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AMBIGUITIES ABOUT INFINITE NUCLEAR MATTER

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Abstract

Exact solutions of the harmonic-oscillator and infinite hyperspherical well are given for the ground state of a infinitely heavy (N=Z) nucleus. The density of matter is a steadily decreasing function. The kinetic energy per particle is 12% smaller than the one predicted by the Fermi sea.

Introduction

In order to obtain predictions about the volume properties in nuclei the uniform nuclear matter filled up with plane waves has been introduced. It is interesting to compare the results obtained with this standard matter to those which can be calculated in trying to solve exactly the many nucleons problem for an infinitely heavy nucleus.

This nucleus is supposed to be constructed in starting from a finite nucleus by adding indefinitely new nucleons in such a way that a balanced (N=Z=A/2) nuclear matter is obtained. The ground state is calculated at each stage of the filling.

The question to which we would like to give an answer is whether the standard nuclear matter has similar properties as the ground state of our infinitely heavy nucleus.

Crudely the most important questions are :

i) does the nuclear matter describe an infinite medium of nucleons in the ground state ?

ii) is the density constant all over the volume of our infinitely heavy nucleus ?

Let us treat to the first beginning as an academic but demonstrative example the case of a harmonic oscillator (H.O.).

Infinite harmonic oscillator system

Let us assume that the nucleons are submitted to the mutual H.O. interaction

$$V(r_{ij}) = \frac{\hbar^2}{2mb^4} r_{ij}^2$$

in which the H.O. parameter b is a function of the total number A of nucleons. b can be chosen in such a way that the r.m.s. radius of the nucleus grows like $A^{1/3}$ when the number of nucleons increases. The ground state of the H.O. Schrödinger equation

$$\left\{ \frac{\hbar^2}{2m} \sum_{i=1}^{A} (-\nabla_i^2 + \frac{x_i^2}{b^4}) - E \right\} \Psi(\vec{x}) = 0 \qquad \vec{x}(\vec{x}_1 \ldots \vec{x}_A)$$

is given by the familiar H.O. Slater determinant in which all the individual H.O. states of the first shells has been filled till the last shell denoted by $\Lambda_m = \ell_{max}$, where ℓ_{max} is the maximum of the individual orbital quantum number in the last filled shell. For the same number of neutrons and protons the shells are equally filled by the two kinds of nucleons.

The total binding energy is

$$E = \sum_{i=1}^{A} (2n_i + \ell_i + \frac{3}{2}) \frac{\hbar^2}{mb^2} \qquad (1)$$

where the sum is taken over all the occupied H.O. states. n_i and ℓ_i are the usual H.O. quantum numbers.

The number of H.O. states (including spin-isospin states) in a completely filled Λ shell is

$$N_\Lambda = 2(\Lambda+1)(\Lambda+2)$$

The contribution of this shell into the sum (1) is $(\Lambda + \frac{3}{2})N_\Lambda$. Adding together the contributions of the first filled shells gives the ground state energy

$$E_{gs} = \sum_{\Lambda=0}^{\Lambda_m} (\Lambda + \frac{3}{2}) N_\Lambda \frac{\hbar^2}{mb^2} = \frac{3}{4} (\Lambda_m+2) A \frac{\hbar^2}{mb^2} \quad (2)$$

The total number of nucleons is given in terms of Λ_m by

$$A = \frac{2}{3} (\Lambda_m+1)(\Lambda_m+2)(\Lambda_m+3) \approx \frac{2}{3} (\Lambda_m+2)^3 \quad (3)$$

leading finally to

$$E_{gs} = \frac{1}{2} (\frac{3}{2} A^{4/3}) \frac{\hbar^2}{mb^2} \quad (4)$$

The average kinetic and potential energies are equally shared in H.O. (virial theorem) therefore the average kinetic energy in the H.O. ground state is $\overline{T} = \frac{1}{2} E_{gs}$.

Using the relation

$$a^2 = \frac{b^2}{A} \sum_{1}^{A} (2n_i + \ell_i + \frac{3}{2})$$

between the H.O. parameter b and the m.s. radius a^2 of our system, the kinetic energy per particle becomes

$$\overline{T}_{H.O.} = \frac{15}{32} \left(\frac{3}{2} \right)^{2/3} \frac{\hbar^2}{mr_0^2} A \quad (5)$$

where r_0 is the parameter of a uniform density providing the m.s. radius

$$a^2 = \frac{3}{5} r_0^2 A^{2/3} \quad \text{of a saturated system .} \quad (6)$$

Reminding that the kinetic energy per particle in the Fermi sea is $E_F = \frac{3}{10}\frac{\hbar^2}{m}k_F^2$ with $k_F = \frac{(9\pi)^{1/3}}{2r_0}$ one notices that when the number A of nucleons increases indefinitely the average kinetic energy per particle in H.O. ground state

$$\frac{\bar{T}_{HO}}{A} = \frac{25}{4}(6\pi)^{-2/3} E_F = 0.88\, E_F \tag{7}$$

is 12% smaller than the one given by a nucleon in the Fermi sea.[1]

The form factor of the H.O. ground state is given by[2]

$$F_{H.O}(k) = \sqrt{4\pi} \sum_{n,\ell} \left[\frac{1}{A} \sum_{(\alpha)} (\alpha|n,\ell) \right] (-1)^{\ell/2}\, Y_\ell^0(\omega_k) \tag{8}$$

$$\left(\frac{Aa^2k^2}{2L_m+D}\right)^{n+\frac{\ell}{2}} \cdot\, e^{-\frac{A-1}{2L_m+D}\frac{a^2k^2}{2}}$$

where $L_m = \sum_{i=1}^{A} (2n_i+\ell_i) = \frac{3}{4}\Lambda_m A$ for ground states. The sum $\sum_{(\alpha)}$ is taken over the spin-isospin and H.O. occupied states and the coefficient $(\alpha|n,\ell)$ is

$$(\alpha|n,\ell) = \sqrt{2\pi^{3/2}}\,[2^{2n+\ell}\, n!\, \Gamma(n+\ell+\tfrac{3}{2})]^{-1/2}$$

$$\langle Y_{\ell_\alpha}^{m_\alpha}|Y_\ell^0|Y_{\ell_\alpha}^{m_\alpha}\rangle \begin{bmatrix} n_\alpha & n_\alpha & n \\ \ell_\alpha & \ell_\alpha & \ell \end{bmatrix} \tag{9}$$

where $(4\pi)^{-1/2} \begin{bmatrix} n_\alpha & n_\beta & n_\gamma \\ \ell_\alpha & \ell_\beta & \ell_\gamma \end{bmatrix}$ is the Gogny coefficient [3] $T_{\alpha\beta}^{\gamma}$.

For closed shell nuclei filled up to the Λ_m shell included, the sum over (α) gives the simple expression for our coefficients :

$$\frac{1}{A}\sum_{(\alpha)}(\alpha|n,\ell) = \frac{(-1)^n}{n!}\binom{\Lambda_m+3}{n+3}\delta_{\ell,0} \tag{10}$$

leading to the form factor

$$F_{H.O}(\vec{k}) = \frac{4}{A}L^3_{\Lambda_m}\left[\frac{A\ a^2k^2}{2L_m+3A}\right] e^{-\frac{A-1}{2L_m+3A}\frac{a^2k^2}{2}} \tag{11}$$

where L_n is a Laguerre polynomial.

When Λ_m (together with A) increases indefinitely one uses the asymptotic properties of Laguerre polynomials [(4)] in order to obtain the asymptotic expression of the form factor :

$$F^{\infty}_{H.O}(k) = \lim_{A\to\infty} F_{H.O}(k) = \frac{6}{\Omega^3}J_3(2\Omega) \qquad \Omega = \sqrt{\frac{2}{3}}\,ak \tag{12}$$

where J_3 is a Bessel function.

The matter density is the properly normalised original of the form factor :

$$R(\vec{x}) = \frac{A}{(2\pi)^3}\int F(\vec{k})\ e^{-i\vec{k}\vec{x}}\ d^3k$$

Using the asymptotic expression $F^{\infty}_{H.O}(\vec{k})$ one obtains

$$R^{\infty}_{H.O}(\vec{x}) = \frac{A}{\pi^2}\left(\frac{3}{2a^2}\right)^{3/2}[1-y^2]^{3/2} \quad , \ y = \frac{x}{2\sqrt{\frac{2}{3}}\ a} \ , \ y \leq 1 \tag{14}$$

while $R^{\infty}_{H.O}(x) = 0$ for $y \geq 1$.

The behaviour of $R^{\infty}_{H.O}(\vec{x})$ is illustrated in fig.1 where the uniform density corresponding to the same r.m.s. radius a has also been plotted.

In conclusion the kinetic energy per particle of the H.O. ground state of an infinitely heavy nucleus is 12% smaller than the corresponding energy in the Fermi sea and the density of matter is not a uniform but a steadily decreasing function. It is interesting to notice that the excitation energy of the hyperradial motion which describes the breathing mode (7) is well reproduced ($E_m = \frac{118}{r_0^2(fm^2)} A^{-1/3}$ Mev) for the usual values of r_0 appearing in saturated nuclear matter ($r_0 \sim 1.1-1.2$ fm).

The infinite hyperspherical well

We intend now to study whether the total energy of a system of free fermions is dependent or not on the boundary conditions which insure its confinement.

When the fermions are contained into a box growing indefinitely with the number of particles in such a way to produce a constant uniform density, the energy per particle in the ground state is given by $E_F = \frac{\hbar^2}{m} k_F^2$ where k_F is the Fermi momentum.

Let us assume instead that the motion of the A free fermions of our system is described by the motion of one point into a 3A dimensional space and that the confinement proceeds from the condition

$$\sum_{i=1}^{A} (\vec{x}_i-\vec{X})^2 \leq R^2/2 \qquad \vec{X} = \frac{1}{A}\sum_{1}^{A}\vec{x}_i \tag{15}$$

where R is the radius of a hypershpere in the 3A dimensional space.

This condition requires that the sum of the square of the distance of the particles with respect to the center of mass cannot exceed a maximum $R^2/2$. The problem is solved in using polar coordinates in which we define a hyperradial coordinate

$$r = \left[2 \sum_{1}^{A} (x_i - X)^2\right]^{1/2}$$

and a set Ω of 3A-4 angular coordinates.

The Schrödinger equation becomes

$$(T-E)\Psi = \left\{-\frac{\hbar^2}{m}\left(\frac{d^2}{dr^2} + \frac{3A-4}{r}\frac{d}{dr} + \frac{L^2(\Omega)}{r^2}\right) - E\right\}\Psi = 0 \qquad (16)$$

The eigenfunctions of the square of the grand orbital operator $L(\Omega)$ are the hyperspherical harmonics (H.H.) $Y_{[L]}(\Omega)$ regular solutions of

$$[L^2(\Omega) + L(L+3A-5)]\ Y_{[L]}(\Omega) = 0$$

The quantum number L is called grand orbital.

Let us denote by $D_L(\Omega)$ an H.H. of grand orbital L including spin-isospin states completely antisymmetric for any exchange of two nucleons.

The normalised eigenfunctions of (16) are [5]

$$\psi_{L,n}(r,\Omega) = D_L(\Omega)\, r^{-\frac{3A-4}{2}}\, u_{L,n}(r)$$

$$u_{L,n}(r) = \frac{1}{J'_\nu(j_{\nu,n+1})}\ \frac{\sqrt{2r}}{R}\ J_\nu\left[j_{\nu,n+1}\frac{r}{R}\right] \qquad r < R \qquad (17)$$

$$u(r) = 0 \qquad r \geq R \quad , \qquad \nu = L + \frac{3A-5}{2}$$

where J_ν is a Bessel function, J'_ν its derivative and $j_{\nu,m}$ is m^{th} zero of J_ν $(J_\nu(j_{\nu,m})=0)$. n is the number of nodes of the hyperradial wave. The energy is given by

$$E_{\nu,n} = \frac{\hbar^2}{m}\frac{j^2_{\nu,n+1}}{R^2} = \frac{1}{3}\,[\,j^2_{\nu,n+1}-2(\nu^2-1)]\;\frac{1}{A}\;\frac{\hbar^2}{m\;a^2_{\nu,n}} \tag{18}$$

where the m.s. radius is

$$a^2_{\nu,n} = \frac{1}{2A} <u_{\nu,n}|r^2|u_{\nu,n}> = \left[1 + \frac{2(\nu^2-1)}{j^2_{\nu,n+1}}\right]\frac{R^2}{6A} \tag{19}$$

The ground state corresponds to a wave without node (n=0) and to L minimum.

For A large the asymptotic expression $j_{\nu,1} = \nu + 1.855\nu^{1/3} +\ldots$ can be used. It leads to asymptotic energy and m.s. radius

$$E_{\nu,0} = \frac{\nu^2}{2A}\,\frac{\hbar^2}{m\;a^2_{\nu,0}} \qquad\qquad a_{\nu,0} = \frac{R^2}{2A} \tag{20}$$

According to (19) the last expression implies that in the hyperspace the matter is concentrated near the limit hypersphere of radius R.

In agreement with the definition of H.H. $r^L\,D_L(\Omega)$ is an harmonic polynomial.

The grand orbital L is minimum (L_m) in ground state. A receipe to construct harmonic polynomials of degree minimum has been proposed by Simonov [6]. It is quite similar to the one applied in the construction of the H.O. ground state because a H.O. ground state wave function is precisely the product of a harmonic polynomial $D_{L_m}(\vec{x})$ of degree minimum L_m homogeneous in the coordinates $\vec{x}_i$ of the nucleons and a gaussian $e^{-\frac{1}{2b^2}\sum_1^A x_i^2}$.

In particular the minimum ν_m of ν is exactly

$$\nu_m = L_m + \frac{3A-5}{2} = \frac{3}{4} \Lambda_m A \qquad (21)$$

where L_m has the value previously calculated.

Introducting (21) and (6) in (20) one finds that the ground state (kinetic) energy is given by (5) which is 12% smaller than the one given by the Fermi sea.

The matter density is calculated in using the form factor [3] for closed shell nuclei

$$F(\vec{k}) = \Gamma(\nu_m+1) \sum_n \left[\frac{1}{A} \sum_{(\alpha)} (\alpha|n,0) \right] \left(\frac{2A}{A-1} \right)^n f_n(q)$$

$$f_n(q) = \int_0^{+\infty} \left(\frac{2}{qr} \right)^{\nu_m - n} J_{\nu_m+n}(qr) \, [u_{L_m,0}(r)]^2 \, dr \qquad (22)$$

$$q = \sqrt{\frac{A-1}{2A}} \; k$$

Assuming that very large values of k are not significant, the Bessel function can be replaced by the asymptotic expression

$$J_N(z) = \frac{1}{\Gamma(N+1)} \left(\frac{z}{2} \right)^N e^{-\frac{1}{N} \left(\frac{z}{2} \right)^2} \qquad (23)$$

leading to an asymptotic density of matter

$$R(x) = \frac{8A}{\pi^2} \left(\frac{\nu_m}{\Lambda_m} \right)^{3/2} \int_0^{x\sqrt{\nu_m/\Lambda_m}} \left[1 - \frac{\nu_m}{\Lambda_m} \left(\frac{x}{r} \right)^2 \right]^{3/2} [u_{L_m,0}(r)]^2 \frac{dr}{r^3} \qquad (24)$$

We have seen in (19) and (20) that $[u_{L_m,0}(r)]^2$ behaves like a singular δ function

$$\delta(r-\sqrt{2A}\,a) \tag{25}$$

Using this property one finds again the density (14) for our gas of free fermions submitted to the boundary condition (15).

One conclude from this example that the ground state (kinetic) energy of a system of free fermions depends on the boundary conditions which confine the gas of particles.

Coupled equations for infinite systems

One can argue that our results have been obtained either with an infinite range interaction or for a system of free fermions.

Unfortunately it does not seem to be possible to have an exact solution of our many body problem for other kinds of potentials.

Nevertheless one can study how the problem can be formulated.

For this purpose the ground state is expanded into H.H.

$$\Psi_{gs} = \sum_{k=0}^{\infty} D_{L_m+2K}(\Omega)\; r^{-\frac{3A-5}{2}}\; u_K(r) \tag{26}$$

where $D_L(\Omega)$ is a normalised H.H. of grand orbital L and $u_K(r)$ the related hyperradial partial wave. If we assume that the first term (K=0) in the H.H. expansion of the potential

$$\sum_{i,j>i} V(r_{ij}) = \sum_{K=0}^{\infty} \mathcal{P}_{2K}(\Omega)\ V_{2K}(r) \qquad r_{ij}=|\vec{x}_i-\vec{x}_j| \tag{27}$$

is largely predominant, the theorem according to which the grand orbital L of the ground state is minimum (L_m) for hypercentral potentials still holds and the first term in (26) is also predominant.

It is this property which justify the use of H.O. wave functions in nuclear calculations because a sum of H.O. Wigner potential is purely hypercentral.

The experimental filling of the shells according to H.O. ground state (corrected by a suitable L.S. interaction) is to some extend a prove of our assumption. We therefore expect that a treatment of the problem in the L_m approximation should give the most important contribution in our expansion. The partial waves $u_K(r)$ in the expansion (26) are solution of an infinite set of coupled differential equations

$$\sum_{K'=0}^{\infty} \langle D_{L_m+2K}(\Omega)|T + \sum_{i,j>i} V(r_{ij})-E|D_{L_m+2K'}(\Omega)\rangle r^{-\frac{3A-5}{2}} u_{K'}(r)=0 \tag{28}$$

We have seen that either for a H.O. interaction or for an infinite hyperspherical well the wave function behaves roughly like a δ function around the point $r_m = \sqrt{2A}$ a where the wave function is maximum.

It seems therefore suitable to use a reduced variable

$$x = r\ A^{-5/6} \tag{29}$$

in such a way that for saturated systems in which the r.m.s. radius grows like $\sqrt{\frac{3}{5}}\, r_0\, A^{1/3}$ the value of x which corresponds to r_m takes the constant value

$$x_m = \sqrt{\frac{6}{5}}\, r_0 \tag{30}$$

With this coordinate the system of coupled equations (28) is transformed into

$$\left\{ -\frac{\hbar^2}{m}\left[A^{-8/3}\frac{d^2}{dx^2} - \frac{(\nu_m+2K)^2}{A\,x^2}\right] + \frac{1}{A}\,U_K^K(x\,A^{5/6}) - \frac{E}{A}\right\} u_K(x) \tag{31}$$

$$+\sum_{K'} \frac{1}{A}\,U_K^{K'}(x\,A^{5/6})\; u_{K'}(x) = 0$$

where $U_K^{K'}(r) = \langle D_{L_m+2K}(\Omega)\,|\,\Sigma V(r_{ij})\,|\,D_{L_m+2K'}(\Omega)\rangle$ is the potential matrix. $\frac{E}{A}$ is the binding energy per particle.

The use of the parabolic approximation [7] is justified for heavy nuclei.[8] The partial waves have a gaussian shape around r_m

$$u_K(r) = \left(\sqrt{\frac{\pi}{2}}\,\rho\right)^{-1/2} e^{-\left(\frac{r-r_m}{\rho}\right)^2} \cdot u_K \tag{32}$$

where u_K is a constant while ρ is related to the monopole excitation energy by

$$E_m = \frac{4\hbar^2}{m\rho^2} \tag{33}$$

Experimentally E_m decreases like $\epsilon_m\, A^{-1/3}$ with $\epsilon_m \simeq 70$ MeV.

Let us put (34) $E_m = \frac{2\hbar^2}{m\rho_0^2}\, A^{-1/3}$, the partial waves become

$$u_k(r) = \left(\sqrt{\frac{\pi}{2}}\rho\right)^{-1/2} e^{-\frac{1}{2}\left(\frac{x-x_m}{\rho_0}\right)^2 A^{4/3}} u_K$$

while the system of coupled equations (31) is transformed into

$$\left\{-\frac{\hbar^2}{m}\left[\left(\frac{x-x_m}{\rho_0}\right)^2 - \frac{(\nu_m+2K)^2}{A\,x^2}\right] + \frac{1}{A}U_K^K(x\,A^{5/6}) - \frac{E}{A}\right\}u_K$$
$$+\frac{1}{A}\sum_{K'\neq K} U_K^{K'}(x\,A^{5/6})u_{K'} = 0 \qquad (36)$$

The solution is obtained in solving the set of linear equations

$$\left[\frac{\hbar^2}{m}\,\frac{(\nu_m+2K)^2}{A\,x^2} + \frac{1}{A}U_K^K(x\,A^{5/6})\right]u_K$$
$$+\frac{1}{A}\sum_{K'\neq K} U_K^{K'}(x\,A^{5/6})\,u_{K'} = W(x)u_K \qquad (37)$$

Around the minimum of W(x) one uses the parabolic approximation

$$W(x) = W_0 + \frac{1}{2}W''(x_m)(x-x_m)^2 \qquad (38)$$

The equation (36) is locally solved with u_K constant in putting

$$\frac{E}{A} = W_0 \qquad \frac{\hbar^2}{m\rho_0^2} = \frac{1}{2}W''(x_m) \qquad (39)$$

This solution exact in the range of validity of the parabolic approximation is practically exact everywhere because $u_K(x)$ becomes infinitely narrow (like a δ function) when $A \to \infty$.

The average kinetic energy is given by

$$\overline{T} = A\,\frac{\hbar^2}{m x_m^2}\sum_{K=0}^{\infty}(\nu_m+2K)^2|u_K|^2 \qquad (40)$$

with $\sum_0^\infty |u_K|^2=1$.

If the convergence is rapid enough the relations

$$\sum_0^\infty K|u_K|^2 = C_1 A^m \qquad \sum_0^\infty K^2|u_K|^2 = C_2 A^n \tag{41}$$

where C_i are constants and $m < \frac{4}{3}$, $n < \frac{8}{3}$ are fulfilled. In this case the kinetic energy per particle is again given by (5). The conditions (41) (with n=m=0) are always fulfilled when the expansion (26) is limited to a finite number of terms.

This expansion is then similar to a limited expansion in terms of H.O. particle-hole states.

Application to zero range interactions

It is especially interesting to investigate our problem with a zero range interaction which is just the opposite of the infinite range H.O. interaction. A solution obtained by Navarro for finite nuclei [9] in the L_m approximation in which the first term of the expansion (26) is kept only has been extended to an infinite number of nucleons [8] for Skyrme [10] and Moskowski [11] potentials.

It has been shown that for these interactions the first diagonal element of the potential matrix is a simple function of x

$$\frac{1}{A} U_0^0(x A^{5/6}) = \frac{C_1}{x^3} + \frac{C_2}{x^5} + \frac{C_3}{x^6} \tag{42}$$

where the constants C_i depend on the potential parameters.

The results obtained for various sets of the Skyrme potentials and Moskowski modified δ interactions are shown in table I.

A comparison with the numbers obtained by Hartree-Fock calculations[12] shows a very good agreement between the Fermi momentum k_F given by Hartree-Fock and the one deduced from the value of x_m using the relation $k_F = \frac{(9\pi)^{1/3}}{2r_0} = \frac{(9\pi)^{1/3}}{2}\sqrt{\frac{6}{5}}\,\frac{1}{x_m}$ according to (30).

Unfortunately, as expected, the agreement is not as good for the binding energy $\frac{E}{A}$. A discrepancy of about 3 MeV which corresponds to 1/10 of the average potential energy is observed.

But we are nevertheless lucky to notice that in spite of our rough approximation the results are not meaningless.

Because in the L_m approximation the expansion is limited only to the first term eq.(22) is still valid and the singular behaviour of $u_0(r)$ around r_m leads again to the density (14).

How the various terms of the H.H. expansion operate to flatten the density of matter has still to be investigated. It is not clear whether a finite number of terms is sufficient to make the density uniform except for a limited surface effect, and whether the ground state of our infinitely heavy nucleus is similar to the one described by the standard nuclear matter.

Conclusion

We have shown that for very particular systems of fermions the uniform nuclear matter does not describe the ground state.

It is not proved that for systems bound by finite range interactions the binding energy per particle is the same for an infinitely heavy nucleus and for nuclear matter. Especially there is an ambiguity about the kinetic energy per particle which should be used.

References

(1) M. Fabre de la Ripelle, Résolution du problème à N corps par le formalisme hypersphérique (La Toussuire 1975) ed. Elbaz.

(2) M. Fabre de la Ripelle, Hyperspherical Harmonics and Shell Model (La Toussuire 1975) ed. Elbaz.

(3) D. Gogny, Nucl. Phys. A237 (1975) 399.

(4) Higher Transcendental functions, H. Bateman T.2, p.199.

(5) M. Fabre de la Ripelle, C.R. Adac. Sc. Paris 270 (1970) 9.

(6) Yu. A. Simonov, Yad. Fiz. 7(1968) 1210.

(7) M. Fabre de la Ripelle, Fizika 6 (1974) 77
M. Sotona and J. Žofka, Phys. Lett. 57B (1975) 27
G.L. Strobel, Nucl. Phys. A271 (1976) 162.

(8) J. Navarro, PH. D. Thesis,University of Valencia (Spain).

(9) J. Navarro, Phys. Lett. 62B (1976) 21.

(10) T.H.R. Skyrme, Nucl. Phys. 9 (1959) 615.

(11) S.A. Moskowski, Phys. Rev. C2 (1970) 402.

(12) D. Vautherin and D.M. Brink, Phys. Rev. C5 (1972) 626.

Potential	E/A H.F	E/A H.H	k_F H.F	k_F H.H	r_0	$\bar{T}/A$
SIII	15.87	12.35	1.29	1.26	1.21	17.4
SIV	16.06	13.03	1.32	1.30	1.17	18.6
MDI 1	15.7	12.54	1.35	1.335	1.14	19.6
MDI 2	16.45	13.25	1.33	1.31	1.16	18.8
MDI 4	16.0	13.56	1.37	1.35	1.13	20.0

Table 1

H.H and H.F refers to hyperspherical harmonics and Hartree-Fock methods. Energies per particle in MeV are given in the two first columns, Fermi momentum in fermi^{-1} in the two next columns, values of r_0 in fm and average kinetic energies per particles in MeV given by H.H method are shown in the last two columns.

Fig. 1