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# A NEW TRAPPED-ION INSTABILITY WITH LARGE FREQUENCY AND RADIAL WAVENUMBER

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The need for theoretical previsions concerning anomalous transport in large Tokomaks, as well as the recent results of PLT, ask the question of the process responsible for non-linear saturation of trapped-ion instabilities. This in turn necessitates the knowledge of the linear behaviour of these waves at large frequencies and large radial wevenumbers.

We study the linear dispersion relation of these modes, in the radially local approximation, but including a term due to a new physical effect, combining finite banana-width and bounce resonances. Limiting ourselves presently to the first harmonic expansion of the bounce motion of trapped ions, we show that the effect of finite banana-width on the usual trapped-ion mode is complex and quite different from what is generally expected.

In addition we show, analytically and numerically, the appearance of a nex branch of this instability. Essentially due to this new effect, it involves large frequencies ($\omega \sim \omega_b$) and is destabilized by large radial wavelengths ($k_x \Lambda \sim 1$, where $\Lambda$ is the typical banana-width). We discuss the nature of this new mode and its potential relevance of the experiments.

## I - INTRODUCTION

Trapped-ions instabilities [1] have long been predicted to exist in large Tokomaks, where ligh temperatures and low collisionality allow for the presence of a significant fraction of ions trapped in the toroidal magnetic field. They could be responsible for a large anomalous loss of particles and energy, but the question of their non-linear saturation, and thus of the induced anomalous diffusion, remains to be solved.

Recently, coherent mode coupling has been proposed as a mechanism for non-linear saturation. It would give low saturation levels but the theoretical analysis, in the model used, cannot solve the problem of stability of the predicted non-linear equilibrium. It could be unstable to the generation of modes with large radial wavenumbers [2].

The term responsible for the generation of these modes had been used formerly by Kadomtsev [3] to predict non-linear isotropization of trapped-ion turbulence, and to derive a much higher anomalous diffusion.

More recently, the results of PLT, with discharges in the expected regime of instability of these waves, indicate the presence of modes with large wavenumbers (kl where $\Lambda$ is the typical banana width).

To study more precisely this non-linear mode coupling, we have already developped a kinetic model, taking into account all important kinetic effects [4]. It has since then been extended to include the radial structure of the waves, but needs to be fed with their linear characteristics. Then we have developped a numerical code to study the linear spectrum, including the effects of finite banana-width and trapped-ion bounce resonances.

II - In order to establish the linear dispersion relation, we begin with the usual equation for the perturbed distribution functions of trapped particles :

$$[\, i(\omega - \omega_D + i\nu) - v_{\parallel} \frac{\partial}{\partial s} \,]\, g = -i(\omega - \omega^{*}) \frac{e}{T} F_0 \, \phi(r, s)$$

where : $\varphi = \phi(r, s)\, e^{i\, l(\varphi - q\theta) - i\omega t}$

s is the coordinate along the field lines

$\varphi$ and $\theta$ the angles respectively the long way and the short way around the torus.

$\omega_D$ is the magnetic curvature and gradient drifts frequency, which we take proportional to u (the particle energy), thus neglecting its dependance on the particle's turning points.

We expand g and $\phi$ in harmonics of the bounce frequency :

$$\Psi = \frac{e\phi}{T} = \sum_m \Psi_m \, e^{im\omega_b \tau(\theta)} \qquad (2)$$

$$\Psi_m = \left[ \oint \frac{d\theta'}{\sqrt{1-\lambda B(\theta')}} \right]^{-1} \oint \frac{d\theta'}{\sqrt{1-\lambda B(\theta')}} \Psi\,[\theta', r'(\theta')]\, e^{-im\omega_b \tau(\theta)}$$

where : $\lambda = \dfrac{v_\perp^2}{2Bv^2}$

$$\tau(\theta) = \int^{s(\theta)} \frac{ds(\theta')}{v_{/\!/}(\theta')} \qquad \tau = \oint \frac{ds}{v_{/\!/}} \qquad \omega_b = \frac{2\pi}{\tau}$$

and $r'(\theta')$ is determined by the second adiabatic invariant :

$$r'(\theta') = r_0(r,\theta) + \alpha 6' \sqrt{1-\lambda B(\theta')}$$

$$r_0(r,\theta) = r - \alpha 6 \sqrt{1-\lambda B(\theta)}$$

$$6 = \pm 1 \qquad \alpha = \frac{\sqrt{u}}{eB} \, \frac{q}{\varepsilon} \sqrt{\frac{2T}{m}}$$

$$\varepsilon = \frac{r}{R}$$

To complete the calculations we take as a realistic longitudinal dependance of $\Psi$ :

$$\Psi\,[r, s(\theta)] = a \cos k r \; \left|\cos \frac{\theta}{2}\right|$$

The calculation is then straight forward, though lengthy. Writing as usual the quasi-neutrality equation, and operating on both sides with $\int_0^{2\pi} \frac{d\theta}{B} \Psi^*$ to get a quadratic form, yields :

$$2\sqrt{\pi}\int_0^{2\pi} \frac{d\theta}{B} \Psi \Psi^* = \int_0^{\infty} du\, u^{1/2} e^{-u} \int_0^{2\pi} \frac{d\theta}{B} \Psi^* \int_{1/B_{max}}^{1/B} \frac{d\lambda\, B}{\sqrt{1-\lambda B}} \Omega_e \Psi$$

$$+ 2\pi \frac{a^2}{B_0} \cos^2 k\, r \quad \sqrt{2\varepsilon}\int_0^{\infty} du\, u^{1/2} e^{-u} \left\{\frac{\omega - \omega^*}{\omega - \omega_D + i\nu_+} S_0(k\, \Lambda \sqrt{u}) \right. \tag{3}$$

$$\left. + \frac{(\omega - \omega^*)(\omega - \omega_D + i\nu_+)}{(\omega - \omega_D + i\nu_+)^2 - \omega_b^2} S_1\ (k\, \Lambda \sqrt{u})\right\}$$

where $\Omega e = \frac{\omega - \omega_e^*}{\omega - \omega_{De} + i\nu_-}$

The first and second terms in the brackets come respectively from the $\Psi_0$ and $\Psi_{\pm 1}$ terms in the harmonic expansion, eq. (2). We neglect here terms of higher orders. $S_2$ has been checked to be smaller than $S_1$, for not too small values of $k\, \Lambda$. $S_0$ and $S_1$ are respectively defined as :

$$S_0(x) = \int_0^1 dt \frac{\pi}{2\kappa(t)}\ J_0^2\ (x \sqrt{t})$$

$$S_0(x) = 2\int_0^1 dt \frac{\pi\ Y^2(x/t)}{2\ \kappa(t)}$$

and $$Y(x/t) = \frac{2}{\pi}\int_0^{\pi/2} dy \cos\left[\frac{\pi\ F(y/t)}{2\ \kappa(t)}\right] \sin\ (x \sqrt{t} \cos\ y)$$

$\kappa$ and F are the complete and incomplete elliptic integrals of the second kind, $J_0$ is the Bessel Function.

Performing the remaining integrals in Eq. (3), and providing for circulating ions Landau damping or growth, we get the final dispersion relation :

$$\frac{1}{\sqrt{\varepsilon}} = \frac{1}{\sqrt{\pi}} \int_0^\infty du\, u^{1/2} e^{-u} \left\{ \frac{\omega - \omega^*}{\omega - \omega_D + i\nu_+} T_0 (k \Lambda \sqrt{u}) \right.$$

$$+ \frac{\omega - \omega_e^*}{\omega - \omega_{De} + i -} T_0(0)$$

$$\left. + \frac{(\omega - \omega^*)(\omega - \omega_D + i\nu_+)}{(\omega - \omega_D + i\nu_+)^2 - \omega_b^2} T_1 (k \Lambda \sqrt{u}) \right\} \tag{4}$$

$$- \frac{\omega_T^*}{2\omega} z \left\{ \kappa [Z(z) - Z(z\sqrt{\varepsilon})] + z(1 - \sqrt{\varepsilon}) + (z^2 - \frac{1}{2}) Z(z) \right.$$

$$\left. - (\varepsilon z^2 - \frac{1}{2}) Z(z\sqrt{\varepsilon}) \right\}$$

where : $\omega^* = \omega_T^* (u - \frac{3}{2} + \kappa)$ $\qquad \kappa = \frac{dLnN}{dLnT}$

$$\omega_T^* = \frac{lq}{r} \frac{T}{eB} \frac{dLnT}{dLn\pi} \quad \omega_D = \tilde{\omega}_D u \quad \omega_b = \tilde{\omega}_b \sqrt{u} \quad \nu_+ = \frac{\tilde{\nu}_+}{u^{3/2}}$$

$z = \frac{\omega}{k_\parallel V_{Th} \sqrt{\varepsilon}}$ and Z is the plasma dispersion function.

$$T_0 = S_0 \sqrt{2} \qquad T_1 = S_1 \sqrt{2}$$

To solve numericaly the dispersion relation, we approximate $T_0$ and $T_1$ by :

$$T_0(x) = a + b\, e^{-cx^2} \qquad T_1 = dx^4 + e\, x^2\, e^{-fx^2}$$

$$a = .1881 \qquad b = .9662 \qquad c = .2913 \qquad d = 2.66.10^{-4} \qquad e = .306$$

$$f = .1649$$

These are precise within 1% up to $x = 5.5$, which is largely enough for our purpose.

III - We have solved eq.(4) numericaly in two cases, corresponding to PLT parameters ; they are defined by :

$$T_i = 4 \text{ keV} \qquad T_e = 2 \text{ keV} \qquad B = 3T \qquad q = 2 \qquad \varepsilon = .1 \qquad R = 135 \text{ cm}$$

and respectively :

$$\frac{dLnT_i}{dLnr} = \frac{dLnT_e}{dLnr} = \frac{1}{3}, \quad \frac{dLnN}{dLnr} = 1 \quad \text{for case 1}$$

$$\frac{dLnT_i}{dLnr} = \frac{dLnT_e}{dLnr} = 1, \quad \frac{dLnN}{dLnr} = .5 \quad \text{for case 2}$$

giving in both cases : $\dfrac{\nu_+}{\omega_D} = \dfrac{.3212}{1} \quad \dfrac{\nu_-}{\omega_D} = \dfrac{155}{1} \quad \dfrac{\omega_D}{\omega_b} = .07090\ 1$

$$\nu^* = \frac{\nu_+}{\omega_b} = .02277$$

The figures show resulting values of $\omega$ and $\gamma$ as functions of $l$, the toroidal mode number. $l \sim 15$ gives $\omega_D \sim \omega_b$ and the limit of validity of our calculations.

The results are quite unexpected. They first show that the effect of finite banana-width on $\omega$ and $\gamma$ is quite different from the usual, $(1 - k^2\Lambda^2)$ assumption.

Second, they show the appearance of a new branch of the instability, due to a combined finite banana-width and bounce resonance effect. This is due to the fact that, neglecting ion collisions, the denominator of the third term in the brackets, in Eq. (4), is a 2nd-order polynomial. This gives two poles, which coalesce for $\gamma = 0$, $\frac{\omega\omega_D}{\omega_b^2} = -.25$. This term is then infinite, or may be arbitrarily large as $\omega$ tends towards this value. This gives the possibility of a new branch, close to $\omega = -\frac{\omega_b^2}{4\omega_D}$, $\gamma = 0$ ; this is the exact result for J = 0, and the mode is more unstable as J grows. What it becomes beyond J $\sim$ 1 remains however to be studied, retaining more terms in the harmonic expansion, eq. (2).

The general features of the results are as follows :

. In case 1, with low $\nabla T$, we get $\frac{\omega}{\omega_D} < 0$ (phase velocity with electron drifts), and $\frac{\omega}{\omega_D} > 0$ in case 2, as expected [5].

. Circulating ions Landau effect is stabilizing at high l values in case 1, destabilizing in case 2.

. Even without banana-width effects, the dispersion relation is dispersive ($\frac{\omega}{\omega_D}$ is not constant), which enhances again the need of large J to be abble to couple modes.

. The "usual" branch is strongly damped in both cases, around l $\sim$ 6-8 ($\frac{\omega_D}{\omega_b} \sim .5$), as the poles come to the domain where their residue is maximum.

. The new branch behaves differently in the two cases. In case 2 (the most relevant to PLT discharges), it is weakly unstable around l $\sim$ 4-6. for J $\gtrsim$ 1, whereas the old branch is strongly damped, even for moderate values of J.

The results seem to indicate that it might be even more unstable for higher J values.

In case 1, the condition $\frac{\omega\,\omega_D}{\omega_b^2} = -.25$ shows that, for $\frac{\omega_D}{\omega_b} \sim .5$, we get $\omega \sim \omega_D$ and the two branches cross, leading to a strong interaction between them. A singular point occurs for J # .5, where $\frac{\partial\omega}{\partial l}$ becomes infinite, as the $\omega$-derivative of the dispersion relation passes through 0 (writing the dispersion relation as $D(l, \;) = 0$, we have : $\frac{d\omega}{dl} = - \frac{\partial D/\partial l}{\partial D/\partial\omega}$ ). Above J = .5, the crossing point in $\omega$ vanishes and a crossing point in $\gamma$ appears, as shown on the figures.

In conclusion, we have shown that proper inclusion of finite banana-width and ion bounce resonance effects considerably affects the picture of trapped-ion instabilities. They strongly damp the usual trapped-ion mode, and generate a new branch around $\omega\,\omega_D = - \frac{\omega_b^2}{4}$ .

This new branch is more unstable as $J = k\,\Lambda$ is increased. In cases with large temperature g radients, it is more unstable than the usual branch. The question of its behaviour at $J > 1$, where it might be even more unstable, remains to be solved.

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