x) \left( \phi - \frac{\omega}{c} \frac{A_\parallel}{k_\parallel(x)} \right) , \qquad (2)$$

where

$$F(x) = \frac{x^2}{x^2 - i x_R^2} \left( \frac{\omega - \omega_*}{\omega} - \frac{x^2}{x_s^2} \right) . \qquad (3)$$

Here $\nabla_\perp^2 = d^2/dx^2 - k_y^2$ , $\omega_* = cT_e k_y/eBL_n$ ,

$$L_n = [d \ln n(x)/dx]^{-1} , \quad \rho_s = c_s/\omega_{ci} , \quad c_s^2 = T_e/m_i ,$$

$$k_\parallel = k_\parallel' x , \quad k_\parallel' = k_y/L_s , \quad x_R^2 = \omega \nu_{ei}/k_\parallel'^2 v_e^2 ,$$

and $x_s^2 = \omega^2/k_\parallel'^2 c_s^2$. In deriving Eqs. (1) and (2), we have assumed the resistive limit $\nu_{ei} > |\omega|$ , $|k_\parallel| v_e$ .

To proceed with our analysis, let us assume that there is an unstable eigenmode with $\mathrm{Im}\,\omega = \gamma > 0$ . The boundary conditions in $x$ are thus outward energy propagating waves viz. waves with asymptotic form $\exp[-i\int(x/x_s)dx]$ . We now introduce a complex transformation of the independent variable viz.

$$s = (x/x_s)\exp(i\pi/4) . \qquad (4)$$

This follows the Antonsen trick[8] of making the electron parallel response term purely real [viz. $x^2/(x^2 - i x_R^2)$ goes to $-s^2/(s^2+1)$], and it also satisfies the requirement that the eigenfunctions fall-off rapidly along the $s$ direction.

We multiply Eqs. (1) and (2) in the transformed variable $s$, respectively, by $\phi^*$ and $(V_A^2/c^2)A_\parallel^*(-i\omega^*)/|\omega|$ (where the superscript $*$ represents the complex conjugate), use partial integrations and add the resulting expressions to obtain

$$\rho_s^2\left[-\int \left|\frac{\partial\phi}{\partial s}\right|^2 ds + ik^2x_R^2\int |\phi|^2 ds - i\frac{\omega^*}{|\omega|}\,\frac{V_A^2}{c^2}\left(\int \left|\frac{\partial A_\parallel}{\partial s}\right|^2 ds - ik^2x_R^2\int |A_\parallel|^2 ds\right)\right]$$

$$= -ix_R^2\int ds\,\frac{s^2}{s^2+1}\left(\frac{\omega-\omega_*}{\omega} + i\,\frac{x_R^2}{x_s^2}s^2\right)\left|\phi - \frac{\omega}{c}\,\frac{A_\parallel}{k_\parallel' s x_R \exp(i\pi/4)}\right|^2 \tag{5}$$

where the boundary terms vanish because of $\exp(-s^2 \times \text{constant})$ dependence of $\phi$, $A_\parallel$, etc.

Writing down the real part of Eq. (5), we obtain

$$\rho_s^2\left(-\int \left|\frac{\partial\phi}{\partial s}\right|^2 - \frac{k^2\gamma\nu}{k_\parallel'^2 v_e^2}\int |\phi|^2 - \frac{\gamma}{|\omega|}\,\frac{V_A^2}{c^2}\int \left|\frac{\partial A_\parallel}{\partial s}\right|^2 - |\omega|\,\frac{k^2\nu}{k_\parallel'^2 v_e^2}\int |A_\parallel|^2 ds\right)$$

$$-\int ds\,\frac{s^2}{s^2+1}\left|\phi - \frac{\omega}{c}\,\frac{A_\parallel}{k_\parallel' s \exp(i\pi/4)}\right|^2\cdot\left(\frac{\gamma\nu_{ei}}{k_\parallel'^2 v_e^2} + \frac{c_s^2}{v_e^2}s^2\nu_{ei}^2\right) = 0\ . \tag{6}$$

Note that for $\gamma > 0$, all terms are negative definite; therefore these terms cannot add up to zero. Thus, there can be no eigenmodes with $\gamma > 0$. This contradicts our assumption and completes the stability proof.

We have thus shown that the coupled equations (1) and (2) describing drift-wave-like eigenmodes in a resistive plasma slab with magnetic shear can have no unstable eigenmodes. The quadratic form for the electrostatic case given in Ref. 2 is a special case of a more general result given here. The finite-$\beta$ results of Tsang et al.[3] and Catto et al.[4] are consistent with our generalized quadratic form. The analytical results of Hsu et al.[10] contradict the quadratic form and are actually incorrect (see Ref. 3 and 4 for a more complete discussion).

We would like to emphasize a few points:

(1) The quadratic form given above is only applicable to modes going asymptotically as $\exp[-i\int(x/x_s)dx]$ . It does not exclude tearing-like instabilities from the coupled equations (1) and (2).

(2) Our electron collisional response terms contain no finite Larmor radius (FLR) effects (because we use a drift kinetic equation). Recent calculations (11) indicate that inclusion of a pitch-angle collision operator and FLR corrections for electrons give additional diffusion-like terms $\nu\rho_e^2[(d/dx)^2 - k_y^2]\phi$ etc., and <u>may</u> alter the stability conclusions; detailed calculations are in progress.

(3) Our quadratic form is so far restricted to the zero ion temperature case. The finite-ion temperature fluid-like modifications make the stability proof intractable.

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