KFKI-1978-45

I. MONTVAY
J. ZIMÁNYI

HADRON CHEMISTRY IN HEAVY ION COLLISIONS

Hungarian Academy of Sciences

CENTRAL RESEARCH INSTITUTE FOR PHYSICS

BUDAPEST

KFKI-1978-45

HADRON CHEMISTRY IN HEAVY ION COLLISIONS

I. Montvay and J. Zimányi

Central Research Institute for Physics
H-1525 Budapest, P.O.B.49. Hungary

Submitted to Nuclear Physics

HU ISSN 0368 5330
ISBN 963 371 426 6

ABSTRACT

The relaxation times necessary to establish chemical equilibrium among different hadrons in hot, dense hadronic matter are investigated in a statistical model. Consequences for heavy ion collisions are exploited in the framework of a simple reaction model. The possibility of Bose-Einstein pion condensation around the break up time of the nuclear fireball is pointed out.

АННОТАЦИЯ

В статистической модели изучаются время релаксации для установления химического равновесия между различными адронами в горячем, плотном адронном веществе. Рассматриваются следствии для процессов столкновения тяжелых ионов в рамках простой модели реакции. Возникает возможность конденсации Бозе-Эйнштейна пионов на концу распада ядерного файербола.

KIVONAT

Forró és sürü anyagban a különböző hadronok közötti kémiai egyensuly beállásához szükséges relaxációs időket vizsgáljuk egy statisztikus modellben. A nehéz ion ütközésekre vonatkozó következményeket egy egyszerü modell keretében használjuk ki. Rámutatunk a Bose-Einstein pion kondenzáció lehetőségére a maganyag-tüzgolyó szétesése körüli pillanatban.

I. INTRODUCTION

In most models for energetic heavy ion reactions it is assumed that during the reaction a hot and dense matter is formed from all or a part of nucleons of the target and projectile nuclei.[1,2,3] For some time in this hot hadron matter there are interactions between the constituent particles, but as the time goes on this fireball explodes and develops into a system of non-interacting fast moving fragments. In the first part of the life of the fireball the energy concentration is enough for the production of pions and resonances, therefore different sorts of hadrons coexist. Even if thermal distribution is /at least approximately/ assumed for the kinetic energies of hadrons it is an interesting question to answer whether the time is enough for establishing chemical equilibrium between the concentrations of different hadrons. /Speaking of hadrons we always have in mind also hadronic resonances produced abundantly at high enough energies./

The main purpose of this paper is to clarify this situation by determining the relaxation times necessary to establish the chemical equilibrium for the different hadrons. In Section II. and III. the relativistic statistical equations are given for the chemical equilibrium and for the time development of systems not being in chemical equilibrium. In Section IV. the concept of the statistical approach is incorporated into a simple heavy ion reaction model. The discussion of the results is contained in Section V.

II. EQUILIBRIUM MIXTURES

Before starting our considerations concerning hadro-chemical reactions we collect in this Section a few remarks about equilibrium mixtures of relativistic ideal gases. The equations of state of an equilibrium mixture of relativistic ideal gases in the Boltzmann limit are the following [4]:

$$P_i \beta = \nu_i = A_i d_i Q_i(\beta) \,, \tag{2.1}$$

$$\varepsilon = \sum_i \varepsilon_i = \sum_i m_i \nu_i R(m_i \beta) \,.$$

Here we use /as in all what follows/ the system of units where $\hbar = c = k_{\text{Boltzmann}} = 1$. The index i denotes the different components of the mixture with mass m_i /below we shall have $i=\pi, N, \Delta$, etc./. $\beta=T^{-1}$ is the inverse temperature, P_i is the partial pressure of the component i and ν_i is its /number/ density. The total relativistic energy density is denoted by ε/ including the contribution of rest masses/, ε_i is the energy density of the component i and A_i is its absolute activity /fugacity/. The conditions of the chemical equilibrium can be formulated in terms of the quantities A_i /c. f. Sections III.-IV./. The quantity d_i is the /spin and isospin/ degeneracy of the particle state with index i : $d_i=(2J_i+1)(2I_i+1)$ where J_i and I_i are the spin and the isospin, respectively. The functions Q_i and R introduced in Eq. (2.1) are defined as

$$
\begin{aligned}
Q_i(\beta) &= \frac{m_i^2}{2\pi^2\beta} K_2(m_i\beta) , \\
R(x) &= \frac{3}{x} + \frac{K_1(x)}{K_2(x)} ,
\end{aligned}
\qquad (2.2)
$$

where $K_\mu(x)$ is the modified Bessel-function of index μ. As it can be seen from Eq. (2.1) $R(m_i\beta)$ is the average energy per particle for the component i measured in the unit m_i.

In the non-relativistic limit $m_i\beta \to \infty$ one can use the asymptotic expansion[5]

$$
K_\mu(x) \approx \sqrt{\frac{\pi}{2x}}\, e^{-x} \left[1 + \frac{1}{2x}\left(\mu^2-\frac{1}{4}\right) + \cdots \right] . \qquad (2.3)
$$

This gives

$$
\begin{aligned}
Q_i(\beta) &\approx \left(\frac{m_i}{2\pi\beta}\right)^{3/2} e^{-m_i\beta} \left(1+\frac{15}{8m_i\beta}+\cdots\right) , \\
R(m_i\beta) &\approx 1 - \frac{3}{2m_i\beta} + \cdots
\end{aligned}
\qquad (2.4)
$$

The Boltzmann limit corresponds to the case of small occupation numbers of states in quantum gases. The quantum corrections to Eq. (2.1) can be determined from the corresponding equations of state for relativistic quantum ideal gases. We note, however, that such equations are not unique as their form depends on the convention for the quantum counting of states. The usual "box quantization" /i.e. periodic boundary conditions/ gives the equations of state[6] /for simplicity, in the case of a single boson component/:

$$
\begin{aligned}
\nu &= m^3 (2\pi m\beta)^{-3/2}\, G_1^{(2)}(A, m\beta)\ , \\
P\beta &= m^3 (2\pi m\beta)^{-3/2}\, G_2^{(2)}(A, m\beta), \\
\varepsilon &= m^4 (2\pi m\beta)^{-3/2} \left[G_1^{(1)}(A, m\beta) + \frac{3}{m\beta} G_2^{(2)}(A, m\beta) \right] .
\end{aligned}
\tag{2.5}
$$

The functions $G_\mu^{(i)}$ are difined like

$$
G_\mu^{(i)}(x,y) = \sqrt{\frac{2y}{\pi}} \sum_{j=1}^{\infty} \frac{x^j}{j^\mu} K_i(jy) \ . \tag{2.6}
$$

For fermions $G_\mu^{(i)}$ has to be replaced by

$$
F_\mu^{(i)}(x,y) = - G_\mu^{(i)}(-x,y) \ . \tag{2.7}
$$

Another convention for state counting based on the Newton-Wigner localization gives instead of Eq. (2.5) the equations of state which can be easily obtained from the partition function in Ref.[7]

$$
\begin{aligned}
\nu &= \frac{m^3}{v_Q(m\beta)}\, G_{3/2}\left(\frac{A}{D(m\beta)}\right) , \\
P\beta &= \frac{m^3}{v_Q(m\beta)}\, G_{5/2}\left(\frac{A}{D(m\beta)}\right) , \\
\varepsilon &= m \left[\nu \frac{d}{d(m\beta)} \ln D(m\beta) + P\beta \frac{d}{d(m\beta)} \ln v_Q(m\beta) \right] .
\end{aligned}
\tag{2.8}
$$

The functions $G_\rho(x)$ are known from the non-relativistic case[4], namely

$$G_\rho(x) = \sum_{j=1}^{\infty} j^{-\rho} x^j \tag{2.9}$$

For fermions we have again

$$F_\rho(x) = -G_\rho(-x) \tag{2.10}$$

instead of $G_\rho(x)$. The functions $D(x)$ and $\vartheta_Q(x)$ /the latter denoted in[7] by $\vartheta_{BE}(x)$ / are defined as

$$D(x) = \sqrt{\frac{\pi}{2}}\,\frac{\left[4K_0(x)+K_1(x)\left(x+\frac{8}{x}\right)\right]^{3/2}}{x^2\left[K_0(x)+\frac{2}{x}K_1(x)\right]^{5/2}} \,,$$

$$\vartheta_Q(x) = (2\pi)^{3/2} x^3 \left[\frac{K_0(x)+\frac{2}{x}K_1(x)}{4K_0(x)+K_1(x)\left(x+\frac{8}{x}\right)}\right]^{3/2} . \tag{2.11}$$

It can be easily seen that in the Boltzmann limit, when in Eqs. (2.5-6) or (2.8-9) the $j=1$ terms dominate both Eqs. (2.5) and (2.8) are reduced to Eq. (2.1). The two forms coincide also in the non-relativistic case $m\beta \to \infty$ hence the non-uniqueness is reflected only by the relativistic part of the quantum corrections.

III. HADRON REACTIONS IN NON-EQUILIBRIUM MIXTURES

In this Section we consider high temperature mixtures of hadrons /nucleons, pions, Δ -resonance, ρ - resonance etc./ in thermal equilibrium. The temperature will be taken high enough for a reasonably high rate of resonance production i.e. of hadronic reactions transforming different hadron states into each other. /This situation is similar to ordinary chemical reactions therefore the name "hadro-chemical reactions" is appropriate for it./ Actually, this means temperatures in the range $T \cong 50-150$ MeV. The lower limit is fixed by the resonance production threshold wheareas the higher limit corresponds roughly to the Hagedorn-temperature T_o [8]. Near T_o the rapid /exponential/ rise of the resonance state density implies the dominance of highly excited, highly degenerate hadron states resulting in a phase-transition-like phenomenon /maximal temperature in the statistical bootstrap model[9] or a second order phase transition from hadronic matter to "quark-soup" in the quark model[10]/. In the present paper we do not consider the $T \cong T_o$ region restricting ourselves only to the lowest resonances /we hope, however, to return to this interesting problem in a future publication/.

As it was stated above we assume thermal equilibrium with hadro-chemical reactions still going on. that is no chemical equilibrium. Our main interest will be just to study different reaction rates and the time development of the densities of different hadrons. Such an approach is legitimate if thermal equilibrium sets in earlie than chemical equilibrium, i.e. among the collisions establishing equilibrium the elastic ones dominate. It can be seen from the equations below that, at least at large baryon number densities /relevant in heavy ion collisions/ and if the momentum distribution is already near to the thermal distribution, this is indeed the case. The reason is the large increase of nucleon-nucleon elastic cross-sections with decreasing energy below 1 GeV/c laboratory momentum.

Mathematically, we approximate the hadron gas by a multicomponent, relativistic ideal Boltzmann-gas /the thermal equilibrium equations of which are given by Eq.(2.1)/. Implied by this assumption is that the gas is sufficiently dilute such that it makes sense to speak about individual collisions with the same cross-sections as measured in hadron scattering experiments. However, this assumption can hardly be fulfilled in situations occuring in heavy ion collisions. In spite of that we believe that one can obtain at least order of magnitude estimates based on this extrapolation. Such estimates may be useful also for the construction of a correct /relativistic, quantum, .../ theory of the processes in high density hadronic matter.

Once the use of the S-matrix for individual collisions is allowed /at least approximately/ the situation is not as bad as one would think at the first sight. Namely, taking into account resonances means to include an essential part of the interaction among hadrons[11,12/]. This is supported by the experimentally verified "duality" property of quasi-two-body reactions[13/]. According to duality the non-diffractive scattering amplitude /dominant in the energy range relevant in nearly equilibrium hadron gases/ can be approximated /in the average/ by the sum of the direct channel resonance contributions. A basic assumption of the statistical bootstrop model is, in fact, that the strongly interacting hadron gas is statistically equivalent to a free /i.e. ideal/ gas of the resonances[9/].

In the present Section the general form of the equations governing hadro-chemical reactions will be derived under the above assumptions. Here only the simplest situations will be considered. The specific particles and reactions relevant in heavy ion collisions will be dealt with in the next Section.

First let us consider a gas consisting of a single sort of neutral ground state hadrons /called " π -meson"/ and a single sort of neutral resonances /called " ρ -meson"/. The only reactions considered /besides elastic scattering/ are the formation and decay of the ρ -resonance:

$$\rho \leftrightarrow n\pi \qquad (n=2,3,\dots)\ . \tag{3.1}$$

The total width of ρ will be denoted by Γ and the probability of its n -pion decay by w_n . For this latter we have the normalization condition

$$1=\sum_{n=2}^{\infty} w_n \quad . \tag{3.2}$$

The summation over n here is, in fact, not infinite as multipion decays for $n > m_\rho / m_\pi$ are kinematically forbidden / m_ρ and m_π denote the masses of ρ - and π -mesons, respectively/. The total width of the ρ -meson is[14, 15]:

$$\Gamma=\sum_{n=2}^{\infty}\Gamma_n=\sum_{n=2}^{\infty}\frac{1}{n!}\int\frac{d^3p_1}{(2\pi)^3}\cdots\frac{d^3p_n}{(2\pi)^3}(2\pi)^4\delta^4(p-p_1-\dots-p_n)\cdot$$

$$\cdot\frac{|T_n|^2}{2p_0\,2p_{10}\cdots 2p_{n0}}\ , \tag{3.3}$$

where p is the four-momentum of ρ and $p_1, \dots, p_n$ are the four-momenta of the decay product pions. The partial width Γ_n gives the n-pion decay probability like

$$w_n = \frac{\Gamma_n}{\Gamma} \quad . \tag{3.4}$$

We use invariant normalization of states, hence the invariant amplitude T_n is defined by

$$\langle p_1 \dots p_n | S | p \rangle = -i(2\pi)^4 \delta^4(p - p_1 - \dots - p_n)\, T_n \quad , \tag{3.5}$$

where S is the S-operator.

We always assume that thermal equilibrium is established faster than chemical equilibrium. /This is due to the dominantly elastic character of the average collisions in the gas./ In the present Section we also keep the temperature fixed. The normalized momentum distribution of particles in thermal equilibrium is the following:

$$w_i(p)\, d^3p = \{(2\pi)^3 Q_i(\beta)\}^{-1} e^{-\beta p_0} \qquad (i = \pi, \rho) . \tag{3.6}$$

Here, and in what follows, the four-momenta will be specified in the rest system of the gas, therefore e.g. p_0 is the energy in this system. /The function $Q_i(\beta)$ was defined in Eq. (2.2)./ The number of $\rho \longrightarrow n\pi$ decays in unit time and volume is /15,16/:

$$\nu_\rho w_\rho(p) \frac{(2\pi)^{4-3n}}{n!} \delta^4(p - p_1 - \dots - p_n) \frac{|T_n|^2}{2p_0 2p_{10} \dots 2p_{n0}} d^3p\, d^3p_1 \dots d^3p_n \quad . \tag{3.7}$$

Similarly, the number of the reversed processes is /assuming time-reversal invariance/:

$$\nu_\pi^n w_\pi(p_1)\cdots w_\pi(p_n)\,\frac{2\pi}{n!}\,\delta^4(p-p_1-\cdots-p_n)\,\frac{|T_n|^2}{2p_0 2p_{10}\cdots 2p_{n0}}\,d^3p\,d^3p_1\cdots d^3p_n\,. \qquad (3.8)$$

In equilibrium these numbers are equal for every n hence the densities satisfy:

$$\nu_i = Q_i(\beta) \qquad\qquad (i=\pi,\rho)\,. \qquad (3.9)$$

Comparing to Eq. (2.1), as we have put now $d_i = 1$, the condition of chemical equilibrium /in the absence of any conserved quantum numbers/ is:

$$A_i = 1 \qquad\qquad (i=\pi,\rho) \qquad (3.10)$$

/Note that for additively conserved quantum numbers, like e.g. baryon number $\mathcal{B}$, the condition of chemical equilibrium is $A_i = A_{\mathcal{B}}$, $i = N, \Delta, \ldots$ ./

Outside chemical equilibrium the ρ- and π- densities are changing in time. From Eqs. (3.7-8) it follows easily that

$$\frac{d\nu_\rho(t)}{dt} = -\nu_\rho(t)\,\Gamma\,\frac{K_1(m_\rho\beta)}{K_2(m_\rho\beta)} + \sum_{n=2}^{\infty}\Gamma_n\,\nu_\pi(t)^n\,\frac{K_1(m_\rho\beta)}{K_2(m_\rho\beta)}\,\frac{Q_\rho(\beta)}{Q_\pi(\beta)^n}\,,$$

$$\frac{d\nu_\pi(t)}{dt} = \nu_\rho(t)\,\Gamma\,\frac{K_1(m_\rho\beta)}{K_2(m_\rho\beta)}\sum_{n=2}^{\infty} n w_n - \sum_{n=2}^{\infty} n\Gamma_n\,\nu_\pi(t)^n\,\frac{K_1(m_\rho\beta)}{K_2(m_\rho\beta)}\,\frac{Q_\rho(\beta)}{Q_\pi(\beta)^n}\,. \qquad (3.11)$$

The other simple case we consider in this Section is that of a single component gas /called again "pion" gas/, where particle creation is possible due to scattering processes like

$$2\pi \leftrightarrow n\pi \qquad (n=2,3,\dots) \;. \tag{3.12}$$

The general case $m\pi \leftrightarrow n\pi$ with $m>2$ will not be considered here for simplicity. /The previous case contains, in fact, also such k nd of processes through resonance intermediate states./

The scattering cross-section of the process $2\pi \to n\pi$ is given by[14/]:

$$d\sigma_n = \frac{(2\pi)^{4-3n}}{2!\,n!}\,\delta^4(p_1+p_2-p_3-\dots-p_{n+2})\,\frac{|T_{2\to n}|^2}{2\sqrt{\lambda_{\pi\pi}(s)}}\;\frac{d^3p_3\dots d^3p_{n+2}}{2p_{30}\dots 2p_{n+2\,0}} \tag{3.13}$$

where s is the centre of mass energy squared: $s=(p_1+p_2)^2$, and the function $\lambda_{\alpha\alpha'}(s)$ is given as

$$\lambda_{\alpha\alpha'}(s) = \left[s-(m_\alpha+m_{\alpha'})^2\right]\left[s-(m_\alpha-m_{\alpha'})^2\right] \;. \tag{3.14}$$

The invariant scattering amplitude $T_{2\to n}$ is defined in analogy with Eq. (3.5).

The number of scattering processes $2\pi \to n\pi$ in the pion gas in unit time and volume is[15/]:

$$\nu_\pi^2 w_\pi(p_1) w_\pi(p_2) \frac{(2\pi)^{4-3n}}{2!\,n!} \delta^4(p_1+p_2-p_3-\cdots-p_{n+2}) |T_{2\to n}|^2 \frac{d^3p_1 \cdots d^3p_{n+2}}{2p_{10}\cdots 2p_{n+2\,0}} . \tag{3.15}$$

The reversed process goes like

$$\nu_\pi^n w_\pi(p_3) \cdots w_\pi(p_{n+2}) \frac{(2\pi)^{-2}}{2!\,n!} \delta^4(p_1+p_2-p_3-\cdots-p_{n+2}) \frac{d^3p_1 \cdots d^3p_{n+2}}{2p_{10}\cdots 2p_{n+2\,0}} . \tag{3.16}$$

The condition for chemical equilibrium is given, of course, again by Eq. (3.9), and the change of the pion density in time is determined by the equation

$$\frac{d\nu_\pi(t)}{dt} = \sum_{n=3}^{\infty} (n-2) \left[\frac{\nu_\pi(t)^2}{Q_\pi(t)^2} - \frac{\nu_\pi(t)^n}{Q_\pi(t)^n} \right] \cdot$$

$$\cdot \int_{(nm_\pi)^2}^{\infty} ds \frac{(s-4m_\pi^2)}{16\pi^2\sqrt{s}} \sigma_n(s) \frac{K_1(\sqrt{s}\beta)}{K_2(\sqrt{s}\beta)} Q_{\sqrt{s}}(\beta) . \tag{3.17}$$

The notation $Q_{\sqrt{s}}(\beta)$ is used here for $Q_i(\beta)$ in Eq. (2.2) when the mass m_i is replaced by $\sqrt{s}$. In deriving Eq. (3.17) from Eqs. (3.15-16) the following identity has to be used:

$$\int \frac{d^3p_1\, d^3p_2}{2p_{10}\, 2p_{20}} e^{-\beta(p_{10}+p_{20})} \delta[(p_1+p_2)^2 - s] =$$

$$= \frac{K_1(\sqrt{s}\beta)}{K_2(\sqrt{s}\beta)} Q_{\sqrt{s}}(\beta) \frac{(2\pi)^3\pi}{2s} \sqrt{\frac{\lambda_{\pi\pi}(s)}{4s}} . \tag{3.18}$$

The appearance of the factor $Q_{\sqrt{s}}(\beta)$ in the integral of the right hand side of Eq. (3.17) is remarkable. For large c.m. energy $\sqrt{s}$ the function $Q_{\sqrt{s}}(\beta)$ behaves according to Eq. (2.4) like $\sim e^{-\sqrt{s}\beta}$. As the cross-sections do not rise appreciably, this means that the integral is cut off exponentially for $\sqrt{s} \gg T = \beta^{-1}$. Therefore, scattering processes with centre of mass energies much larger than the average thermal energy are unimportant in hadro-chemical reactions /at least when the momentum distribution is nearly thermal/. This leads e.g. to the dominance of elastic $N-N$ scattering in the temperature range $T \sim$ 50–150 MeV we are considering.

In the following Section equations like Eqs. (3.11) and (3.17) will be adapted to the physical situation in a simple heavy ion collision model. A numerical study of the time development of the solutions will also be performed there.

IV. THE HEAVY ION REACTION MODEL

The main purpose of this paper is the investigation of time development of the compressed and hot nuclear matter. For the description of the nuclear reaction mechanism part of the heavy ion collision process a very simple model is used. Only central collisions between heavy ions of equal masses are considered. The exact treatment of the problem is naturally impossible. The model presented here contains crude approximations but it is believed to describe the main properties of the reaction.

The reaction is described as the collision of two interpenetrating spheres originally filled with cold nucleon gas. The assumptions of the model are summarized as follows:

a/ The target and projectile nuclei having $A_p = A_t = A$ nucleons originally are represented by moving spheres of volumes $V_p = V_t$ /constant in time/. Their sum is denoted by $V_o = V_p + V_t$. Before the collision the number density of the cold nucleons ρ_{oo} is uniform within the two spheres, thus $\rho_{oo} V_o = 2A$. As the reaction proceeds the two spheres begin to overlap. The overlap volume is denoted by $V_{pt}(t)$. It is assumed that nucleons outside $V_{pt}(t)$ /the "collision zone"/ are not influenced, thus retain the original ρ_{oo} density. The cold nucleons within the collision zone are assumed to have a spatially uniform time dependent density $\rho_o(t)$ in the whole volume $V_{pt}(t)$.

b/ As the spheres representing the target and projectile nuclei begin to overlap the nucleons in V_{pt} begin to collide with each other. There are elastic scatterings as well as Δ - resonance production. The scattered out nucleons and produced Δ's are considered as the constituents of a hot gas cloud at rest in the c.m. system and with given temperature T and volume V_g . The overlap of volume V_g with the volumes of nuclei is denoted by V_{gpt} . In the first period of the reaction $V_g(t) = V_{pt}(t) = V_{gpt}(t)$. The particles of the cloud collide with each other and with the fast moving cold nucleons, too. During the collisions resonances are also produced, hence the hot gas consists of nucleons (N) , Δ-resonances (Δ) , π -mesons (π) and ρ -mesons (ρ) .Denoting the "cold" nucleons in the original nuclei by N_o the list of different "inelastic" /from the point of view of the model/ processes we take into account is the following:

$$
\begin{array}{lll}
N_o N_o \rightarrow NN & N_o N_o \rightarrow N\Delta & \\
N_o N \rightarrow NN & N_o N \rightarrow N\Delta & N_o \pi \rightarrow N\pi \\
NN \leftrightarrow N\Delta & \Delta \leftrightarrow N\pi & \rho \leftrightarrow \pi\pi
\end{array}
\qquad (4.1)
$$

c/ The hot gas cloud is described as a multicomponent ideal relativistic Boltzmann gas. However, the interaction between the particles is accounted for to a large extent by allowing the production of resonances /c.f. previous Section/. The gas is assumed to be in thermal but not in a chemical equilibrium. /This corresponds to the assumed predominance of elastic collisions./ The time evolution of the densities of different particles are described in terms of statistical equations of type given in the previous Section taking into account the effect of changing volumes. The time dependence of the temperature of the gas is determined from an equation expressing energy conservation.

d/ At the moment t_m of maximum overlap of the colliding spheres $\left(V_g(t_m) = V_{pt}(t_m) = V_{gpt}(t_m) = V_p = V_t \right)$ the gas decouples from the incident nuclei and the volume $V_g(t)$ of the spherical gas cloud begins its adiabatic expansion. For the approximate description of the expansion the time dependence of the radius $R(t)$ of the sphere is borrowed from a simple hydrodynamic model[17/]. The kinetic energy of the hydrodynamic flow is subtracted from the total thermic energy. The densities and temperature are kept spatially constant within V_g also in the expansion period.

The division of the process into an initial "ignition period" /when nucleons are scattered out from the original cold nuclei and the hot gas is constrained to the overlap and a subsequent "expansion period" when cold nucleons are already ignored is, of course, somewhat artificial. In reality the two processes go over into each other smoothly and there is some overlapping period. Our strategy is to consider the two dominant processes separately for simplifying things.

Collecting these ideas one obtains for the description of heavy ion collision process the following set of equations:

$$\frac{d\nu_0(t)}{dt} = \frac{1}{V_g(t)}\frac{dV_g(t)}{dt}\left(\nu_{00}-\nu_0(t)\right) - \frac{V_{pt}(t)}{V_0}\frac{\nu_0(t)}{d_0}\frac{\nu_0(t)}{d_0}C_{0,00} -$$

$$-\frac{V_{gpt}(t)}{V_0}\frac{\nu_0(t)}{d_0}\frac{\nu_N(t)}{d_N}C_{0,0N} - \frac{V_{gpt}(t)}{V_0}\frac{\nu_0(t)}{d_0}\frac{\nu_\pi(t)}{d_\pi}C_{0,N\pi} \qquad (1.2)$$

$$\frac{d\nu_N(t)}{dt} = -\frac{1}{V_g(t)}\frac{dV_g(t)}{dt}\nu_N(t) + \frac{V_{pt}(t)}{V_g(t)}\frac{\nu_0(t)}{d_0}\frac{\nu_0(t)}{d_0}C_{N,00} +$$

$$+\frac{V_{gpt}(t)}{V_g(t)}\frac{\nu_0(t)}{d_0}\frac{\nu_N(t)}{d_N}C_{N,0N} + \frac{V_{gpt}(t)}{V_g(t)}\frac{\nu_0(t)}{d_0}\frac{\nu_\pi(t)}{d_\pi}C_{N,0\pi} +$$

$$+\left(\frac{\nu_\Delta(t)}{d_\Delta Q_\Delta(\beta)} - \frac{\nu_N(t)}{d_N Q_N(\beta)}\frac{\nu_\pi(t)}{d_\pi Q_\pi(\beta)}\right)C_{N,\Delta N\pi} +$$

$$+\left(\frac{\nu_N(t)}{d_N Q_N(\beta)}\frac{\nu_\Delta(t)}{d_\Delta Q_\Delta(\beta)} - \frac{\nu_N(t)}{d_N Q_N(\beta)}\frac{\nu_N(t)}{d_N Q_N(\beta)}\right)C_{N,N\Delta} \qquad (1.3)$$

$$\frac{d\nu_\Delta(t)}{dt} = -\frac{1}{V_g(t)}\frac{dV_g(t)}{dt}\nu_\Delta(t) + \frac{V_{pt}(t)}{V_g(t)}\frac{\nu_0(t)}{d_0}\frac{\nu_0(t)}{d_0}C_{\Delta,00} +$$

$$+\frac{V_{gpt}(t)}{V_g(t)}\frac{\nu_0(t)}{d_0}\frac{\nu_N(t)}{d_N}C_{\Delta,0N} -$$

$$-\left(\frac{\nu_\Delta(t)}{d_\Delta Q_\Delta(\beta)} - \frac{\nu_N(t)}{d_N Q_N(\beta)}\frac{\nu_\pi(t)}{d_\pi Q_\pi(\beta)}\right)C_{\Delta,\Delta N\pi} -$$

$$- \left(\frac{\nu_N(t)}{d_N Q_N(\beta)} \frac{\nu_\Delta(t)}{d_\Delta Q_\Delta(\beta)} - \frac{\nu_N(t)}{d_N Q_N(\beta)} \frac{\nu_N(t)}{d_N Q_N(\beta)} \right) C_{\Delta, N\Delta}$$

(4.4)

$$\frac{d\nu_\pi(t)}{dt} = - \frac{1}{V_g(t)} \frac{dV_g(t)}{dt} \nu_\pi + \left[\frac{\nu_\rho(t)}{d_\rho Q_\rho(t)} n_{av} - \right.$$

$$\left. - 2 w_2 \left(\frac{\nu_\pi(t)}{d_\pi Q_\pi(\beta)} \right)^2 - 4 w_4 \left(\frac{\nu_\pi(t)}{d_\pi Q_\pi(\beta)} \right)^4 \right] c_{\pi, \rho\pi} +$$

$$+ \left(\frac{\nu_\Delta(t)}{d_\Delta Q_\Delta(\beta)} - \frac{\nu_N(t)}{d_N Q_N(\beta)} \frac{\nu_\pi(t)}{d_\pi Q_\pi(\beta)} \right) c_{\pi, \Delta N \pi}$$

(4.5)

$$\frac{d\nu_\rho(t)}{dt} = - \frac{1}{V_g(t)} \frac{dV_g(t)}{dt} \nu_\rho(t) -$$

$$- \left[\frac{\nu_\rho(t)}{d_\rho Q_\rho(\beta)} - w_2 \left(\frac{\nu_\pi(t)}{d_\pi Q_\pi(\beta)} \right)^2 - w_4 \left(\frac{\nu_\pi(t)}{d_\pi Q_\pi(\beta)} \right)^4 \right] c_{\rho, \rho\pi}$$

(4.6)

the conservation of energy yields:

$$\frac{d\beta}{dt} = \Big[\sum_{i=N,\Delta,\pi,\varrho} \frac{d\nu_i(t)}{dt}\, m_i R(m_i\beta) - \sqrt{s_1}\left(\frac{1}{V_g(t)} \frac{dV_g(t)}{dt} (\nu_{\infty} - \nu_0(t)) - \frac{d\nu_0(t)}{dt} \right) + $$

$$+ \frac{1}{V_g(t)} \frac{dV_g(t)}{dt}\, \varepsilon(t) + \frac{1}{V_g(t)} \frac{dE_{hyd}(t)}{dt} \Big] \Big/ \Big[\sum_{i=N,\Delta,\pi,\varrho} m_i^2 R(m_i\beta)' \nu_i(t) \Big] . \qquad (1.7)$$

The definition of symbols in these equations are the following:

$\nu_0(t)$: number density of cold, fast moving nucleons in the projectile and target spheres of volume V_0 ;

$\nu_N(t), \nu_\Delta(t), \nu_\pi(t), \nu_\varrho(t)$: number densities of hot nucleons, Δ's , pions and ϱ's in the $V_g(t)$ gas volume;

$d_0 = d_N, d_\Delta, d_\pi, d_\varrho$: degeneracies of the components;

$$d_N = 4\,, \quad d_\Delta = 16\,, \quad d_\pi = 3\,, \quad d_\varrho = 9\,;$$

$\beta = \frac{1}{kT}$: with T being the temperature of the gas and k the Boltzmann constant;

$E_{hyd}(t)$: the kinetic energy of the hydrodynamic flow connected with the expansion of the gas sphere in the decaying phase of the reaction;

$R(m_i\beta)$: the average energy of a particle in units of its mass;

$R'(x)$: derivative of R with respect of its argument;

$Q_i(\beta) = m_i^3 (2\pi^2 m_i \beta)^{-1} K_2(m_i\beta)$: the single particle partition function;

$K_m(x)$: the Bessel function with imaginary argument;

n_{av} : average number of pions originating in the decay of ρ's ;

w_n : probability of the n pion decay of a ρ ;

$$C_{o,oo} = 2\left(2\sigma_1^{NN} + 2\sigma_1^{N\Delta}\right)\sqrt{(s_1 - 4m_N^2)/s_1} \quad ,$$

$$C_{o,oN} = \left(\sigma_2^{NN} + \sigma_2^{N\Delta}\right)\sqrt{(s_{2N} - 4m_N^2)\, s_{2N}/(s_1 m_N^2)} \quad ;$$

$$C_{o,N\pi} = \sigma_2^{N\pi}\sqrt{\left[s_{2\pi} - (m_N + m_\pi)^2\right]\left[s_{2\pi} - (m_N - m_\pi)^2\right]/(s_1 m_\pi^2)}$$

$$C_{N,oo} = 2\left(2\sigma_1^{NN} + \sigma_1^{N\Delta}\right)\sqrt{(s_1 - 4m_N^2)/s_1} \quad ,$$

$$C_{N,oN} = \sigma_2^{NN}\sqrt{(s_{2N} - 4m_N^2)\, s_{2N}/(m_N^2 s_1)} \quad ,$$

$$C_{N,0\pi} = \sigma_2^{N\pi} \sqrt{\left[s_{2\pi} - (m_N + m_\pi)^2\right]\left[s_{2\pi} - (m_N - m_\pi)^2\right]/(m_\pi^2 s_1)} \quad ,$$

$$C_{N,\Delta N\pi} = \overline{\Gamma}_\Delta \; K_1(m_\Delta \beta) / K_2(m_\Delta \beta) \quad ,$$

$$C_{N,N\Delta} = \frac{1}{16\pi^4 \beta} \int\limits_{(m_N+m_\Delta)^2}^{\infty} \frac{ds}{2\sqrt{s}} \, (s - 4m_N^2)\, s\, K_1(\sqrt{s}\beta)\, \sigma_\Delta(s) \, ,$$

$$C_{\Delta,00} = 2\sigma_1^{N\Delta} \sqrt{(s_1 - 4m_N^2)/s_1} \quad ,$$

$$C_{\Delta,0N} = \sigma_2^{N\Delta} \sqrt{(s_{2N} - 4m_N^2)\, s_{2N} / (m_N^2 s_1)} \quad ,$$

$$C_{\Delta,\Delta N\pi} = C_{N,\Delta N\pi} \; ,$$

$$C_{\Delta,N\Delta} = C_{N,N\Delta} \; ,$$

$$C_{\pi,\rho\pi} = \overline{\Gamma}_\rho \, Q_\rho(\beta)\, K_1(m_\rho \beta) / K_2(m_\rho \beta) \quad ,$$

$$C_{\pi,\Delta N\pi} = C_{N,\Delta N\pi} \quad ,$$

$$C_{\rho,\rho\pi} = C_{\pi,\rho\pi} \quad ;$$

$\sqrt{s_1} = \{2m_N(2m_N + E_{kin})\}^{1/2}$ c. m. energy in the $N_0 + N_0$ collision,

$\sqrt{s_{2N}} = (2m_N^2 + m_N\sqrt{s_1})^{1/2}$ c. m. energy in the $N_0 + N$ collision,

$\sqrt{s_{2\pi}} = (m_N^2 + m_\pi^2 + m_\pi\sqrt{s_1})^{1/2}$ c. m. energy in the $N_0 + \pi$ collision,

$\bar{\Gamma}_\Delta = d_\Delta \Gamma'_\Delta$ degeneracy times the width of Δ resonance,

$\bar{\Gamma}_\rho = d_\rho \Gamma'_\rho$ degeneracy times the width of ρ resonance,

$\sigma_1^{NN} = 8(\sigma_{pp\to pp} + \sigma_{pn\to pn})$ calculated at energy $\sqrt{s_1}$

$\sigma_1^{N\Delta} = 16(\sigma_{pp\to pn\pi^+} + \sigma_{pp\to pp\pi^0})$ calculated at energy $\sqrt{s_1}$,

$\sigma_2^{NN} = 8(\sigma_{pp\to pp} + \sigma_{pn\to pn})$ calculated at energy $\sqrt{s_{2N}}$,

$\sigma_2^{N\Delta} = 16(\sigma_{pp\to pn\pi^+} + \sigma_{pp\to pp\pi^0})$ calculated at energy $\sqrt{s_{2N}}$,

$\sigma_2^{N\pi} = 8\sigma_{\pi^+p\to\pi^+p}(T=\frac{3}{2})$ calculated at energy $\sqrt{s_{2\pi}}$.

The numerical values of the cross sections were taken from /18/.

$V_0 = 2\frac{4\pi}{3}R_0^3$, $R_0 = 1.12\,A^{1/3}$ fm with A being the target mass number;

$V_{pt}(t)$: the overlap of the target and projectile spheres;

$V_{gpt}(t)$: the overlap volume of the hot gas with the fast moving target and projectile spheres;

$V_g(t)$: the volume of the hot gas;

$t_m = R_o/v_{rel}$: the time at which the maximal overlap of target and projectile spheres occour; v_{rel} is the relative speed of the two spheres.

$v_{rel} = \{E_{kin}/(2m_N + E_{kin})\}^{1/2}$

According to assumption d), the time dependence of volume $V_g(t)$ is given as follows:

$$V_g(t) = \begin{cases} V_{pt}(t) & \text{if} \quad t \leq t_m , \\ \frac{4\pi}{3} R\,(t - t_m)^3 & \text{if} \quad t \geq t_m . \end{cases} \tag{4.8}$$

The radius of the isentropically expanding sphere of uniform density has the time dependence [17]

$$R(t) = \frac{R_o}{t_o} \left(t^2 + t_o^2\right)^{1/2} . \tag{4.9}$$

The kinetic energy associated with this hydrodynamic flow is [17]

$$E_{hyd}(t) = E_{tot} \frac{t^2}{t^2 + t_o^2} , \tag{4.10}$$

where E_{tot} is the total energy content of the gas sphere which can be transformed into kinetic energy, and

$$t_o = \left(0.3\, A\, m_N R_o^2 / E_{tot}\right)^{1/2} . \tag{4.11}$$

The kinetic equations were integrated numerically by Runge-Kutta method. The initial values of the densities and of the temperature were determined by expansion of the equations for small t values and by the prescription that the time derivative of the temperature should be zero at $t=0$. The densities multiplied by the corresponding volumes yielded the number of different particles as a function of time. They are displayed in Figs. 1-2. The chemical potentials μ_i for all the particles were also calculated on the basis of the expression valid for Boltzmann gases:

$$e^{\mu_i(t)\beta_i(t)} = A_i(t) = \frac{\nu_i(t)}{Q_i(\beta(t))} . \tag{4.12}$$

In order to check the consistency of the Boltzmann gas assumption the functions $D(x)$ and $v_Q(x)$ as given by Eq. (2.11) were determined, too.

According to assumption d/ the development of the reaction is described by a somewhat different mechanism in the formation /or "ignition"/ period and in the subsequent explosion period. This change in the reaction dynamics is emphasised in the Figures by inserting gaps between the two parts of the curves /which are calculated, of course, continuously/. The dotted curves in the second part of the Figures show the development of the system in the constant volume case: $V_g(t) = V_p = V_t$ for $t \geq t_m$. The continuation by an arrow connects these curves with the corresponding equilibrium values /attained practically in all cases before $t = 8 \cdot 10^{-23}$ s /.

V. RESULTS AND DISCUSSION

The analysis of the reaction model

Figure 5 shows that, while the pions play a negligible role at $E_{lab} \leqslant 400$ MeV/nucleon bombarding energy, they have to be taken into account from about $E_{lab} = 800$ MeV/nucleon. Above the energy of about 2 GeV/nucleon the highly excited nucleon and meson states become presumably more and more important. Their excitation may lead to a maximum temperature.

The inspection of Figs. 1-2 shows that the Δ resonances and pions are produced mainly in the "ignition period" of the reaction and their sum does not change appreciably during the explosion period. The ratio of pions to Δ's /or to ρ -mesons/, however, varies strongly during the expansion. This ratio - if it were possible to measure it - would give the break up time of the fireball.

The greatest part of the fast moving cold nucleons /especially in the U+U case/ suffers scattering for the time the spheres interpenetrated each other completely. This suggests that in central collisions of heavy ions of equal masses all the nucleons participate in some way in the formation of the fireball. Peripheral collisions or unequal mass nuclei are clearly less advantageous from this point of view.

On Figs. 1 and 2 it can be seen, that even before the complete overlap of the spheres the density of "gas" exceeds that of the "cold nucleons". Besides, the cross sections are larger for the "cold nucleon" - "gas" scattering becouse of the lower energy. These facts show, that the collisions of the "cold nucleons" with the constituents of the hot gas play an important role in the "ignition" of the fireball.

Throughout the whole calculation the Boltzmann limit was used for the momentum distribution within the gas. To check the consistency of this approximation, the $x = A/D(m\beta)$ quantity which appears in eq. (2.8) was calculated for each particle type and for all time steps. As long as $x \ll 1$, so that in calculating $G_p(x)$ one can neglect the higher order terms besides the first term in Eq. (2.9), the Boltzmann limit is a good approximation. The calculated values of x were less than 0.1 for most of the time of the reaction. /Large x values appeared only in the progressed phase of the expansion of the fireball./ The Boltzmann approximation can be used therefore consistently in the description of energetic heavy ion reactions.

The chemical equilibrium

The Figs 1 and 2 show, that the time necessary to reach the chemical equilibrium is of the same order of magnitude as the total reaction time. Therefore, although the ratios of particle numbers of different "chemical products" don't reach the equilibrium value, they are not very far from them.

At the time when the number of Δ -s plus pions plus twice the number of ρ mesons arrives to a constant level, the thermic coupling ceases among the constituent particles. This time can be regarded roughly as the break up time. At this point the density in the present model is about 0.25 times the overlap density, i.e. about half the normal nuclear density.

<u>Is a Bose-Einstein pion condensate formed in the reaction?</u>

Inspecting the inserts in Figs. 1-2 one can observe a very interesting point on the plot of the pion chemical potential μ_π versus time. Namely, near the break-up time it reaches the value $\mu_\pi = 0.14$ GeV i.e. $\mu_\pi = m_\pi$! This is a singular point in the present description. If the gas mixture were large enough and it were spending long enough time in this state then it would correspond to a phase transition implying the creation of a pion condensate. In ideal quantum gases this is the Bose-Einstein condensation. At this point pions could be created without energy investment. It is important to note that this condensate is a hot one! Its existence is not restricted to near zero temperatures. A remarkable feature is that this condensation /if it occurs/ is just in the last part of the fireball's history, therefore, directly observable. /Events that occur earlier in the fireball's life are "washed out" to a large extent from its "memory" by the later thermal history./ This may provide us with a rather unique tool to study the properties of dense hot and condensed hadronic matter.

The appearence of the condensate can be understood here as follows. In the collision many hot pions are produced. At this time the system can be described approximately as a Boltzmann gas. During the expansion, however, the pions have to cool down but for the lower temperature there are too many of them in the gas phase. As the temperature is dropping the "pion consuming" processes /like $\pi + N \to \Delta$; $\Delta + N \to N + N$ / slow down very much. Therefore the pions have to be removed by the formation of a condensate. The characteristic feature of such a condensation is the clusterisation of pions in the momentum space. One has to realize, however, that the intermediate state in the energetic heavy ion reactions has a short lifetime. Therefore the formation of this new type of pion condensate /different from the much discussed pion condensation in cold nuclear matter[19, 20]/, is to be regarded presently more as a question towards experiments than a firm theoretical prediction. The question, how strongly this tendency of momentum space clusterization of pions will manifest itself in the heavy ion reaction is to be answered by further theoretical and experimental investigations.

REFERENCES

1/ G.D. Westfall, J. Gosset, P.J. Johansen, A.M. Poskanzer, W.G. Meyer, H.H. Gutbrod, A. Sandoval and R. Stock; Phys. Rev. Lett. 37, /1976/ 1202

2/ A. Mekjian, Phys. Rev. Lett. 38, /1977/ 640
A. Mekjian, Phys. Rev. C17, /1978/ 1051

3/ for recent compilations of the topic see e.g. M. Gyulassy in Proc. Int. Symp. on Nuclear Collisions and Their Microscopic Description, Bled, Yugoslavia, 1977, Physica /to be published/; J.R. Nix, Theory of High Energy Heavy Ion Collisions, LA - UR - 77 - 2952;
H. Feshbach, Relativistic Heavy Ions, Lectures at les Houches Summer School, Aug. 1977.

4/ A. Münster, Statistical Thermodynamics, Vol. I. Berlin, 1969.

5/ I.S. Gradshtein, I.M. Ryzhik, Tables of integrals, sums, series and products, Moscow, 1962.

6/ M. Chaichian, R. Hagedorn, M. Hayashi, Nucl. Phys. 92B, /1975/ 445

7/ I. Montvay, Nuovo Cim. 41A /1977/ 287

8/ R. Hagedorn, Thermodynamics of strong interactions, CERN Lecture Notes 71-12

9/ R. Hagedorn, Suppl. Nuovo Cim. 3 /1965/ 147;
S. Frautschi, Phys. Rev. D3 /1971/ 2821

10/ N. Cabibbo, G. Parisi, Phys. Letters 59B /1974/ 67

11/ E. Beth, G.E. Uhlenbeck, Physica, 4 /1937/ 915

12/ S.Z. Belenkij, Nucl. Phys. 2 /1956/ 259

13/ R. Dolen, D. Horn, C. Schmid, Phys. Rev. 166 /1968/ 1768;
H. Harari, Annals of Physics 63 /1971/ 432;
B. Schrempp, F. Schrempp, Nucl. Phys. B54 /1973/ 525;
G. Veneziano, Nuovo Cim. 57A /1968/ 190

14/ S.S. Schweber, Introduction to relativistic quantum field theory, New York, 1961, chap. 14.

15/ M.L. Goldberger, K.M. Watson, Collision Theory, John Wiley, New York, 1964.

16/ R.E. Mickens, Lettere Nuovo Cim. 20 /1977/ 377

17/ J.P. Bondorf, S.I.A. Garpman, J. Zimányi, Nucl. Phys. A296 /1978/ 320

18/ NN and ND interactions, Particle Data Group, UCRL - 20000 NN /1970/

19/ A.B. Migdal, Nucl. Phys. A210 /1973/ 421

20/ R.F. Sawyer, D.J. Scalapino, Phys. Rev. D7 /1973/ 353

FIGURE CAPTIONS

Figure 1.

The dynamics of the $U+U$ heavy ion reaction at 2.1, 1.4, 0.8 and 0.4 GeV/nucleon bombarding energies. The number of particles and the gas temperature in the overlap region is plotted as a function of time. The shaded spheres on the top of the Figures indicate the geometry of the process: the interpenetration and eventually the expansion of the projectile and target nuclei. The vertical dashed lines separate the "ignition" part from the expansion part. The particle numbers and temperature shown by the dashed branch of the curves correspond to the case, when no expansion was allowed after the complete overlap of the two spheres. The arrows at the end of these curves point to the equilibrium values of the corresponding quantities. The insert in the upper right part show the chemical potential, μ_π and activity, A_π for pions as a function of time. The horizontal line marks the $\mu_\pi = m_\pi$ value, where the possibility of Bose-Einstein pion condensation appears.

Figure 2.

The dynamics of the $Ar+Ar$ heavy ion reaction at 2.1, 1.4, 0.8 and 0.4 GeV/nucleon bombarding energies. The explanation of the details is the same as in Fig. 1.

Figure 3.

The pion to nucleon ratio as a function of bombarding energy in the $U+U$ and $Ar+Ar$ central collisions.

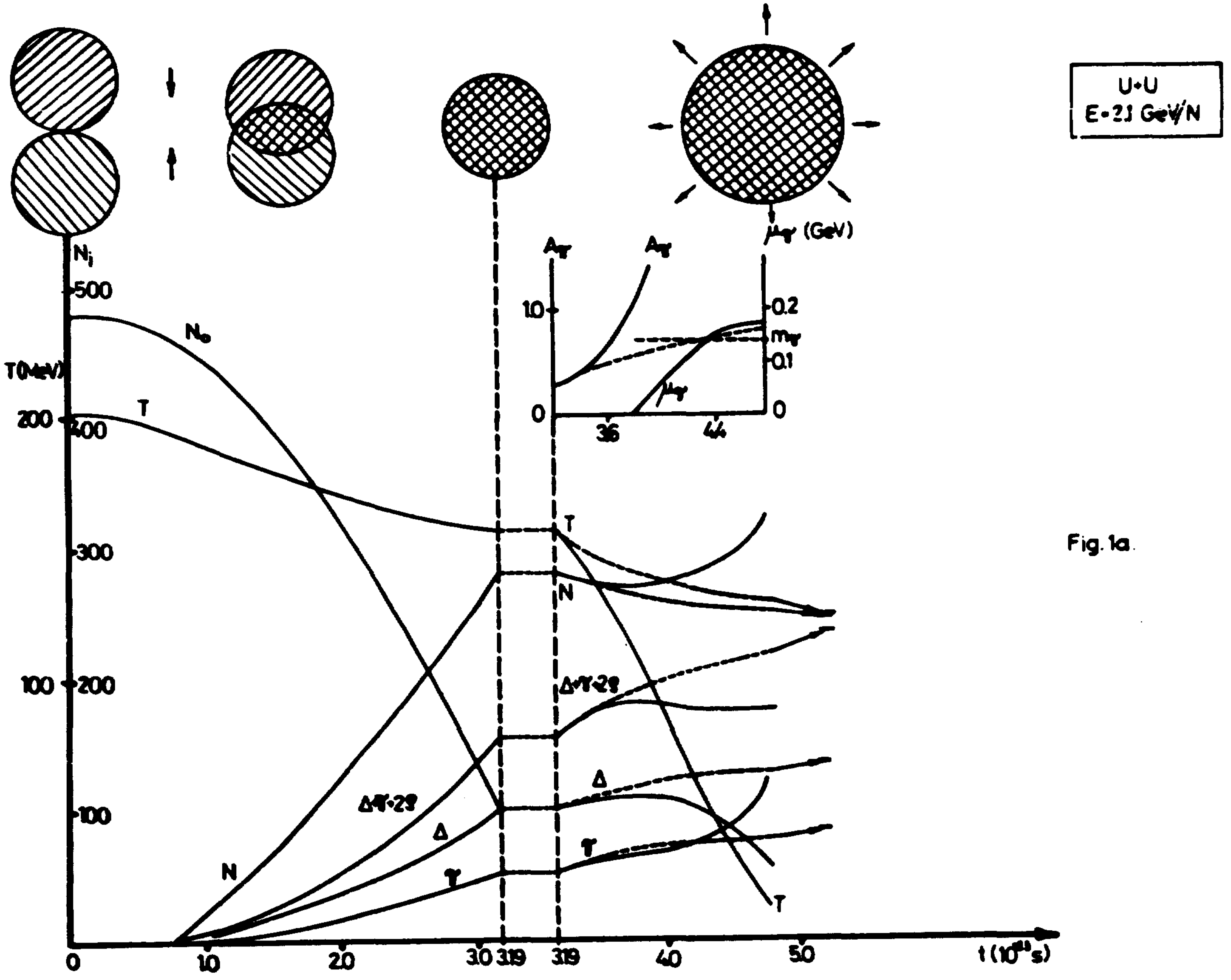
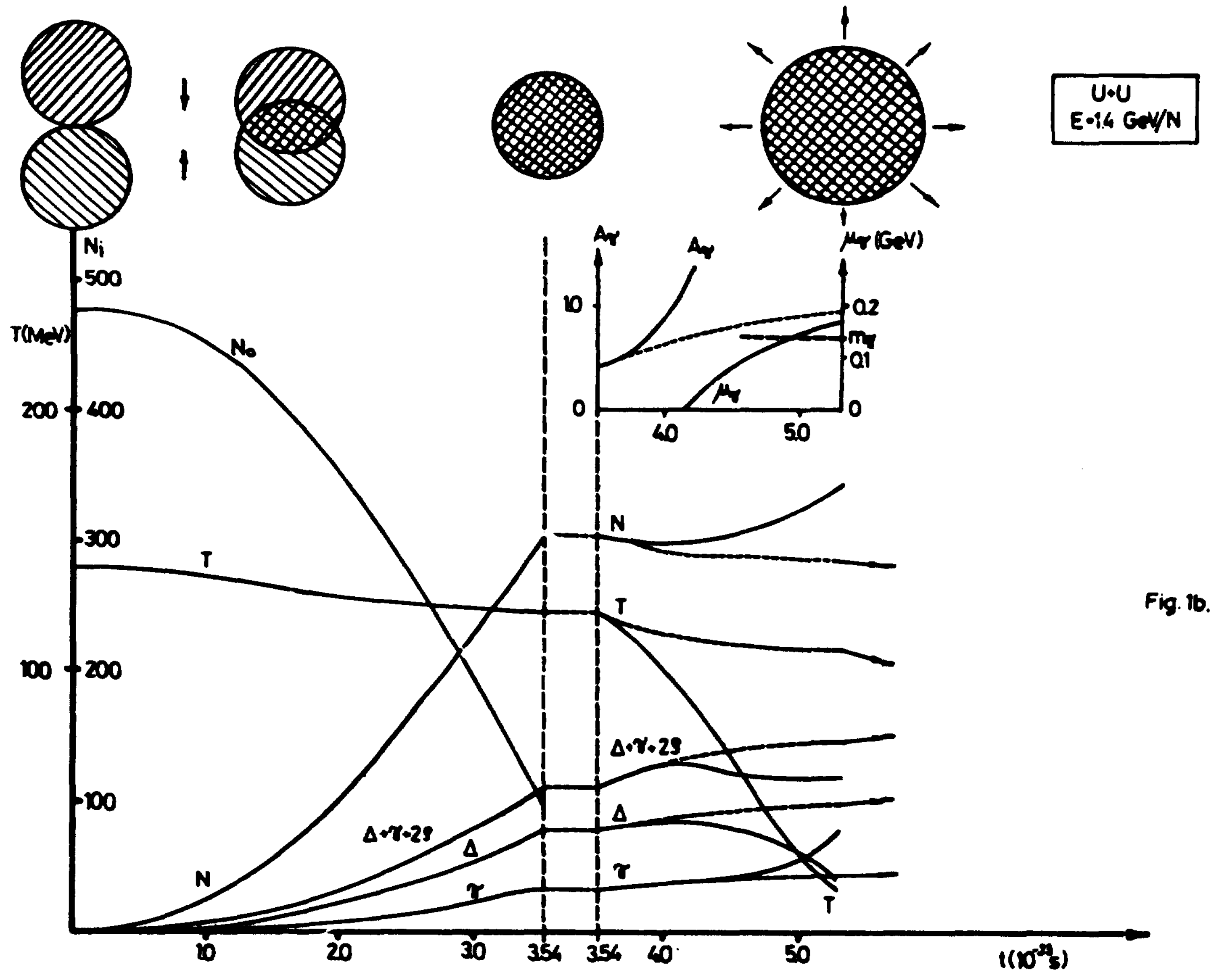

U+U
E=2.1 GeV/N
N_i
500
400
300
200
100
T(MeV)
200
100
N_o
T
N
Δ
$\Delta + Y^* + \Xi$
Y^*
$A_{q\bar{q}}$
10
0
36
44
$\mu_{q\bar{q}}$ (GeV)
0.2
$m_{q\bar{q}}$
0.1
$\mu_{q\bar{q}}$
0
1.0
2.0
3.0
3.19
4.0
5.0
t (10^{-23} s)

Fig. 1a.



U+U
E=1.4 GeV/N
N_i
T(MeV)
500
400
300
200
100
N_o
T
N
$\Delta+\Upsilon+2\Sigma$
Δ
Υ
10
20
30
3.54
3.54
40
50
t(10^{-23}s)
A_{η}
μ_{η} (GeV)
0.2
m_{η}
0.1
0

Fig. 1b.

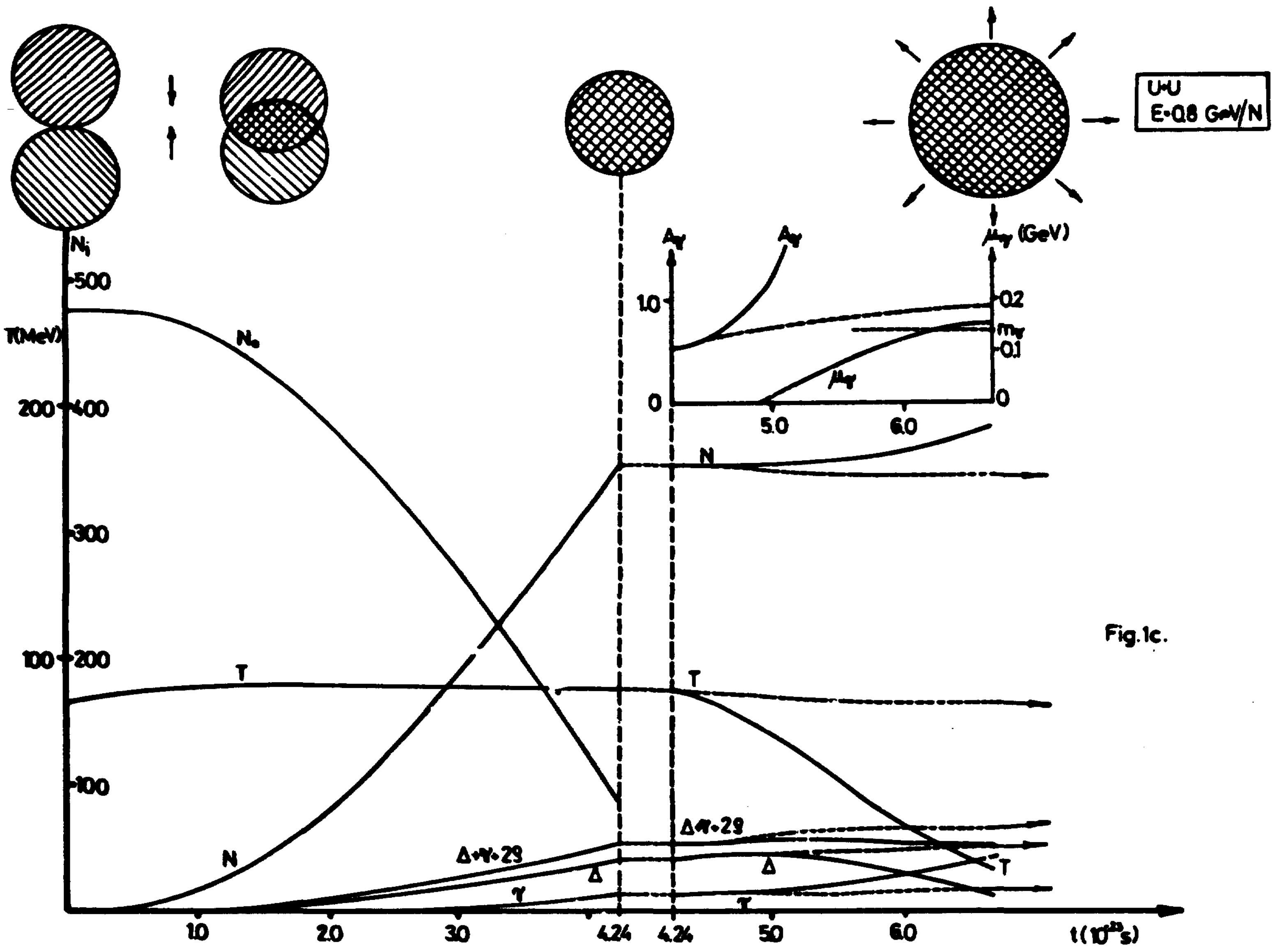
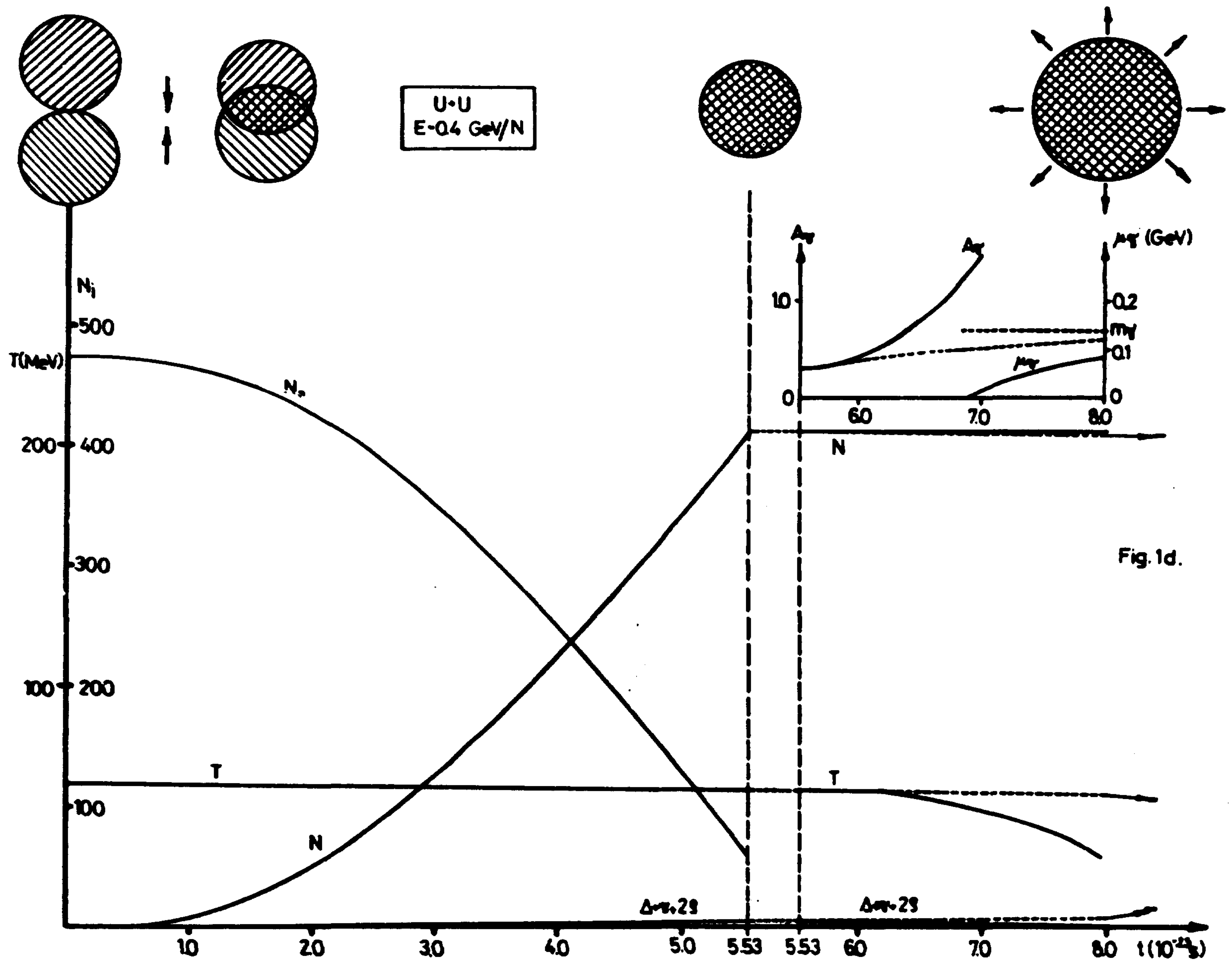
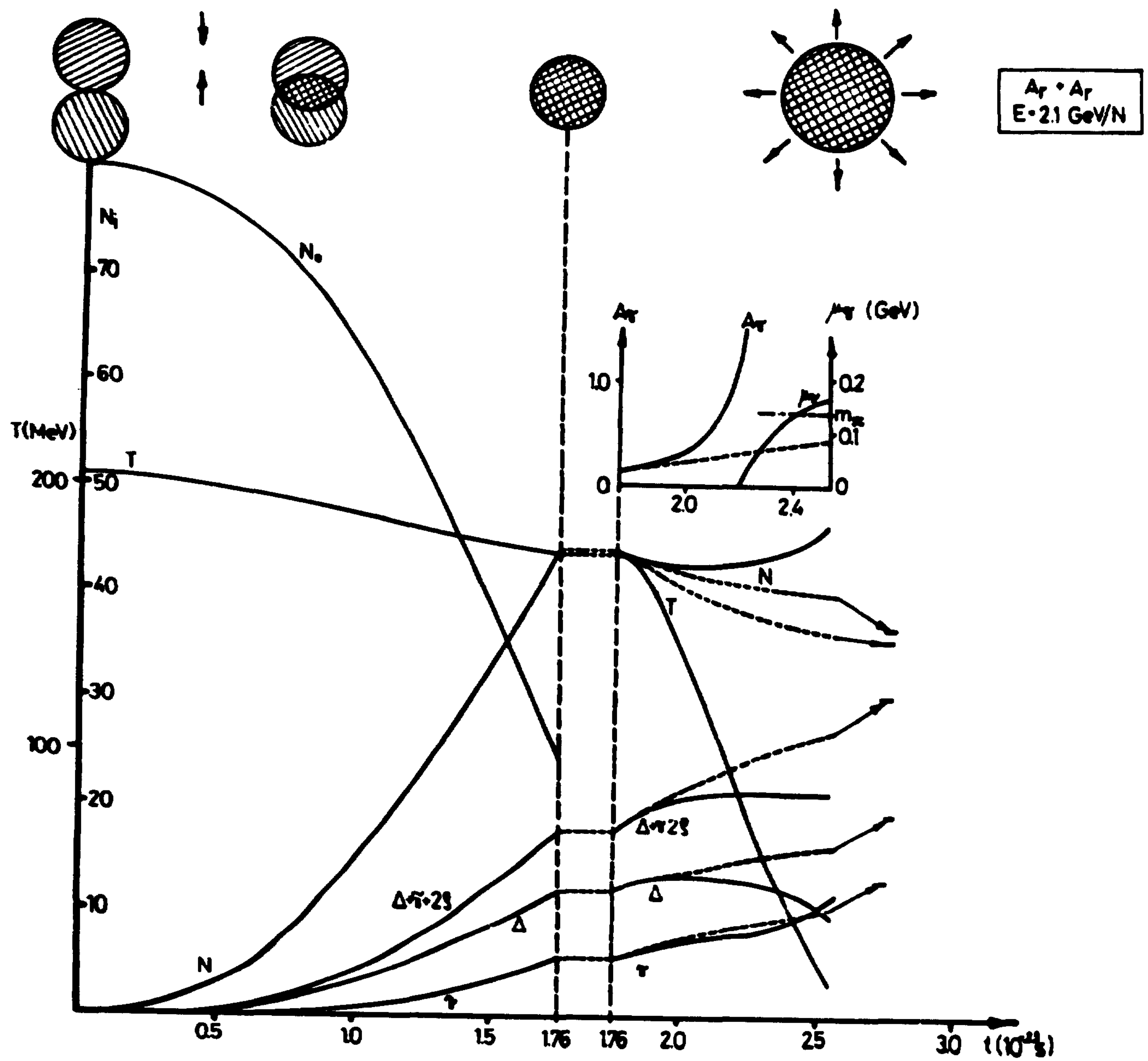

U+U
E=0.8 GeV/N
N_i
T(MeV)
500
400
300
200
100
200
100
N_o
N
T
$\Delta+\Upsilon=2.9$
Δ
γ
A_{qr}
μ_{qr} (GeV)
1.0
0
0.2
m_{qr}
0.1
5.0
6.0
1.0
2.0
3.0
4.24
$t(10^{-23}s)$

Fig.1c.



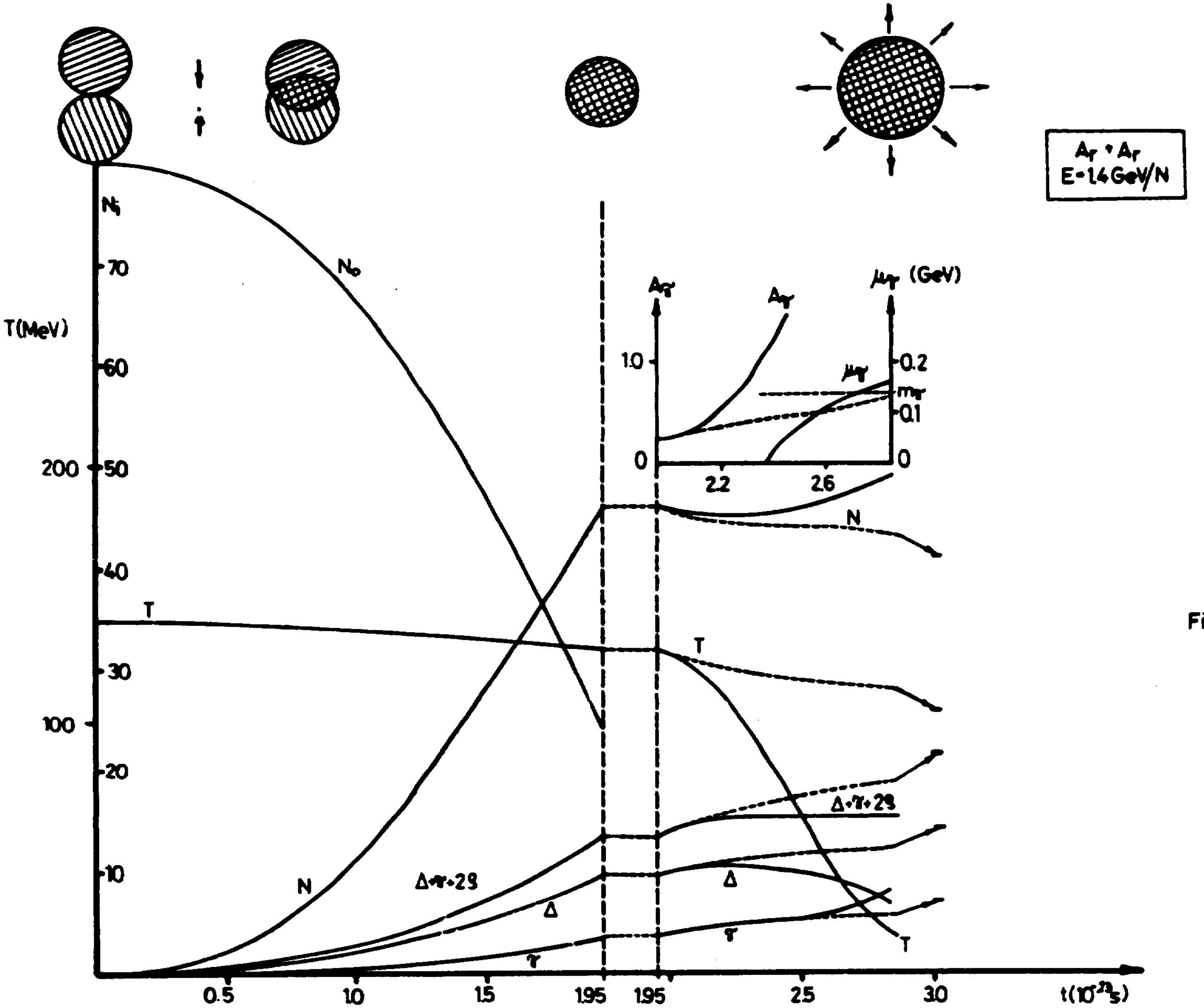
U+U
E=0.4 GeV/N
N_i
500
400
300
200
100
T(MeV)
200
100
N_π
N
T
1.0
2.0
3.0
4.0
5.0
5.53
5.53
6.0
7.0
8.0
t(10^{-23}s)
1.0
0
0.2
0.1
0
μ_q (GeV)
m_q
μ_q

Fig. 1d.



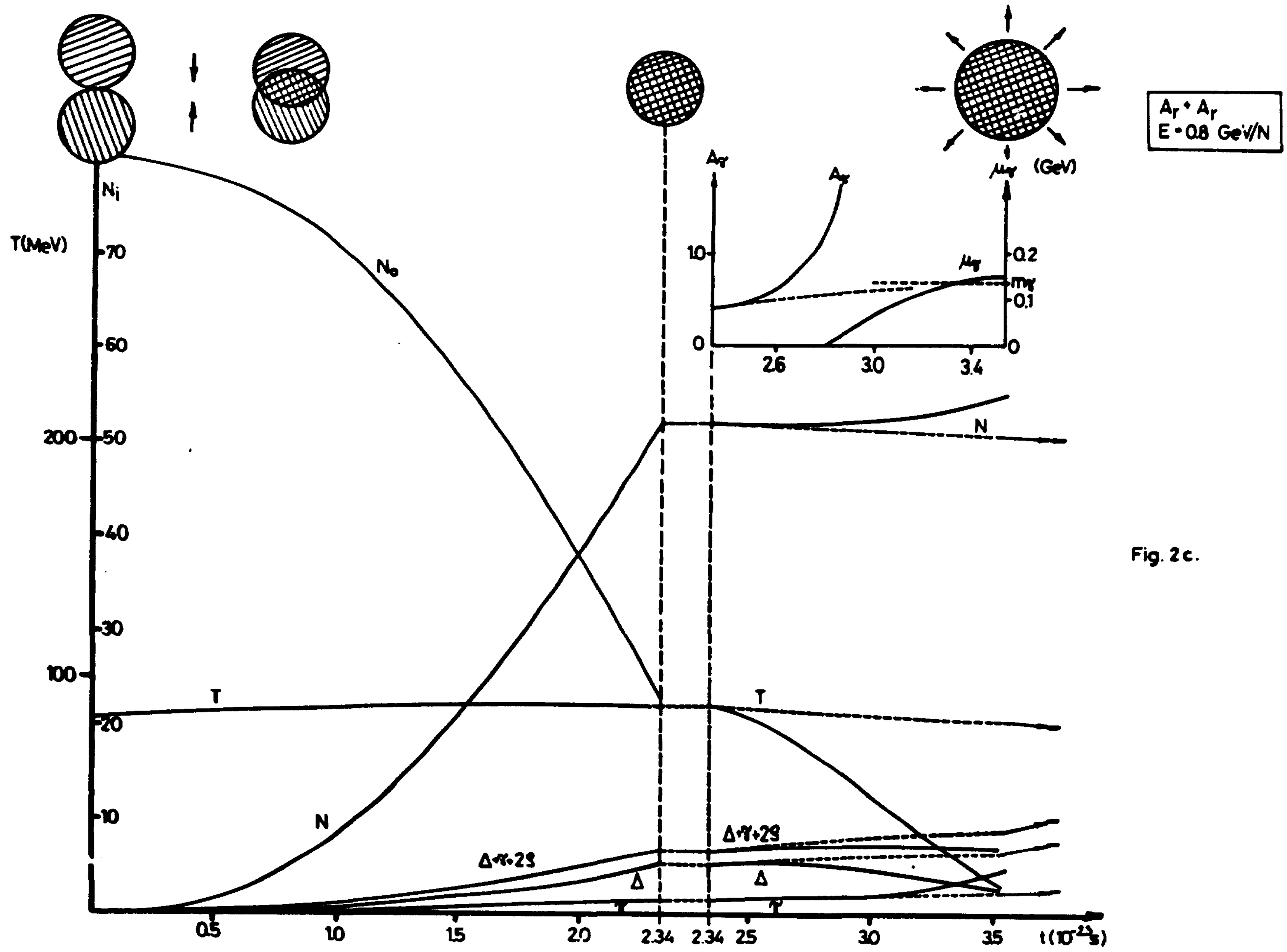
$A_r + A_r$
E = 2.1 GeV/N
N_i
N_o
T(MeV)
T
N
$\Delta+\pi+2\rho$
Δ
π
A_π
μ_π (GeV)
m_π
t (10^{-22}s)

Fig. 2a.



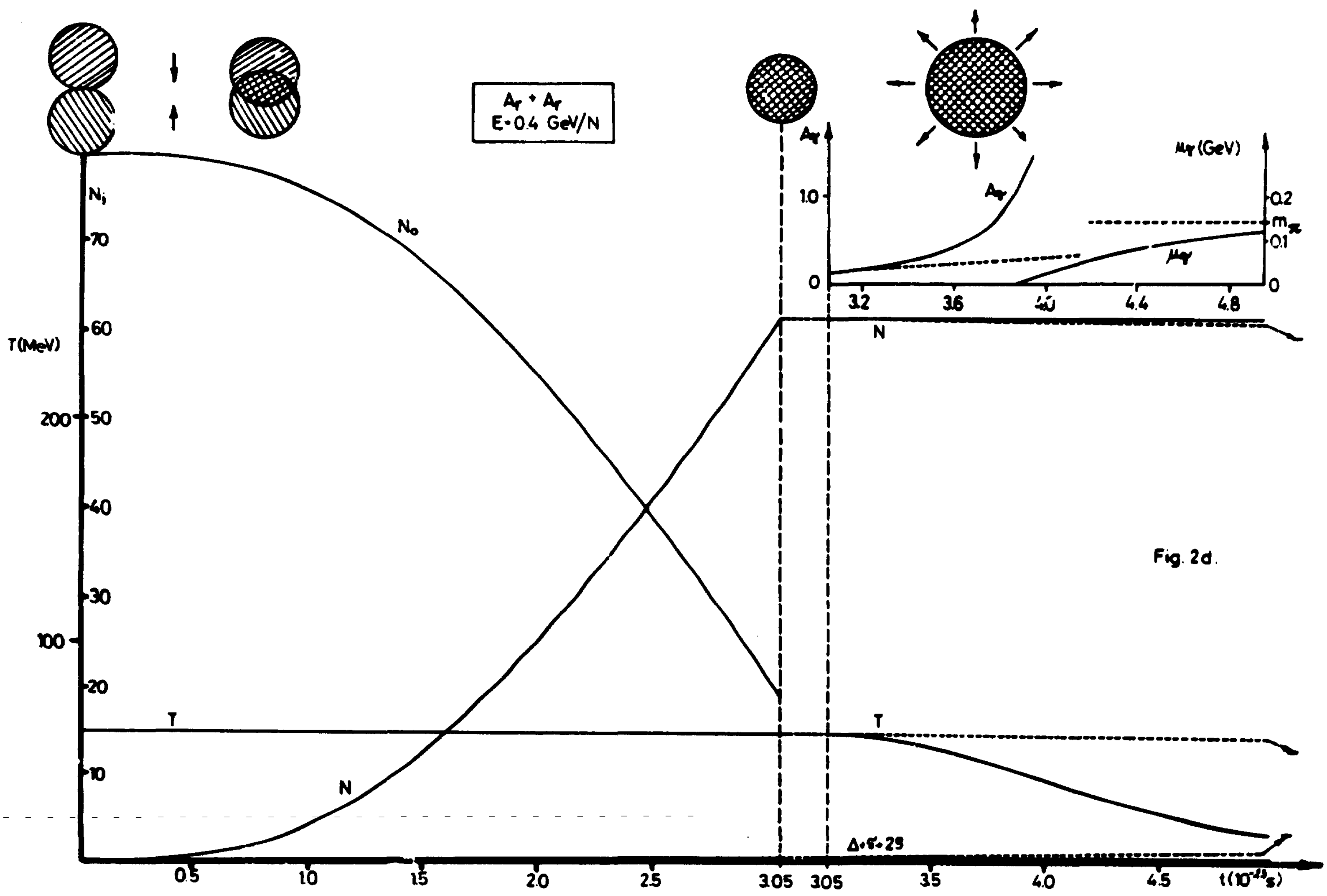
$A_r + A_r$
$E = 1.4$ GeV/N
T(MeV)
200
100
N_i
70
60
50
40
30
20
10
N_0
T
N
$\Delta + \pi + 2\vartheta$
Δ
π
$A_{q\bar{q}}$
$\mu_{q\bar{q}}$ (GeV)
1.0
0
0.2
0.1
m_π
2.2
2.6
0.5
1.0
1.5
1.95
1.95
2.5
3.0
$t(10^{-23}s)$

Fig. 2b.



$A_r + A_r$
$E = 0.8$ GeV/N
T(MeV)
N_i
N_0
N
T
$\Delta+\pi+2S$
Δ
π
A_π
μ_π (GeV)
m_π
$t(10^{-23}s)$

Fig. 2c.



Ar + Ar
E = 0.4 GeV/N
T(MeV)
200
100
Ni
70
60
50
40
30
20
10
N0
N
T
Aq
1.0
0
3.2
3.6
4.0
4.4
4.8
μq (GeV)
0.2
mπ
0.1
μq
Δ = 5 = 2.9
0.5
1.0
1.5
2.0
2.5
3.05
3.5
4.0
4.5
t(10^-23 s)

Fig. 2d.

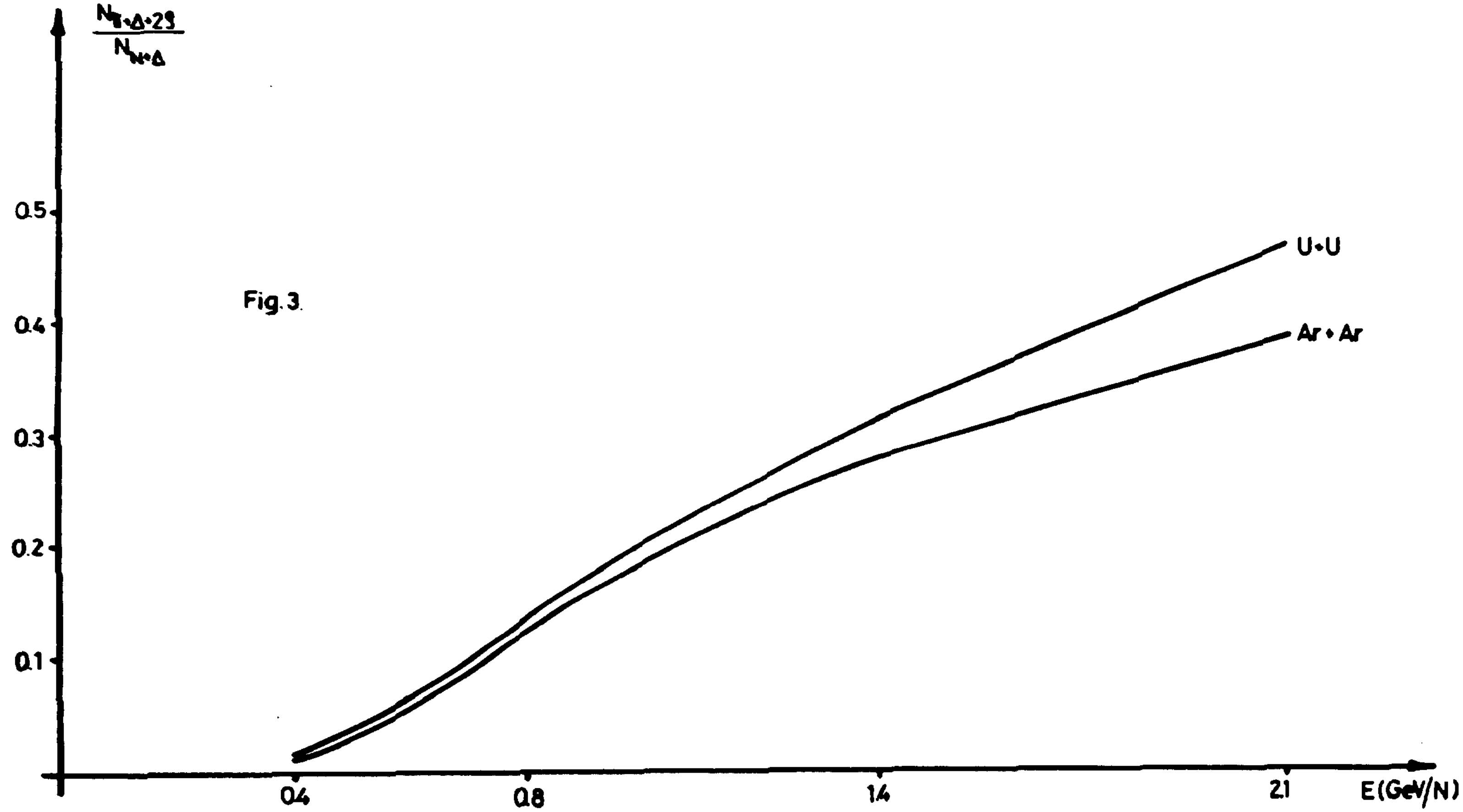


Fig. 3

Kiadja a Központi Fizikai Kutató Intézet
Felelős kiadó: Szegő Károly
Szakmai lektor: Lovas István
Nyelvi lektor: Lukács Béla
Példányszám: 460 Törzsszám: 1978-646
Készült a KFKI sokszorosító üzemében
Budapest, 1978 junius hó