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Field Reconstruction for the KEK Large-Aperture-Spectrometer-Magnet " TOKIWA "

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Abstract

Field reconstruction has been performed for the KEK large-aperture-magnet "TOKIWA". The magnetic field components are determined point-by-point by an iteration method in which the output voltage from the Hall probes placed in three dimensional directions are used simultaneously. The field components are thus reconstructed accurately within 32 G everywhere in the magnet volume.

1. Introduction

Recent experiments in high energy physics frequently require large-aperture-spectrometer magnets. Since the field of these magnets is in general strongly non-uniform, we have to determine the distribution of field strength on a three dimensional grid quite densely.

For the large spectrometer magnets BENKEI and TOKIWA at KEK, a computer controlled mapping system with Hall plate assemblies was constructed, having the following features;

1) complete automation with short measuring time,

2) high accuracy in spite of the simplicity, and

3) ability to measure non-uniform field.

These features were realized with our carefully designed on-line and mechanical systems described in references 1 and 2.

We used several Hall plates to measure the three-dimensional field components simultaneously. Since the magnetic field distribution we are handling is highly non-uniform, the normal Hall voltages are affected by tangential field components, which we can not neglect to obtain the precise distribution of field value. Therefore, one applies the following procedure to get rid of this difficulty[3]:

1) Three-dimensional calibration of a Hall plate.

2) One-dimensional field measurement. (The vertical field is scanned only in a plane parallel to the median.)

3) Reconstruction of the preliminary vertical component by using the output voltage.

4) Introduction of magnetic scalar potentials by using the

vertical component.

5) To obtain a solution of a static field equation (Laplace eq.) when the correction for tangential field components taken into consideration.

In this case, however, the field reconstruction is rather complicated because of cumbersome numerical treatment of the field equation. It can be considerably simplified, if the three-dimensional measurement is performed. Therefore, we have adopted a following method:

1) To make three-dimensional field calibration of a Hall plate.
2) To carry out three-dimensional field measurements. (The Hall plates are set in three mutually perpendicular planes.)
3) Field reconstruction performed only by using the measured three output voltages from the Hall plates without a help of static magnetic field equation.

This method simplifies the field reconstruction to a considerable extent avoiding the field equation.

2. Genaral principle

When a Hall plate is placed in a magnetic field (Fig. 1), an electric field appears in a direction perpendicular to the supplied current flow. The output voltage of the Hall plate is represented by

$$V = V_o + R\, B_y\, I - P\, B_T^2\, I \sin(2\phi), \tag{1}$$

V_o : residual voltage without field,

R : Hall coefficient,

I : Hall current,

P : planar Hall coefficient.

The second term of this equation represents the normal Hall effect and the third one from the planar Hall effect due to the tangential field component[4]. This planar Hall effect term cannot be neglected in high-accuracy three-dimensional reconstruction which we intend to do. The expression (1) is slightly modified for a practical use:

$$V = V_o + R\,B\,I\cos\theta - R\,B\,I\,\varepsilon\sin\theta\sin(\phi + \phi_1) + P\,B^2\,I\sin^2\theta\sin(2\phi + \phi_2), \qquad (2)$$

where ε, ϕ_1 and ϕ_2 are the angular misalignments of the Hall plates. In addition to the normal Hall constant R, we must determine the constants P, ϕ_1, ϕ_2 and ε in order to correct for the planar Hall effect. Once all of these constants are determined, the observed Hall voltage becomes a function of only B, θ and ϕ ;

$$V = H_o(B, \theta, \phi,) .$$

Setting the Hall plates having a common center point in three mutually perpendicular planes, we get three output voltages from Hall plates and, therefore, three independent equations, from which we can easily reconstruct the three-dimensional field components B_x, B_y and B_z.

3. Determination of calibration constants

a) Normal Hall constant

The calibration of the normal Hall constant was performed θ = 0.0 ± 0.5 mrad in a uniform field. In this case, eq. (2) becomes

$$V = V_o + R\,B\,I, \qquad (3)$$

which depends only on B. The V-B relation was measured in an interval from 1.0 to 13.5 kG with a step of 200 G.

The magnetic field was determined to an accuracy of 0.2 G using an NMR probe. The dependence of the Hall voltage on the field was fitted by a seventh-order polynomial:

$$V = f_o(B) = \sum_{i=0}^{7} a_i B^i, \tag{4. 1}$$

which is shown in Fig. 2. The reverse relation is also necessary for the field reconstruction:

$$B = g_o(V) = \sum_{i=0}^{7} b_i V^i. \tag{4. 2}$$

The results calculated from (4. 2) reproduce the observed ones within an accuracy of 0.5 G.

b) Planar Hall constant

The calibration of the planar Hall constant was performed at $\theta = 90°$ within 0.5 mrad. Under this condition, eq. (2) is reduced to

$$V = K_o - K_1 B \sin(\phi + \phi_1) - K_2 B^2 \sin(2\phi + \phi_2), \tag{5}$$

where

$$K_o = V_o \tag{5. 1}$$

$$K_1 = R I \varepsilon \tag{5, 2}$$

$$K_2 = P I. \tag{5, 3}$$

In a uniform field, the voltage depends only on ϕ. If the coefficients and the phases in eq. (5) are determined, therefore, the planar Hall constant P and ε can be obtained from the equation (5. 2) and (5. 3).

In order to determine these coefficients, a special device was built to vary the angles ϕ within an accuracy of 1.0 mrad[2].

The measurement was performed in an interval from 0.0° to 360.0° with a step of 5.0° at a constant field of 13.5 kG.

The results are shown in Fig. 3. The coefficients and phases were determined by using a Fourier analysis method where the harmonics higher than three were neglected. The planar Hall constant P was found to be slightly depending on B. The resulting accuracy in terms of field value is better than 3.0 G over the range of the field.

4. Field reconstruction

After the calibration, the Hall plate was set on a mapping machine and measurements were performed point by point. The three output voltages of the Hall plate at a given spatial point are expressed as

$$V_x = V_o + R\,B\,I\cos(\theta_x) - R\,B\,I\,\varepsilon\sin(\theta_x)\sin(\phi_x + \phi_1) - P\,B^2\,I\sin^2(\theta_x)\sin(2\phi_x + \phi_2) \tag{6. 1}$$

$$V_y = V_o + R\,B\,I\cos(\theta_y) - R\,B\,I\,\varepsilon\sin(\theta_y)\sin(\phi_y + \phi_1) - P\,B^2\,I\sin^2(\theta_y)\sin(2\phi_y + \phi_2) \tag{6. 2}$$

$$V_z = V_o + R\,B\,I\cos(\theta_z) - R\,B\,I\,\varepsilon\sin(\theta_z)\sin(\phi_z + \phi_1) - P\,B^2\,I\sin^2(\theta_z)\sin(2\phi_z + \phi_2)\,, \tag{6. 3}$$

where (θ_x, ϕ_x), (θ_y, ϕ_y) and (θ_z, ϕ_z) represent the field direction on the coordinate when the Hall plate is placed on y-z, z-x and x-y plane respectively. From equations (6. 1, 2, 3) with the three measured voltages V_x, V_y and V_z, we can reconstruct B, θ and ϕ. For this computation, an iteration method was used. The iteration was performed as follows: At first, we obtain preliminary values for B_x, B_y and B_z using eq. (4. 2), under the hypothesis that V_x,

V_y and V_z depend only on the normal Hall effect. After the coordinate transformation from (B_x, B_y, B_z) to B, (θ_x, ϕ_x), (θ_y, ϕ_y) and (θ_z, ϕ_z), the field values and the direction are put into the equations (6. 1), (6. 2) and (6, 3) and voltages are calculated using eq. (4. 1). The raw data V_x, V_y and V_z differ from the calculated voltages because of the tangential field effect. At this point, however, the effect can be numerically calculated through the third and forth terms of the equations (6. 1), (6. 2) and (6. 3). After the numerical correction, we can get more accurate values for the normal Hall voltages. Using these new voltages, better values for B_x, B_y and B_z are calculated. This process is repeated until the calculated voltages converge to the measured value.

The iteration was done point by point at all of the measured points. Typically, 5th iteration satisfied the convergence condition for individual points. An average time for the computation was about 4 msec per point.

5. Precision of the result

a) Errors in the calibration

As already described in 3), the error in the normal Hall calibration is ± 0.5 G, while it is ± 3.0 G for the planar Hall calibration.

b) Errors due to mechanical accuracy

There are two sources of mechanical errors; position and orientation of the Hall plate.

The positioning accuracy of the Hall center is within ± 0.24 mm. Therefore, for a maximum field gradient of 700 G/cm, the error in the field reconstruction is 15.4 G.

The accuracy of orientation when plates are set on the machine is within ± 1.0 mrad. It causes an error less than 12.8 G even at the maximum field of 12.8 kG.

The instability of the well controlled temperature of the Hall plates and the electronics system produces errors far less than 1.0 G [2]. Therefore, we can neglect them comparing with the errors due to the mechanical accuracy.

If we consider the maxmum value of the combined effect of all these errors, the accuracy of the field reconstruction is better than 31.7 G.

To check the self-consistency of the results, field distributions B_x, B_y and B_z were determined by solving the Laplace equation under the boundary conditions that the potentials, which were calculated using measured B_y components, were fixed on the su ce of the effective volume of the magnet. Field distributions thus obtained have a good agreement with measured ones within the errors discussed above.

6. Conclusion

For the magnet TOKIWA, the corrections due to the tangential field component are from 30 to 170 G in the 10% region of the whole magnet volume. By a simultaneous measurement of three field components and a precise calibration of normal and planar Hall coefficients, the use of the static magnetic equation was avoided, allowing a simple and reliable correction for the effect of the tangential component. The resulting field values are, therefore, accurate within 32 G even in a space where the field gradient has the largest value. The result has shown that the precise mechanical and the versatile on-line control systems permitting efficient and accurate mapping of non-uniform field were successfully constructed, and that, as a result of this, the magnetic field values reconstructed by a simple iteration method were accurate enough for most of the conceivable experiments.

Typical field distribution thus obtained are shown in Figs. 4, 5 and 6. The results of the three-dimensional field reconstruction (field components at each position) are available as the facility data stored on a magnetic tape.

Acknowledgements

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2) K. Amako et al., KEK-EXP. FACILITIES-78-8

3) F. Schaff, CERN ISR-MA/74-48

4) C. Goldberg and R. E. Davis, Phys. Rev. Vol.94, No.5 (54)1121.

Figure caption

Fig. 1. : Hall plate coordinate system.

Fig. 2. : Hall output voltages due to normal Hall effect.

Fig. 3. : Hall output voltages due to planar Hall effect.

Fig. 4. : Two-dimensional view of the field.

Fig. 5. : Three-dimensional view of the field on the median plane.

Fig. 6. : Three dimensional view of the field near the pole piece.

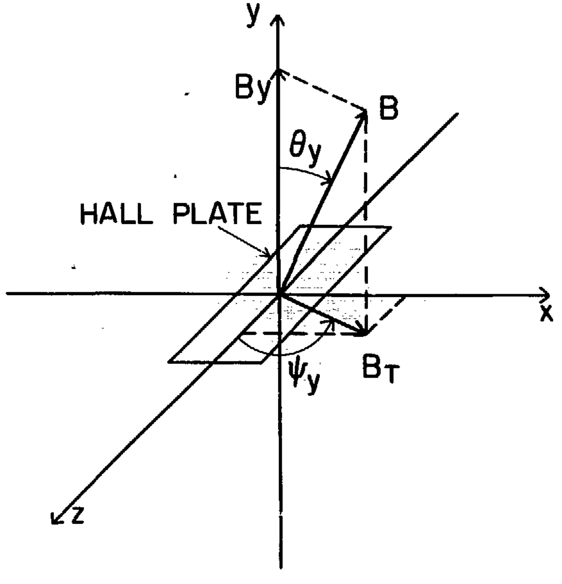

Fig . 1

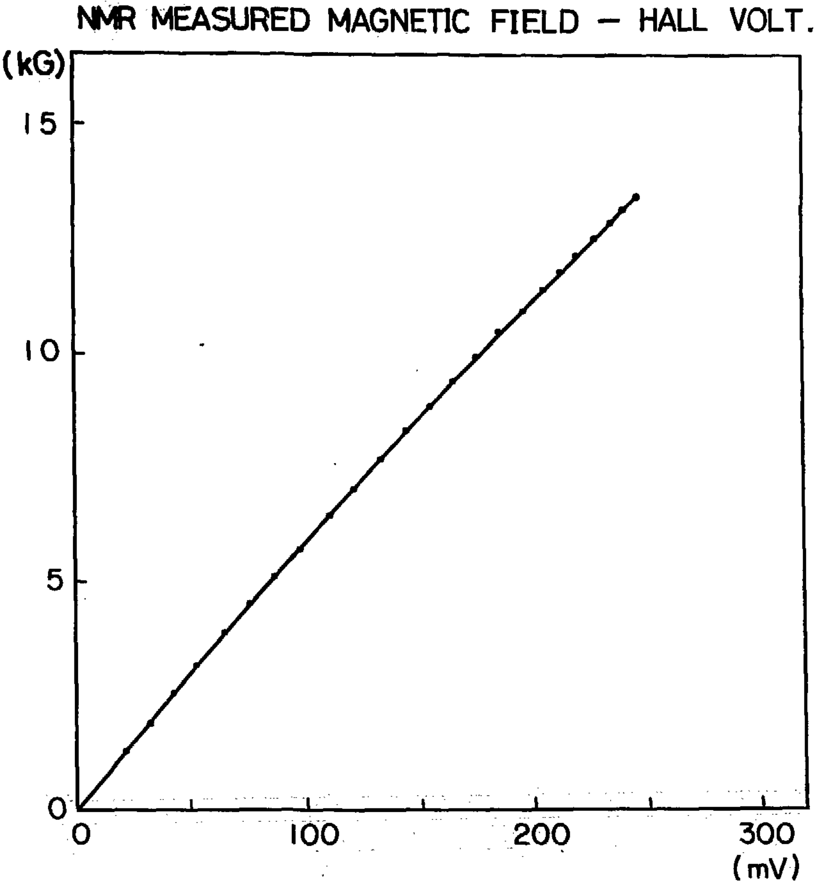
NMR MEASURED MAGNETIC FIELD — HALL VOLT.
(kG)
15
10
5
0
0
100
200
300
(mV)

Fig. 2

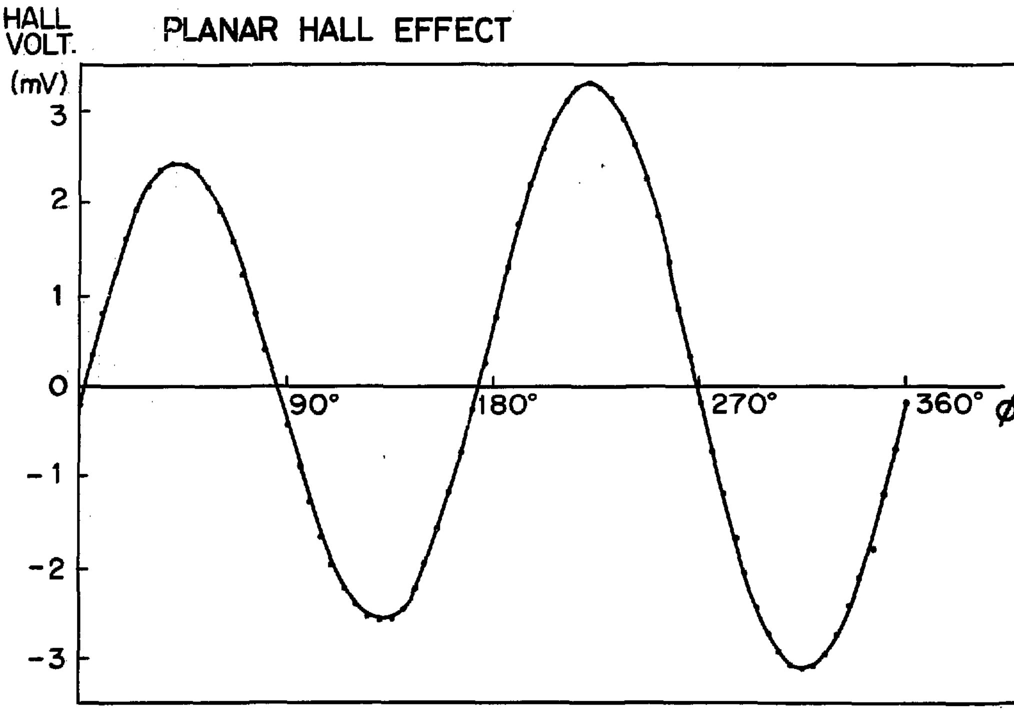

Fig . 3

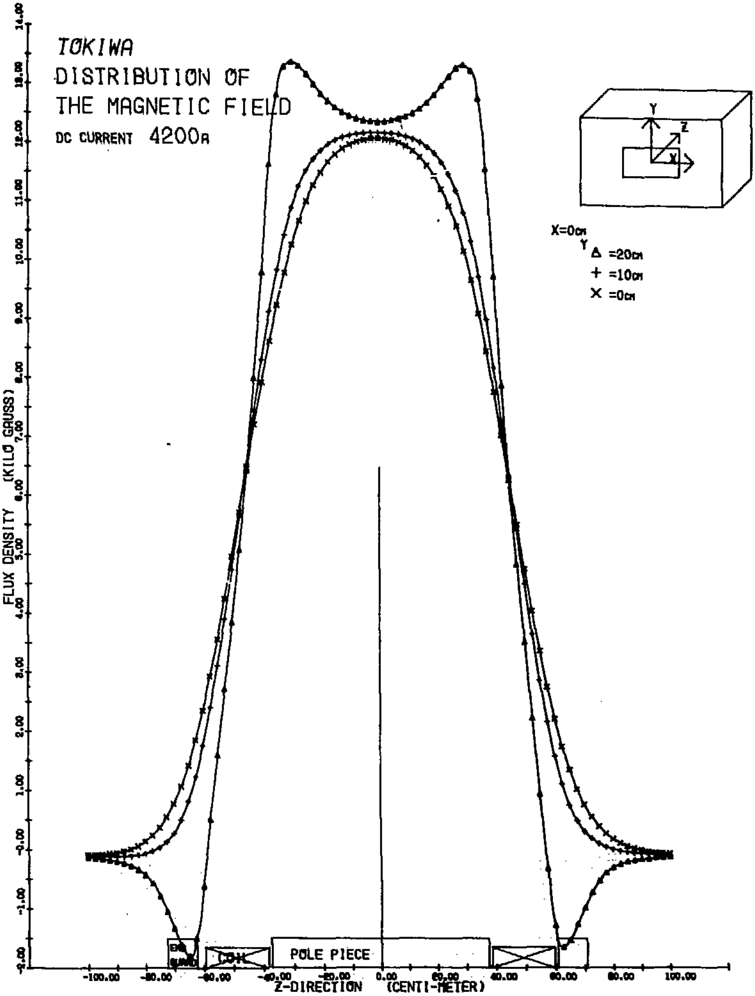
TOKIWA
DISTRIBUTION OF
THE MAGNETIC FIELD
DC CURRENT 4200A
X=0cm
Y
△ =20cm
+ =10cm
× =0cm
Y
Z
X
FLUX DENSITY (KILO GAUSS)
Z-DIRECTION (CENTI-METER)
POLE PIECE
COIL
-100.00
-80.00
-60.00
-40.00
-20.00
0.00
20.00
40.00
60.00
80.00
100.00

Fig. 4

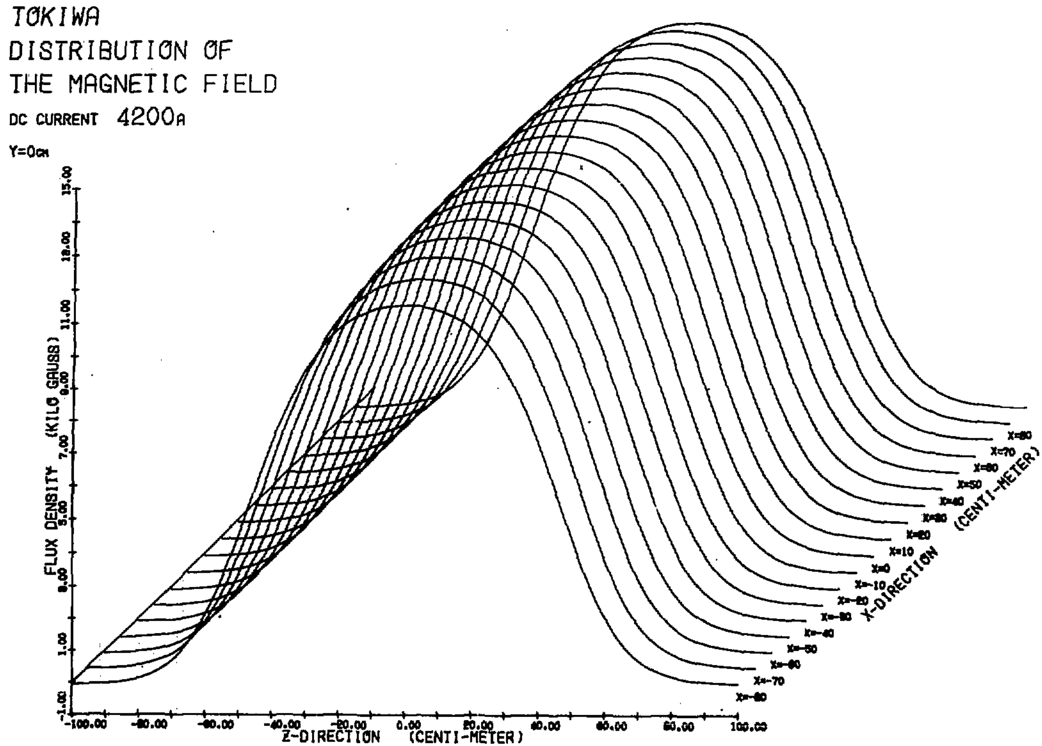
TOKIWA
DISTRIBUTION OF
THE MAGNETIC FIELD
DC CURRENT 4200A
Y=0cm
FLUX DENSITY (KILO GAUSS)
15.00
13.00
11.00
9.00
7.00
5.00
3.00
1.00
-1.00
-100.00
-80.00
-60.00
-40.00
-20.00
0.00
20.00
40.00
60.00
80.00
100.00
Z-DIRECTION (CENTI-METER)
X=80
X=70
X=60
X=50
X=40
X=30
X=20
X=10
X=0
X=-10
X=-20
X=-30
X=-40
X=-50
X=-60
X=-70
X=-80
X-DIRECTION (CENTI-METER)

Fig. 5

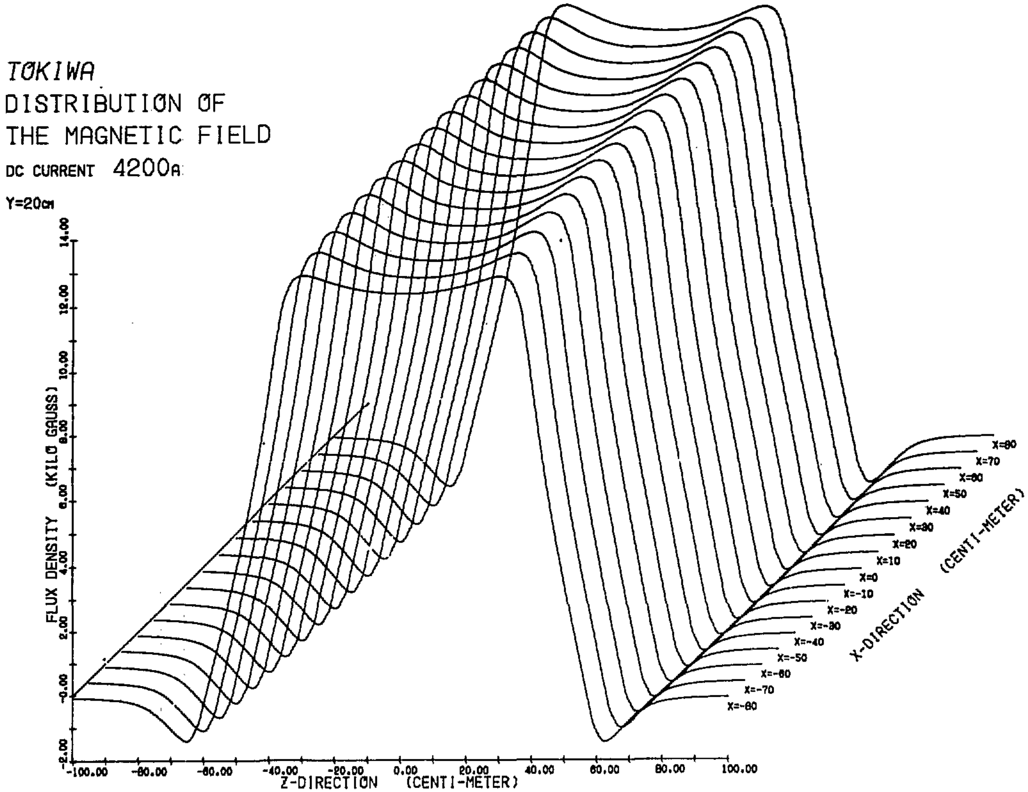
TOKIWA
DISTRIBUTION OF
THE MAGNETIC FIELD
DC CURRENT 4200A
Y=20cm
FLUX DENSITY (KILO GAUSS)
-2.00
-0.00
2.00
4.00
6.00
8.00
10.00
12.00
14.00
-100.00
-80.00
-60.00
-40.00
-20.00
0.00
20.00
40.00
60.00
80.00
100.00
Z-DIRECTION (CENTI-METER)
X=80
X=70
X=60
X=50
X=40
X=30
X=20
X=10
X=0
X=-10
X=-20
X=-30
X=-40
X=-50
X=-60
X=-70
X=-80
X-DIRECTION (CENTI-METER)

Fig.6