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THE UNIVERSAL MODE REVISITED

BY

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The Universal Mode Revisited

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ABSTRACT

Our present understanding of the theory of universal eigenmodes in a collisionless plasma slab with magnetic shear, is reviewed and critically examined.

The theory of universal drift waves in a plasma slab with magnetic shear has recently attracted renewed attention.[1-5] Numerical and analytical work has led to the surprising discovery that drift wave eigenmodes in a collisionless plasma with Maxwellian electrons are always stable.[2,3] Similar conclusions have been obtained for collisional plasmas.[4] Numerical work on the current-driven problem has shown that a drifted Maxwellian electron distribution function can drive the collisionless drift eigenmode unstable, provided the parallel electron current is sufficiently strong.[5]

In this note we reiterate that for monotonic electron distributions, the stability of the universal eigenmode in a plasma slab with magnetic shear is strictly tied to the assumed even symmetry $[f_o(-v_{||}) = f_o(v_{||})]$ of the function. Departures from even symmetry may lead to instability, irrespective of whether the distribution carries any net current or not. This reflects the fact that in the eigenmode problem there is a delicate competition between the growth effects due to locally resonant electrons and the shear damping effects due to the global mode structure and that this balance is determined not by the first velocity moment of f_o but by a more complex velocity weighting. It appears then, that a detailed knowledge of the shape of $f_o(v_{||})$ may be crucial in determining stability of the universal eigenmode. Parenthetically, we note here that it is quite likely that the electron distribution function in a tokamak is far from a simple drifting Maxwellian (cf., runaway effects due to $E_{||}$,[6] interesting effects due to heat flow along stochastic field lines[7] etc., and some experimental evidence in this direction[8]).

We consider the simplest model of drift waves in a sheared magnetic field. The equilibrium magnetic field is given by $\underset{\sim}{B} = B_0(\hat{e}_z + x/L_s\ \hat{e}_y)$, the ions are treated as a cold species and the electrons are described by the drift kinetic equation. The ion density fluctuation (obtained from equations of continuity and motion for ions) is given by

$$\frac{\tilde{n}_i}{n_o} = \rho_s^2\ \nabla^2\left(\frac{e\phi}{T}\right) + \left(\frac{\omega_*}{\omega} + \frac{k_{||}^2 c_s^2}{\omega^2}\right)\frac{e\phi}{T} \tag{1}$$

where $\rho_s^2 = T/M\omega_{ci}^2$, $\omega_* = cT/eBL_n$, $c_s^2 = T/M$, we have assumed $\exp(ik_y y - i\omega t)$ dependence and $k_{||}(x) = k_y x/L_s$. The first term in Eq. (1) comes from divergence of ion polarization drift, the second term comes from $E \times B$ motion of ions along the density gradient, and the last term comes from the parallel ion motion. The electron response, obtained from the drift kinetic equation, is given by

$$\frac{\tilde{n}_e}{n_o} = \int dv_{||}\ f = \left(\frac{\omega_*}{\omega} + \int dv_{||}\ \frac{[(\omega_*/\omega)v_{||}\ f_o + (T/m)(\partial f_o/\partial v_{||})]}{(\omega/k_{||}) - v_{||}}\right)\frac{e\phi}{T}\ . \tag{2}$$

Using quasineutrality, we get the mode equation

$$\nabla^2\phi + \int dv_{||}\frac{(1/\omega)v_{||}f_o + (1/2)(\partial f_o/\partial v_{||})}{v_{||} - x_e/x}\ \phi + \frac{x^2}{x_s^2}\ \phi = 0 \tag{3}$$

where x is normalized to ρ_s, ω to ω_*, ϕ to T/e, $v_{||}$ to $(2T/m)^{1/2}$, $x_e = (\omega/\omega_*)(m/2M)^{1/2}(L_s/L_n)$ and $x_s = (\omega/\omega_*)(L_s/L_n)$. Together with the boundary conditions at $\pm\infty$ (outward going waves for growing modes), this equation defines the eigenvalue problem. For a Maxwellian f_o, it reduces to the standard form.[1]

We first demonstrate that the stability of drift wave eigenmodes is only determined by the even symmetry of $f_o(v_{||})$. Following a procedure due to Antonsen,[3] we introduce a variable η by the equation $x = -i\omega\eta$. Changing from x to η in (3), multiplying by ϕ^*, integrating from $-\infty$ to $+\infty$, separating the real and imaginary parts and manipulating them, we get the equation

$$\int d\eta \left[\frac{1}{|\omega|^2}\left|\frac{\partial\phi}{\partial\eta}\right|^2 + \left(k^2 + \frac{\omega^2}{x_s^2}\right)\eta^2|\phi|^2 - |\phi|^2 \int dv_{||} \frac{(v_{||}/2)(\partial f_o/\partial v_{||})}{v_{||}^2 + (x_e^2/\omega^2\eta^2)}\right]$$

$$- \frac{\Omega}{2\gamma} \int d\eta\, |\phi|^2 \int dv_{||} \frac{(x_e/\omega\eta)[(\partial f_o/\partial v_{||}) + (2v_{||}f_o/\Omega)]}{v_{||}^2 + (x_e^2/\omega^2\eta^2)} = 0$$

where $\omega = \Omega + i\gamma$, and it is to be noted that ω/x_e and ω/x_s are independent of ω.

If $f_o(v_{||}) = f_o(-v_{||})$, the last term vanishes. Furthermore, for monotonically decreasing $f_o(v_{||})$, all the other terms are positive definite and cannot add up to zero. This is the Antonsen proof of eigenmode stability.[3] If $f_o(v_{||}) \neq f_o(-v_{||})$, the last term is finite and of indifferent sign and the stability proof breaks down. Thus, for monotonic functions, the absolute stability of drift waves is only dependent on the even symmetry of $f_o(v_{||})$ and has nothing to do with its detailed functional form.

Numerical investigation of Eq. (3) using the shooting and WKB methods has established[2] that for Maxwellian electrons, the eigenmodes are always stable ($\mathrm{Im}\,\omega < 0$). We have recently confirmed[9] that this is indeed true for any assumed even and monotonic form of $f_o(v_{||})$. Furthermore, we have found that for $M_{||} \to \infty$ (i.e., on ignoring parallel ion response), the eigenmodes are always marginally stable ($\mathrm{Im}\,\omega = 0$); this latter result is independent of $k\rho_s$, L_s/L_n and form of shear function and only depends on evenness of $f_o(v_{||})$. Here then is the provocative mystery stripped to its barest essentials. Why, for even $f_o(v_{||})$, are there exact concellations taking place in this model? We still do not know the answer and would like to deepen the mystery by a physical description of the cancellations.

We multiply Eq. (3) with $(n_o e^2/T)\omega\phi^*$ integrate in x from -L to L (where L is some large distance) and take the real part of the resulting equation; this leads us to the following equation:

$$2\gamma \int_{-L}^{L} dx \left[\frac{\omega_{pi}^2}{\omega_{ci}^2}\left(\frac{1}{8\pi}\left|\frac{\partial\phi}{\partial x}\right|^2 + \frac{k^2|\phi|^2}{8\pi}\right) + \frac{\omega_{pi}^2}{|\omega|^2}\,\frac{k_{||}^2|\phi|^2}{8\pi}\right] - \frac{\omega_{pi}^2}{\omega_{ci}^2}\,\mathrm{Im}\left(\omega\phi^*\,\frac{\partial\phi}{\partial x}\right)\Bigg|_{-L}^{L}$$

$$= \frac{n_o e^2}{T}\,\mathrm{Im}\int_{-L}^{L} dx\,|\phi|^2 \int_{-\infty}^{\infty} dv_{||}\,\frac{v_{||} f_o \omega_* + (\omega T/m)(\partial f_o/\partial v_{||})}{v_{||} - (\omega/k_{||})}\,. \qquad (4)$$

we can readily interpret the various terms in Eq. (4). The terms proportional to γ (viz., the first three terms on the left side and a portion of the right side) can be interpreted as the rate of change of the mode energy; mode energy is dominantly in the form of kinetic energy of perpendicular and parallel ion sloshing and the parallel electron sloshing (viz., nonresonant electron effects). The fourth term on the left side can be interpreted as the Poynting flux of drift waves out of the ends $\pm L$. [To verify this, note that the x-Poynting flux $\equiv \mathrm{Re}(c/4\pi) E^*_{||} B_y$ and $B_y = (i/k_{||})(4\pi/c)J_x = -(4\pi NMc^2/B^2)(\omega/ck_{||})(\partial\phi/\partial x)$ etc.] This term is responsible for the shear damping of drift waves. The remaining term (viz., the leftover portion of the right side) may, using the drift kinetic equation, be shown to be the $\mathrm{Re}(J_{||}E^*_{||})$ work done by resonant electrons on the wave. This is the term which could drive the wave unstable. Equation (4) is thus a power balance equation. The rate of J·E work done by resonant electrons is balanced by the rate of growth of mode energy minus the energy flux out of the ends. The real mystery of the recent drift wave work is that for even $f_o(v_{||})$, the frequency $\mathrm{Re}\,\omega$ and mode-structure adjust in such a way that for large $k\rho_s$, there is a near exact cancellation[10] of the J·E work done by resonant electrons and the energy flux out of the ends; the cancellation is <u>exact</u> for the model $(M_{||} \rightarrow \infty)$ in which outward energy flux and shear damping is dominated by nonresonant electron effects. We do not see any obvious physical reason for this cancellation. So the mystery remains: Is there anything deeper in this exact cancellation for symmetric f_o ?!

Once it is appreciated that the stability of drift wave eigenmode is tied to the even symmetry of $f_0(v_{||})$, the next obvious task is to investigate stability for more general electron distributions. One such effort is the calculation for a drifted Maxwellian[5]; this shows that the instability is recovered when the current exceeds a critical value. In the discussion given below, we present the results of a numerical investigation (using WKB and shooting methods) of the eigenmode stability for a number of nonsymmetric electron distributions.[9] Our first choice is the model

$$f_+ = (C_1/v_e)/[1 + (v_{||}/v_e)^2]^2 \quad v_{||} > 0$$

$$f_- = (C_1/v_e)/[1 + (v_{||}R/v_e)^2]^N \quad v_{||} < 0 \qquad (5)$$

with $N \geq 2$ and C_1 determined by the normalization condition $\int f\, dv_{||} = 1$. R controls the width of the distribution for $v_{||} < 0$. For $R = R_c = 1/(N - 1)^{1/2}$, the distribution carries no net current; any other choice of R gives a net parallel electron current. This model may be treated as a simple analytical approximation to the runaway distribution function in a plasma with parallel electric field. Both the WKB and shooting methods show that such nonsymmetric distributions give unstable eigenmodes. Table I presents the results of γ versus $k^2\rho_s^2$ obtained from the shooting code for $N = 8$, $R = 1/(7)^{1/2}$, $L_s/L_n = 10^2$ and $(M/m) = 1837$. Significant growth rates are observed. The growth rate does not change very much when R is made slightly different from R_c so that a net current flows. Thus, it is the asymmetry of f_o rather than the current carried by it which determines the growth rate. We have also considered the effect of peaking the distribution function away from $v_{||} = 0$. This is accomplished by replacing $v_{||}$ with $(v_{||} - v_o)$ in Eq. (5). R and v_o may again be adjusted so that no net current flows; this model could correspond to a physical situation where parallel temperature gradients and electric fields distort f_o to give a large heat flow but not net current. Table II shows the variation of γ/ω_* vs v_o/v_e. The growth rate can be substantially increased by peaking the distribution away from $v_{||} = 0$.

So what do we conclude? Stated briefly, we conclude that stability of the universal eigenmode is tied to symmetries of $f_o(v_{||})$. For nonsymmetric f_o, unstable modes can be obtained with a growth rate determined by the detailed shape of f_o. Finally, we believe that from a physical point of view, it is still a mystery why for symmetric f_o, the local growth terms are always overpowered by the shear damping terms.

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TABLE I

$L_s/L_N = 100$, $m/M = 1/1837$, $v_o/v_e = 0$, $R = 1/(7)^{1/2}$

$k_y^2\rho_s^2$	0.01	0.05	0.1	0.5	1
γ/ω_*	0.014	0.031	0.041	0.014	0.009

TABLE II

$L_s/L_N = 100$, $m/M = 1/1837$, $J_{||} = 0$, $k^2 = 0.1$

v_o/v_e	0.01	0.03	0.08	0.1
γ/ω_*	0.046	0.057	0.081	0.091

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10. More accurately, for arbitrary $k\rho_s$, we have the inequality that $J\cdot E$ work done is always less than the outward energy flux.

TABLE CAPTIONS

Table I. Variation of normalized growth rate γ/ω_* with $k^2\rho_s^2$ for a current-free asymmetric f_0 [given by Eq. (5)] with peak at $v_{||} = 0$.

Table II. Variation of γ/ω_* with v_0/v_e for a current-free asymmetric distribution peaked at $v_{||} = v_0$.