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FALLACIES IN SOME THEORIES OF THE RENORMALIZED DIELECTRIC

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Fallacies In Some Theories of The Renormalized Dielectric

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ABSTRACT

Several popular theories of the renormalized dielectric are examined and shown to be logically flawed. A recent conclusion that the "weak-coupling" approximation to the "renormalized quasilinear dielectric" is divergent is shown to be misleading because of an improper definition of the dielectric. The usual "resonance-broadened" dielectric is shown to be in error because the approximation neglects subtle correlations of the same order and physical importance as the terms retained. The problem is traced specifically to an erroneous application of statistical averaging and to the often-ignored difference between the infinitesimal response function and the single particle propagator. The procedure of "resonance-broadening the non-adiabatic response" is discussed, but no justification for the usual form of this approximation is found.

I. INTRODUCTION

In essentially every nonlinear theory of plasma turbulence, some quantity arises which is identified with *the nonlinear dielectric function* ε. Because this identification has generally been based more on intuition rather than on rigor, many different, often inconsistent approximations have appeared in the literature. In view of the considerable complexity of the problem, the more mathematical of the early renormalized theories[1,2] can perhaps be excused for overlooking various nonlinear terms of potential importance. However, in recent years certain aspects of those theories seem to have been distilled and codified into a "physically intuitive" prescription which yields a certain canonical approximation [Eq. (8)] to the dielectric — the implication being that, although the recipe is perhaps non-rigorous in some ill-specified way, the final result is "clearly" essentially correct. Intuition notwithstanding, this conclusion must be wrong since the form in question disagrees with modern, systematic renormalizations as well as with perturbation theory. Nevertheless, the "physical" arguments can be very compelling. One purpose of this paper is to identify the logical flaws in the usual "physical" arguments and, to a lesser extent, to argue for the modern approach.

Although the usual forms of the dielectric are incomplete, it is important to understand their consequences, as they are often used for practical computations. A second purpose of the paper is to discuss some startling conclusions which have been drawn recently from an approximation superficially very similar to Eq. (8). In particular, Misguich[3] concluded that a certain "weak coupling" approximation to the "renormalized quasilinear dielectric" is divergent. He then argued that more subtle physics, such as non-linear frequency shifts or non-Markovian effects, must be included to provide a sensible, convergent dielectric. In fact, however, we will show that the divergence results from an improper definition of the dielectric. Both the correct dielectric [Eq. (44)] as well as the approximate form (8) appear to be convergent even in the simplest Gaussian-Markov approximation with constant diffusion coefficient.

A third purpose of the paper is to clarify the approximations needed to obtain the recipe of "resonance-broadening the non-adiabatic response" as conventionally employed.[4] It appears that the approximations are severe; to date, they have not been justified.

The remainder of the paper is organized as follows. In Sec. II we present our version of the "physical" derivation of the dielectric. In Sec. III we relate this approximation to the recent work of Catto[4] and Misguich,[3] showing in detail how Misguich was incorrectly led to a divergence. In Sec. IV we discuss the flaws in the physical derivation. We discuss the non-adiabatic response in Sec. V, where we also summarize the major points of the paper.

II. A POPULAR "PHYSICAL" DERIVATION of the DIELECTRIC

It is surprisingly difficult to give a single reference to all aspects of the physical argument which leads to Eq. (8), which seem to be so much a part of the lore that some authors[5] assume them to be self-evident. As we understand it, one version of the argument proceeds as follows. Consider for simplicity a one-dimensional, spatially homogeneous, temporally stationary, turbulent Vlasov plasma in the electrostatic approximation. The governing equations are

$$(\partial_t + v\nabla + E\partial) f = 0 \quad , \tag{1a}$$

$$E = -\nabla\phi \ , \ \nabla^2\phi = -4\pi\sum nq\int dv\, f \quad . \tag{1b,c}$$

Here $\nabla \equiv \partial/\partial x$, $\partial \equiv (q/m)\partial/\partial v$, and the sum is over species. We shall often write the solution of Eqs. (1b,c) as $E = \mathcal{E} f$, thus defining the operator $\mathcal{E}$. We may assume that the averaged field $\langle E\rangle$ vanishes. In this case, the plasma fluctuations obey

$$(\partial_t + v\nabla)\delta f + \delta E\partial\langle f\rangle = -(\delta E\partial\delta f - \langle\delta E\partial\delta f\rangle) \quad . \tag{2}$$

Because of spatial homogeneity, the mean of any quantity dependent on only a single space point x must be independent of x, or have only the k = 0 Fourier component as non-vanishing. Conversely, δf is nonvanishing only for $k \neq 0$. It is then argued that the k = 0 term $\langle\delta E\delta f\rangle$ in Eq. (2) does not contribute to δf and can be ignored in the equation for δf. One then finds

$$(\partial_t + v\nabla + \delta E\partial)\delta f = -\delta E\partial\langle f\rangle \quad , \tag{3}$$

where we have chosen to treat the term in $\langle f\rangle$ as a source term. This is particularly appropriate for the stationary case, in which $\langle f\rangle$ is time-independent and can be assumed to be a known function. (δE is, of course, unknown at this point.) The characteristics of the operator in parentheses in Eq. (3) are the actual orbits of a test particle in the turbulence. It is thus convenient to introduce a propagator $\tilde{U}$ which is the stochastic Green's function for the operator in question[6]:

$$(\partial_t + v\nabla + \delta E\partial)\tilde{U}(x,v,t;\ x',v',t') = \delta(t-t')\delta(x-x')\delta(v-v') \quad , \tag{4a}$$

or

$$\tilde{U}(x,v,t;x',v',t') = \delta[\tilde{x}(t';x,v,t) - x']\delta[\tilde{v}(t';x,v,t) - v'] \quad . \tag{4b}$$

Here $\tilde{x}$ and $\tilde{v}$ are the actual time-reversed orbits which pass through x and v, respectively, at time $t = t'$. In terms of $\tilde{U}$, Eq. (3) can be "solved" to give, in a compressed notation which omits integrations over phase space coordinates,

$$\delta f(t) = -\int_{-\infty}^{t} dt'\,\tilde{U}(t;t')\,\delta E(t')\partial'\langle f\rangle \quad , \tag{5}$$

where we followed standard practice by ignoring initial conditions.

A self-consistency condition for the field then follows upon integrating Eq. (5) appropriately over velocity:

$$\delta E(t) = -\int_{-\infty}^{t} dt' \mathscr{E}\tilde{U}(t;t')\,\partial' \langle f \rangle\, \delta E(t') \quad . \tag{6}$$

Upon averaging Eq. (6) over the turbulent particle motions, one then replaces $\tilde{U}$ by $\langle \tilde{U} \rangle \equiv U$. This quantity is translationally invariant in space-time, so Fourier analysis is appropriate. We finally arrive at

$$\varepsilon(k,\omega)\, \delta E_{k,\omega} = 0 \quad , \tag{7}$$

where the "dielectric" is defined as

$$\varepsilon(k,\omega) \equiv 1 - i \sum \frac{\omega_p^2}{k^2} \int dv d\bar{v}\, U_{k,\omega}(v;\bar{v}) k \frac{\partial}{\partial \bar{v}} \langle f(\bar{v}) \rangle \tag{8}$$

$(\omega_p^2 \equiv 4\pi n q^2/m)$. This is the canonical result in question. We placed the word dielectric in quotes because we have not shown (and, indeed, will be unable to show, that the quantity in question satisfies the defining relations for a dielectric function.

A further common approximation is the Gaussian-Markov hypothesis[7] with velocity-independent diffusion coefficient D , for which case

$$U_{k,\omega} = \int_{-\infty}^{\infty} dx \int_{0}^{\infty} d\tau \exp[-i(kx - \omega\tau)]\, \Gamma(x,v,\tau;\bar{x},\bar{v}) \quad , \tag{9}$$

P being a jointly normal distribution in x and v with means $\langle x \rangle = \bar{x} + \bar{v}\tau$, $\langle v \rangle = \bar{v}$, and dispersions $\langle \delta x^2 \rangle = \frac{2}{3} D\tau^3$, $\langle \delta x \, \delta v \rangle = D\tau^2$, $\langle \delta v^2 \rangle = 2D\tau$. For completeness, we note that when velocity dispersion is inconsistently ignored so that $U_{k,\omega}(v;\bar{v}) \propto \delta(v-\bar{v})$, the well-known form

$$U_{k,\omega}(v;\bar{v}) \simeq \int_0^\infty d\tau \exp[i(\omega - kv)\tau - \tfrac{1}{3}k^2 D\tau^3] \, \delta(v - \bar{v}) \tag{10}$$

emerges. We shall not make use of this form, but shall consider in the next section some consequences of the approximation (9).

III. INCONSISTENCIES in SOME RECENT DISCUSSIONS of the RENORMALIZED DIELECTRIC

The form (8) is an approximation, as we discuss in more detail in the next section. However, it does describe some of the physics contained in the more complete theories. Recently, some confusing discussions of this resonance-broadening approximation have appeared which we wish to clarify. The problems are concerned with the proper action of the mean propagator U on the velocity derivative of the mean distribution — that is, they are concerned with the velocity dependence of U.

The velocity dependence of the mean propagator was already stressed by Benford and Thomson.[7] Catto[4] returned to this point and argued (in somewhat different language) that the proper velocity dependence was essential in order that the "adiabatic" response be treated correctly. [For a Maxwellian, for which $k\partial\langle f\rangle/\partial v = -kv\langle f\rangle/v_t^2$, Catto would say that one must propagate the factor kv as well as the usual factor exp(ikx).] Misguich[3] argued, in effect, that Catto's form was divergent, and that a more intricate renormalization was required for a sensible theory. In fact, thre are misconceptions and inconsistencies in both works.

We first determine the correct simplification of Eq. (8) in the Gaussian-Markov approximation with constant D and Maxwellian $\langle f\rangle$. We have

$$\varepsilon(k,\omega) \simeq 1 + \sum \frac{1}{(k\lambda_D)^2} \int dv\, d\bar{v}\, U_{k,\omega}(v;\bar{v})\, ik\bar{v}\, \langle f(\bar{v})\rangle \quad , \tag{11}$$

where

$$U_{k,\omega}(v;\bar{v}) \equiv \int_0^\infty d\tau\, \exp[i(\omega - kv)\tau - \tfrac{1}{3} k^2 D\tau^3] P_v(v - \bar{v} - ikD\tau^2, \tau) \quad , \tag{12}$$

$$P_v(z,\tau) \equiv (2\pi|\sigma_v^2(\tau)|)^{-1/2} \exp[-z^2/2\sigma_v^2(\tau)] \quad , \tag{13}$$

$$\sigma_v^2(\tau) \equiv 2D\tau \quad , \tag{14}$$

and $\lambda_D^2 \equiv T/4\pi nq^2$. (The important term in $ikD\tau^2$ arises from the cross-correlation between position and velocity.) To evaluate (11) and (12), it is convenient to change variables from $(v,\bar{v})$ to

$(u \equiv v - \bar{v} - ikD\tau^2, \bar{v})$:

$$I \equiv \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} d\bar{v}\ \exp(-ikv\tau) P_V(v - \bar{v} - ikD\tau^2)\, ik\bar{v} \langle f(\bar{v}) \rangle$$

$$= \exp(k^2 D\tau^3) \int_{-\infty}^{\infty} d\bar{v}\ \exp(-ik\bar{v}\tau)\, ik\bar{v} \langle f(\bar{v}) \rangle$$

$$\times \int_{-\infty - ik^2 D\tau}^{\infty - ik^2 D\tau} du\ \exp(-iku\tau) P_V(u) \quad . \tag{15}$$

A standard application of Cauchy's theorem enables one to shift the u contour upwards onto the real axis, so we have

$$\int_{-\infty}^{\infty} du\ \exp(-iku\tau)\ P_V(u) = \exp(-\tfrac{1}{2} k^2 \sigma_V^2 \tau)$$

$$= \exp(-k^2 D\tau^3) \quad . \tag{16}$$

This factor cancels with the first term in Eq. (15), whereupon

$$I = \int_{-\infty}^{\infty} d\bar{v}\ \exp(-ik\bar{v}\tau)\, ik\bar{v} \langle f(\bar{v}) \rangle$$

$$= -\frac{\partial}{\partial\tau} \int d\bar{v}\ \exp(-ik\bar{v}\tau) \langle f(\bar{v}) \rangle \quad . \tag{17}$$

Because <f> is Maxwellian, the remaining velocity integral can be performed if desired. However, for comparison with standard forms in the literature, we find it more convenient to retain the form (17). Then, upon using (17) in (11), we find

$$\varepsilon(k,\omega) = 1 + \sum \frac{1}{(k\lambda_D)^2} \int_0^\infty d\tau \exp(i\omega\tau - \tfrac{1}{3}k^2 D\tau^3)$$

$$\times \left(-\frac{\partial}{\partial\tau}\right) \int d\bar{v} \exp(-ik\bar{v}\tau) \langle f(\bar{v})\rangle$$

$$= 1 + \sum \frac{1}{(k\lambda_D)^2} + \sum \frac{1}{(k\lambda_D)^2} \int d\bar{v} \langle f(\bar{v})\rangle$$

$$\times \int_0^\infty d\tau (i\omega - k^2 D\tau^2) \exp[i(\omega - k\bar{v})\tau - \tfrac{1}{3}k^2 D\tau^3] \quad , \quad (18)$$

where in obtaining the last line we integrated by parts in τ .

Catto makes subsidiary approximations to be discussed later, which lead him to neglect the $k^2 D\tau^2$ term. The resulting approximation,

$$\varepsilon \simeq 1 + \sum \frac{1}{(k\lambda_D)^2} + \sum \frac{i\omega}{(k\lambda_D)^2} \int d\bar{v} \langle f(\bar{v})\rangle$$

$$\times \int_0^\infty d\tau \exp[i(\omega - k\bar{v})\tau - \tfrac{1}{3}k^2 D\tau^3] \quad , \quad (19)$$

is often further approximated by

$$\varepsilon \simeq 1 + \sum \frac{1}{(k\lambda_D)^2} + \sum \frac{i\omega}{(k\lambda_D)^2} \int d\bar{v} \langle f(\bar{v})\rangle \int_0^\infty d\tau \exp[i(\omega - k\bar{v})\tau - \tau/\tau_d] \quad (20a)$$

$$= 1 + \sum \frac{1}{(k\lambda_D)^2} + \sum \frac{i\omega}{(k\lambda_D)^2} \left(\frac{1}{\sqrt{2}\,kv_t}\right) Z\left(\frac{\omega + i\tau_d^{-1}}{\sqrt{2}\,kv_t}\right) \quad , \quad (20b)$$

where

$$\tau_d \equiv (\tfrac{1}{3}k^2 D)^{-1/3} \tag{21}$$

and

$$Z(z) \equiv \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} du \; \frac{e^{-u^2}}{u - z} \qquad (\mathrm{Im}\, z < 0) \quad . \tag{22}$$

In the form (20b), ω is not replaced everywhere by $\omega + i\tau_d^{-1}$. However, with the same approximation for the propagator, Eq. (18) can be reduced to

$$\varepsilon = 1 + \sum \frac{1}{(k\lambda_D)^2} + \sum \frac{i(\omega + i\tau_d^{-1})}{(k\lambda_D)^2} \left(\frac{1}{\sqrt{2}\, kv_t}\right) Z\left(\frac{\omega + i\tau_d^{-1}}{\sqrt{2}\, kv_t}\right) \quad , \tag{23}$$

which is the usual approximation of resonance-broadening theory, in which ω is replaced everywhere by $\omega + i\tau_d^{-1}$. We shall return to this point.

Misguich also obtained the result (19). [His Eq.(12) is readily reduced to (19) upon integration by parts in τ .] However, he also considered an alternative form, obtained by an integration by parts in velocity space, which was obviously divergent yet, according to Misguich, equivalent to Eq. (19) to dominant order in D . Since Eq. (19) is convergent, Misguich's arguments are logically inconsistent. In fact, in arriving at the form (19), Misguich made two compensating errors.

The first problem lies with Misguich's form [his Eq. (5)] for the dielectric, which is incorrect. In an attempt to be more precise and include non-Markovian corrections, Misguich and Balescu[8] argued that Eq. (8) should be replaced by

$$\varepsilon(k,\omega;t) = 1 - i \sum \frac{\omega_p^2}{k^2} \int_0^\infty d\tau\, e^{i\omega\tau}$$

$$\times \int dv\, d\bar{v}\, U_k(v,\tau;\bar{v}) \frac{\partial}{\partial \bar{v}} \langle f(\bar{v},t-\tau)\rangle \quad . \tag{24}$$

They write

$$\langle f(\bar{v},t-\tau)\rangle \simeq \int dv'\, U_{k=0}(\bar{v},-\tau;v')\langle f(v',t)\rangle \tag{25}$$

and then take $\langle f\rangle$ to be stationary. It is, however, unclear why $\langle f(t-\tau)\rangle$ is any less stationary than $\langle f(t)\rangle$. Apparently Misguich and Balescu ignore the point that the dielectric describes the result of probing the system after the turbulent state is set up. Both they and we assume stationarity, so $\langle f\rangle$ is unchanging before the probe is applied and (25) is incorrect.

Though (24) with (25) is in error, it is instructive to understand its consequences, in view of the startling conclusions drawn by Misguich. We have

$$U_{k=0}(v,-\tau;\bar{v}) \simeq P_v(v-\bar{v},-\tau) \quad .$$

Misguich (in a separate, also inconsistent approximation) passes this through the $\partial/\partial\bar{v}$ operator, after which it partially cancels with $P_v(v-\bar{v}-ikD\tau^2)$ in such a way that $P_v(v-\bar{v}-ikD\tau^2)$ is effectively replaced in (12) by $\Delta(v-\bar{v}-ikD\tau^2)$, where $\Delta(z)$ is the Dirac delta function analytically continued from real to complex values of z: e.g.,

$$\Delta(z) = \lim_{\sigma\to 0^+} (2\pi\sigma^2)^{-1/2} \exp(-z^2/2\sigma^2) \quad . \tag{27}$$

The manipulations leading up to (16) still hold; however, because of the delta function approximation to P_v, (16) is replaced by unity, the term $\exp(k^2D\tau^3)$ in (15) is not cancelled, and in (12) a net factor of $\exp(^2/_3\, k^2D\tau^3)$ remains. The resulting time integral is divergent, as Misguich noted.

In an attempt to circumvent the divergence and obtain Catto's result, Misguich returned to the form (11) (with P_v still replaced by Δ) and integrated over $\bar{v}$:

$$\varepsilon = 1 + \sum \frac{1}{(k\lambda_D)^2} \int_{-\infty}^{\infty} dv \int_0^{\infty} d\tau \exp[i(\omega-kv)\tau - {}^1/_3\, k^2D\tau^3]$$
$$\times (ikv + k^2D\tau^2)\langle f(v - ikD\tau^2)\rangle \quad , \tag{28}$$

where this result can be justified by Cauchy's theorem. This form is obviously still equivalent to the divergent result discussed above, as a change of variables to $v' \equiv v-ikD\tau^2$ reveals.

However, Misguich now neglects the factor of $-ikD\tau^2$ inside (but not outside) $\langle f \rangle$, arguing inconsistently that the action of the propagator on $\langle f \rangle$ results in "higher order contributions" in D. The result,

$$\varepsilon \simeq 1 + \sum \frac{1}{(k\lambda_D)^2} \int dv \langle f(v) \rangle$$

$$\times \int_0^\infty d\tau \exp[i(\omega - kv)\tau - \tfrac{1}{3}k^2 D\tau^3] \, (ikv + k^2 D\tau^2) \quad , \tag{29}$$

is convergent. It is equivalent to Catto's result and, upon integration by parts in τ , to (19).

It should already be clear that the derivation of a consistent nonlinear dielectric is somewhat tricky, being sensitive to the precise point at which nonlinear terms are neglected. Consider, for example, the difference between Eq. (18) and Catto's result (19). The neglect of the term $-k^2 D\tau^2$ is apparently made by Catto at the point where he retains "only the leading F_M and $\overline{G}$ contributions to g". Indeed, one might be tempted to argue that the term in $k^2 D\tau^2$ in Eq. (18) is higher-order in D and can hence be neglected. However, this approximation neglects a term of the same order as that of those retained, as a comparison of Esq. (20b) and (23) clearly reveals; it is not justifiable *a priori*.

That (23) emerges without approximation from Eq. (11) vitiates the logic of Catto's conclusion: namely, that when one correctly propagates the factor kv, only the non-adiabatic part of the distribution is resonance-broadened. In fact, the approximation leading to the latter conclusion is separate from the propagation question and is, within Catto's formalism, essentially ad hoc. However, the question remains whether a more systematic renormalization might nevertheless justify a form similar to (19). To address this point, we must discuss to what extent Eq. (8) is an adequate approximation. We do this in the next section, where we show that Eq. (8) is asystematic because it neglects terms of the same order as that of those retained.

IV. PROBLEMS WITH the PHYSICAL DERIVATION

Two related features of formula (8) suggest that all is not well. First, (8) cannot be reduced to weak turbulence theory (except in the trivial limit of linear theory). Second, U is described in terms of test particle quantities only, and shows no evidence of the physically-expected shielding effect which should arise from polarization of the medium by the test particles. It is worth emphasizing that these severe problems are not consequences of either the Gaussian hypothesis, Markovian approximation, or the neglect of velocity dispersion, but are more fundamental. Both can be traced to a misconceived notion of the statistical averaging process. Let us discuss this in detail.

Consider first the somewhat imprecise sentence immediately following Eq. (6). How can it be that the particle propagator is averaged while δE remains unscathed? As we understand the lore, this is often justified by an argument based on the stochastic instability of the particle orbits. Because stochastic instability develops exponentially rapidly, it is argued that the "waves" $\delta E(t)$ are essentially static on the time scale of $\tilde{U}$, so that a two-time-scale procedure can be used to justify averaging $\tilde{U}$ only, on the fast time scale. Physically as well as mathematically, however, it is clear that δE describes not only coherent, wave-like oscillations but also the microscopic fluctuations associated with the stochastic evolution of phase space elements. The time scale of these fluctuations is clearly the same as that of the velocity integral of $\tilde{U}$, as can be seen by writing

$$\mathcal{E}\tilde{U}(t;t_o) = \sum nq \int \frac{dk}{2\pi} \exp[ik\,\tilde{x}\,(t_o;t)] \quad , \tag{30a}$$

$$\delta E(t) = \int \frac{dk}{2\pi}\, \delta E_k(t) \exp[ik\,\tilde{x}(t_o;t)] \quad . \tag{30b}$$

Hence, the average of $\tilde{U}\,\delta E$ does not obviously factor. (It is irrelevant that $U(t;t')$ and $E(t')$, required in Eq. (6), seem to involve different time intervals $t > t'$ and $t < t'$, since the motion of a single particle for $t > t'$ is rigidly (deterministically) correlated to its motion for $t < t'$.) Furthermore, upon rigorously averaging Eq. (6) and recalling the definition of δE, one finds

$$0 = -\int_{-\infty}^{t} dt' \; \langle \mathcal{E}\tilde{U}(t;t')\, \partial' \langle f\rangle \; \delta E(t')\rangle \quad . \tag{31}$$

Instead of finding a consistency relations for δE, we find a statement about the correlations between $\mathcal{E}\tilde{U}$ and δE .

Equation (31) states (surprisingly, in view of our previous arguments) that $\mathcal{E}\tilde{U}$ and δE are uncorrelated. Unfortunately, Eq. (31) is incorrect.

The error arises in cavalierly neglecting the term $\langle\delta E\delta f\rangle$ in passing from Eq. (2) to Eq. (3), while retaining <u>all</u> of the term $\delta E\delta f$, which, of course, also has a component $\langle\delta E\delta f\rangle$. Returning to Eq. (2), we have rigorously

$$(\partial_t + v\nabla + \delta E\partial)\,\delta f = -\delta E\,\partial\langle f\rangle + \langle\delta E\partial\delta f\rangle \quad ,$$

or

$$\delta f(t) = \int_{-\infty}^{t} dt'\,\tilde{U}(t;t')\,[-\delta E(t')\partial'\langle f\rangle + \langle\delta E(t')\partial'\delta f(t')\rangle] \quad . \tag{32}$$

Upon averaging this equation, we find

$$0 = \int_{-\infty}^{t} dt'\,[-\langle\tilde{U}(t;t')\delta E(t')\rangle\,\partial'\langle f\rangle + U(t;t')\,\langle\delta E(t')\,\partial'\delta f(t')\rangle] \quad . \tag{33}$$

Inasmuch as $\delta f = O(\delta E)$, Eq. (33) shows that the correlations between $\tilde{U}$ and δE do not vanish but are of order $\langle\delta E^2\rangle$ — the same order as that of the terms which were previously retained. In fact, one can show that it is precisely the new correlations which describe polarization effects and permit the successful reduction to weak turbulence theory.

The proper definition of the dielectric has been discussed in Refs. 9-11; it is the proportionality factor in the relation between the mean response of the nonlinear, turbulent plasma to an infinitesimal external perturbing field E_e:

$$E_e + \left\langle \mathcal{E}\left(\frac{\delta f}{\delta E_e}\right)\Big|_{E_e=0} \right\rangle E_e = \epsilon^{-1} E_e \quad . \tag{34}$$

We have in schematic notation (for details, see Ref. 10)

$$(\partial_t + v\nabla)\left(\frac{\delta f}{\delta E_e}\right) + \mathcal{E}\left(\frac{\delta f}{\delta E_e}\right)\delta f + E\partial\left(\frac{\delta f}{\delta E_e}\right) = -\partial f \quad . \tag{35}$$

Equation (35) can be solved by introducing the stochastic infinitesimal response function $\tilde{R}$, defined by

$$\tilde{R} \equiv \delta f/\delta\hat{\eta}\big|_{\hat{\eta}=o} \quad ,$$

where $\hat{\eta}$ is a non-random source added to the right-hand side of Eq. (1a). In fact, $\tilde{R}$ is a stochastic Green's function for the left-hand side of Eq. (35), so that

$$\frac{\delta f}{\delta E_\epsilon} = -\tilde{R}\,\partial f \quad . \tag{36}$$

The quantity $\tilde{R}$ is distinct from the stochastic propagator $\tilde{U}$.

We can determine the relation between them by noting that $\tilde{R}$ obeys

$$(\partial_t + v\nabla + E\partial)\tilde{R} = 1 - \partial f \mathcal{E} \tilde{R} \quad , \tag{37}$$

whereupon, noting Eq. (4a),

$$\begin{aligned} \tilde{R} &= \tilde{U} - \tilde{U}\,\partial f\, \mathcal{E}\, \tilde{R} \\ &= \tilde{U} - \tilde{U}\,\partial f\,(1 + \mathcal{E}\tilde{U}\partial f)^{-1} \mathcal{E}\, \tilde{U} \quad . \end{aligned} \tag{38}$$

One than arrives at the form (8) by the following approximations. First, one ignores the correlations between $\tilde{R}$ and f in Eq. (36) so that

$$\langle \tilde{R}\partial f \rangle \simeq R\,\partial \langle f \rangle \tag{39}$$

and, from (34),

$$\varepsilon^{-1} = 1 - \mathcal{E} R\,\partial \langle f \rangle \quad . \tag{40}$$

Next, one replaces all quantities in formula (38) by their means:

$$R \simeq U - U\,\partial \langle f \rangle\, \varepsilon^{-1} \mathcal{E}\, U \quad , \tag{41}$$

where

$$\varepsilon \equiv 1 + \mathcal{E} U \partial \langle f \rangle \quad . \tag{42}$$

The formula (42) is compatible with (40) beacuse of the identity

$$\mathcal{E}R = \varepsilon^{-1}\mathcal{E}U \quad ,$$

and also identical to Eq. (8).

Of course, it is inconsistent to neglect the correlations between $\tilde{R}$ and f; just as $<\tilde{U}\delta E> = O(E^2)$, it readily follows that $<\tilde{R}\delta f> = O(E^2)$. Since we are retaining terms of $O(E^2)$ in the renormalization, these correlations cannot be ignored. Furthermore, there is no obvious justification for the approximation (41). Even if we were to adopt the often-used but seldom justified procedure[2] of expanding $\tilde{U}$ about its mean, so that $\tilde{U} \simeq U$ in lowest order, a further approximation would still have to be made because of the highly nonlinear way in which the stochastic distribution enters Eq. (39) — that is,

$$<\partial f\,(1 + \mathcal{E}U\,\partial f)^{-1}> \neq \partial<f>\,(1 + \mathcal{E}U\partial<f>)^{-1} \quad . \tag{43}$$

The correction terms which are required to make (43) an identity are again of $O(E^2)$ and must be retained in a consistent renormalization.

Systematic techniques for approximating the average $<\tilde{R}\delta f>$, required in Eq. (34), are discussed in Refs. 10 and 11. It is not within the scope of the present article to discuss the details. However, we may record that the proper dielectric is of the form

$$\varepsilon(k,\omega) = 1 - i \sum \frac{\omega_p^2}{k^2} \int dv\, d\bar{v}\, g_{k,\omega}(v;\bar{v})\, k \frac{\partial}{\partial \bar{v}} \bar{f}_{k,\omega}(\bar{v}) \quad . \tag{44}$$

Here

$$g_{k,\omega}(v;\bar{v}) \equiv [-i(\omega - kv)\delta(v-\bar{v}) + \Sigma_{k,\omega}(v,\bar{v})]^{-1} \quad , \tag{45a}$$

$$\Sigma \equiv \Sigma^{(d)} + \Sigma^{(p)} \quad , \tag{45b}$$

$$\bar{f} \equiv \langle f \rangle + \delta\bar{f}^{(d)} + \delta\bar{f}^{(p)} \quad , \tag{45c}$$

where $\Sigma^{(d)}_{k,\omega}$ is the non-Markovian version of the usual $-D\partial^2/\partial v^2$ operator of resonance-broadening theory, and where $\Sigma^{(p)}$, $\delta\bar{f}^{(d)}$, and $\delta\bar{f}^{(p)}$ are additional $O(E^2)$ terms arising from the correlations neglected in the usual arguments and describing polarization effects [$\Sigma^{(p)}$ and $\delta\bar{f}^{(p)}$], back-reaction of the test particles on the medium [$\delta\bar{f}^{(d)}$], etc. The similarity in form between Eqs. (44) and (3) may be noted. However, the difference in physical content between the two forms is profound. [In Ref. 12 we introduced a function similar in form to both Eqs. (8) and (44), but differing from both. However, we were careful to stress that the function was not a dielectric as defined by Eq. (34).]

V. DISCUSSION and SUMMARY

With the form (44) in hand, one may now ask to what extent the approximation (19) can be justified. The obvious way to proceed is to write

$$-ik\frac{\partial}{\partial\bar{v}}\,\bar{f}(\bar{v}) = i\frac{k\bar{v}}{v_t^2}<f(\bar{v})> - ik\frac{\partial}{\partial\bar{v}}\,\delta\bar{f}_{k,\omega} \tag{46}$$

and to add and subtract $-i\omega+\Sigma$ so that

$$-ik\frac{\partial}{\partial\bar{v}}\bar{f}_{k,\omega}(\bar{v}) = \frac{1}{v_t^2}\left(\int dv'\, g_{k,\omega}^{-1}(\bar{v};v')<f(v')>\right.$$

$$\left. -[-i\omega\delta(\bar{v}-v') + \Sigma_{k,\omega}(\bar{v},v')]<f(v')> - ikv_t^2\frac{\partial}{\partial\bar{v}}\delta\bar{f}_{k,\omega}\right) \quad , \tag{47}$$

whereupon

$$\varepsilon = 1 + \sum\frac{1}{(k\lambda_D)^2} + \sum\frac{1}{(k\lambda_D)^2}\int dv\, d\bar{v}\, g_{k,\omega}(v;\bar{v})$$

$$\times\left(\int dv'[-i\omega\delta(\bar{v}-v') - \Sigma_{k,\omega}(\bar{v},v')]<f(v')> + kv_t^2\frac{\partial}{\partial\bar{v}}\delta\bar{f}_{k,\omega}(\bar{v})\right) \quad . \tag{48}$$

A result of the general form (20a) would emerge from the result (48) if the term in $\delta\bar{f}$ would cancel the explicit term in Σ. Upon appealing to the explicit forms for Σ and $\delta\bar{f}$ as given, for example, in Ref. 11, it becomes clear that the cancellation is not exact. Although an approximate cancellation may in principle occur for certain classes of turbulence, we tend to doubt it, particularly for the generalization of (48) to such interesting modes as the universal instability. Clearly, much further work is called for.

[Part of the problem with assessing (48) arises because we have not explicitly included the generalization of adiabatic response to the nonlinear regime (e.g., the effective shielding length is modified nonlinearly). However, such a generalization, which merely amounts to a certain rearrangement of Eq. (48), would not in itself answer the question of the importance of the additional nonlinear terms.]

We must also emphasize that a theory of the nonlinear dielectric is not synonymous with a theory of strong (or weak!) turbulence, as seems sometimes to be assumed. A complete theory requires analysis of the so-called incoherent noise source $\tilde{F}$, defined and discussed in Refs. 11 and 13. For many practical problems, it would seem that this term is of equal importance to the nonlinear terms in the dielectric.

In conclusion, let us summarize the main points we have discussed.

(1): The definition of Misguich and Balescu of ε includes a spurious "non-Markovian correction". The divergence they find in the "renormalized weak-coupling quasilinear" approximation is therefore non-physical and irrelevant.

(2): Propagating the factor kv is not sufficient to justify "resonance-broadening of only the non-adiabatic response."

(3): The explicit term $\langle \delta E \delta f \rangle$ in Eq. (2) cannot be ignored.

(4): $\tilde{U}$ and δE have similar time scales and are correlated; stochastic instability is not a sufficient reason for ignoring those correlations.

(5): The dielectric is defined as the mean response of the plasma to an infinitesimal external perturbation; self-consistency conditions for fluctuating fields are meaningless in a turbulent plasma.

(6): The mean response function R, in terms of which the dielectric is rigorously defined, differs from the mean propagator U by the presence of additional terms which describe polarization effects and other correlations. These effects are not obviously negligible.

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REFERENCES

[1] T.H. Dupree, Phys. Fluids 9, 1773 (1966).

[2] J. Weinstock, Phys. Fluids 12, 1045 (1969).

[3] J.H. Misguich, Phys. Fluids 22, 1589 (1979).

[4] P.J. Catto, Phys. Fluids 21, 147 (1978); 22, 1591 (1979).

[5] As one example of many, we may mention the interesting work of S.P. Hirschman and K. Molvig, Phys. Rev. Lett. 42, 648 (1979).

[6] T.J. Birmingham and M. Bornatici, Phys. Fluids 14, 2239 (1971).

[7] J.J. Thomson and G. Benford, Phys. Rev. Lett. 28, 590 (1972).

[8] J.H. Misguich and R. Balescu, J. Plasma Phys. 13, [illegible]85 (1975).

[9] D. DuBois and M. Espedal, Plasma Phys. 20, 1209 (1978).

[10] J.A. Krommes and R. G. Kleva, Phys. Fluids (November 1979).

[11] J.A. Krommes, Princeton Plasma Physics Laboratory Report PPPL-1568 (1979); to appear in Handbook of Plasma Physics, edited by R.N. Sudan and A.A. Galeev (North-Holland, Amsterdam).

[12] J.A. Krommes, in Theoretical and Computational Plasma Physics (international Atomic Energy Agency, Vienna, 1978), p. 405.

[13] J.A. Krommes, Princeton Plasma Physics Laboratory Report PPPL-1603 (1979).

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