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ABSTRACT

It is shown that by suitably pulsing a magnetic mirror end plug of a linear theta pinch, one can substantially reduce the end loss. A system consisting of a linear pinch with sequentially pulsed multimirror end plugs is proposed which shows promise of fusion feasibility.

It is now generally believed that the achievement of a significant reduction in end losses from a linear theta pinch can make it an attractive fusion device. Efforts are being made to stopper the end of a theta pinch by a single mirror and multiple mirrors among other schemes. The action of a single d.c. mirror is understood in terms of reducing the end loss by flow constriction in a magnetic nozzle. The action of multiple d.c. mirrors can be described in terms of transformation of the simple flow into a diffusive flow. Here we present a new scheme involving pulsed mirrors in which an additional axial plugging force is generated. Let us consider pulsing an end plugging mirror with a time constant considerably longer than the implosion and compression time scales. This will induce an azimuthal current in a resistive skin which will not be very thin due to the long time scale of pulsing, specially when one considers the anomalous resistivity of a typical theta pinch plasma. This diffusing induced current in conjunction with the diffusing radial magnetic field of the mirror will produce an axial force. This force in the half-mirror region closer to the end of the pinch will be in a direction opposite to the flow and will augment the end plugging. It is emphasized that this force is caused by the time varying nature of the magnetic mirror and therefore represents an end plugging effect which is clearly additional to the usual effect of a d.c. mirror.

In order to substantially enhance the end plugging effect and to clearly delineate the end plugging roles of a d.c. mirror and that of this new dynamic end stoppering force due to the pulsed nature of the mirror, we consider the system shown in Fig.1. In this figure we place one pulsed and one d.c. mirror consecutively at the end of a linear theta pinch. For simplicity we consider the plasma flow constriction to be dictated principally by the d.c. mirror, while the pulsed mirror is considered to provide only the additional end stoppering axial force f_z. This simplification clearly errs on the pessimistic side. The d.c. mirror has mirror ratio R_0, while a pulsed parabolic mirror is generated by the following time dependent magnetic field at the plasma surface.

$$B_z(z,r_p,t) = B_0[1 + (R_0-1)(1-e^{-t/\tau})(\frac{2z}{L})^2] ,$$

where $\frac{L}{2}$ is the pulsed mirror cell half length, τ is the time constant of pulsing, r_p is the plasma radius and R_0 is the final mirror ratio. The exact solution of radial magnetic diffusion due to this surface field is very complex and is not necessary for our present purpose. Therefore we consider τ to be a quasi-static time scale (although it is not quite true) and write the one dimensional (x) diffusion solution very approximately as

$$B_z(z,x,t) \simeq B_0(R_0-1)(1-e^{-t/\tau})(\frac{2z}{L})^2(1 - \text{erf}\,\frac{x}{2\sqrt{\kappa\ t}}) + B_0$$

where $x=r_p-r$, r_p is the plasma radius, r the radial coordinate and the diffusivity $\kappa = \frac{1}{\mu_0\ \sigma_{an}}$. For $\frac{x}{2\sqrt{\kappa t}} << 1$, the above expression can be simplified to

$$B_z(z,r,t) \simeq B_0(R_0-1)(1-e^{-t/\tau})(\frac{2z}{L})^2[1-\frac{2}{\sqrt{\pi}}(\frac{r_p-r}{\delta})\sqrt{\frac{\tau}{t}}] + B_0$$

$$\text{for } \frac{r_p-r}{\delta} \leq \frac{\sqrt{\pi}}{2}\sqrt{\frac{t}{\tau}}, \quad t \gtrsim .5\tau$$

$$\simeq 0 \quad \text{otherwise.} \qquad (1)$$

where $\delta \doteq 2\sqrt{\kappa\tau}$ is the skin thickness.

Now the induced current J_θ can be derived from $\nabla \times \overline{H} = \overline{J}$ and Eq.(1) as

$$J_\theta(z,r,t) \simeq \frac{B_0(R_0-1)}{\mu_0\delta} \frac{2}{\sqrt{\pi}} \sqrt{\frac{\tau}{t}} (1 - e^{-t/\tau}) \text{ for } \frac{r_p-r}{\delta} \leq \frac{\sqrt{\sigma}}{2} \sqrt{\frac{t}{\tau}}$$

$$= 0, \text{ otherwise}$$

From $\nabla \cdot \overline{B}=0$ and Eq.(1) one can derive the diffusing radial magnetic field B_r, very approximately as

$$B_r(z,r,t) \simeq -4B_0(R_0-1)\frac{rz}{L^2}(1-e^{-t/\tau})[1 - \frac{2}{\sqrt{\pi}}(\frac{r_p-r}{\delta})\sqrt{\frac{\tau}{t}}] \quad (3)$$

From Eqs.(2) & (3) one can derive the end plugging axial force f_z and the corresponding pressure P_z at the central plane of the mirror as

$$P_z(L/2,r,t) = \int_0^{L/2} f_z dz = -\int_0^{L/2} J_\theta B_r dz = - \frac{B_0{}^2(R_0-1)^2 r}{\mu_0\delta \quad 2\sqrt{\pi}} \sqrt{\frac{\tau}{t}}$$

$$(1 - e^{-t/\tau})^2(1 - \frac{2(r_p-r)}{\sqrt{\pi}\ \delta} \sqrt{\frac{\tau}{t}}$$

The space (r) time (t) average end plugging pressure $\langle P_z \rangle$ is obtained as

$$\langle P_z(L/2)\rangle = [\pi r_p^2 \ (t_{max}-.5\tau)]^{-1} \int_{.5\tau}^{tmax} dt \int_{(r_p - \frac{\sqrt{\pi}}{2}\sqrt{\frac{t}{\tau}}\delta)}^{r_p} 2\pi r dr\ P_z(L/2,r,t)$$

where $t_{max} \simeq (\frac{a}{\delta})^2 \frac{4}{\pi} \tau$.

Performing the above integration we find approximately,

$$\langle P_z(L/2)\rangle \simeq -\frac{B_0{}^2(R_0-1)^2}{2\mu_0}\left[1-\frac{1.5\pi}{4}\left(\frac{\delta}{a}\right)^2\right] \text{ for } \frac{\delta}{a}<1. \quad (4)$$

For numerical estimates we consider fusion type conditions: $T \sim 5$keV, $n \sim 3\times10^{16}$cm^{-3}, $B \sim 100$kG, $L \sim 100$m, $r_p \sim 3$cm. The free streaming 5 keV ion escape time from the pinch is $\sim 1.4\times10^{-4}$ sec. Any end stoppering will prolong this time and the axial end plugging force should have a time constant τ of the order of this reduced time, which we conservatively take as $\tau \sim 3 \times 10^{-4}$ sec. For these parameters, we estimate the anomalous conductivity σ_{an} due to lower hybrid drift instability[1] as

$$\sigma_{an} \sim 10^6\ \Omega^{-1}.\ m.$$

Again for a conservative estimate, we use 4 times this value and find the skin thickness δ as

$$\delta \sim 2\sqrt{\tau/\mu_0\sigma_{an}} \sim 1.5\text{cm} \sim \frac{1}{2}\ r_p. \quad (5)$$

Using this result in Eq.(4), we find

$$\langle P_z(L/2)\rangle \simeq -.7\ \frac{B_0{}^2(R_0-1)^2}{2\ \mu_0} = -.7\ \frac{p_0}{\beta_0}(R_0-1)^2 \quad (6)$$

where p_0 and β_0 are the plasma pressure and beta in the undisturbed central region of the theta pinch. This is clearly a very significant end stoppening pressure for values of $R_0 \sim 2$ to 3.

In order to solve for the end loss problem we use a steady isentropic, ideal MHD parallel flow model[2]. The equations of continuity, isentropy, transverse pressure balance, axial momentum balance and flux conservation are written consecutively as follows.

$$\rho a v = \rho_0 a_0 v_0 = J, \quad p/p_0 = (\rho/\rho_0)^{\gamma}$$

$$p + \frac{B_i^2}{2\mu_0} = \frac{B_e^2}{2\mu_0}, \qquad \rho v\frac{\partial v}{\partial z} + \frac{\partial p}{\partial z} = f_z; \; a=\pi r_p^2, \quad a_0=\pi r_0^2 \tag{7}$$

$$aB_i = a_0 B_{i_0}; \qquad (A-a)B_e = (A_0 - a_0)B_{e_0}$$

where p, ρ, a, A, v, J, B are the pressure, density, plasma area, coil area, flow velocity, efflux and magnetic field respectively. The subscript 0 indicates an undisturbed state in the central region of the pinch, the subscripts e and i indicate external and internal relative to the plasma, respectively. The mirror ratio is $R_0=A_0/A_{min}$. The only difference in the above system of equation from Ref.2 is the appearance of the new axial force f_z. Integrating the axial momentum equation and using other equations in Eq.(7) one can find approximately for a << A,

$$\frac{1}{1-B_0}\left(\frac{A_0}{A}\right)^2 = \frac{\beta_0}{1-\beta_0}\left(\frac{p}{p_0}\right) + \left[1 + \frac{2\gamma p_0}{\rho_0 v_0^2(\gamma-1)}\right]\left(\frac{p}{p_0}\right)^{2/\gamma}$$

$$- \frac{2\gamma p_0}{\rho_0 v_0^2(\gamma-1)}\left(\frac{p}{p_0}\right)^{(\gamma+1)/\gamma} - \langle P_z(z)\rangle\left(\frac{\gamma-1}{\gamma}\right)\frac{\gamma}{\rho_0 v_0^2(\gamma-1)}\left(\frac{p}{p_0}\right)^{2/\gamma} \tag{8}$$

The throat condition[2] is written as

$$\frac{1}{2}\frac{B_e^2}{\mu_0 p} = \frac{\gamma \rho v^2}{2(\gamma p - \rho v^2)} \tag{9}$$

With the help of Eqs.(7) and γ=2, this can be reduced to

$$\frac{J^2}{\rho_0^2 a_0^2 c_0^2} = \frac{(1-\beta_0)(p/p_0)^{3/2}}{R_0^2}: \qquad c_0^2 = \gamma p_0/\rho_0 \tag{10}$$

Eq.(8) at the throat for $\gamma=2$ can be written as

$$R_0^2 (p/p_0)^{1/2} = (p/p_0)^{3/2} + 2R_0^2 [1-(p/p_0)^{1/2}] - \frac{1}{2} \frac{\langle P_z(L/2)\rangle R_0^{3/2}}{\rho_0} \tag{11}$$

Simultaneous solution of Eqs.(10) and (11) yield

$$\frac{J^2}{\rho_0^2 a_0^2 c_0^2} = 8R_0(1-\beta_0)\cos^3\{\frac{1}{3}\cos^{-1}\{\frac{\sqrt{1+\langle P_z(L/2)\rangle/4p_0}}{R_0}\} + \frac{\pi}{3}]$$

Substituting the value of $\langle P_z(L/2)\rangle$ from Eq.(6) in this equation and approximating for large R_0, we can find

$$J/\rho_0 a_0 c_0 \approx .54[1 - .175(R_0-1)^2/\beta_0]^{3/4} \sqrt{1-\beta_0}/R_0 \quad . \tag{12}$$

The second term in the square braces is due to pulsing and represents significant reduction in the end loss for values of $R_0 \sim 2$ to 3.

It is noted that the axial plugging force due to pulsing lasts only for a time of the order of skin penetration time which in our case is $\sim 10^{-3}$ sec. Therefore in order to have an effective system with considerably longer confinement time, one should consider a series of sequentially pulsed multiple mirrors at either end of the pinch. Ofcourse it will be necessary to superpose linked quadrupoles on the multiple mirrors in order to achieve stabilization in a minimum-B configuration. We propose a scheme whereby alternate mirrors are pulsed sequentially, leaving the other mirrors at d.c. excitation. The time interval T of sequential pulsing can be chosen as the skin penetration time so that $T \sim 4\tau$. Furthermore, we propose a cyclic pulsing schedule, where after sequential pulsing, the pulsed mirrors are returned to the R=1 state. This switching should be done with a longer time constant of the order of 10τ in order to minimize the generation of reversed axial forces. For example, one can visualize a system consisting of 100m linear pinch section with 10 mirrors at each end. The mirror cell length can be chosen to be in 3 to 4 m range, so that the length between alternate d.c. mirrors is roughly equal to the mean free path in our case. Then the following sequential and cyclic pulsing and switching off schedule for the mirrors is proposed. Here i refers to the ith mirror, the subscript n indicates the cycle number and one complete cycle time is 10T.

At t=0, i=even,off; i=odd,on.

For $t \geq 0$: i=even, pulsed at $t_n^{on} = [(\frac{i}{2} - 1) + 10(n-1)]T$

switched off at $t_n^{off} = [\frac{i}{2} + 4) + 10(n-1)]T$

i=odd, pulsed at $t_n^{on} = [\frac{i-1}{2} + 10(n-1)]T$

switched off at $t_n^{off} = [\frac{i+9}{2} + 10(n-1)]T$

We choose the number of cycles n=5 which will yield a total effective time for axial plugging force of $50T \sim 200\tau \sim .06$sec. Using the result of Eq. (12) we estimate the confinement time τ_{conf} as

$$\tau_{conf} \sim .064 \text{ sec for } R_0 \sim 3.25,\ \beta_0 \sim .9,\ T \sim 5\text{KeV},\ n \sim 3\times10^{16}\ \text{cm}^{-3}.$$

The number of cycles was chosen above to give a total effective time for axial plugging force which matches the confinement time. The corresponding $n\tau_{conf} \sim 1.9 \times 10^{15}$ which indicates fusion feasibility.

Throughout the operation of the system one has an effective d.c. multimirror end plug in addition to sustained effective axial plugging force due to pulsing of alternate mirrors. It has been pointed out[3,4] that the d.c. multimirror will result in a diffusion loss process where the mean freepath is longer than the mirror field scale length, but is of the order of mirror cell length, which approximately holds in our case. The corresponding confinement time scales as square of the system length, instead of linearly in the MHD flow case which has been used here. Therefore one can expect even substantially better confinement than indicated in the above MHD flow analysis. In fact this is part of the motivation for the scheme of alternate d.c. and pulsed mirror outlined above.

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CAPTION OF FIGURE

Fig.1 A Schematic of Pulsed Mirror End Plug. Only one pair of pulsed and d.c. mirrors is shown. The wavy lines with arrows indicate magnetic field lines. The radius of the cylindrical plasma under the pulsed mirror and in the undisturbed central region are indicated by r_p and r_0, respectively.

REFERENCES

R.C.Davidson and N.T.Gladd, Phys. Flds. 18, 1327 , (1975).

J.B. Taylor and J.A. Wesson, Nucl. Fusion, 5, 159, (1965).

B.G. Logan, A.J.Lichtenberg, M.A.Lieberman and A.Makhijani, Phys. Rev. Lett. 28, 144 (1972).

A. Makhijani, A.J.Lichtenberg, M.A.Lieberman, B.Grant Logan, Phys. Fluids. 17, 1291, (1974).

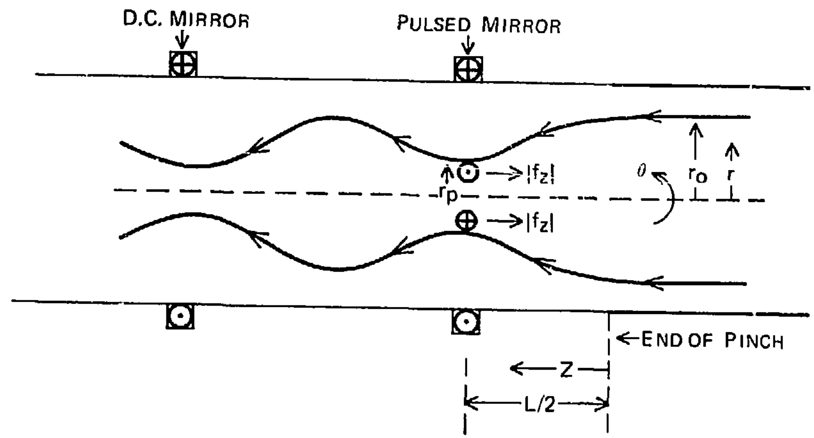

Fig. 1