FR 10 0 2024

APPLICATION OF THE HYPERSPHERICAL FORMALISM

TO THE TRINUCLEON BOUND STATE PROBLEMS

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IPNO-TH 79-12 March 1979

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Abstract :

A method for solving the Schrödinger equation for the ground state of any number of bosons or for the trinucleon system or α-particle is formulated in the framework of the hyperspherical harmonic expansion method. It is applied to the trinucleon system for nucleons interacting through realistic soft core potentials. The convergence of the method is carefully studied. Binding energies, electric form factors, one body densities, two body correlation functions and two body photo-desintegration are calculated for various potentials.

I. INTRODUCTION

In studying the three body system we are confronted with a problem very much more difficult than for the two body system because :

i) The three body problem cannot be solved exactly in the state of the modern computational facilities

ii)The full two body nuclear potential contributes to the motion.

The evolution of the methods enabling to solve the many body problem has been strongly related to the computers capabilities and the quality of the two body interactions. At the beginning simple standard variational methods and more sophisticated including Monte Carlo integrations were used (3). But only the so called completely symmetric state was used. On the other hand during the 60th the Faddeev equations were tractable only for separatable potentials.

In the first introduction of the hyperspherical harmonics (H. H) expansion method Delves (1, 2) had in mind to describe the three body channels in nuclear reactions. Banville (4) in his attempt to solve the trinucleon bound state with an H. H expansion, has been confronted with the difficulty arising from the large degeneracy of the H. H basis preventing to obtain a satisfactory convergence with realistic potentials without a suitable selection of the H. H involved in a specific problem. To avoid this difficulty Simonov (5) introduced a set of H. H including the symmetry required to construct a complete H. H basis antisymmetric with respect to the exchange of a pair of fermions. The number of independent Simonov harmonics necessary to solve the trinucleon bound state with a good enough accuracy has been studied by Erens and Van Wageningen (54). They found that the convergence was still to slow for enabling the treat realistic potentials with a tractable number of coupled equations. The number of H. H needed to treat the problem has been reduced again by Fabre (10), who introduced the potential basis and the related optimal subset. These

subsets select out of the complete H. H basis the elements giving non negligible contributions to the wave functions.

The convergence rate of the H. H expansion method using the optimal subsets has been studied with various kinds of potentials by Erens et al (55), by Beiner and Fabre (12) and by Baliot et al (13, 41, 53). Erens (4) has shown that for central potentials the H. H orthogonal to the elements of the optimal subset bring negligible contributions to the wave function and the binding energy of the trinucleon.

This method has been subsequently applied to the calculation of the properties of the trinucleon bound state with realistic N-N potentials by J. Bruinsma et al (52) and by Demin and Efros (16). At the same time Gignoux and Laverne (34, 21) solved Faddeev equations in configuration space with realistic local potentials and carefull variational calculations with a harmonic oscillator basis was also performed by Sauer and Strayer (56) with realistic potentials.

Our aim in this work is to give a comprehensive survey of the H. H expansion method applied to the trinucleon.

This paper is divided into four parts.

The first part is a general review of the hyperspherical formalism in which some mathematical properties of hyperspherical functions are discussed and general formulas related to the expansion of local realistic potentials into hyperspherical harmonics are derived.

In the second part devoted to the three body bound systems we are confronted to the problem of determining an optimal subset in order to reduce the number of significant terms in the hyperspherical expansion of the wave function. A side of the local potential, the velocity dependent potential is also studied.

The third part is devoted to the applications of the formalism to the calculation of electric charge form factors, one body density, two body correlation functions and two body photodesintegration of ^{3}He.

In the last part we discuss the results obtained in three-nucleon observables and we emphasis their sensitivity to the details of nuclear forces in analyzing various s-wave potentials and also realistic potentials.

II. General features

II.1 Jacobi coordinates

Let σ be a system of A identical particles with spatial coordinates $\vec{x}_i$ (i=1,...A). The system is described in the centre of mass frame of reference by a set of N=A-1 coordinates $\vec{\xi}_j$ (j=1,...N) which are linear combinations of the $\vec{x}_i$. One chooses these linear combinations in such a way that the Laplace operator becomes :

$$\frac{1}{2}\sum_{i=1}^{A} \nabla^2_{\vec{x}_i} = \sum_{j=1}^{N=A-1} \nabla^2_{\vec{\xi}_j} + \frac{1}{2A}\nabla^2_{\vec{X}} \qquad \text{(II.1)}$$

where $\vec{X} = \frac{1}{A}\sum_{i=1}^{A} \vec{x}_i$ is the centre of mass coordinate. Among various possibilities one chooses to use the Jacobi coordinates defined by :

$$\vec{\xi}_j = \left\{\frac{2j}{j+1}\right\}^{\frac{1}{2}} \left(\vec{x}_{j+1} - \frac{1}{j}\left(\vec{x}_1+\ldots+\vec{x}_j\right)\right) \quad . \qquad \text{(II.2)}$$

The evolution of the system σ can be studied by following the motion of one point in the D=3N dimensional space in polar coordinates. The polar coordinates of this point are given by a set (Ω) of 3N-1 angles which in the F. Zernike and H.C. Brinkman (17) representation are :

i) the 2N polar angles $\hat{\xi}_j$ of each vector $\vec{\xi}_j$,

ii) the N-1 hyperspherical angles ϕ_j defined in terms of the length ξ_j of $\vec{\xi}_j$ by

$$\xi_j = \xi \sin\phi_N \ldots \sin\phi_{j+1} \cdot \cos\phi_j \qquad \text{(II.3)}$$

with $\phi_1 \equiv 0$ and $0 < \phi_j < \frac{\pi}{2}$.

The hyperradial coordinate ξ is defined by :

$$\xi^2 = \sum_{j=1}^{N} \xi_j^2 = 2 \sum_{i=1}^{A} (\vec{x}_i - \vec{X})^2 = \frac{2}{A} \sum_{i>j}^{A} (\vec{x}_i - \vec{x}_j)^2 \ . \qquad \text{(II.4)}$$

Note that ξ is symmetric with respect to all permutation of the particle coordinates.

II.2 The kinematic rotation vector

In order to study the effect of the permutation operators on the coordinates, we introduce a tridimensional vector

$$\vec{z}(\varphi) = \sum_{j=1}^{N} \sin\varphi_N \ldots \sin\varphi_{j+1} \cos\varphi_j \vec{\xi}_j \qquad \text{(II.5)}$$

called the "kinematic rotation vector" (36) which is a linear combination of the vectors $\{\vec{\xi}_1, \vec{\xi}_2 \ldots \vec{\xi}_N\}$. These vectors are function of N-1 angular parameter φ_j (j=2...N), $\varphi_1 \equiv 0$.

Thus any linear combination of the vectors $\{\vec{\xi}_j\}$

$$\vec{A}(\vec{\xi}) = \sum_{j=1}^{N} a_j \vec{\xi}_j \quad , \qquad \text{(II.6)}$$

can be expressed in terms of the kinematic rotation vector

$$\vec{A}(\vec{\xi}) = C \vec{z}(\varphi) \quad , \qquad \text{(II.7)}$$

with

$$C^2 = \sum_{j=1}^{N} a_j^2 \quad , \tag{II.8}$$

the angular parameter being determined by

$$\cos^2\varphi_j = \frac{a_j^2}{\sum_{i=1}^{j} a_i^2} \quad . \tag{II.9}$$

The notations $\varphi^{(i)}$ and $\varphi^{(i,j)}$ will be used for the set of parameters $\{\varphi\}$ for which,

$$\vec{z}(\varphi^{(i)}) = \sqrt{\frac{2(N+1)}{N}} \quad (\vec{x}_i - \vec{X}) \tag{II.9bis}$$

and

$$\vec{z}(\varphi^{(i,j)}) = \vec{x}_i - \vec{x}_j \quad .$$

II.3 Kinetic energy and hyperspherical harmonics

In polar coordinates, the kinetic energy operator T_ξ is ,

$$T_\xi = -\frac{\hbar^2}{m} \sum_{j=1}^{N} \nabla_{\xi_j}^2 = -\frac{\hbar^2}{m}\left\{\frac{\partial^2}{\partial\xi^2} + \frac{3N-1}{\xi}\frac{\partial}{\partial\xi} + \frac{L^2(\Omega)}{\xi^2}\right\}, \tag{II.10}$$

where $L^2(\Omega)$ is an angular operator in the 3N dimensional space (36). Its analytical expression depends o the specific choice made for the angular coordinates. The elements of the complete basis

eigenfunctions of $L^2(\Omega)$ and the quantum numbers involved in its definitions will also depend on the choice of these coordinates. Yu. A. Simonov (6,7,37) has used for example in the six dimensional space one of the possible basis defined in terms of the specific symmetries of the three body problem. Vilenkin, Kouznetzov and Smorodinsky have given a general prescription to construct the eigenfunctions of the operator $L^2(\Omega)$ (5) .

In this work we use the Zernike, Brinkman representation (17) in terms of which the operator $L^2(\Omega)$ is :

$$L^2(\Omega) = \sum_{i=1}^{N} \left\{ \prod_{j=i+1}^{N} \sin^2\phi_j \right\}^{-1} \cdot \left\{ \frac{\partial^2}{\partial\phi_j^2} + ((3i-4)\cot g\phi_i - 2tg\phi_i) \frac{\partial}{\partial\phi_i} + \frac{\ell^2(\hat{\xi}_i)}{\cos^2\phi_i} \right\} , \qquad (II.11)$$

where $\ell^2(\hat{\xi}_i)$ is the usual angular momentum operator corresponding to the vector $\hat{\xi}_i$. The eigenfunctions of the operator $L^2(\Omega)$ are the hyperspherical harmonics (H.H.)

$$Y_{[L]}(\Omega) = Y^{m_1}_{\ell_1}(\hat{\xi}_1) \cdot \prod_{j=2}^{N} Y^{m_j}_{\ell_j}(\hat{\xi}_j) \cdot {}^{(i)}P^{\ell_j, L_{j-1}}_{L_j}(\phi_j) , \qquad (II.12)$$

solution of the equation,

$$\{L^2(\Omega) + L(L+3N-2)\} Y_{[L]}(\Omega) = 0. \qquad (II.13)$$

In the expression of the H.H. (II.12), $Y^{m_j}_{\ell_j}(\hat{\xi}_j)$ are the spherical harmonics and

$$ {}^{(j)}P^{\ell_j, L_{j-1}}_{L_j}(\phi_j) = \left\{ \frac{2\nu_j\, \Gamma(\nu_j - n_j)\Gamma(n_j+1)}{\Gamma(\nu_j - n_j + \ell_j - \frac{1}{2})\Gamma(n_j + \ell_j + \frac{3}{2})} \right\}^{\frac{1}{2}} (\cos\phi_j)^{\ell_j} (\sin\phi_j)^{L_{j-1}} \cdot $$

$$ P^{\nu_{j-1}, \ell_j + \frac{1}{2}}_{n_j}(\cos 2\phi_j) \quad , \qquad \text{(II.14)} $$

with $\quad \nu_j = L_j + \frac{3j}{2} - 1$

$$ L_j = \sum_{i=1}^{j} (2n_i + \ell_i) \qquad n_1 = 0 $$

$P_n^{\alpha,\beta}$ stands for the Jacobi polynomial.

The set [L] of the 3N-1 quantum numbers defining a H.H. in our choice of angular coordinates (Ω) is constituted by

i) the 2N orbital and magnetic quantum numbers ℓ_j and m_j for each vector $\vec{\xi}_j$ (j=1,...N),

ii) the N-1 hyperspherical quantum numbers n_j, (j=2,..N) related to the hyperspherical angle ϕ_j.

L is the grand orbital related to the ℓ_i and n_i quantum numbers by

$$ L = \sum_{i=1}^{N} (2n_i + \ell_i) \qquad (n_1 \equiv 0) \quad . \qquad \text{(II.15)} $$

The parity of a H.H. in the change of $\vec{\xi}$ into $-\vec{\xi}$ is

$$ (-1)^{\sum_{i=1}^{N} \ell_i} = (-1)^L \qquad \text{(II.16)} $$

II.4 Expansion of a plane wave in hyperspherical harmonics

One starts from the expansion of a plane wave in spherical harmonics,

$$\exp(i\sum_{j=1}^{N}\vec{k}_j.\vec{\xi}_j) = (2\pi)^{\frac{3N}{2}}\prod_{j=1}^{N}\sum_{\ell_j m_j}(i)^{\ell_j}Y^{m_j}_{\ell_j}(\hat{\xi}_j).Y^{m_j*}_{\ell_j}(\hat{k}_j).$$

$$\frac{J_{\ell_j+\frac{1}{2}}(k_j\xi_j)}{(k_j\xi_j)^{\frac{1}{2}}} \qquad \text{(II.17)}$$

where k_j and $\hat{k}_j$, (ξ_j and $\hat{\xi}_j$) are respectively the length and polar angles of the vector $\vec{k}_j$, ($\vec{\xi}_j$). Using eqs. (II.3) one express the components k_j and ξ_j respectively in terms of a length k and ξ and N-1 hyperspherical angles $\{\varphi_j\}$ and $\{\phi_j\}$. Inserting these expressions in (II.17) and using the Bateman formula (48) one finds,

$$\exp(i\sum_{j=1}^{N}\vec{k}_j.\vec{\xi}_j) = (2\pi)^{\frac{3N}{2}}\sum_{[L]=0}^{\infty}i^L Y^{*}_{[L]}(\Omega_k)Y_{[L]}(\Omega)\frac{J_{L+\frac{3N}{2}-1}(k\xi)}{(k\xi)^{\frac{3N}{2}-1}}, \qquad \text{(II.18)}$$

in which (k, Ω_k) are the polar coordinates of the vector $\vec{k}$ $(\vec{k}_1 \ldots \vec{k}_N)$ in the 3N dimensional space defined by the 2N polar angles $\hat{k}_j$ of each vector $\vec{k}_j$ and the N-1 hyperspherical angles $\{\phi_{k_j}\}$

$$k_j = k \sin\phi_{k_N}\ldots\sin\phi_{k_{j+1}}.\cos\phi_{k_j} \qquad \phi_{k_1} \equiv 0.$$

The symbol $\sum_{[L]}$ denotes a summation over all the quantum numbers $[L]$ for which $\sum_{i=1}^{N}(2n_i+\ell_i) = L$, $(n_1 \equiv 0)$.

Now let us turn all the vector $\vec{k}_j$ in the same direction $\vec{q}(q,\hat{q})$ then $\hat{k}_j=\hat{q}$ and

$$k_j = q \sin\varphi_N \cdots \sin\varphi_{j+1} \cos\varphi_j \quad ,$$

$$\sum_{j=1}^{N} \vec{k}_j . \vec{\xi}_j = \sum_{j=1}^{N} \sin\varphi_N \cdots \sin\varphi_{j+1} \cos\varphi_j \, \vec{q} . \vec{\xi}_j = \vec{q} . \vec{z}(\varphi) . \qquad \text{(II.19)}$$

For any linear combination $\vec{z}(\varphi)$ of the vectors $\vec{\xi}_j$, the expansion of the plane wave $\exp(i\,\vec{q}.\vec{z}(\varphi))$ in the 3N dimensional space is :

$$\exp(i\,\vec{q}.\vec{z}(\varphi)) = (2\pi)^{\frac{3N}{2}} \sum_{[L]} i^L Y^*_{[L]}(\varphi,\hat{q}) Y_{[L]}(\Omega) \frac{J_{L+\frac{3N}{2}-1}(q\xi)}{(q\xi)^{\frac{3N}{2}-1}} \qquad \text{(II.20)}$$

where

$$Y^*_{[L]}(\varphi,\hat{q}) = Y^{m_1^*}_{\ell_1}(\hat{q}) \prod_{j=2}^{N} Y^{m_j^*}_{\ell_j}(\hat{q}) \, {}^{(i)}P^{\ell_j, L_j - 1}_{L_j}(\varphi_j) \qquad \text{(II.21)}$$

is a function of φ and of the angular coordinates $\hat{q}$ of the vector $\vec{q}$ only.

II.5 The multipoles of realistic local N-N potentials

For realistic local soft core two body potentials, the following interactions are used [20, 22, 35, 38, 40] :

α) $$V(\vec{r}_{ij}) = V_c(r_{ij}) + V_{L2}(r_{ij}) . \vec{L}^2_{ij} + V_{LS}(r_{ij}) \vec{L}_{ij} . \vec{S}_{ij} + V_T(r_{ij}) . \hat{S}_{ij}(\hat{r}_{ij}) + V_Q(r_{ij}) . Q_{ij}$$

$$\beta) \quad V(\vec{r}_{ij}) = V_C(r_{ij}) + V_{LL}(r_{ij}) LL_{ij} + V_{LS}(r_{ij}) \vec{l}_{ij}.\vec{S}_{ij} + V_T(r_{ij}) \hat{S}_{ij}(\hat{r}_{ij})$$

$$\gamma) \quad V(\vec{r}_{ij}) = V_C(r_{ij}) + V_{L2}(r_{ij}) \vec{l}^2_{ij} + V_{LS}(r_{ij}) \vec{l}_{ij}.\vec{S}_{ij} + V_T(r_{ij}).\hat{S}_{ij}(\hat{r}_{ij})$$

(II.22)

with

$$\hat{S}_{ij}(\hat{r}_{ij}) = \frac{3}{r^2_{ij}} (\vec{\sigma}_i.\vec{r}_{ij})(\vec{\sigma}_j.\vec{r}_{ij}) - \vec{\sigma}_i.\vec{\sigma}_j$$

$$LL_{ij} = (\vec{\sigma}_i.\vec{\sigma}_j)\vec{l}^2_{ij} - (\vec{l}_{ij}.\vec{\sigma}_i)(\vec{l}_{ij}.\vec{\sigma}_j) = \{\delta_{l_{ij}j_{ij}} + 2S^2_{ij} - 3\}\vec{l}^2_{ij}$$
$$- (\vec{l}_{ij}.\vec{S}_{ij})^2$$

$$Q_{ij} = 3(\vec{\sigma}_i.\vec{l}_{ij})(\vec{\sigma}_j.\vec{l}_{ij}) - (\vec{\sigma}_i.\vec{\sigma}_j)\vec{l}^2_{ij} = 2(2S^2_{ij} - 3)\vec{l}^2_{ij} - 3LL_{ij}$$

(II.23)

where for the pair (ij)

$\vec{l}_{ij}$ is the angular momentum vector

$\vec{S}_{ij}$ is the spin vector

$\vec{j}_{ij}$ is the total spin vector

while $\vec{\sigma}_i$ are the Pauli matrix of the particle i.

In order to perform the expansion of the potential in H.H. one writes the components in terms of spherical harmonics

$$V_\alpha(\vec{r}_{ij}) = \sum_{\ell m}^{\alpha} a^{m}_{\ell}(ij)\, Y^m_\ell(\hat{r}_{ij})\, v^\ell_\alpha(r_{ij}) \qquad (II.24)$$

where α labels the various components (central, LS, tensor...) of the interaction. $\ell=0$ for (C, LS, LL, Q, L^2) components and $\ell=2$ for the tensor force. All orbital and spin operators of each α

component are contained in the operator ${}^{\alpha}a_{\ell}^{m}(ij)$.
For the components $\alpha=(C, LS, LL, Q, L^2)$ we have

$$ {}^{\alpha}a_{0}^{0}(ij) = \sqrt{4\pi} \cdot \Xi^{\alpha} \quad , $$

where $\Xi^{\alpha}=(1, \vec{t}_{ij}.\vec{S}_{ij}, LL_{ij}, Q_{ij}, \vec{l}^{2}_{ij})$ respectively, while

$$ {}^{T}a_{2}^{m}(ij) = \sqrt{24\pi} \sum_{\nu} \sigma_{i}^{\nu-m}(i) \begin{pmatrix} 1 & 1 & 2 \\ m-\nu & \nu & -m \end{pmatrix} \sigma_{i}^{-\nu}(j), \qquad (II.26) $$

for tensor force and where $\sigma_{i}^{\nu}(k)$ is the Pauli tensor of rank one in the standard representation.

The Fourier transform $\mathcal{V}_{\alpha}(\vec{q})$ of the two body potential

$$ V_{\alpha}(\vec{r}_{ij}) = \int d^{3}q \ \mathcal{V}_{\alpha}(\vec{q}) \exp(i \vec{q}.\vec{z}(\varphi^{(i,j)}) \quad , \qquad (II.27) $$

is defined by

$$ \mathcal{V}_{\alpha}(\vec{q}) = (2\pi)^{-\frac{3}{2}} \sum_{\ell m} (-i)^{\ell} \, {}^{\alpha}a_{\ell}^{m}(ij) \, Y_{\ell}^{m}(\hat{q}) \int_{0}^{\infty} dx \, x^{2} v_{\alpha}^{\ell}(x) \frac{J_{\ell+\frac{1}{2}}(qx)}{(qx)^{\frac{1}{2}}} \, . \qquad (II.28) $$

Introducing this expression in (II.27) in which the plane wave has been expanded into H.H. according (II.20), one finds

$$ V_{\alpha}(\vec{r}_{ij}) = 2\pi^{\frac{3(N-1)}{2}} \sum_{\ell m} {}^{\alpha}a_{\ell}^{m}(ij) \sum_{K=0}^{\infty} (-1)^{K} \frac{\Gamma(K+\ell+\frac{3}{2})}{\Gamma(\ell+\frac{3}{2})\Gamma(K+\frac{3N}{2}-\frac{3}{2})} \cdot $$

$$ \sum_{[2K+\ell]} Y_{[2K+\ell]}^{(\Omega)} \int d\hat{q} \, Y_{\ell}^{m}(\hat{q}) \, Y_{[2K+\ell]}^{*}(\varphi^{(i,j)},\Omega) \int_{0}^{1} v_{\alpha}^{\ell}(u\xi) . u^{\ell+2} . (1-u^{2})^{\frac{3N-5}{2}} $$

$$ {}_{2}F_{1}(-K, \ell+K+\frac{3N}{2}-1, \ell+\frac{3}{2}, u^{2}) \, du. \qquad (II.29) $$

where as usual the sum $\sum_{[2K+\ell]}$ is taken over all the quantum numbers excluding the grand orbital $2K+\ell$.

Introducing the multipoles

$$\alpha V^{(3N,\ell)}_{2K}(\xi) = 2.\ \frac{\Gamma(\frac{\ell}{2}+\frac{3N}{2})\Gamma(K+\ell+\frac{3}{2})}{\Gamma(\frac{\ell+3}{2})\Gamma(K+\frac{3N}{2}-\frac{3}{2})\Gamma(\ell+\frac{3}{2})} \int_0^1 V^{\ell}_{\alpha}(\xi u) u^{\ell+2} (1-u^2)^{\frac{3N-5}{2}}$$

$$2F_1 (-K, \ell+K+\frac{3N}{2}-1, \ell+\frac{3}{2}; u^2)\ du \quad , \qquad \text{(II.30)}$$

with the normalisation

$$\alpha V_0^{(3N,\ell)}(0) = V_{\alpha}(0) \quad , \qquad \text{(II.31)}$$

a similar expression is obtained for the interaction $V_{\alpha}(\vec{r}_{ij})$:

$$V_{\alpha}(\vec{r}_{ij}) = \pi^{\frac{3(N-1)}{2}} \sum_K {}^{\alpha}a^{m}_{\ell}(ij)\ {}^{\alpha}V^{(3N,\ell)}_{2K}(\xi).\ A^{\ell,m}_{2K}(\varphi^{(i,j)},\Omega) \qquad \text{(II.32)}$$

where

$$A^{\ell,m}_{2K}(\varphi^{(i,j)},\Omega) = \frac{\Gamma(\ell+\frac{3}{2})}{\Gamma(\frac{3N}{2}+\frac{\ell}{2})} \int d\hat{q}\, Y^{m}_{\ell}(\hat{q}) \sum_{[2K+\ell]} Y^{*}_{[2K+\ell]}(\varphi^{(i,j)},\hat{q})\, Y_{[2K+\ell]}(\Omega) \qquad \text{(II.33)}$$

is a linear combination of H.H. $Y_{[2K+\ell]}(\Omega)$. In this linear combination the quantum numbers ℓ and m are related to the nature of the α component of the potential. The hyperangular functions $A^{\ell,m}_{2K}(\varphi^{(i,j)},\Omega)$ will be used in the construction of the optimal subset. Integrating over the angles $\hat{q}$ in eqs. (II.33) one obtains,

$$A^{\ell,m}_{2K}(\varphi^{(i,j)},\Omega) = \frac{\Gamma(\ell+\frac{3}{2})}{\Gamma(\frac{3N}{2}+\frac{\ell}{2})} \sum_{\lambda_1\lambda_2} (-1)^{\lambda_1+\lambda_2} \frac{\Delta(\ell\lambda_1\lambda_2)}{\sqrt{4\pi(2\ell+1)}}$$

$$P^{(2)\lambda_2\lambda_1}_{2K+\ell}(\varphi^{(i,j)}) \cdot P^{(2)\lambda_2\lambda_1}_{2K+\ell}(\phi) \cdot Y^{m}_{\ell(\lambda_1\lambda_2)}(\hat{\xi}_1,\hat{\xi}_2) \qquad \text{(II.34)}$$

where Δ is given in terms of the 3J symbols by

$$\Delta(\ell\lambda_1\lambda_2) = [(2\ell+1)(2\lambda_1+1)(2\lambda_2+1)]^{\frac{1}{2}} \begin{pmatrix} \ell & \lambda_1 & \lambda_2 \\ 0 & 0 & 0 \end{pmatrix} \qquad \text{(II.35)}$$

while

$$Y^{m}_{\ell(\lambda_1\lambda_2)}(\hat{\xi}_1,\hat{\xi}_2) = \sum_{\mu_1\mu_2} \sqrt{2\ell+1}\,(-1)^{\lambda_2-\lambda_1-m} \begin{pmatrix} \lambda_1 & \lambda_2 & \ell \\ \mu_1 & \mu_2 & -m \end{pmatrix} Y^{\mu_1}_{\lambda_1}(\hat{\xi}_1)\, Y^{\mu_2}_{\lambda_2}(\hat{\xi}_2). \qquad \text{(II.36)}$$

The multipoles $V^{\alpha\,(3N,\ell)}_{2K}(\xi)$ given by eqs. (II.30) have the following properties :

i) When the potential $V_\alpha(\vec{r}_{ij})$ has a Fourier transform regular near the origin the multipoles behave asymptotically like

$$V^{(3N,0)}_{2K}(\xi) \underset{\xi\to\infty}{\sim} 16\pi \frac{\Gamma(K+\frac{3}{2})\Gamma(\frac{3N}{2})}{\Gamma(K+3\frac{(N-1)}{2})} \frac{V_\alpha(0)}{\xi^3} \qquad \text{(II.37)}$$

with $V_\alpha(0) = \frac{1}{2\pi^2}\int_0^\infty V_\alpha(x)x^2dx$

ii) The multipole $V^{\alpha\,(3N,0)}_{0}(\xi)$ has the behavior of the potential $V_\alpha(\vec{r}_{ij})$ near the origin, for instance according to (II.30) a potential $(r_{ij})^n$ generates multipoles proportional to ξ^n, (n=-1 for the Coulomb potential).

iii) When $V_\alpha(r_{ij})$ is finite at the origin then according to (II.31) $\overset{\alpha\ (3N,\ell)}{V_0}(0) = V_\alpha(0)$ and $^{\alpha}V_{2K}(\xi)_{\xi\to 0} \to 0$.

III. Bound states of three body systems

III.1 Basic equations

For describing the position of three identical particles with respect to the centre of mass one uses the Jacobi coordinates $\vec{\xi}_1$ and $\vec{\xi}_2$ given in terms of the coordinates $\vec{x}_i$ of the particles (i=1,2,3) by

$$\begin{aligned} \vec{\xi}_1 &= \vec{x_2 x_1} \\ \vec{\xi}_2 &= \frac{2}{\sqrt{3}} \left(\vec{x}_3 - \frac{1}{2}\left(\vec{x_1 + x_2}\right)\right) = \sqrt{3}\ (\vec{x}_3 - \vec{X}) \\ \vec{X} &= \frac{1}{3}\ (\vec{x}_1 + \vec{x}_2 + \vec{x}_3) \end{aligned} \qquad \text{(III.1)}$$

We introduce the hyperspherical coordinates (ξ, Ω) of the six dimensional vector $\vec{\xi}$ like in (II.3). The hyperradius is

$$\xi^2 = \vec{\xi}_1^{\,2} + \vec{\xi}_2^{\,2} \qquad 0 \leqslant \xi \leqslant \infty \quad , \qquad \text{(III.2)}$$

Ω is a set of five angular coordinates : one hyperspherical angle ϕ ($\mathrm{tg}\phi = \frac{\xi_1}{\xi_2}$) and the four angular coordinates $\hat{\xi}_1$ and $\hat{\xi}_2$ in the three dimensional space. With this choice the kinetic energy operator is

$$T = -\frac{\hbar^2}{m}\left(\frac{\partial^2}{\partial \xi^2} + \frac{5}{\xi}\frac{\partial}{\partial \xi} + \frac{L^2(\Omega)}{\xi^2}\right) \qquad \text{(III.3)}$$

with

$$L^2(\Omega) = \frac{\partial^2}{\partial \phi^2} + 4\mathrm{cotg}2\phi\ \frac{\partial}{\partial \phi} - \frac{\vec{l}_1^{\,2}(\hat{\xi}_1)}{\sin^2\phi} - \frac{\vec{l}_2^{\,2}(\hat{\xi}_2)}{\cos^2\phi} \ . \qquad \text{(III.4)}$$

The eigenfunctions of this "grand angular" operator are the orthonormalized hyperspherical harmonics (H.H)

$$Y_{[L]}(\Omega) = Y^{m_1}_{\ell_1}(\hat{\xi}_1)\, Y^{m_2}_{\ell_2}(\hat{\xi}_2)\, {}^{(2)}P^{\ell_2,\ell_1}_{2n+\ell_1+\ell_2}(\phi) \quad . \qquad \text{(III.5)}$$

The symbol [L] stands for the five angular quantum numbers {n ℓ_1 ℓ_2 m_1 m_2} appearing in (III.5). Note that for a grand orbital L ($L=2n+\ell_1+\ell_2$) the degeneracy is

$$D(L) = \frac{L+2}{12} \cdot \frac{(L+3)!}{L!} \quad . \qquad \text{(III.6)}$$

The Schrödinger equation written in hyperspherical coordinates becomes :

$$(H-E)\Psi(\xi,\Omega) = \left\{ -\frac{\hbar^2}{m}\left\{ \frac{d^2}{d\xi^2} + \frac{5}{\xi}\frac{d}{d\xi} + \frac{L^2(\Omega)}{\xi^2} \right\} + V(\xi,\Omega) - E \right\} \Psi(\xi,\Omega) = 0 \qquad \text{(III.7)}$$

where $V(\xi,\Omega)$ is the interaction.

In expanding the wave function $\Psi(\xi,\Omega)$ in the H.H basis

$$\Psi(\xi,\Omega) = \sum_{[L]} \xi^{-\frac{5}{2}}\, u_{[L]}(\xi) \cdot Y_{[L]}(\Omega) \quad , \qquad \text{(III.8)}$$

and integrating over the angular set Ω one transforms the Schrödinger equation (III.7) into an infinite set of second order coupled differential equations :

$$\langle Y_{[L]} |\ H-E\ | \sum_{[L']} \xi^{-\frac{5}{2}}\, u_{[L']}(\xi)\, Y_{[L']}(\Omega) \rangle = 0 \qquad \text{(III.9)}$$

$$\left\{ -\frac{\hbar^2}{m}\left[\frac{d^2}{d\xi^2} - \frac{(L+2)^2-1/4}{\xi^2}\right] - E \right\} U_{[L]}(\xi)$$

$$+ \sum_{[L']} \langle Y_{[L]} | V(\vec{\xi},\Omega) | Y_{[L']} \rangle U_{[L']}(\xi) = 0 \quad . \qquad \text{(III.10)}$$

The radial partial waves $u_{[L]}(\xi)$ are normalized according to

$$\sum_{[L]} \int_0^\infty d\xi |U_{[L]}(\xi)|^2 = 1 \quad . \qquad \text{(III.11)}$$

For large values of the grand orbital L the number D(L) of independent H.H is very large. In order to solve numerically the three body problem we have to reduce the number of partial waves involved in the expansion of the wave function $\Psi(\xi,\Omega)$ to the minimum number of significant terms.

The first step is to couple the spherical harmonics in order to obtain a basis of definite total angular momentum ℓ of projection m. It is made as usual with the 3J coefficients :

$$Y^{m}_{(\ell_1\ell_2)\ell}(\Omega) = \sum_{m_1 m_2} \hat{\ell}\,(-1)^{\ell_2-\ell_1-m} \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ m_1 & m_2 & -m \end{pmatrix} Y_{[L]}(\Omega). \qquad \text{(III.12)}$$

The ground state of the trinucleon system for a potential without tensor force is completely described by the elements ℓ=m=0 only. In this case the number of H.H of grand orbital L=2K is K+1 because the parity conservation selects only the even values of L. In section III.2 it will be shown how to select an optimal subset of the complete H.H basis taking into account the most important features of a given potential and including the permutation symmetry of the three particles introduced by the Pauli principle.

III.2 Antisymmetrization of the wave function

The spatial wave function of the three body system $\Psi(z(\varphi), z(\varphi-\pi/2))$ is written in terms of the kinematic rotation vector,

$$\vec{z}(\varphi) = \vec{\xi}_1 \sin\varphi + \vec{\xi}_2 \cos\varphi \quad . \tag{III.13}$$

For $\varphi=\pi/2$, $\pi/2-2\pi/3, \pi/2+2\pi/3$, one obtains respectively the interdistances between two particles $(\vec{x}_2-\vec{x}_1$, $\vec{x}_3-\vec{x}_2$, $\vec{x}_1-\vec{x}_3)$ and, for the same values of φ, $z(\varphi-\pi/2)$ give $\sqrt{3}(\vec{x}_3-\vec{X})$, $\sqrt{3}(\vec{x}_1-\vec{X})$ and $\sqrt{3}(\vec{x}_2-\vec{X})$.

We define two types of wave functions $\Psi^{\pm}_{\alpha}(\vec{z}(\varphi), \vec{z}(\varphi-\pi/2))$ (anti)symmetric (superscript (-), +) according to the parity with respect to $\vec{z}(\varphi) \to -\vec{z}(\varphi)$. Since the cyclic permutation operator acting on a symmetric (or antisymmetric) function under the exchange of two particles produces a fully symmetric (or antisymmetric) function under the exchange of any particle, we construct the completely antisymmetric states by taking the cyclic permutation of the isospin-spin state with the spatial function exhibiting two conjugate symmetry features. For convenience we construct the fully antisymmetric state from wave functions exhibiting definite symmetries in the pair (1,2) i.e. for $\varphi=\pi/2$.

The antisymmetric wave function has the general form :

$$\begin{aligned}
\Psi(\xi,\Omega) = {} & \left[{}^{m_T m_S}_{\frac{1}{2}\frac{1}{2}(A)}\right] \sum_C \Psi^{(+)}_S(\vec{z}(\varphi), \vec{z}(\varphi-\tfrac{\pi}{2})) + \left[{}^{m_T m_S}_{\frac{1}{2}\frac{1}{2}(S)}\right] \sum_C \Psi^{(-)}_A(\vec{z}(\varphi), \vec{z}(\varphi-\tfrac{\pi}{2})) \\
& + \sum_C \Big\{ \sin 2\varphi \left[{}^{m_T m_S}_{\frac{1}{2}\frac{1}{2}(M+)}\right] - \cos 2\varphi \left[{}^{m_T m_S}_{\frac{1}{2}\frac{1}{2}(M-)}\right] \Big\} \Psi^{(+)}_M(\vec{z}(\varphi), \vec{z}(\varphi-\tfrac{\pi}{2})) \\
& - \sum_C \Big\{ \sin 2\varphi \left[{}^{m_T m_S}_{\frac{1}{2}\frac{1}{2}(M-)}\right] + \cos 2\varphi \left[{}^{m_T m_S}_{\frac{1}{2}\frac{1}{2}(M+)}\right] \Big\} \Psi^{(-)}_M(\vec{z}(\varphi), \vec{z}(\varphi-\tfrac{\pi}{2})) \\
& + \sum_C \Big\{ \sin\varphi \left[{}^{m_T m_S}_{\frac{1}{2}\frac{1}{2}(M-)}\right] + \cos\varphi \left[{}^{m_T m_S}_{\frac{1}{2}\frac{1}{2}}\right] \Big\} \Psi^{(+)}_{M'}(\vec{z}(\varphi), \vec{z}(\varphi-\tfrac{\pi}{2}))
\end{aligned}$$

$$+\sum_C \Big\{-\cos\varphi\ \Gamma^{m_T m_S}_{\frac{1}{2}\frac{1}{2}}(M-)+\sin\varphi\ \Gamma^{m_T m_S}_{\frac{1}{2}\frac{1}{2}}(M+)\Big\}\,\Psi^{(-)}_{M'}(\vec{z}(\varphi),\vec{z}(\varphi-\tfrac{\pi}{2}))$$

$$+\sum_C \Big\{\ \sin\varphi\ \Gamma^{m_T m_S}_{\frac{1}{2}\frac{1}{2}}(M-)+\cos\varphi\ \Gamma^{m_T m_S}_{\frac{1}{2}\frac{1}{2}}(M+)\Big\}\,\Psi^{(+)}_{M''}(\vec{z}(\varphi),\vec{z}(\varphi-\tfrac{\pi}{2}))$$

$$+\sum_C \Big\{-\cos\varphi\ \Gamma^{m_T m_S}_{\frac{1}{2}\frac{1}{2}}(M-)+\sin\varphi\ \Gamma^{m_T m_S}_{\frac{1}{2}\frac{1}{2}}(M+)\Big\}\,\Psi^{(-)}_{M''}(\vec{z}(\varphi),\vec{z}(\varphi-\tfrac{\pi}{2}))$$

$$+\Gamma^{m_T m_S}_{\frac{1}{2}\frac{1}{2}}(S)\ \sum_C\ \Psi^{(-)}_{A'}(\vec{z}(\varphi),\vec{z}(\varphi-\tfrac{\pi}{2}))\,. \qquad \text{(III.14)}$$

In this expression, $\sum_C$ is the cyclic permutation operator and $\Gamma^{m_T m_S}_{TS}(k)$ are the nine orthonormal irreductible representation of spin isospin states for trinucleon system.

We define three operators Σ_0 Σ_+ Σ_- acting on a function of the angular parameter φ via the relations :

$$\Sigma_0 f(\varphi) = \frac{1}{3}\{f(\varphi) + f(\varphi-\frac{2\pi}{3}) + f(\varphi+\frac{2\pi}{3})\}$$

$$\Sigma_+ f(\varphi) = \frac{1}{3}\{2f(\varphi) - f(\varphi-\frac{2\pi}{3}) - f(\varphi+\frac{2\pi}{3})\} \qquad \text{(III.15)}$$

$$\Sigma_- f(\varphi) = \frac{1}{\sqrt{3}}\{f(\varphi-\frac{2\pi}{3}) - f(\varphi+\frac{2\pi}{3})\}\,.$$

We perform the cyclic permutation in (III.14) by using (III.15).

The completely antisymmetric wave function becomes :

$$\begin{aligned}\Psi(\xi,\Omega) = {} & \Gamma_{\frac{1}{2}\frac{1}{2}}(A).\phi^{(0,+)}_S(\xi,\Omega) + \Gamma_{\frac{1}{2}\frac{1}{2}}(S)\,\phi^{(0,-)}_A(\xi,\Omega)\\ & + \Gamma_{\frac{1}{2}\frac{1}{2}}(M-)\{\phi^{(++)}_M(\xi,\Omega) + \phi^{(-,-)}_M(\xi,\Omega)\}\\ & + \Gamma_{\frac{1}{2}\frac{1}{2}}(M+)\{-\phi^{(-,+)}_M(\xi,\Omega) + \phi^{(+,-)}_M(\xi,\Omega)\}\\ & + \Gamma_{\frac{1}{2}\frac{1}{2}}(M-)\{\phi^{(+,+)}_M(\xi,\Omega) - \phi^{(-,-)}_M(\xi,\Omega)\}\\ & + \Gamma_{\frac{1}{2}\frac{1}{2}}(M+)\{\phi^{(-,+)}_M(\xi,\Omega) + \phi^{(+,-)}_M(\xi,\Omega)\}\end{aligned}$$

$$+ \Gamma_{\frac{1}{2}\frac{1}{2}}(M-) \{\phi_M^{(+,+)}(\xi,\Omega) - \phi_M^{(-,-)}(\xi,\Omega)\}$$
$$+ \Gamma_{\frac{1}{2}\frac{1}{2}}(M+) \{\phi_M^{(-,+)}(\xi,\Omega) + \phi_M^{(+,-)}(\xi,\Omega)\} \qquad \text{(III.16)}$$
$$+ \Gamma_{\frac{1}{2}\frac{1}{2}}(S) \phi_A^{(0,-)}(\xi,\Omega) \quad ,$$

with the definitions

$$\phi_\alpha^{(0,\varepsilon)}(\xi,\Omega) = 3\Sigma_0 \Psi_\alpha^{(\varepsilon)}(\vec{z}(\varphi),\vec{z}(\varphi-\tfrac{\pi}{2}))$$
$$\phi_\alpha^{(\varepsilon',\varepsilon)}(\xi,\Omega) = \frac{3}{2}\Sigma_\varepsilon \Psi_\alpha^{(\varepsilon)}(\vec{z}(\varphi),\vec{z}(\varphi-\tfrac{\pi}{2})), \quad \varepsilon'=\pm \quad . \qquad \text{(III.17)}$$

III.3 General method for the construction of the hyperspherical basis

To calculate the completely antisymmetric wave function $\Psi(\xi,\Omega)$ (eq. III.16), we have to construct a function $\Psi_\alpha^{(\varepsilon)}(\vec{z}(\varphi),\vec{z}(\varphi-\frac{\pi}{2}))$ which has a definite symmetry (ε) in the $\vec{z}(\varphi) \to -\vec{z}(\varphi)$. Since according to (II.4) the hyperradius ξ is symmetric, the symmetry characters of the wave function is contained in the angular part which can be extracted from a Fourier transform analysis. Let be $\phi(\vec{k}_1,\vec{k}_2)$ the Fourier transform of $\Psi(\vec{z}(\varphi),\vec{z}(\varphi-\frac{\pi}{2}))$:

$$\Psi(\vec{z}(\varphi),\vec{z}(\varphi-\tfrac{\pi}{2})) = \int d^3k_1 d^3k_2 \phi(\vec{k}_1,\vec{k}_2) \exp(i[\vec{k}_1.\vec{z}(\varphi)+\vec{k}_2.\vec{z}(\varphi-\tfrac{\pi}{2})]) \qquad \text{(III.18)}$$

where
$$\vec{z}(\varphi) = \vec{\xi}_1 \sin\varphi + \vec{\xi}_2 \cos\varphi = \vec{z}_1$$
$$\vec{z}(\varphi-\tfrac{\pi}{2}) = -\vec{\xi}_1 \cos\varphi + \vec{\xi}_2 \sin\varphi = \vec{z}_2 \quad . \qquad \text{(III.19)}$$

We introduce two six dimensional vectors $\vec{k}$ and $\vec{z}$ constructed from the vectors $(\vec{k}_1,\vec{k}_2)$ and $(\vec{z}_1,\vec{z}_2)$ respectively. The vector $\vec{k}$ has for components $k=[\vec{k}_1^2+\vec{k}_2^2]^{\frac{1}{2}}$ and the five angles $\Omega_k=(\hat{k}_1,\hat{k}_2,\phi_k)$ with $tg\phi_k = \frac{|\vec{k}_1|}{|\vec{k}_2|}$. The hyperspherical coordinates of the vector $\vec{z}$ are constituted by $\xi=[\vec{z}_1^2+\vec{z}_2^2]^{\frac{1}{2}}=[\xi_1^2+\xi_1^2]^{\frac{1}{2}}$ and the five angles $\Omega_z=(\hat{z}_1,\hat{z}_2,\phi_z)$ with $tg\phi_z = \frac{|\vec{z}_1|}{|\vec{z}_2|}$.
We expand $\phi(\vec{k}_1,\vec{k}_2)$ and the plane wave in hyperspherical harmonics,

$$\exp i\{\vec{k}_1.\vec{z}_1+\vec{k}_2.\vec{z}_2\} = (2\pi)^3 \sum_{[L]} i^L Y^*_{[L]}(\Omega_k) Y_{[L]}(\Omega_z) \frac{J_{L+2}(k\xi)}{(k\xi)^2} \qquad \text{(III.20)}$$

and

$$\phi(\vec{k}_1,\vec{k}_2) = \sum_{[L']} \phi_{[L']}(k)\, Y_{[L']}(\Omega_k) \qquad \text{(III.21)}$$

The wave function (III.18) becomes :

$$\Psi(\vec{z}_1,\vec{z}_2) = (2\pi)^3 \sum_{[L]} i^L Y_{[L]}(\Omega_z) \int k^5 dk\, \phi_{[L]}(k) \frac{J_{L+2}(k\xi)}{(k\xi)^2} \qquad \text{(III.22)}$$

where all the angular dependance is contained in $Y_{[L]}(\Omega_z)$. The H.H $Y_{[L]}(\Omega_z)$ may be projected (appendix 2) on the complete H.H basis $\{Y_{[L']}(\Omega)\}$ as follows :

$$Y_{[L]}(\Omega_z) = \sum_{[L']} C^{[L']}_{[L]}(\varphi)\, Y_{[L']}(\Omega) \ , \qquad \text{(III.23)}$$

where

$$C^{[L']}_{[L]}(\varphi) = \delta_{LL'} \frac{(2\pi)^3}{4(L+4)(L+3)} \sum_{\substack{[\Lambda][\Lambda'] \\ L=\Lambda+\Lambda'}} (\cos\varphi)^{\Lambda'}.(\sin\varphi)^{\Lambda}(-1)^{n'+\lambda'}$$
$$\cdot \frac{(L+4)!}{(L-\Lambda+2)!(\Lambda+2)!} \left\{\int d\Omega_k Y^*_{[\Lambda]}(\Omega_k) Y^*_{[\Lambda']}(\Omega_k) Y_{[L]}(\Omega_k)\right\} \cdot$$
$$\cdot \int d\Omega\ Y^*_{[L']}(\Omega) Y_{[L]}(\Omega) Y_{[\Lambda']}(\Omega). \qquad \text{(III.24)}$$

By coupling the two angular momenta ℓ_1 and ℓ_2 contained in the set $|L|$ to the angular momentum ℓ, the coefficients $C^{[L']}_{[L]}(\varphi)$ becomes independant of the magnetic quantum numbers leading to

$$Y^{m}_{L(\ell_1\ell_2)\ell}(\Omega,\varphi) = N \sum_{\ell_1'\ell_2'} a^{[L]}_{\ell_1'\ell_2'}(\varphi)\, {}^{(2)}P^{\ell_2'\ell_1'}_{L}(\phi)\, Y^{m}_{(\ell_1'\ell_2')\ell}(\hat{\xi}_1,\hat{\xi}_2) \quad \text{(III.25)}$$

where N is a normalization factor and

$$Y^{m}_{(\ell_1'\ell_2')\ell}(\hat{\xi}_1,\hat{\xi}_2) = \sum_{m_1'm_2'} \overline{\quad\quad} \; (-1)^{\ell_1'-\ell_2'-m} \begin{pmatrix} \ell_1' & \ell_2' & \ell \\ m_1' & m_2' & -m \end{pmatrix} \sqrt{\quad} \; Y^{m_1'}_{\ell_1'}(\xi_1)\, Y^{m_2'}_{\ell_2'}(\xi_2) \,. \quad \text{(III.26)}$$

The coefficiants are

$$a^{[L]}_{\ell_1'\ell_2'}(\varphi) = \frac{\pi}{2} \sum_{\substack{\Lambda\Lambda'\lambda_1'\lambda_2' \\ \lambda_1\lambda_2}} (-1)^{n'+\lambda_2'} \frac{(L+4)!}{(L-\Lambda+2)!\,(\Lambda+2)!} (\cos\varphi)^{\Lambda'} (\sin\varphi)^{L-\Lambda'} \cdot$$

$$\cdot \begin{Bmatrix} \ell_1 & \lambda_1 & \lambda_1' \\ \ell_2 & \lambda_2' & \lambda_2 \\ \ell & \ell_1' & \ell_2' \end{Bmatrix} \cdot \Delta(\lambda_1\lambda_1'\ell_1)\Delta(\lambda_2\lambda_2'\ell_2)\Delta(\ell_1'\lambda_1\lambda_2')\Delta(\ell_2'\lambda_2\lambda_1') \cdot$$

$$\cdot < P^{\lambda_2\lambda_1}_{\Lambda} | P^{\lambda_2'\lambda_1'}_{\Lambda} | P^{\ell_2\ell_1}_{L} > < P^{\ell_2'\ell_1'}_{L} | P^{\ell_2'\ell_1'}_{L'} | P^{\lambda_1'\lambda_2'}_{\Lambda'} > \quad \text{(III.27)}$$

with $\Delta(abc) = [(2a+1)(2b+1)(2c+1)]^{\frac{1}{2}} \begin{pmatrix} a & b & c \\ 0 & 0 & 0 \end{pmatrix}$

and

$$< P^{\lambda_2\lambda_1}_{\Lambda} | P^{\lambda_2'\lambda_1'}_{\Lambda'} | P^{\ell_2\ell_1}_{L} > = \int_0^{\frac{\pi}{2}} d\phi \; {}^{(2)}P^{\lambda_1\lambda_2}_{\Lambda}(\phi)\, {}^{(2)}P^{\lambda_1'\lambda_2'}_{\Lambda'}(\phi)\, P^{\ell_1\ell_2}_{L}(\phi)\, (\sin\phi)^2 (\cos\phi)^2 \quad \text{(III.28)}$$

The coefficients $a^{[L]}_{\ell_1\ell_2}$ are known as Raynal-Revai coefficients

The expansion of the wave function becomes

$$\Psi(\vec{z}(\varphi),\vec{z}(\varphi-\frac{\pi}{2})) = \sum_{\substack{L\ell_1\ell_2\\ \ell m}} \phi_{[L]}(\xi)\ Y^{m}_{(\Omega,\varphi)\,L(\ell_1\ell_2)\ell} \quad . \qquad \text{(III.29)}$$

The expansion of $\Psi(-\vec{z}(\varphi),\vec{z}(\varphi-\frac{\pi}{2}))$ is deduced from (III.29) by replacing the coefficients $a^{[L]}_{\ell_1'\ell_2'}(\varphi)$ by $(-1)^{\ell_1'}\ a^{[L]}_{\ell_1'\ell_2'}(\varphi)$. According to the parity with respect to $\vec{z}(\varphi)$ one defines the wave function

$$\Psi^{(\varepsilon)}(\vec{z}(\varphi),\vec{z}(\varphi-\frac{\pi}{2})) = \sum_{\substack{L\ell_1\ell_2\\ \ell m}} \phi_{[L]}(\xi)\ {}^{(\varepsilon)}Y^{m}_{(\Omega,\varphi)\,L(\ell_1\ell_2)\ell} \quad , \qquad \text{(III.30)}$$

with $${}^{(\varepsilon)}Y^{m}_{(\Omega,\varphi)\,L(\ell_1\ell_2)\ell} = N \sum_{\ell_1'\ell_2'} a^{[L]}_{(\varphi)\,\ell_1'\ell_2'}\ {}^{(2)}P^{\ell_2'\ell_1'}_{L}(\phi)\ Y^{m}_{(\tilde{\xi}_1,\tilde{\xi}_2)\,(\ell_1'\ell_2')\ell} \qquad \text{(III.31)}$$

where $\varepsilon=\pm$ according to the even or odd parity of ℓ_1'. To obtain the expression of the completely antisymmetric wave function (III.16) it is sufficient to know the effect of the operators Σ_0, Σ_+ and Σ_- on the coefficients $a^{[L]}_{\ell_1'\ell_2'}(\varphi)$. One finds

$$\Sigma_0 a^{[L]}_{\ell_1'\ell_2'}(\varphi) = \Sigma_0 \sum_{\Lambda\Lambda'} (\sin\varphi)^{L-\Lambda'}\ (\cos\varphi)^{\Lambda'}\ D^{[L]\,\Lambda\Lambda'}_{\ell_1'\ell_2'}$$

$$= \sum_{\Lambda'} D^{[L]\,0\Lambda'}_{\ell_1'\ell_2'} + \frac{(-1)^L}{2^{L-1}} \sum_{\Lambda'\,\Lambda\text{even}\neq 0} 3^{\frac{\Lambda}{2}-1} D^{[L]\,\Lambda\Lambda'}_{\ell_1'\ell_2'}$$

$$\Sigma_+ a^{[L]}_{\ell_1'\ell_2'}(\varphi) = (-1)^{L+1}.2^{1-L}. \sum_{\Lambda'\,\Lambda\text{even}} 3^{\frac{\Lambda}{2}-1} D^{[L]\,\Lambda\Lambda'}_{\ell_1'\ell_2'}$$

$$\Sigma_- a^{[L]}_{\ell_1'\ell_2'}(\varphi) = (-1)^{L+1}.2^{1-L}. \sum_{\Lambda'\,\Lambda\text{odd}} 3^{\frac{\Lambda}{2}-1} D^{[L]\,\Lambda\Lambda'}_{\ell_1'\ell_2'} \quad , \qquad \text{(III.32)}$$

in terms of the coefficients (see eqs. III.27)

$$D^{L\,\Lambda\Lambda'}_{\ell_1\ell_2\,\ell_1'\ell_2'} = \frac{\pi}{2} \sum_{\substack{\lambda_1\lambda_2 \\ \lambda_1'\lambda_2'}} (-1)^{n'+\lambda_2'} \frac{(L+4)!}{(L-\Lambda+2)!\,(\Lambda+2)!} \begin{Bmatrix} \ell_1 & \lambda_1 & \lambda_1' \\ \ell_2 & \lambda_2' & \lambda_2 \\ \ell & \ell_1' & \ell_2' \end{Bmatrix} \Delta(\lambda_1\lambda_1'\ell_1)\Delta(\lambda_2\lambda_2'\ell_2)$$

$$\Delta(\ell_1'\lambda_1\lambda_2')\Delta(\ell_2'\lambda_2\lambda_1') \langle P^{\lambda_2\lambda_1}_{\Lambda} | P^{\lambda_2'\lambda_1'}_{\Lambda'} | P^{\ell_2\ell_1}_{L} \rangle \langle P^{\ell_2'\ell_1'}_{L} | P^{\ell_2'\ell_1'}_{L'} | P^{\lambda_1'\lambda_2'}_{\Lambda'} \rangle . \qquad \text{(III.32)}$$

III.4 The optimal subset and potential basis

A potential independent of Ω is hypercentral. It is a function of coordinate the ξ only. A sum of two body interactions is not hypercentral in the six dimensional space (except for harmonic oscillator potential) because it contains hyperspherical deformations described by the various components of the H.H expansion of the potential. These components in turn generate hyperspherical deformations in the wave function. When the deformation of the potential is small with respect to the hypercentral term, it seems justified to use a perturbation expansion of the wave function in terms of the deformed part of the potential. Assume that a state $\Psi(\vec{\xi},s,t)$ is described to a good approximation by a wave function $\Psi_0(\vec{\xi},s,t)$ in such a way that $\langle\psi|\psi_0\rangle \simeq 1$. One defines an optimal subset with respect to this state by stating that any element $\Psi_k(\vec{\xi},s,t)$ orthogonal to $\psi_0(\vec{\xi},s,t)$ must fulfill the condition

$$\langle \Psi_k | \mathcal{V}(\vec{\xi}) | \Psi_0 \rangle = \int d\Omega \; \Psi_k^*(\vec{\xi},s,t) \; \mathcal{V}(\vec{\xi}) \Psi_0(\vec{\xi},s,t) \neq 0 \; . \qquad \text{(III.34)}$$

In this case the exclusive use of the $\Psi_k(\vec{\xi},s,t)$ subset enables one to take completely into account a perturbative calculation up to the third order included. In the hyperspherical formalism $\Psi_0(\vec{\xi},s,t)$ is the product of $\phi_0(\xi)$ and a H.H $B_0(\Omega,s,t)$ fully antisymmetric. The elements $B_k(\Omega,s,t)$ of a hyperspherical optimal subset with respect to $B_0(\Omega,s,t)$ must fulfill the conditions :

$$\begin{aligned} &\int d\Omega \; B_k^*(\Omega,s,t) \, . \, B_{k'}(\Omega,s,t) = \delta_{kk'} \\ &\int d\Omega \; B_k^*(\Omega,s,t) \quad V(\xi\Omega) B_0(\Omega,s,t) \neq 0. \end{aligned} \qquad \text{(III.35)}$$

For determining the "hyperspherical state" $B_0(\Omega,s,t,)$ describing the ground state one uses the general theorem which states, that in the case of hypercentral potentials the grand orbital L is minimal for the ground state. This property remains valid when the potential is slightly deformed i.e. when the first term K=0 in the expansion (II.32) of the potentials is largely predominant. In the trinucleon system the minimal value of L is reached when all the nucleons are in the S state. In this case $L_{min}=0$ and the hyperspherical ground state $B_0(\Omega,s,t)$ is a product of the completely antisymmetric spin-isospin state and $\mathcal{Y}_{[0]}(\Omega)$ i.e. :

$$B_0(\Omega,s,t) = \Gamma_{\frac{1}{2}\frac{1}{2}}(A) \; \mathcal{Y}_{[0]}(\Omega) = \pi^{-3/2} \; \Gamma_{\frac{1}{2}\frac{1}{2}}(A) \, . \qquad \text{(III.36)}$$

It is clear from the conditions (III.35) that the optimal subset depends essentially upon the nature of the interaction (i.e. two body or many body forces with or without exchange properties,

central or tensor interaction...). The potential is an even function therefore the H.H expansion,

$$V(\xi,\Omega) = \sum_{K=0}^{\infty} a_{2K} \; P_{2K}(\Omega) \; \mathcal{V}_{2K}(\xi) \qquad \text{(III.37)}$$

contains only even H.H (i.e with grand orbital even), a_{2K} are constants. The H.H $P_{2K}(\Omega)$ of grand orbital 2K which include spin-isospin operators $(\vec{\sigma},\vec{\tau})$ is an element of the so called "potential basis" and $\mathcal{V}_{2K}(\xi)$, the multipoles of the potential are scalar functions of ξ.

The conditions (III.35) lead immediately for the ground state $\Gamma_{\frac{1}{2}\frac{1}{2}}(A)\; Y_{[0]}(\Omega)$ to the H.H optimal subset,

$$B_{2K}(\Omega,s,t) = C_{2K} \; P_{2K}(\Omega,\vec{\sigma},\vec{\tau}) \; \Gamma_{\frac{1}{2}\frac{1}{2}}(A) \qquad \text{(III.38)}$$

where C_{2K} are normalization constants.

III.5 Optimal subsets for central and tensor forces

The H.H expansion of the completely antisymmetric wave function $\Psi^{TM_T}_{(LS)JM_J}(\xi,\Omega)$ of the trinucleon is made by using the potential H.H operator (II.34) symmetric under the interchange $i \leftrightarrow j$ for ℓ even (ℓ=0 and ℓ=2 respectively for central and tensor potentials) therefore the completely antisymmetric wave function (III.14) is restricted to the components,

$$\Psi(\xi,\Omega) = \Gamma_{\frac{1}{2}\frac{1}{2}}(A)\ \overset{(0,+)}{\phi_S}(\xi\Omega) + \Gamma_{\frac{1}{2}\frac{1}{2}}(M-)\ \overset{(+,+)}{\phi_M}(\xi\Omega) - \Gamma_{\frac{1}{2}\frac{1}{2}}(M+)\ \overset{(-,+)}{\phi_M}(\xi\Omega)$$

$$+ \Gamma_{\frac{1}{2}\frac{1}{2}}(M-)\ \overset{(+,+)}{\phi_{M'}}(\xi\Omega) + \Gamma_{\frac{1}{2}\frac{1}{2}}(M+)\ \overset{(-,+)}{\phi_{M'}}(\xi\Omega) + \Gamma_{\frac{1}{2}\frac{1}{2}}(M-)\ \overset{(+,+)}{\phi_{M''}}(\xi\Omega)$$

$$+ \Gamma_{\frac{1}{2}\frac{1}{2}}(M+)\ \overset{(-,+)}{\phi_{M''}}(\xi\Omega)\ . \qquad \text{(III.39)}$$

The isospin-spin functions $\Gamma(R)$ for $R=(A,M+,M-)$ must be combined with spatial wave functions of conjugate symmetry $R^*=(0,-,+)$ generated by the operator defined by eq. (III.15). Then we define the orthonormalized elements of the optimal subset,

$$\overset{R^*}{B}{}^{m}_{2K+\ell}(\Omega) = \overset{R^*}{N}_{2K+\ell} \sum_{\ell_1\ell_2} (-1)^{\ell} \sqrt{(2\ell_1+1)(2\ell_2+1)} \begin{pmatrix} \ell & \ell_1 & \ell_2 \\ 0 & 0 & 0 \end{pmatrix} \left(\sum_{R^*} \overset{(2)}{P}{}^{\ell_2\ell_1}_{(\varphi)\,2K+\ell} \right)$$

$$\cdot\ \overset{(2)}{P}{}^{\ell_2\ell_1}_{(\phi)\,2K+\ell} \cdot Y^{m}_{(\ell_1\ell_2)\ell}(\hat{\xi}_1,\hat{\xi}_2) \quad , \qquad \text{(III.40)}$$

where $\overset{R^*}{N}_{2K+\ell}$ is a normalization factor. In the $(\ell S)JM$ subspace the completely antisymmetric function becomes

$$\Psi^{TM_T}_{(\ell S)JM_J}(\xi,\Omega) = \sum_{KR} \rho_{TS}(R)\, u^{RTS}_{2K+\ell}(\xi)\ \xi^{-5/2}\ [\Gamma_{TS}(R) \otimes \overset{R^*}{B}{}^{M_J}_{2K+\ell}(\Omega)]^{J}_{M_J} \qquad \text{(III.41)}$$

the coefficient $\rho_{TS}(R)=\pm 1$ according to the construction (III.39). To explicit the tensorial product appearing in eq. (III.41), we use the following expansion of the isospin-spin states $\Gamma^{m_T m_S}_{TS}(R)$

$$\Gamma^{m_T m_S}_{TS}(R) = \sum_{st} b^{t}_{s}(R)\, |(s\tfrac{1}{2})Sm_S\rangle |(t\tfrac{1}{2})Tm_T\rangle \qquad \text{(III.42)}$$

where the coefficients $b^{t}_{s}(R)$ are given in eqs. (A9-A17).

The elements of the optimal subset in the (ℓS)JM subspace take the general form :

$$|| \Gamma_{TS}(R) \otimes {}^{R^*}B_{2K+\ell}(\Omega) \; |_J^{M_J} >= {}^{R^*}N_{2K+\ell} \sum_{\ell_1 \ell_2 s t m m_S} b_s^t(R) \; (-1)^{S-M_J} \hat{\ell}_1 \hat{\ell}_2 \hat{J} \; .$$

$$\begin{pmatrix} \ell & \ell_1 & \ell_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell & S & J \\ m & m_S & -M_J \end{pmatrix} \left\{ \sum_{R^*} {}^{(2)}P^{\ell_2 \ell_1}_{2K+\ell (\varphi)}(1,j) \right\} \left\{ {}^{(2)}P^{\ell_2 \ell_1}_{2K+\ell (\phi)} \right| (\ell_1 \ell_2) \ell m ; (s\tfrac{1}{2}) S m_S ;$$

$$(t\tfrac{1}{2}) T m_T > \; . \qquad \text{(III.43)}$$

The use of this optimal subset enables one to calculate the trinucleon states $^{2,2}S_{\frac{1}{2}}$ for $\ell=0$ and $^{2,4}D_{\frac{1}{2}}$ for $\ell=2$.
The contribution of the two components of the $^{2,2}S_{\frac{1}{2}}$ state proceeding from the space completely symmetric and mixed symmetry states (in choosing the pair (1,2) as a reference $\varphi^{(1,2)}=\pi/2$) is :

$$\Psi^{\frac{1}{2}m_T}_{(0\frac{1}{2})\frac{1}{2}\frac{1}{2}}(\xi\Omega) = \sum_K \Gamma_{\frac{1}{2}\frac{1}{2}}(A) \cdot P^{(0)}_{2K}(\Omega) \cdot u^{S\frac{1}{2}\frac{1}{2}}_{2K}(\xi) \; \xi^{-\frac{5}{2}} +$$

$$\sum_K \frac{1}{\sqrt{2}} \left\{ \Gamma_{\frac{1}{2}\frac{1}{2}}(M-) P^{(+)}_{(\Omega)\,2K} - \Gamma_{\frac{1}{2}\frac{1}{2}}(M+) P^{(-)}_{(\Omega)\,2K} \right\} u^{M\frac{1}{2}\frac{1}{2}}_{2K}(\xi) \; \xi^{-\frac{5}{2}} \qquad \text{(III.44)}$$

in terms of the orthonormal H.H

$$P^{(\varepsilon)}_{(\Omega)\,2K} = {}^{(\varepsilon)}B_{2K}(\Omega) = {}^{\varepsilon}N_{2K} \sum_{\ell=0}^{X} {}^{\varepsilon}F^{\ell\ell}_{2K}(\pi/2) \; {}^{(\varepsilon)}P^{\ell,\ell}_{2K\,(\phi)} \sum_{m=-\ell}^{+\ell} Y^m_\ell(\hat{\xi}_1) Y^{m^*}_\ell(\hat{\xi}_2) \qquad \text{(III.45)}$$

in which the symmetry is introduced by the function

$${}^{\varepsilon}F^{\ell,\ell}_{2K\,(\varphi)} = \sum_{\varepsilon} {}^{(2)}P^{\ell,\ell}_{2K\,(\varphi)} \; . \qquad \text{(III.46)}$$

$${}^{\varepsilon}N^{-2}_{2K} = \sum_{\ell=0}^{K} (2\ell+1) [{}^{\varepsilon}F^{\ell,\ell}_{2K}]^2 \qquad \text{(III.47)}$$

is a normalization constant.

A similar expression is obtained for the wave function of the $^{2,4}D_{\frac{1}{2}}$ state :

$$\Psi^{\frac{1}{2}m_T}_{(2\frac{1}{2})\frac{1}{2}}(\xi,\Omega) = \sum_K \xi^{-\frac{5}{2}}\, u^{M\frac{1}{2}\frac{1}{2}}_{2K+2}(\xi)\,\frac{1}{\sqrt{2}}\,\{\,{}^{(+)}\mathcal{D}^{(\Omega)}_{2K+2}\,|(0\tfrac{1}{2})\tfrac{1}{2}m_T\rangle + \\ + {}^{(-)}\mathcal{D}^{(\Omega)}_{2K+2}\,|(1\tfrac{1}{2})\tfrac{1}{2}m_T\rangle\} \qquad \text{(III.48)}$$

with

$${}^{(\epsilon)}\mathcal{D}^{(\Omega)}_{2K+2} = {}^{\epsilon}N_{2K+2}\sum_{\ell_1\ell_2}\hat{\ell}_1\hat{\ell}_2\begin{pmatrix}2 & \ell_1 & \ell_2\\ 0 & 0 & 0\end{pmatrix}{}^{\epsilon}F^{\ell_2\ell_1}_{2K+2}(\pi/2)\,{}^{(2)}P^{\ell_2\ell_1}_{2K+2}(\phi)\,\langle\hat{\xi}_1\hat{\xi}_2| \\ |(\ell_1\ell_2)2;(1\tfrac{1}{2})\tfrac{5}{2};\tfrac{1}{2}\tfrac{1}{2}\rangle \quad . \qquad \text{(III.49)}$$

and the normalization factor.

$${}^{\epsilon}N^{-2}_{2K+2} = \sum_{\ell_1\ell_2}(2\ell_1+1)(2\ell_2+1)\begin{pmatrix}2 & \ell_1 & \ell_2\\ 0 & 0 & 0\end{pmatrix}{}^{\epsilon}F^{\ell_2\ell_1}_{2K+2}(\pi/2)]^2 \qquad \text{(III.50)}$$

III.6 The coupled equations for local realistic potentials

The partial waves $u^{RTS}_{2K+\ell}(\xi)$ of the H.H expansion of the trinucleon wave function

$$\Psi^{JM_J}(\xi\Omega) = \sum_{ST\ell}\Psi^{TM_T}_{(\ell S)JM_J}(\xi\Omega) = \sum_{KRST\ell}\rho_{TS}(R)\,u^{RTS}_{2K+\ell}(\xi)\,\xi^{-\frac{5}{2}}\,[\,\Gamma^{(R)}_{TS}\otimes\mathcal{B}^{K^*}_{2K+\ell}(\Omega)\,]^{M_J}_J \qquad \text{(III.51)}$$

are solution of the coupled differential equations (III.17)

$$\left\{-\frac{\hbar^2}{m}\left[\frac{d^2}{d\xi^2}-\frac{(2K+\ell+2)^2-\frac{1}{4}}{\xi^2}\right]-E\right\}\ U^{RTS}_{2K+\ell}(\xi)\ +$$

$$\sum_{K'R'T'S'\ell'} \rho_{TS}(R)\,\rho_{T'S'}(R')\,\langle[\Gamma_{TS}(R)\otimes {}^{R^*}B_{2K+\ell}]^1_J\ V(\xi\Omega)\,|[\Gamma_{T'S'}(R')\otimes {}^{R'^*}B_{2K'+\ell'}]^1_J\rangle \cdot U^{R'T'S'}_{2K'+\ell'}(\xi) \qquad \text{(III.52)}$$

The expressions of $\Gamma^{m_T m_S}_{TS}(R)$, ${}^{R^*}B_{2K+\ell}(\Omega)$ and $V(\xi,\Omega)$ given respectively by equations (III.42), (III.40), (II.32) lead to the set of coupled equations

$$\left\{-\frac{\hbar^2}{m}\left[\frac{d^2}{d\xi^2}-\frac{(2K+\ell+2)^2-\frac{1}{4}}{\xi^2}\right]-E\right\}\ U^{RTS}_{2K+\ell}(\xi)\ +$$

$$\sum_{\substack{K''K'R'T'S'\\ sts't'\ell''\gamma}} (-)^{K''}\ C^{(2K'+\ell')R'J}_{(K'',\gamma,\ell'')(2K+\ell)RJ}\ V^{\gamma,\ell''}_{2K''}(\xi,R,R')\ U^{R'T'S'}_{2K'+\ell'}(\xi) = 0 \qquad \text{(III.53)}$$

in terms of the coupling coefficients $C^{(2K'+\ell')R's'S'J}_{(K'',\gamma,\ell'')(2K+\ell)RsSJ}$ given in appendix B. The potential matrix $V^{\gamma,\ell''}_{2K''}(\xi,R,R')$ is given in appendix C.

III.7 Coupled equations for a velocity-dependent potential

During the past and more recently in Orsay several authors (40) have shown that velocity dependent potentials could give as good fit in the relevant two body data and nuclear matter in nuclear physics as the hard core potentials (38,35) and also the class of super soft core potentials (35,22,20). It is then interesting to analyze the static and dynamic properties of the three body problem with these less conventional interactions. However the corresponding equations in the hyperspherical formalism are not obvious.

In the general case we must determine the matrix elements of the two body interaction $V(\xi,\Omega)$ appearing in the equation (III.52) with a velocity dependent term which reads :

$$\vec{\nabla}^2_{ij} \quad \omega\,(r_{ij}) + \quad \omega\,(r_{ij})\vec{\nabla}^2_{ij} \quad . \tag{III.54}$$

To avoid cumbersome calculations we give the method for the part of the optimal subset corresponding to a central potential (III.45). Then we have to analyze the action of the operator (III.54) on the set of hyperspherical harmonics contained in the $Y^{0}_{2K(\ell\ell)0}(\Omega)$:

$$Y^{0}_{2K(\ell\ell)0}(\Omega) = \frac{1}{\sqrt{2\ell+1}} \; {}^{(2)}P^{\ell\ell}_{2K}(\phi) \sum_{m=-\ell}^{+\ell} Y^{m}_{\ell}(\hat{\xi}_1)\, Y^{m*}_{\ell}(\hat{\xi}_2) \,. \tag{III.55}$$

Choosing the particles 1 and 2 as reference pair, $\vec{\xi}_1=\vec{x}_2-\vec{x}_1$ the velocity dependent operator becomes,

$$\vec{\nabla}^2_{\xi_1} \quad \omega(\xi_1)+\omega(\xi_1)\vec{\nabla}^2_{\xi_1} = \frac{d^2\omega}{d\xi_1^2} + \frac{2}{\xi_1}\,\frac{d\omega}{d\xi_1} + \frac{d\omega}{d\xi_1}\,\frac{\partial}{\partial\xi_1} + 2\,\omega(\xi_1)\vec{\nabla}^2_{\xi_1} \quad . \tag{III.56}$$

We symmetrize the two last terms of the previous formula according,

$$\frac{d\omega}{d\xi_1}\frac{\partial}{\partial\xi_1}+2\omega(\xi_1)\nabla^2_{\xi_1}=\frac{1}{2}\frac{d}{d\xi_1}\omega(\xi_1)\;\xi_1\left\{\left(\frac{1}{\xi_1}\frac{d}{d\xi_1}+\frac{1}{\xi_2}\frac{d}{d\xi_2}+\frac{1}{\xi_1}\frac{d}{d\xi_1}-\frac{1}{\xi_2}\frac{d}{d\xi_2}\right)\right.$$

$$\left.+\;\omega(\xi_1)\;\left[(\nabla^2_{\xi_1}+\nabla^2_{\xi_2})+(\nabla^2_{\xi_1}-\nabla^2_{\xi_2})\right]\right. \qquad \text{(III.57)}$$

With this expression the action of the operator (III.56) on the H.H (III.55) is :

$$\left[\nabla^2_{\xi_1}\;\omega(\xi_1)+\omega(\xi_1)\nabla^2_{\xi_1}\right]Y^{0\ (\Omega)}_{2K(\ell\ell)0}=\frac{\sqrt{\pi}}{4}\sum_{K'=0}^{\infty}(K'+1)\;{}^{(2)}P^{00}_{2K'}(\phi).$$

$$\left\{Y^{0\ (\Omega)}_{2K(\ell\ell)0}\left[{}^{(2)}\omega_{2K'}(\xi)+{}^{(1)}\omega_{2K'}(\xi)\;\xi\;\frac{\partial}{\partial\xi}+{}^{(0)}\omega_{2K'}(\xi)\;\left(\frac{\partial^2}{\partial\xi^2}+\frac{5}{\xi}\frac{\partial}{\partial\xi}-\frac{4K(K+2)}{\xi^2}\right)\right]\right.$$

$$-a^{\ell}_{K}\;Y^{0\ (\Omega)}_{2(K+1)(\ell\ell)0}\left[{}^{(1)}\omega_{2K'}(\xi)\left(\xi\frac{\partial}{\partial\xi}+2K\right)+{}^{(0)}\omega_{2K'}(\xi)\;\left\{\frac{\partial^2}{\partial\xi^2}\right.\right.$$

$$\left.\left.-\frac{4K+1}{\xi}\frac{\partial}{\partial\xi}+\frac{4K(K+1)}{\xi^2}\right\}\right]$$

$$-a^{\ell}_{K-1}\;Y^{0\ (\Omega)}_{2(K-1)(\ell\ell)0}\left[{}^{(1)}\omega_{2K'}(\xi)\;\left(\xi\frac{\partial}{\partial\xi}-2(K+2)\right)+{}^{(0)}\omega_{2K'}(\xi)\left(\frac{\partial^2}{\partial\xi^2}+\frac{4K+7}{\xi}\frac{\partial}{\partial\xi}\right.\right.$$

$$\left.\left.+\frac{4(K+1)(K+2)}{\xi^2}\right)\right]\right\} \qquad \text{(III.58)}$$

with

$$a^{\ell}_{K}=\frac{1}{2}\left[\frac{(K+\ell+2)(K-\ell+1)}{(K+1)(K+2)}\right]^{\frac{1}{2}} \qquad K\geq\ell$$

$${}^{(0)}\omega(\xi_1)=\omega(\xi_1)$$

$${}^{(1)}\omega(\xi_1)=\frac{1}{2\xi_1}\frac{d}{d\xi_1}\;{}^{(0)}\omega(\xi_1)$$

$${}^{(2)}\omega(\xi_1)=\left(\frac{d^2}{d\xi_1^2}+\frac{2}{\xi_1}\frac{d}{d\xi_1}\right){}^{(0)}\omega(\xi_1)$$

where $\omega^{(i)}_{2K}(\xi)$ are the multipoles of the corresponding potentials $\omega^{(i)}(\xi_1)$. The matrix elements of the velocity dependent potential are :

$$
\begin{aligned}
&\langle Y^{0}_{2K(\ell\ell)0} | \vec{\nabla}^2_{\xi_1} \omega(\xi_1) + \omega(\xi_1) \vec{\nabla}^2_{\xi_1} | Y^{0}_{2K'(\ell\ell)0} \rangle \xi^{-\frac{5}{2}} \; u_{2K'}(\xi) = \\
&\frac{\sqrt{\pi}}{4} \xi^{-\frac{5}{2}} \sum_{K''=0}^{\infty} (K''+1) \left\{ \omega^{(0)}_{2K''}(\xi) \left[\langle {}^{(2)}P^{\ell\ell}_{2K} | {}^{(2)}P^{00}_{2K''} | {}^{(2)}P^{\ell\ell}_{2K} \right.\right. \\
&- a^{\ell}_{K'} \, {}^{(2)}P^{\ell\ell}_{2(K'+1)} - a^{\ell}_{K'-1} \, {}^{(2)}P^{\ell\ell}_{2(K'-1)} \rangle \frac{\partial^2}{\partial \xi^2} + \langle {}^{(2)}P^{\ell\ell}_{2K} | {}^{(2)}P^{00}_{2K''} | \\
&a^{\ell}_{K'}(2b_{K'}+3) \, {}^{(2)}P^{\ell\ell}_{2(K'+1)} - a^{\ell}_{K'-1}(2b_{K'}-1) \, {}^{(2)}P^{\ell\ell}_{2(K'-1)} \rangle \frac{1}{\xi} \frac{\partial}{\partial \xi} \\
&- \langle {}^{(2)}P^{\ell\ell}_{2K} | {}^{(2)}P^{00}_{2K''} | b_{K'}(b_{K'}+1) \, {}^{(2)}P^{\ell\ell}_{2K'} + a^{\ell}_{K}(b_{K'}+1)(b_{K'}+3) \, {}^{(2)}P^{\ell\ell}_{2(K'+1)} \\
&+ a^{\ell}_{K'-1} \, b_{K'}(b_{K'}-2) \, {}^{(2)}P^{\ell\ell}_{2(K'-1)} \rangle \frac{1}{\xi^2} \Big] + \omega^{(1)}_{2K''}(\xi) \langle {}^{(2)}P^{\ell\ell}_{2K} | {}^{(2)}P^{00}_{2K''} | \\
&{}^{(2)}P^{\ell\ell}_{2K'} - a^{\ell}_{K'} \, {}^{(2)}P^{\ell\ell}_{2(K'+1)} - a^{\ell}_{K'-1} \, {}^{(2)}P^{\ell\ell}_{2(K'-1)} \rangle \xi \frac{\partial}{\partial \xi} \\
&- \langle {}^{(2)}P^{\ell\ell}_{2K} | {}^{(2)}P^{00}_{2K''} | \frac{5}{2} \, {}^{(2)}P^{\ell\ell}_{2K'} + a^{\ell}_{K'}(b_{K'}-4) \, {}^{(2)}P^{\ell\ell}_{2(K'+1)} - \\
&\qquad - a^{\ell}_{(K'-1)}(b_{K'}+5) \, {}^{(2)}P^{\ell\ell}_{2(K'-1)} \rangle \Big] \\
&\left. + \omega^{(2)}_{2K''}(\xi) \langle {}^{(2)}P^{\ell\ell}_{2K} | {}^{(2)}P^{00}_{2K''} | {}^{(2)}P^{\ell\ell}_{2K'} \rangle \right\} u_{2K'}(\xi \quad , \qquad \text{(III.59)}
\end{aligned}
$$

where $b_K = 2K + \frac{3}{2}$.

To obtain the coupled equations one reconstructs the optimal subset (III.45) in terms of the $Y^{0}_{2K(\ell\ell)0}(\Omega)$,

$$< P^{(\varepsilon)}_{2K}(\Omega) | \nabla^2_{\xi_1} \omega(\xi_1) + \omega(\xi_1) \nabla^2_{\xi_1} | P^{(\varepsilon')}_{2K'} > \xi^{-\frac{5}{2}} \; u_{2K}(\xi) =$$

$$c N_{2K} \; \varepsilon' N_{2K'} \sum_{\ell=0}^{\min(K,K')} (2\ell+1) \; \varepsilon^{\ell\ell}_{2K} \; \varepsilon'^{\ell\ell}_{2K'} < Y^0_{2K(\ell\ell)0} | \nabla^2_{\xi_1} \omega(\xi_1) +$$

$$\omega(\xi_1) \nabla^2_{\xi} | Y^0_{2K'(\ell\ell)0} > \xi^{-\frac{5}{2}} \; u_{2K'}(\xi) \quad , \qquad \text{(III.60)}$$

leading to new coupled equations.

IV.1 Convergence of the hyperspherical harmonic expansion

The convergence trend depends on the nature of the shape of the N-N interaction. For studying the convergence in terms of the number of solved coupled equations, the completely symmetric S state only will be investigated for the sake of simplicity. It can be extended without ambiguity to the other S' and D states. In the coupled equations (III.53) relative to a central two body potential with the fully symmetric wave function only, we separate out the main K=0 equation writting :

$$\left\{-\frac{\hbar^2}{m}\left[\frac{d^2}{d\xi^2}-\frac{\nu_0^2-1}{\xi^2}\right]+\mathcal{V}_0(\xi)-E\right\}u_0(\xi)+\sum_{K\neq 0}(-1)^K\langle K|K|0\rangle\ \mathcal{V}_{2K}(\xi)u_{2K}(\xi)=0$$

$$\left\{-\left[\frac{\hbar^2}{m}\frac{d^2}{d\xi^2}-\frac{\nu_K^2-1}{\xi^2}\right]+\mathcal{U}_K^K(\xi)-E\right\}u_{2K}(\xi)+\sum_{\substack{K'\neq K\\ K''}}(-1)^{K''}\langle K|K''|K'\rangle\ \mathcal{V}_{2K''}(\xi)u_{2K'}(\xi)=0 \qquad \text{(IV.1)}$$

where

$$\nu_K = 2(K+1)$$

while $\langle K|K''|K'\rangle = C_{2K}^{2K'}(K'',0,0)$ is a coupling coefficient and $\mathcal{U}_K^K(\xi)=\sum_{K''}(-1)^{K''}\langle K|K''|K\rangle\ \mathcal{V}_{2K''}(\xi)$ is a diagonal term of the potential matrix. One introduces the change of function,

$$u_{2K}(\xi) = G_{2K}(\xi)\ u_0(\xi) \qquad \text{(IV.2)}$$

with $G_0(\xi)\equiv 1$

leading to the new equation equivalent to (IV.1)

$$G_{2K}(\xi)\left\{-\frac{\hbar^2}{m}\left[\frac{d^2}{d\xi^2}-\frac{\nu_0^2-1}{\xi^2}\right]+W(\xi)-E\right\}u_0(\xi)-\frac{2\hbar^2}{m}\frac{d}{d\xi}G_{2K}(\xi)\cdot\frac{d}{d\xi}u_0(\xi)$$

$$+u_0(\xi)\left\{\left(-\frac{\hbar^2}{m}\left[\frac{d}{d\xi^2}-\frac{\nu_K^2-\nu_0^2}{\xi^2}\right]+\left[U_K^K(\xi)-W(\xi)\right]\right)G_{2K}(\xi)+\right.$$

$$\left.+\sum_{\substack{K''\\K'\neq K}}\langle K|K''|K'\rangle\, V_{2K''}(\xi)G_{2K'}(\xi)\right\}=0 \quad , \qquad \text{(IV.3)}$$

where $W(\xi)$ is an effective potential acting on the main $K=0$ partial wave according to

$$\left\{-\frac{\hbar^2}{m}\left[\frac{d^2}{d\xi^2}-\frac{\nu_0^2-1}{\xi^2}\right]+W(\xi)-E\right\}u_0(\xi)=0 \qquad \text{(IV.4)}$$

In the equation (IV.3) one makes the following approximations :

i) the term $-\frac{2\hbar^2}{m}\frac{d}{d\xi}G_{2K}(\xi)\cdot\frac{d}{d\xi}u_0(\xi)$ is neglected because the major contribution in $u_{2K}(\xi)$ comes from the vicinity of the point ξ where the partial wave $u_0(\xi)$ is maximum i.e. where $\frac{d}{d\xi}u_0(\xi)=0$, (notice that according to the numerical analysis the partial wave $u_0(\xi)$ contributes around 98% to the total wave function).

ii) the terms $\frac{d^2}{d\xi^2}G_{2K}(\xi)$ and $\left(U_K^K(\xi)-W(\xi)\right)G_{2K}(\xi)$ are neglected with respect to the centrifugal term $\frac{\hbar^2}{m}\frac{\nu_K^2-\nu_0^2}{\xi^2}$ for large K values.

Taking into account these two approximations (IV.3) becomes

$$\frac{\hbar^2}{m}\frac{(\nu_K^2-\nu_0^2)}{\xi^2}\cdot G_{2K}(\xi)+\sum_{\substack{K''\\K'\neq K}}\langle K|K''|K'\rangle\, V_{2K''}(\xi)G_{2K'}(\xi)=0 \qquad \text{(IV.5)}$$

with

$$G_0(\xi)\equiv 1 \quad \text{and} \quad \nu_K^2-\nu_0^2=4K(K+2).$$

If one neglects $G_{2K'}(\xi)$ in the last term of the previous equation with respect to the unity, one obtains :

$$G_{2K}(\xi) = -\frac{m}{\hbar^2}\,\xi^2\,\frac{\langle K|X|0\rangle}{4K(K+2)}\,\mathcal{V}_{2K}(\xi)\,. \qquad (IV.6)$$

introducing $G_{2K}(\xi)$ in (IV.3) one obtains (IV.4) where the effective potential,

$$W(\xi) = \mathcal{V}_0(\xi) - \frac{\xi^2 m}{4\hbar^2}\sum_{K\neq 0}\frac{\langle K|X|0\rangle^2}{K(K+2)}\,\mathcal{V}^2_{2K}(\xi) \qquad (IV.7)$$

contains an estimate of the effect of the partial waves $K\neq 0$ on the binding energy E and on the main partial wave $u_0(\xi)$.

Assuming that the system of coupled equations has been solved numerically up to a value K_{max}, the missing energy with respect to the exact value is given by :

$$\Delta E = -\langle u_0|\frac{m}{4\hbar^2}\sum_{K=K_{max}+1}^{\infty}\frac{\langle K|X|0\rangle^2}{K(K+2)}\,(\xi\,\mathcal{V}_{2K}(\xi))^2|u_0\rangle\,. \qquad (IV.8)$$

The convergence trend proceed from the variation with K_{max} of

$$\sum_{K=K_{max}+1}^{\infty}\frac{\langle K|X|0\rangle^2}{K(K+2)}\,(\xi\,\mathcal{V}_{2K}(\xi))^2 \qquad (IV.9)$$

which depends on the shape of the two body interaction through the multipoles

$$\mathcal{V}_{2K}(\xi) = \frac{32\pi}{\xi^2}\int_0^{\infty} J_{2K+2}(q\xi)\,\mathcal{V}(q)\,dq \qquad (IV.10)$$

where $\mathcal{V}(q)$ is the Fourier transform of the two body potential.

α - Yukawa potential and Coulomb potential

The Fourier transform,

$$V_y(q) = \frac{V_0}{2\pi^2} \cdot \frac{1}{q^2+\mu^2} = \frac{V_0}{2\pi^2} \frac{1}{q^2} \left(1 - \frac{\mu^2}{q^2} + \ldots \right) \qquad \text{(IV.11)}$$

of the Yukawa potential

$$V_y(r_{ij}) = V_0 \frac{e^{-\mu r_{ij}}}{r_{ij}} \qquad \text{(IV.12)}$$

generates an expansion of the multipole in term of μ^2

$$\xi \, \mathcal{V}_{2K}^{(y)}(\xi) = \frac{4V_0}{\pi} \left\{ \frac{1}{(K+\frac{1}{2})(K+\frac{1}{2})} - \frac{\mu^2\xi^2}{4(K+\frac{1}{2})(K+\frac{1}{2})(K+\frac{1}{2})(K-\frac{1}{2})} + \ldots \right\} \qquad \text{(IV.13)}$$

which in turn provide a convergence trend

$$\sum_{K=K_{max}+1}^{\infty} \frac{\langle K|K|0\rangle^2}{K(K+2)} \frac{16V_0^2}{\pi^2} \left[\frac{1}{(K+\frac{1}{2})(K+\frac{1}{2})} - \frac{\mu^2\xi^2}{4(K+\frac{5}{2})(K+\frac{1}{2})(K+\frac{1}{2})(K-\frac{1}{2})} \right]^2 \qquad \text{(IV.14)}$$

the first term corresponds to a Coulomb contribution because

$$V_c(r_{ij}) = \lim_{\mu \to 0} V_y(r_{ij}) \quad .$$

The convergence trend of Yukawa potentials is therefore very similar to the one of the Coulomb potential because the second term proportional to ξ^2 decrease like K^{-4} and gives weak correction with regard to the Coulomb term. The coupling coefficient between the first and the K^{th} equation is roughly (12)

$$\langle K|K|0\rangle^2 \simeq \frac{1}{3}\,(K+1)^2$$

leading to an extrapolated missing energy

$$\Delta E \simeq -\frac{4mV_0^2}{3\pi^2\hbar^2}\,\langle u_0|u_0\rangle^2 \sum_{K=K_{max}+1}^{\infty} \frac{1}{(K+1)^4} \qquad \text{(IV.15)}$$

ß - Gaussian potential

The multipole

$$V_{2K}^{(G)}(\xi) = \frac{2V_0}{\rho}\, e^{-\rho}\, I_{K+1}(\rho) \qquad \text{(IV.16)}$$

of the Gaussian potential

$$V_G(r_{ij}) = V_0\, e^{-r_{ij}^2/b^2} \qquad \text{(IV.17)}$$

is expressed in terms of a modified Bessel function of the first kind $I_{K+1}(\rho)$ with $\rho = \frac{\xi^2}{2b^2}$.

The potential proceeding from the terms $K>K_{max}$ in the main equation is

$$\sum_{K=K_{max}+1}^{\infty} \frac{\langle K|K|0\rangle^2}{K(K+v_2)}\,(\xi V_{2K}^{(G)})^2 = \sum_{K=K_{max}+1}^{\infty} \frac{\langle K|K|0\rangle^2}{K(K+2)}\cdot 8b^2\frac{V_0^2}{\rho}\, e^{-2\rho} I_{K+1}^2(\rho) \qquad \text{(IV.18)}$$

using the approximate ratio

$$\frac{\langle K|K|0\rangle^2}{K(K+2)} \simeq \frac{1}{3}$$

and the relation

$$I_0(2\rho) = I_0^2(\rho) + 2\sum_{n=1}^{\infty} I_n^2(\rho) \quad , \qquad \text{(IV.19)}$$

the convergence trend becomes

$$\Delta E = -\frac{2}{3}\frac{m}{\hbar^2} b^2 V_0^2 \quad \langle u_0|I_0(2\rho)-I_0^2(\rho)|u_0\rangle - 2\langle u_0|\sum_{n=1}^{K_{max}+1} I_n^2(2\rho)|u_0\rangle \qquad \text{(IV.20)}$$

γ - Shell δ potential

The shell δ potential (52)

$$V_\delta(r_{ij}) = V_0\ a\ \delta(r_{ij}-a)$$

generates multipoles

$$V_{2K}^{(\delta)}(\xi) = \begin{cases} 0 & \text{for } r < a \\ \dfrac{8}{\sqrt{\pi}} \dfrac{K!}{(\frac{1}{2}+K)} V_0 (a\xi^{-1})(1-a^2\xi^{-2})^{\frac{1}{4}} P_K^{\frac{1}{2},\frac{1}{2}}(1-2a^2\xi^{-2}) & r > a \end{cases} \qquad \text{(IV.21)}$$

which contain a Jacobi polynomial $P_K^{\frac{1}{2},\frac{1}{2}}$.

For large value of K one uses the asymptotic expansion of the Jacobi polynomial,

$$P_K^{\frac{1}{2},\frac{1}{2}}(1-2a^2\xi^{-2}) \underset{K\to\infty}{\sim} 2\sqrt{\frac{K}{\pi}} \frac{\sin 2K\, a\xi^{-1}}{2aK\,\xi^{-1}} = \frac{1}{\sqrt{K\pi}} \frac{\xi}{a} \sin 2K\, a\xi^{-1}. \qquad (IV.22)$$

Therefore for K large enough the multipoles become

$$\lim_{K\to\infty} (\xi V_{2K}^{(\delta)}(\xi))^2 \sim \frac{1}{(K+1)K} \left(\frac{8V_0 a}{\pi}\right)^2 (a\xi^{-1})^2 (1-a^2\xi^{-2}) \sin^2 2Ka\xi^{-1}.$$

Using $\sin^2 2Ka\xi^{-1} = \frac{1}{2}(1-\cos 4Ka\xi^{-1})$, one notices that the last oscillating term does not give any significant contribution in the average. The convergence trend is given by the first term only.

$$\lim_{K\to\infty} (\xi V_{2K}^{(\delta)}(\xi))^2 = \frac{1}{2K(K+1)} \left(\frac{8V_0 a}{\pi}\right)^2 (a\xi^{-1})^2 (1-a^2\xi^{-2}) \qquad (IV.23)$$

$$= \left(\frac{8V_0 a}{\pi}\right)^2 (a\xi^{-1})^2 (1-a^2\xi^{-2}) \left\{\left[\zeta(2) - \zeta(3)\right] - \sum_{K=1}^{K_{max}} \left(\frac{1}{K^2} - \frac{1}{K^3}\right)\right\} \qquad (IV.24)$$

where $\zeta(N)$ is the Riemann function.

The missing energy is

$$\Delta E = -8\left(\frac{V_0 a}{\pi\hbar}\right)^2 \langle u_0 | (a\xi^{-2})^2 (1-a^2\xi^{-2}) | u_0 \rangle \sum_{K_{max}+1}^{\infty} \frac{\langle K|K|0\rangle}{K(K+2)} \frac{1}{K(K+1)}$$

$$\Delta E = -\frac{8}{3}\left(\frac{V_0 a}{\pi\hbar}\right)^2 \langle u_0 | (a\xi^{-1})^2 (1-a^2\xi^{-2}) | u_0 \rangle \sum_{K_{max}+1}^{\infty} \frac{K+1}{K^2(K+2)}. \qquad (IV.25)$$

δ - Practical extrapolated missing energy formula

In order to estimate the extrapolated binding energy, we denote by E(K) the energy corresponding to the integration of coupled equations up to L=2K. It has been shown (12) that the behaviour of the hyperspherical coefficients generates an increasing of the binding energy which is of the same order of magnitude for the K=3n+1, K=3n+2 and K=3n+3 partial waves. Then we define the increase of binding energy

$$\Delta E(K) = E(K) - E(K-3)$$

For Gaussian potential (V^R, G2, GPDT,...) the exponential trend of convergence is (13)

$$\Delta E(K) = E(K) - E(K-3) = C \quad e^{-\alpha K} \quad ,$$

leading to extrapolation formula

$$E = E(K_{max}) - \Delta E(K_{max}) \left\{ \frac{\Delta E(K_{max}-3)}{\Delta E(K_{max})} - 1 \right\}^{-1} \quad . \quad \text{(IV.20bis)}$$

For the Coulomb and Yukawa potentials, the increase of binding energy behaves like (13,53)

$$\Delta E(K) = E(K) - E(K-3) = -(AK+B)^{-4} \quad ,$$

leading again to the binding energy

$$E = E(K_{max}) + \frac{8}{81} \Delta E(K_{max}) (1-\rho)^{-1} (1-\frac{\rho}{3})^{-3} \qquad \text{(IV.15bis)}$$

with $\rho = [\Delta E(K_{max})/\Delta E(K_{max}-3)]^{\frac{1}{4}}$.

IV.2 One body density and two body correlations

The one body density $R_1(\vec{\xi}_2)$ and the two body correlation function $R_2(\vec{\xi}_1)$ are respectively given by

$$R_i(\vec{\xi}_j) = \int |\psi(\vec{\xi}_1,\vec{\xi}_2)|^2 d^3\xi_i \qquad i\neq j \ (i,j=1,2). \qquad \text{(IV.26)}$$

One uses the trinucleon wave function with $^{2,2}S_1$ and $^{2,4}D_1$ states. Taking into account the orthogonality of the spin-isospin state of the trinucleon wave function one writes the densities $R_i(\vec{\xi}_j)$ as the sum of five components,

$$R_i(\vec{\xi}_j)=R_i^{(0)}(\vec{\xi}_j)+\frac{1}{2}\left\{R_i^{(+)}(\vec{\xi}_j)+R_i^{(-)}(\vec{\xi}_j)+R_i^{(D+)}(\vec{\xi}_j)+R_i^{(D-)}(\vec{\xi}_j)\right\} \qquad \text{(IV.27)}$$

where

$$R_i^{(\varepsilon)}(\vec{\xi}_j)=\int_0^\infty \xi_i^2 d\xi_i \sum_{KK'} {}^{\varepsilon}N_{2K}\,{}^{\varepsilon}N_{2K'}\,\xi^{-5}\,u_{2K}^{\alpha\frac{1}{2}\frac{1}{2}}(\xi)\,u_{2K'}^{\alpha\frac{1}{2}\frac{1}{2}}(\xi)\sum_{\ell=0}^{\min(K,K')}\frac{2\ell+1}{4\pi}$$

$${}^{\varepsilon}F_{2K}^{\ell\ell}{}_{(\pi/2)}\;{}^{\varepsilon}F_{2K'}^{\ell\ell}{}_{(\pi/2)}\;{}^{(2)}P_{2K}^{\ell,\ell}{}_{(\phi)}\;{}^{(2)}P_{2K'}^{\ell,\ell}{}_{(\phi)}\ , \qquad \text{(IV.28)}$$

for the S and S' states in which the parity of ℓ in the sum is even for ε=0,+ and odd for ε=-; α=S for ε=0 and α=m for ε=±. The contributions of the D states to the densities are,

$$R_i^{(D\eta)}(\vec{\xi}_j)=\int_0^\infty \xi_i^2 d\xi_i \sum_{KK'} {}^{T_N\eta}_{2K+2}\;{}^{T_N\eta}_{2K'+2}\sum_{\ell_1\ell_2}\frac{(2\ell_1+1)(2\ell_2+1)}{16\pi^2}\begin{pmatrix}2 & \ell_1 & \ell_2\\ 0 & 0 & 0\end{pmatrix}^2$$

$${}^{\eta}F_{2K+2}^{\ell_2\ell_1}{}_{(\pi/2)}\;{}^{\eta}F_{2K'+2}^{\ell_2\ell_1}{}_{(\pi/2)}\;\xi^{-5}\;u_{2K+2}^{m\frac{1}{2}\frac{3}{2}}(\xi)\;u_{2K'+2}^{m\frac{1}{2}\frac{3}{2}}(\xi)\;{}^{(2)}P_{2K+2}^{\ell_2,\ell_1}{}_{(\phi)}\;{}^{(2)}P_{2K'+2}^{\ell_2,\ell_1}{}_{(\phi)}\ , \qquad \text{(IV.29)}$$

for η= +,- .

Introducing the two integrals,

$$I^{\ell_1\ell_2}_{L,L'}(\alpha,\xi_1)=\int_{\xi_1}^{\infty} {}^{(2)}P_L^{\ell_1,\ell_2}\left(\frac{2\xi_1^2}{\xi^2}-1\right) {}^{(2)}P_{L'}^{\ell_1,\ell_2}\left(\frac{2\xi_1^2}{\xi^2}-1\right)\frac{[\xi^2-\xi_1^2]}{\xi^4}\, u^{\alpha}_{L}(\xi)u^{\alpha}_{L'}(\xi)d\xi$$

$$I^{\ell_2,\ell_1}_{L,L'}(\alpha,\xi_2)=\int_{\xi_2}^{\infty} {}^{(2)}P_L^{\ell_2,\ell_1}\left(\frac{2\xi_2^2}{\xi^2}-1\right) {}^{(2)}P_{L'}^{\ell_2,\ell_1}\left(\frac{2\xi_2^2}{\xi^2}-1\right)\frac{[\xi^2-\xi_2^2]}{\xi^4}\, u^{\alpha}_{L}(\xi)u^{\alpha}_{L'}(\xi)d\xi\ , \qquad \text{(IV.30)}$$

the components of the one body density and two body correlations functions become,

$$R_i^{(\varepsilon)}(\xi_j)=\frac{1}{4\pi}\sum_{KK'}\varepsilon_{N_{2K}}\varepsilon_{N_{2K'}}\sum_{\ell}^{\min(K,K')}(2\ell+1)\,\varepsilon F^{\ell,\ell}_{2K}(\pi/2)\,\varepsilon F^{\ell,\ell}_{2K'}(\pi/2)\,J^{\ell\ell}_{2K,2K'}(\alpha,\xi_j)$$

$$R_i^{D\eta}(\xi_j)=\frac{1}{16\pi^2}\sum_{KK'}T_{N^{\eta}_{2K+2}}T_{N^{\eta}_{2K'+2}}\sum_{\ell_1\ell_2}(2\ell_1+1)(2\ell_2+1)\begin{pmatrix}2 & \ell_1 & \ell_2\\ 0 & 0 & 0\end{pmatrix}^2 {}^{\eta}F^{\ell_2\ell_1}_{2K+2}(\pi/2)$$

$$\cdot\,{}^{\eta}F^{\ell_2,\ell_1}_{2K'+2}(\pi/2)\,J^{\ell_2\ell_1}_{2K+2,2K'+2}(M\tfrac{1}{2}\tfrac{1}{2};\xi_j) \qquad \text{(IV.31)}$$

with
$$J^{\ell_2\ell_1}_{L,L'}(\alpha,\xi_2) = I^{\ell_2\ell_1}_{L,L'}(\alpha,\xi_2)$$

$$J^{\ell_2\ell_1}_{L,L'}(\alpha,\xi_1) = (-1)^{K+K'}\, I^{\ell_1,\ell_2}_{L,L'}(\alpha,\xi_1)\,. \qquad \text{(IV.32)}$$

IV.3 The electromagnetic form factors of ^3H and ^3He

The non relativistic charge and magnetic nuclear form factor $F_c(q)$ and $F_m(q)$ are given by

$$(\tfrac{3}{2}+T_z)F_c^{T_z}(q)=\langle\Psi|\sum_{i=1}^{3}[\tfrac{1}{2}(1+\tau_z(i))f_c^p(q)+\tfrac{1}{2}(1-\tau_z(i))f_c^n(q)]\,.\exp\, i\vec{q}(\vec{x}_i-\vec{X})\,|\Psi\rangle, \tag{IV.33}$$

and

$$\Gamma_{T_z}.F_m^{T_z}(q)=\langle\Psi|\sum_{i=1}^{3}[\tfrac{1}{2}\sigma_z(i)\,(1+\tau_z(i))\Gamma_p\, f_m^p(q)+ \\ +\tfrac{1}{2}\,\sigma_z(i)\;(1-\tau_z(i))\Gamma_n f_m^n(q)]\exp\, i\,\vec{q}(\vec{x}_i-\vec{X})\,|\Psi\rangle \tag{IV.34}$$

where $T_z=\frac{1}{2}$ for ^3He and $T_z=-\frac{1}{2}$ for ^3H. Γ_{T_z} is the magnetic moment of the nucleus. $f_{c(m)}^{p(n)}q$ are the charge (magnetic) proton (neutron) form factors.

The electric form factor is given in terms of the scalar $G_{ES}(q)$ and vector $G_{EV}(q)$ nucleon charge form factors by

$$(\tfrac{3}{2}+T_z)F_c^{T_z}(q)\;=\;3\langle\Psi|(G_{ES}(q)\;+\;G_{EV}(q)\tau_z(3))e^{i\vec{q}.\vec{\xi}_2/\sqrt{3}}\,|\Psi\rangle, \tag{IV.35}$$

where $\vec{q}(\vec{x}_3-\vec{X})\;=\;\vec{q}.\vec{\xi}_2/\sqrt{3}$.

Using the completely antisymmetric normalized wave function (III.44)

$$\Psi(\xi,\Omega)=\phi^{0\,\frac{1}{2}\,\frac{1}{2}}(\xi\Omega).\Gamma_{\frac{1}{2}\frac{1}{2}}(A)\;+\;\frac{1}{\sqrt{2}}\left[\phi^{+\frac{1}{2}\,\frac{1}{2}}(\xi\Omega)\;\Gamma_{\frac{1}{2}\frac{1}{2}}(M-)-\phi^{-\frac{1}{2}\,\frac{1}{2}}(\xi\Omega)\;\Gamma_{\frac{1}{2}\frac{1}{2}}(M+)\right] \\ +\frac{1}{\sqrt{2}}\left\{\phi^{+\frac{3}{2}\,\frac{1}{2}}(\xi,\Omega)\Gamma_{\frac{3}{2}\frac{1}{2}}(M-)\;+\;\phi^{-\frac{3}{2}\,\frac{1}{2}}(\xi\Omega)\Gamma_{\frac{3}{2}\frac{1}{2}}(M+)\right\}\;+ \\ +\frac{1}{\sqrt{2}}\left\{\phi^{+\frac{1}{2}\,\frac{1}{2}}(\xi,\Omega)\Gamma_{\frac{1}{2}\frac{1}{2}}(M-)\;+\;\phi^{-\frac{1}{2}\,\frac{1}{2}}(\xi,\Omega)\Gamma_{\frac{1}{2}\frac{1}{2}}(M+)\right\}. \tag{IV.36}$$

The electric form factor becomes,

$$(\tfrac{3}{2}+T_z)F_c^{T_z}(q)=(3G_{ES}(q)+2T_z.G_{EV}(q))\langle\psi|e^{i\frac{\vec{q}\vec{\xi}_2}{\sqrt{3}}}|\psi\rangle$$
$$-2T_zG_{EV}(q)\,(2\sqrt{2}\,\langle\phi^{+\frac{1}{2}\frac{1}{2}}|e^{i\frac{\vec{q}\vec{\xi}_2}{\sqrt{3}}}|\phi^{0\frac{1}{2}\frac{1}{2}}\rangle-\langle\phi^{+\frac{1}{2}\frac{1}{2}}|e^{i\frac{\vec{q}\vec{\xi}_2}{\sqrt{3}}}|\phi^{+\frac{1}{2}\frac{1}{2}}\rangle$$
$$+\langle\phi^{-\frac{1}{2}\frac{1}{2}}|e^{i\frac{\vec{q}\vec{\xi}_2}{\sqrt{3}}}|\phi^{-\frac{1}{2}\frac{1}{2}}\rangle)-2T_zG_{EV}(q)\,(2\sqrt{2}\langle\phi^{+\frac{1}{2}\frac{1}{2}}|e^{i\frac{\vec{q}\vec{\xi}_2}{\sqrt{3}}}|\phi^{0\frac{1}{2}\frac{1}{2}}\rangle$$
$$-2\langle\phi^{-\frac{1}{2}\frac{1}{2}}|e^{i\frac{\vec{q}\vec{\xi}_2}{\sqrt{3}}}|\phi^{-\frac{1}{2}\frac{1}{2}}\rangle+2\langle\phi^{+\frac{1}{2}\frac{1}{2}}|e^{i\frac{\vec{q}\vec{\xi}_2}{\sqrt{3}}}|\phi^{+\frac{1}{2}\frac{1}{2}}\rangle)\,. \qquad \text{(IV.37)}$$

The two last brackets of this formula contributes only to the ^3He electric form factor. The various amplitudes $\langle\phi^{R^*TS}|e^{i\frac{\vec{q}\vec{\xi}_2}{\sqrt{3}}}|\phi^{R'^*T'S'}\rangle$ are simple expressions of the hyperspherical coefficients calculated previously for central forces (B14).

$$\langle\phi^{R^*TS}|e^{i\frac{\vec{q}\vec{\xi}_2}{\sqrt{3}}}|\phi^{R'^*T'S'}\rangle=8\sum_{KK'K''}(-1)^{K''}\,C^{(2K'+\ell)R'sS\frac{1}{2}}_{(K'',C,0)\,(2K+\ell)RsS\frac{1}{2}}$$
$$\langle u^{R^*TS}_{2K+\ell}|\frac{J_{(q'\xi)}^{2K''+2}}{(q'\xi)^2}|u^{R'^*T'S}_{2K'+\ell}\rangle \qquad \text{(IV.38)}$$

where $q'=\frac{q}{\sqrt{3}}$ and

$$\langle u^{R^*TS}_{2K+\ell}|\frac{J_{(q'\xi)}^{2K''+2}}{(q'\xi)^2}|u^{R'^*T'S}_{2K'+\ell}\rangle=\int_0^\infty d\xi\;u^{R^*TS}_{2K+\ell}(\xi)\;u^{R'^*T'S}_{2K'+\ell}(\xi)\;\frac{J_{(q'\xi)}^{2K''+2}}{(q'\xi)^2}\,. \qquad \text{(IV.39)}$$

It is worthwhile to notice that in (IV.35) only the spherical term $\ell=0$ of the expansion of the plane wave in spherical harmonics contribute.

IV.4 Two body photodesintegration of the three body system

The two body photodesintegration in which the trinucleon is broken into a deuteron and a nucleon is treated in the dipole approximation. In the Born approximation the final state w.ve function is the product of a deuteron wave function and a plane wave describing the outgoing nucleon. When the tensor force is taken into account the deuteron wave function contains a D state coupled in the final state with the spin of the single nucleon to generate a J=½ total angular momentum. In the plane wave approximation the final state in the C.M. system is :

$$\Psi_f(\xi,\Omega)=\sum_C \psi_f(\vec{\xi}_{ij},\vec{\xi}_k)=\frac{1}{\sqrt{3}}\sum_C\left\{|(0\tfrac{1}{2})\tfrac{1}{2}m_T\rangle|(1\tfrac{1}{2})\tfrac{1}{2}M_J\rangle\frac{u(\xi_{ij})}{\xi_{ij}}Y_0^0(\hat{\xi}_{ij})\right.$$

$$\left.+\sum_{mm'}(-1)^{-\frac{1}{2}+M_J}\sqrt{2}\begin{pmatrix}2 & \frac{1}{2} & \frac{1}{2}\\ m & m' & -M_J\end{pmatrix}\frac{w(\xi_{ij})}{\xi_{ij}}Y_2^m(\hat{\xi}_{ij})|(0\tfrac{1}{2})\tfrac{1}{2}m_T\rangle|(1\tfrac{1}{2})\tfrac{1}{2}m'\rangle\right\}$$

$$.\exp(i\,\vec{k}.\vec{\xi}_k) \quad , \qquad \text{(IV.40)}$$

where $\vec{\xi}_{ij}=\vec{x}_i-\vec{x}_j$ and $\vec{\xi}_k=\sqrt{3}(\vec{x}_k-\vec{X})$. The sum is taken over the cyclic permutation of (ijk) and the final wave function is normalized asymptotically to one free particle per unit volume.
The dipole operator written in Jacobi coordinates is

$$D = ie\left[\frac{\pi E_\gamma}{6}\right]^{\frac{1}{2}}\vec{e}.\{\vec{\xi}_2\tau_+ + \vec{\xi}_1\tau_-\} \qquad \text{(IV.41)}$$

where the isospin operators $\tau_\pm$ are combinations of the individual τ_z of the nucleons

$$\tau_+ = \tau_z(k) - \frac{1}{2}(\tau_z(i)+\tau_z(j))$$
$$\tau_- = \frac{\sqrt{3}}{2}(\tau_z(j) - \tau_z(i)) \quad , \qquad \text{(IV.42)}$$

e is the proton charge and $\vec{e}$ is the polarization vector which is taken along the z axis. The initial state is fully antisymmetrized therefore the final state wave function may be specified for the couple (i=1, j=2) which corresponds to the Jacobi coordinates $\vec{\xi}_1$ and $\vec{\xi}_2$.

The dipole photodesintegration differential cross section is

$$d\sigma = \frac{2\pi}{\hbar c} \mid D_{fi} \mid^2 \rho(E) = \frac{mkd\hat{k}}{8\pi^2 ch^3} \mid D_{fi} \mid^2 \quad , \qquad \text{(IV.43)}$$

where D_{fi} is the dipole matrix element between the initial trinucleon ground state and the final D+N state. The energy density of final states is

$$\rho(E) = \frac{d^3k}{(2\pi)^3 dE} = \frac{mkd\hat{k}}{2\hbar^2(2\pi)^3} \quad , \qquad \text{(IV.44)}$$

and the total energy of the system is conserved according to

$$E = E_d + \frac{\hbar^2k^2}{m} = E_T + E_\gamma \quad , \qquad \text{(IV.45)}$$

where E_d is the deuteron binding energy and $\frac{\hbar^2k^2}{m}$ is the kinetic energy of the outgoing nucleon in the center of mass while E_T is the binding energy of the trinucleon at rest and E_γ the photon energy.

The effect of the dipole operator D on the function $\psi_f(\vec{\xi}_1,\vec{\xi}_2)$ is :

$$D\psi_f(\vec{\xi}_1,\vec{\xi}_2)=6T_z\ ie\left[\frac{\pi E_\gamma}{6}\right]^{\frac{1}{2}}\left\{\xi_{2z}\psi_f(\vec{\xi}_1,\vec{\xi}_2)+\frac{\xi_{1z}}{\sqrt{3}}\left[\frac{u(\xi_1)}{\xi_1}Y_0^0(\hat{\xi}_1)\cdot|(0\tfrac{1}{2})\tfrac{1}{2}m_T>|(1\tfrac{1}{2})\tfrac{1}{2}M_J>+\sum_{mm'}(-1)^{\frac{1}{2}-M_J}\sqrt{2}\begin{pmatrix}2&\frac{1}{2}&\frac{1}{2}\\m&m'&-M_J\end{pmatrix}\left[\frac{w(\xi_1)}{\xi_1}\right]Y_2^m(\hat{\xi}_1)|(0\tfrac{1}{2})\tfrac{1}{2}m_T>|(1\tfrac{1}{2})\tfrac{1}{2}m'>\right]\right\}.\exp(i\,\vec{k}.\vec{\xi}_2). \qquad (IV.46)$$

where $u(\xi_1)$ and $w(\xi_1)$ are respectively the S and D state deuteron wave function. The trinucleon wave function contains the fully symmetric S state, the mixed symmetry S' state and the mixed symmetry D state. It may be written in the simplified form :

$$\psi_T(\vec{\xi}_1,\vec{\xi}_2)=\Gamma(A)\ \phi_0^{(0)}(\vec{\xi}_1,\vec{\xi}_2)+\frac{1}{\sqrt{2}}\left\{\Gamma(M-)\ \phi^{(+)}(\vec{\xi}_1,\vec{\xi}_2)-\Gamma(M+)\ \phi^{(-)}(\vec{\xi}_1,\vec{\xi}_2)\right\}$$
$$+\frac{1}{\sqrt{2}}\left\{|(0\tfrac{1}{2})\,m_T>\phi_D^{(+)}(\vec{\xi}_1,\vec{\xi}_2)+|(1\tfrac{1}{2})\tfrac{1}{2}m_T>.\ \phi_D^{(-)}(\vec{\xi}_1,\vec{\xi}_2)\right\}, \qquad (IV.47)$$

leading to the following expressions of the dipole matrix elements :

$$<\Psi_T|D|\Psi_f>=3(2T_z)ie\left[\frac{\pi E_\gamma}{6}\right]^{\frac{1}{2}}\left\{<\Psi_T|\xi_{2z}|\Psi_f>+\frac{1}{\sqrt{3}}<\Psi_T|\xi_{1z}|\right.$$
$$\left.\frac{u(\xi_1)}{\xi_1}Y_0^0(\hat{\xi}_1)|(1\tfrac{1}{2})\tfrac{1}{2}m_T>|(0\tfrac{1}{2})\tfrac{1}{2}M>+\sum_{mm'}\sqrt{2}\begin{pmatrix}2&\frac{1}{2}&\frac{1}{2}\\m&m'&-M\end{pmatrix}(-1)^{\frac{1}{2}-M}Y_2^m(\hat{\xi}_1)\frac{w(\xi_1)}{\xi_1}\right.$$
$$\left.|(0\tfrac{1}{2})\tfrac{1}{2}m_T>|(1\tfrac{1}{2})\tfrac{1}{2}\,m'>\right\}e^{i\vec{k}.\vec{\xi}_2}>\Big\}. \qquad (IV.48)$$

A more compact formula is obtained in terms of overlaping functions :

$$<\Psi_T|D|\Psi_f>=\pi^2\ \frac{4}{\sqrt{3}}\ Y_1^0(\hat{k})\ \sqrt{e^2E_\gamma}.(2T_z).\int_0^\infty \xi_2^{\frac{3}{2}}\,d\xi_2\ j_1(k\xi_2)\ R\ (\xi_2) \qquad (IV.49)$$

where

$$R(\xi_2) = R_1^0(\xi_2) - \frac{1}{\sqrt{2}}\left(R_1^+(\xi_2)+R_1^-(\xi_2)+R_2^+(\xi_2)\right) \qquad \text{(IV.50)}$$

is a sum of the overlapping functions between the deuteron and the trinucleon system :

$$R_1^{\eta}(\vec{\xi}_2) = \frac{1}{\sqrt{4\pi}}\int d^3\xi_1 \frac{u(\xi_1)}{\xi_1}\ \phi^{(\eta)}_{\frac{1}{2}}(\vec{\xi}_1,\vec{\xi}_2)$$

$$= \frac{1}{\sqrt{4\pi}}\sum_{K=0}^{\infty}{}^{\eta}N_{2K}\ {}^{\eta}F^{00}_{2K}(\pi/2)\int_0^{\infty}\xi_1 d\xi_1\ {}^{(2)}P^{0,0}_{2K}(\phi)\ u(\xi_1)u^{p\frac{1}{2}\frac{1}{2}}_{2K}(\xi)\xi^{-\frac{5}{2}}, \eta=Q+(\beta=S,m)$$

$$R_1^-(\vec{\xi}_2) = \frac{1}{\xi_2\sqrt{4\pi}}\sum_K {}^{-}N_{2K}\ {}^{-}F^{1,1}_{2K}(\pi/2)\int_0^{\infty}\xi_1^2 d\xi_1\ {}^{(2)}P^{1,1}_{2K}(\phi)\ u(\xi_1)u^{m\frac{1}{2}\frac{1}{2}}_{2K}(\xi)\xi^{-\frac{5}{2}}$$

$$R_2^+(\vec{\xi}_2) = \frac{1}{4\pi}\sum_K {}^{+}N_{2K+2}\ {}^{+}F^{0,2}_{2K+2}(\pi/2)\int_0^{\infty}\xi_1 d\xi_1\ {}^{(2)}P^{0,2}_{2K+2}(\phi)\ u^{m\frac{1}{2}\frac{1}{2}}_{2K+2}(\xi)\ w(\xi_1)\xi^{-\frac{5}{2}}\ . \qquad \text{(IV.51)}$$

The differential dipole photodesintegration cross section in the Born approximation becomes :

$$\frac{d\sigma}{d\hat{k}} = \frac{m}{\hbar^2}\left(\frac{e^2}{\hbar c}\right)kE_\gamma\ \frac{2\pi^2}{3}\left|Y_1^0(\hat{k})\right|^2\left[\int_0^{\infty}\xi_2^3 d\xi_2 j_1(k\xi_2)\,R(\xi_2)\right]^2. \qquad \text{(IV.52)}$$

The total cross section is deduced by integrating over $\hat{k}$,

$$\sigma = \frac{m}{\hbar^2}\left(\frac{e^2}{\hbar c}\right)kE_\gamma\ \frac{2\pi^2}{3}\left[\int_0^{\infty}\xi_2^3 d\xi_2 j_1(k\xi_2)\,R(\xi_2)\right]^2 \qquad \text{(IV.53)}$$

V. Results and discussion

A large number of trinucleon calculations have been done with wave functions extracted by various accurate numerical method from the non relativistic Schrödinger or Faddeev equations using realistic conventional two body forces. The various "realistic" wave functions obtained by this model are not similar and do not reproduce accurately the experimental binding energies and r.m.s. radii of ^{3}H and ^{3}He. The most accurate test of the quality of a wave function may be provided by the comparison between the Fourier transform of the charge density and the experimental form factor in spite that it does not give any information about the two body correlations. It is difficult to decide whether the discrepancy with experimental data is due to the unability of the non relativistic Schrödinger equation to describe the nature or to the conventional approach of the two body interaction.

For these reasons a part of this section is devoted to an analysis of the influence of the strength of the repulsive soft core upon the form factor, the charge density, the two body correlation function and the photodesintegration cross section. For this purpose two body model interactions with quite different soft core have been chosen giving the experimental binding energy and the size of the trinucleon. Doing this way we do not expect to deal with interaction in agreement with the two body data nevertheless our model potentials are close enough to the realistic interaction to be able to predict the effect of a similar variation of the core occuring for realistic potentials.

The other part of this section is devoted to an analysis of the binding energies, sizes, charge form factors and dipole photonuclear effects provided by the wave functions extracted from conventional realistic soft core potentials (20,22).

V.1 Wave functions and electric form factors

V.1.1 S wave potentials G2 and V^x

To study the influence of the core we have chosen the model potentials G2 and V^x (13) ; both potentials are central and constituted by a sum of two gaussians for each of the triplet and singlet even states (fig.1). In contrast to the V^x potential, the G2 interaction has a very strong repulsive core and to simplify the three body calculations, the tensor component is omitted. The H.H. expansion is limited to the use of the optimal subset including only the first partial wave of the mixed symmetry state (L=2). This restriction leads to neglect about 0.1% of the mixed symmetry state. A total of 13 coupled equations have been solved in order to obtain the three body binding energy within and accuracy of less than .1 Mev.

In fig.2 and fig.3, the ^3H and ^3He charge form factors are plotted for both potentials. The slope of the form factors for zero momentum transfer (q=0) and therefore the r.m.s. radii are in good agreement with experimental data. However the core of V^x is too weak to enable one to reproduce the first minimum

of the charge form factor at the experimental point in contrast with the result obtained with G2 potential. We have to notice that the good order of magnitude of the second maximum (around $q=16$ fm^{-2}) is not obtained. Nevertheless the stronger is the core the smaller is the position of the minimum and the larger is the magnitude of the second maximum of the charge form factor.

In fig.4, the partial wave L=0 (fully space symmetric) and L=2 (mixed symmetric) for G2 and V^x are shown together. The asymptotic behavior determined only by the binding energy is the same for all these partial waves which are also very similar around the maximum near 3 fm, probably because both potentials produce the same mean square radius. The difference appearing for small ξ proceeds from the strength of the core.

The very similar behavior of both the L=0 and L=2 partial waves subjected to the constraints of the binding energy and of the size of the trinucleon system suggest that the difference between the wave functions proceeds from the higher order partial waves L>2. Indeed the contribution in the ground state wave function of the two first partial waves amount 99% for V^x and 96% only for G2. It is therefore expected that the differences between the results obtained with potentials giving the same binding energy and r.m.s. radii will be sensitive to the core of the interaction when in the analyzed phenomenon the contribution of the cross terms between the small ($L\neq0$) and the main (L=0) partial waves will be significant. In order to show clearly this effect we have choosen to calculate the one body densities and the two body correlations functions in which cross terms exist for both potentials and on the other hand the dipole photodesintegration in

Born approximation for which the cross terms are excluded by the dipole selection rule.

V.1.2 Wave functions and elastic electron scattering with shell δ potentials

In order to compare the previous results to those obtained with a simple central potential giving accurately the $^3S_1$ and $^1S_0$ phase shifts and the correct deuteron binding energy we have constructed a shell δ potential,

$$V_i(\xi_i) = A_i . r_i . \delta(\xi_i - r_i) \qquad (V.1)$$

with $$A_i = -\frac{\hbar^2}{m}\frac{\alpha_i}{r_i^2}$$

The triplet even (i=t) and singlet even (i=s) parameters are given in table (2). The n-p phase shifts $^1S_0$ and $^3S_1$ are plotted together with Mac Gregor et al. (47,60) phase shifts in fig. 6,7. The agreement up to E_{lab}=300 Mev is quite good and similar to those obtained with realistic potentials but we have to notice that the tensor force has not been included in our analysis.

The convergence of the ^3H and ^3He binding energies has been investigated by integrating up to 14 coupled equations for the completely symmetric S states and 10 coupled equations for the mixed symmetric S' states. Table (3, 4) gives the ^3H and ^3He binding energies $-E_K$ (Mev) as a function of K. The rms radii

$R_{^3H} = 1.478$ fm and $R_{^3He} = 1.5$ fm

are too small. The first minimum of the ^{3}He charge form factor around $q^2 \approx 19$ fm^{-2} (fig. 8) is too far.

The shell ó potential provides an example of interaction giving quite good two body s phase shifts but poor results in the trinucleon system. One can of course argue that the introduction of a tensor force should improve our results.

V.1.3 Soft core realistic potentials

The potential models have shown that the calculated trinucleon observables are sensitive to the characteristics of the two body interaction. To exhibit this aspect we have chosen to investigate the trinucleon static and dynamic properties with four realistic local interactions proposed by Gogny-Pires-de Tourreil (GPDT) (22), and by Sprung and de Tourreil (SSCA, SSCB, SSCC) (20). These four potentials reproduce quite well the two-nucleon data and are classified as super soft core potentials. However they differs from each other by the relative strength of the central, LS, Tensor or L^2 components. We have nevertheless to notice that the GPDT potential overbind the deuteron. This property affect strongly the trinucleon binding energy which then becomes in agreement with the experimental value.

In table (5 , 6) we give the static results for ^3H and ^3He nuclei compared to the experimental ones. Our results (see Table (7)) are in close agreement with those of Laverne and Gignoux (34) and Kim and Tubis (31). However we note that our P(S') is always weaker than their ones, but in agreement with those of Demin-Efros (15, 16) whose calculations have been done with the hyperspherical formalism.

The trinucleon binding energies $E(^3H)$ and $E(^3He)$ depend strongly on the two nucleon interaction in the $^1S_0$ and $^3S_1$-$^3D_1$ partial waves. However the contribution from higher partial waves is not negligible and increase the binding energy by about 0.4 Mev. In our formalism nearly all the partial waves of the two body interaction are taken into account. Indeed in integrating coupled equations up to K=14 we use two-nucleon orbitals up to $\ell_{max}=16$ in the $^{2,4}D_{\frac{1}{2}}$ trinucleon state for instance.

The importance of the non potential harmonics has been analyzed. The first non potential partial waves appear at different values of K following the symmetry of the state. For instance in the fully symmetric state it appears from K=6,8..., in the mixed symmetry S' from K=4 and in mixed symmetry D state from K=2. In his thesis Erens (14) has shown that the first non potential partial waves in S and S' state contribute by about 0.1 Kev. We calculated the contribution of the first non potential partial wave (K=2,3) in $^{2,4}D_{\frac{1}{2}}$ state and found ΔE = 7. Kev for *GPDT* and ΔE=37 Kev for *SSCC* potentials.

The electric form factors generated by realistic ^3He wave functions obtained in solving either the Faddeev equations or the coupled equations of the hyperspherical formalism fig. have a too small secondary maximum and a too far minimum. Phenomenologically this dicrepancy proceed from realistic N-N interac-

tions which are not repulsive enough at short distance. The introduction of meson exchange current corrections tend to improve the position of the dip and the magnitude of the second maximum. However these corrections are not large enough to agree with experiments.

V.1.4 The ^3H-^3He energy difference

The Coulomb energy $E_c = E(^3He) - E(^3H)$ is given in the last column of Table (5). It has been shown previously (58) that the Coulomb energy proceeding from the Coulomb interaction e^2/r between the two protons can be obtained in a nearly model independent way in starting from the trinucleon charge form factors. The close relation between the proton distribution and the Coulomb energy is responsible for the sensitivity of E_c to the trinucleon r.m.s. radius. Practically two waves functions generating the same radius lead to the same Coulomb energy therefore the difference between our results and those obtained for the same potentials with other methods reflect the difference in radii proceeding from the wave functions. The model independent formula has been obtained in assuming the same wave function for both elements of the iso doublet. In fact the repulsive Coulomb interaction in reducing the binding energy of ^3He increase its radius of about .03 fm with respect to the one of ^3H. An estimate of the Coulomb energy correction for ^3He is given by

$$\Delta E_c < - \frac{e^2}{R^2} \Delta R$$

with $\frac{e^2}{R} \simeq E_c$ i.e. $\Delta E_c < - E_c \frac{\Delta R}{R} \simeq - \frac{1}{50} E_c$

which is a negligible percentage of E_c.

V.2 One body density and two body correlation functions

The one body density $R_1(\vec{\xi}_2)$ and the two body correlation functions $R_2(\vec{\xi}_1)$ are respectively given by eq. (IV.26)

$$R_1(\vec{\xi}_j) = \int |\Psi(\vec{\xi}_1,\vec{\xi}_2)|^2 d^1\xi_1 \qquad i\neq j \quad (i,j=1,2) \ .$$

The densities $R_1(\vec{\xi}_2)$ and $R_2(\vec{\xi}_1)$ are the same when the two first partial waves are used only fig. (10) . The difference proceeds from the cross term between the higher partial waves and the first wave L=0. For wave functions described by the first fully symmetric term L=0 only, like for the early Irving (61), Irving-Gunn or Gaussian functions (57), the first derivative,

$$\frac{d}{d\xi_i} R_j(\xi_i) = -\frac{2}{\pi^2}\,\xi_i \int_0^\infty (\xi_i^2+\xi_j^2)^{-\frac{5}{2}} \left|U_0\left((\xi_i^2+\xi_j^2)^{\frac{1}{2}}\right)\right|^2 d\xi_j$$

is always negative. The densities are smoothly decreasing functions. The first term cannot therefore take the correlation originating from a strong repulsive core into account. This core should indeed prevent two nucleons to come close together and generate a hole into the two body correlation function.

In figures 11, 12, 13 one shows the two densities calculated with the full wave function for S wave potentials G2 and V^x and for the realistic GPDT potential.
For each potential the two densities have the same asymptotic tail for $\xi_i \to \infty$. The influence of the core is clearly seen. The

stronger is the core the deeper is the hole in the correlation function. The one body density does not exhibit any minimum near the origin. In terms of a H.H expansion only the partial waves $u_{2K}(\xi)$ for odd K values contribute to the difference between the one body density and the correlation function as a consequence of eqs. (IV.31) and (IV.32), because $P^{(\epsilon)}_{2K}(\Omega)$ (eq. III.45) have $(-1)^K$ parity in the exchange of the set $(\vec{x}_i-\vec{x}_j)$ and $\sqrt{3}(\vec{x}_k-\vec{X})$ $(k\neq j\neq i)$. The differences appearing in fig. 11,12 proceed from the odd K partial waves which contribute only 0.2% and 1.6% to the ground state for V^x and G2 potential respectively. One sees how large can be the effect of small components of the wave function on the behaviour of the correlation function at short distance. The comparison between the one body densities shows that the stronger is the core the flatter is the density near the origin which in turn produces a minimum of the form factor for smaller momentum. The proton density differs from the matter density by a cross term between the S and S' symmetry components generated by the $\tau_z(3)$ operator occuring in the proton density $\rho_p(\vec{r})$

$$\rho_p(\vec{r}) = 3\sqrt{3}\ R_p(\sqrt{3}\ \vec{r})$$

with

$$R_p(\vec{\xi}_2) = \int \psi^*(\vec{\xi}_1,\vec{\xi}_2) \sum_{j=1}^{3} \frac{(1+\tau_z(j))}{2}\ \psi(\vec{\xi}_1,\vec{\xi}_2)\, d^3\xi_1 .$$

the density $R_p(\xi)$ is plotted in fig. (14,15) for the potentials V^x, G2 and GPDT. For the G2 potential the contribution of the S-S' terms flattens strongly the density at small distances but asymptotically the proton and the matter densities are the same.

V.3 Photodesintegration of the three body system

Concerning the two body photodesintegration

$$\gamma + {}^3He \rightarrow p + D$$
$$\gamma + {}^3H \rightarrow n + D$$

at low photon energies E_γ<100 Mev many calculations have been done (41, 42, 43, 44, 45, 46). In this energy range the reactions proceed mainly from an electric dipole transition and most of the studies were done in this approximation. However the earlier calculations were restricted to phenomenological wave functions and approximate treatment of final state. Only recently the solution of the three body problem with realistic potential has been used to calculate the photodesintegration cross section. In this section we discuss the results obtained with our S wave potential models and with realistic local potentials.

The effect of the core on the two body dipole photodesintegration has been studied in keeping the same deuteron wave function (40) and the Born approximation for the final state. The overlapping functions between the deuteron and the trinucleon i.e $R_1^0(\xi_2)$, $R_1^+(\xi_2)$ and $R_1^-(\xi_2)$ (eqs. IV.51) are shown in figs. (16) for G2 and V^x potentials. These overlapping functions are nearly the same and lead to very similar dipole photodesintegration cross sections as it is shown on fig. (17). For both potentials the calculated cross sections underestimate the data up to E_γ=14 Mev and overestimate the data for E_γ>14 Mev. We clearly see the influence of the core at low energies. The

calculated cross section in fig. (18) using the three body wave function corresponding to the GPDT interaction show explicitely the contributions of the s-S transitions (total symmetry and mixed symmetry) and the contributions of the d-D transitions. The s-S transitions with the S symmetry state gives a cross section which underestimate the data. The contribution of the mixed symmetry is large and generates together with the total symmetry a cross section which overestimates the data. The introduction of the d-D transitions is quite necessary to reproduce the experimental data. The d-D transitions strongly affects the magnitude of the cross section, but does not change its shape. We do not find the "effect" of Craver et al. (45, 46) producing a peak very flat. In our calculations the d-D transitions have a destructive effect on the total cross section contrary to the result obtained by Craver.

In fig. (19) the cross sections calculated with the Sprung-de Tourreil super soft core potentials are given. The agreement with experimental data is good for all these local potentials the results of which are very similar.

VI. Comments and conclusion

We have developed a method enabling to solve the Schrödinger equation for any number of bosons in the ground state. This method can also be applied for fermion systems when all particles are in the 1s state like for the ground state of ^{3}He, ^{3}H, ^{4}He or helium like atom (53). In this work the three nucleon bound system has been analyzed. We found by comparison with other method for local realistic N-N potential that the hyperspherical harmonic expansion method is at least as well reliable than the numerical solutions obtained with the Faddeev equations. Our binding energies are sometimes over and sometimes under the values obtained in solving the Faddeev equation. But one must notice that in constrast to the Faddeev equation our method fulfill the Rayleigh-Ritz variational principle which state that in solving a truncated system of equations one obtains a binding energy situated over the exact value . The contribution of the various terms of the wave function (symmetry S, S' and D state) are roughly similar to those given by Gignoux-Laverne (34) except for the weight of the S' state which is always about half the percentage of their solutions.It is to some extend surprising to find the percentage of the S' state given by the variational method quite smaller than the one given by the Faddeev equations. At this point one must stress that our basis is completely antisymmetric and that no ambiguity can arise from the symmetrization process. The convergence of the method has been carefully analyzed and a consequence of eq. (IV.2,IV.6) is that the partial wave $u_{2K}(\xi)$ should vanish together with the multipole $V_{2K}(\xi)$. The exact numerical analysis is in agreement with this

prediction giving confidence to our approximation (eq.IV.8). In the framework of the optimal subset all the two body orbitals ℓ_{ij} has been taken into account in our analysis of the two body potential till the value $\ell_{ij}=K_{max}$, ($K_{max}=14$ in our calculation). This means in particular that the odd states have been taken into account. By omitting the odd potentials in the coupled equations we found that the contribution of the odd waves amounts to -0.6 MeV in the binging energy calculated with the GPDT potential.

In order to reduce the number of significant coupled equations we assumed that the contribution of the H.H orthogonal to the optimal subset are negligible. The smallness of this contribution has been previously shown by G. Erens (Thesis) for the S and S' state. Including the D state we found an increase of binding of a few ten of keV proceeding from the "non-potential" elements.

Practically the wave function is concentrated in a small number of partial waves but the amount of binding proceeding from high ℓ partial wave are not negligible because they contribute to the energy through the cross terms with the main partial wave K=0. It is therefore a mistake to use the partial waves of the solution of a truncated problem which is not yet completely converged. This is especially true when we are dealing with a phenomenon in which the tail brings a large contribution like for the photodesintegration, because then the exponential asymptotic behaviour $\exp -\sqrt{\frac{-m}{\hbar^2} E}\, r$ of the wave function is reproduced with the inaccuracy of E and the tail can extend very much farther that it actually do when the experimental value

(-8.48 MeV) is used. From this point of view the wave functions solution of realistic potentials are not accurately "realistic wave functions" because their tail (except for the GPDT potential) extend to far away. We have shown in our analysis with the model potentials V^x and G2 that the shape of the two first partial waves K=0 (full symmetry) and K=1 (mixed symmetry) are to some extend insensitive to the strength of the core except at small distances when the potential has been chosen in such a way to produce the same binding energy and the same size of the trinucleon. Therefore one can guess that many differences between results obtained with various potentials proceed from discrepancies between binding energies and radii produced by these interactions.

The best probe to test the quality of a wave function solution of the Schrödinger equation may be the comparison between the Fourier transform of the charge density and the charge form factor of the iso doublet ^{3}H-^{3}He. We found with our model potential that even with a very strong repulsive core which produce a zero of the form factor at the right value ($q^2 \simeq 11.5$ fm^{-2}) the amplitude of the second maximum of the form factor is missed by a factor 2. This effect is still more pronounced with the super soft core realistic potentials analyzed, in which neither the position of the zero nor the amplitude of the maximum are obtained.

Even the difference between the form factors of ^{3}H and ^{3}He at small momentum transfer which is related to the charge radii is not obtained. This last result proceed from a too small amount of mixed symmetry component in the wave function which

in turn results mainly from a too small difference between the singlet and triplet even potentials which generate this component.

All the realistic potential giving the same result a question arises about either the accuracy of the experiment on ^3H or the behaviour of the realistic even central potentials.

An accord with the experimental charge form factor has been found by Sick with a charge density exhibiting a hole near the origin. On the fig.14 we have plotted the point like proton density for the G2 potential together with the same density deduced by Sick (59) from the experimental charge form factors. In spite of a good agreement between the two densities at large enough distances we did not find any hole in the proton density. We therefore believe that the hole simulate effects proceeding from other physical contributions including exchange currents in nuclei.

An attempt has been done to introduce a three body force in order to see the influence on the form factor. This force has been approximated by its first hyperspherical term, function of ξ only, and we retained the asymptotical shape in ξ^{-6} cut by a repulsive core at a distance chosen in such a way to obtain the experimental binding energy. The effect on the amplitude of the second maximum is not sufficient to give a result better than the one obtained with the G2 potential. Brayshaw (63) did a calculation from which he claims the existence of a strong three body force. In his calculation he retained only the terms K=0, 1, 2 in the H.H expansion missing completely a converged solution. This kind of calculation is a

typical example of what is forbidden to do with the expansion method. Firstly in neglecting the H.H for $K \geq 3$ one miss the correlations generated by the odd K harmonics . On the other hand to the truncation of the wave function corresponds similar truncation in the expansion of the potential which then cannot longer represent the expansion of two body potential especially when the truncation is done for too small K. It is then not surprising to find a large missing part in the interaction which can be misinterpreted as a three body force, indeed writting the first of the coupled equations for K=0

$$\left[-\frac{\hbar^2}{m}\frac{d}{dr^2} + W(r) - E \right] u_0(r) = 0 \text{ with}$$

$$W(r) = \frac{15}{4}\frac{\hbar^2}{mr^2} + V_0(r) + \frac{1}{u_0(r)} \sum_{K'=1}^{\infty} U_0^{K'}(r) u_{2K'}(r)$$

the sum of the neglected components $\sum_{K=3}^{\infty} U_0^{K'}(r) \frac{u_{2K}(r)}{u_0(r)}$

may be also wrongly identified to a three body central force.

We want to stress the efficiency of the small components K odd of the wave function responsible for the correlations, indeed these components contribute respectively 0.33 % and 0.53 % to the norm of the wave function for the GPDT and supersoft core Sprung-de Tourreil potentials. The photodesintegration is another example in which small components cannot be neglected. We have seen that for a contribution of about 0.8 % in the wave function the S' state increases by 20% the two body ^{3}He photodesintegration cross section. As a consequence of the tensorial character of the dipole operator which is a vector in the 6 dimensional space only a small number of H.H is needed

in order to obtain a good accuracy in the calculated cross section. Nevertheless most of the magnitude of the cross section proceed from the overlap of the tail of ^{3}He wave function with the final state. The tail is clearly well described only in a completely converged solution. Therefore the claim of Jibuti (50), that it is a good approximation to use wave functions integrated with a truncation at K=4 for nuclear reactions is certainly quite unreasonable, because the higher partial waves modify the shape of the first partial waves during the process of integration in which the binding energy, and consequently the tail, decrease.

We found that the introduction of the D state has a destructive effect on the photodesintegration cross section. This result is in contradiction with the Caver, Kim and Tubis (46) calculations. Especially we did not find the flattening effect of the D state on the cross section described by these authors.

The H.H expansion method appears to be a power full mean for integrating the Schrödinger equation for bound states, but one must be carefull to include a large enough number of partial waves in order to obtained a reliable solution. A guide to this truncation is given by the trend of the convergence which is quite definite by the shape of the potential. In contrast to the integral equations, it is not more difficult to apply the H.H expansion method to the four nucleon bound state (^{4}He). It requires only the calculation of other coupling coefficients. This method seems therefore promissing for solving at least the few-body bound state problems.

Appendix A

Antisymmetrization of three body isospin-spin states

We use the normalized spin functions in which the total spin s_{ij} of a pair of nucleons (i,j) is coupled to the spin of the last nucleon k in order to give a total spin S with projection m_S :

$$|(s_{ij}\tfrac{1}{2})Sm_S\rangle_k = \sum_{\nu_1\nu_2\nu_3} (-1)^{\frac{1}{2}-s_{ij}-\nu-m_S}\, \hat{S}\hat{s}_{ij} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & s_{ij} \\ \nu_1 & \nu_2 & -\nu \end{pmatrix} \begin{pmatrix} s_{ij} & \frac{1}{2} & S \\ \nu & \nu_3 & -m_S \end{pmatrix} s_{\frac{1}{2}}^{\nu_1}(i) s_{\frac{1}{2}}^{\nu_2}(j) s_{\frac{1}{2}}^{\nu_3}(k) \qquad (A.1)$$

with $\hat{S} = \sqrt{2S+1}$.

A similar treatment of the isospin functions leads to a state of total isospin T with projection m_T :

$$|(t_{ij}\tfrac{1}{2})Tm_T\rangle_k = \sum_{\mu_1\mu_2\mu_3} (-1)^{\frac{1}{2}-t_{ij}-\mu-m_T}\, \hat{T}\hat{t}_{ij} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & t_{ij} \\ \mu_1 & \mu_2 & -\mu \end{pmatrix} \begin{pmatrix} t_{ij} & \frac{1}{2} & T \\ \mu & \mu_3 & -m_T \end{pmatrix} t_{\frac{1}{2}}^{\mu_1}(i) t_{\frac{1}{2}}^{\mu_2}(j) t_{\frac{1}{2}}^{\mu_3}(k) \qquad (A.2)$$

where the subscript k of the ket labels the single particle coupled to the pair (i,j).

α) - S=½ T=½ states

In eqs. A.1 or A.2, the two normalized spin (or isospin) states $|(0\tfrac{1}{2})\tfrac{1}{2}m\rangle_k$ and $|(1\tfrac{1}{2})\tfrac{1}{2}m\rangle_k$ are respectively antisymmetric and symmetric for the exchange i ⇔ j. We introduce the spin (or isospin) vector,

$$|W(\varphi)> = \sin\varphi\,|(0\tfrac{1}{2})\tfrac{1}{2}m>_k + \cos\varphi\,|(1\tfrac{1}{2})\tfrac{1}{2}m>_k \qquad (A.3)$$

similar to the kinematic rotation vector (II.5). A suitable set of angles φ can simulate the effects of spin (or isospin) exchange operators. We have the relations :

$$\begin{aligned}
&|W(\tfrac{\pi}{2})> = |(0\tfrac{1}{2})\tfrac{1}{2}m>_k = |->_k && \text{Antisym. in } i \leftrightarrow j\\
&|W(\tfrac{\pi}{2} - \tfrac{2\pi}{3})> = |(0\tfrac{1}{2})\tfrac{1}{2}m>_i = \tfrac{1}{2}\{\sqrt{3}|+>_k - |->_k\} && \text{Antisym. in } j \leftrightarrow k\\
&|W(\tfrac{\pi}{2} + \tfrac{2\pi}{3})> = |(0\tfrac{1}{2})\tfrac{1}{2}m>_j = -\tfrac{1}{2}\{\sqrt{3}|+>_k + |->_k\} && \text{Antisym. in } k \leftrightarrow i\\
&|W(0)> = |(1\tfrac{1}{2})\tfrac{1}{2}m>_k = |+>_k && \text{sym. in } i \leftrightarrow j\\
&|W(-\tfrac{2\pi}{3})> = |(1\tfrac{1}{2})\tfrac{1}{2}m>_i = -\tfrac{1}{2}\{|+>_k + \sqrt{3}|->_k\} && \text{sym. in } j \leftrightarrow k\\
&|W(\tfrac{2\pi}{3})> = |(1\tfrac{1}{2})\tfrac{1}{2}m>_j = \tfrac{1}{2}\{-|+>_k + \sqrt{3}|->_k\} && \text{sym. in } k \leftrightarrow i
\end{aligned} \qquad (A.4)$$

In order to construct the basic functions of isospin-spin, we consider linear combinations of products $|W(\varphi_1)W(\varphi_2)>$ where the left and right part of the ket stand for isospin and spin functions respectively. Using eqs. (A.4) we construct two isospin-spin vectors,

$$|W(\varphi)W(\varphi - \tfrac{\pi}{2})> = -\sin\varphi\cos\varphi\{|-,->-|+,+>\} + \sin^2\varphi|-,+> - \cos^2\varphi|+,-> \qquad (A.5)$$

$$|W(\varphi - \tfrac{\pi}{2})W(\varphi)> = -\sin\varphi\cos\varphi\{|-,->-|+,+>\} + \sin^2\varphi|+,-> - \cos^2\varphi|-,+> \qquad (A.6)$$

which for values of $\varphi = \frac{\pi}{2}, \frac{\pi}{2} - \frac{2\pi}{3}, \frac{\pi}{2} + \frac{2\pi}{3}$ are antisymmetric in the corresponding interchange ($i \leftrightarrow j$), ($j \leftrightarrow k$), ($k \leftrightarrow i$). The two isospin-spin vectors

$$|W(\varphi),W(\varphi)> = \sin\varphi\cos\varphi\{|+,->+|-,+>\}+\sin^2\varphi|-,->+\cos^2\varphi|+,+> \qquad (A.7)$$

$$|W(\varphi-\tfrac{\pi}{2}),W(\varphi-\tfrac{\pi}{2})> = -\sin\varphi\cos\varphi\{|+,->+|-,+>\}+\sin^2\varphi|+,+>+\cos^2\varphi|-,-> \qquad (A.8)$$

are symmetric in the interchange (i↔j, j↔k, k↔i) for the same values of $\varphi = \frac{\pi}{2}, \frac{\pi}{2} - \frac{2\pi}{3}, \frac{\pi}{2} + \frac{2\pi}{3}$ respectively.
From these vectors two linear orthonormal combinations independent of φ can be constructed. The first

$$\frac{1}{\sqrt{2}}\{-|W(\varphi),W(\varphi-\tfrac{\pi}{2})>+|W(\varphi-\tfrac{\pi}{2}),W(\varphi)>\} = \frac{1}{\sqrt{2}}\{|+,->-|-,+>\} = \Gamma_{\frac{1}{2}\frac{1}{2}}(A) \quad (A.9)$$

is completely antisymmetric under any exchange of nucleons, while

$$\frac{1}{\sqrt{2}}\{|W(\varphi)W(\varphi)>+ W(\varphi-\tfrac{\pi}{2})W(\varphi-\tfrac{\pi}{2})>\} = \frac{1}{\sqrt{2}}\{|+,+>+|-,->\} = \Gamma_{\frac{1}{2}\frac{1}{2}}(S) \quad (A.10)$$

is completely symmetric.
We now construct two mixed symmetry combinations :

$$\frac{1}{\sqrt{2}}\{|W(\varphi),W(\varphi-\tfrac{\pi}{2})>+ |W(\varphi-\tfrac{\pi}{2}),W(\varphi)>\} = -\frac{1}{\sqrt{2}}\{|-,->-|+,+>\}\sin2\varphi - \frac{1}{\sqrt{2}}\{|+,->+|-,+>\}\cdot\cos2\varphi$$

$$= \sin2\varphi\ \Gamma_{\frac{1}{2}\frac{1}{2}}(M+) - \cos2\varphi\ \Gamma_{\frac{1}{2}\frac{1}{2}}(M-), \qquad (A.11)$$

and

$$\frac{1}{\sqrt{2}}\{-|W(\varphi)W(\varphi)>+ W(\varphi-\tfrac{\pi}{2})W(\varphi-\tfrac{\pi}{2})>\} = -\frac{1}{\sqrt{2}}\{|+,->+|-,+>\}\sin2\varphi - \frac{1}{\sqrt{2}}\{|+,+>-|-,->\}\cdot\cos2\varphi$$

$$= -\sin2\varphi\ \Gamma_{\frac{1}{2}\frac{1}{2}}(M-) - \cos2\varphi\ \Gamma_{\frac{1}{2}\frac{1}{2}}(M+), \qquad (A.12)$$

which like previously for $\varphi = \frac{\pi}{2}, \frac{\pi}{2} - \frac{2\pi}{3}, \frac{\pi}{2} + \frac{2\pi}{3}$ are respectively antisymmetric and symmetric under the i↔j, j↔k, k↔i exchange. We have denoted by $\Gamma_{TS}(R)$ the orthonormal irreductible representations of the isospin-spin states for trinucleon systems.

β) $S=\frac{1}{2}$ $T=\frac{3}{2}$ states

The isospin states $|(1\frac{1}{2})\frac{1}{2}\ m>_K$ completely symmetric under any exchange of two nucleons is denoted by $|0>$. The symmetry properties with respect to isospin spin are fixed by the spin function only. Two kinds of functions can be constructed

$$|0,W(\varphi)> = \sin\varphi\,|0,-> + \cos\varphi\,|0,+> = \sin\varphi\ \Gamma_{\frac{1}{2}\frac{1}{2}}(M-) + \cos\varphi\ \Gamma_{\frac{1}{2}\frac{1}{2}}(M+) \qquad (A.13)$$

$$|0,W(\varphi-\frac{\pi}{2})> = -\cos\varphi\,|0,-> + \sin\varphi\,|0,+> = -\cos\varphi\ \Gamma_{\frac{3}{2}\frac{1}{2}}(M-) + \sin\varphi\ \Gamma_{\frac{3}{2}\frac{1}{2}}(M+) \qquad (A.14)$$

They are respectively antisymmetric and symmetric according to φ in the exchange $(i\leftrightarrow j,\ j\leftrightarrow k,\ k\leftrightarrow i)$.

γ) $S=\frac{3}{2}$ $T=\frac{1}{2}$ states

In a quite similar way two kinds of isospin-spin functions are constructed :

$$|W(\varphi),0> = \sin\varphi\,|-,0> + \cos\varphi\,|+,0> = \sin\varphi\ \Gamma_{\frac{1}{2}\frac{3}{2}}(M-) + \cos\varphi\ \Gamma_{\frac{1}{2}\frac{3}{2}}(M+) \qquad (A.15)$$

$$|W(\varphi-\frac{\pi}{2}),0> = -\cos\varphi\,|-,0> + \sin\varphi\,|+,0> = -\cos\varphi\ \Gamma_{\frac{1}{2}\frac{3}{2}}(M-) + \sin\varphi\ \Gamma_{\frac{1}{2}\frac{3}{2}}(M+) \qquad (A.16)$$

They are respectively antisymmetric and symmetric for suitable φ in the exchange $(i\leftrightarrow j,\ j\leftrightarrow k,\ k\leftrightarrow i)$.

6) $S=\frac{1}{2}$ $T=\frac{1}{2}$ states

There is only one state

$$|0,0\rangle = \Gamma_{\frac{1}{2}\,\frac{1}{2}}(S) \qquad (A.17)$$

fully symmetric under any exchange.

Appendix B

Derivation of the hyperspherical coefficients

When we expand the trinucleon wave function in the potential basis, the elements of the potential matrix are :

$$\langle [\Gamma_{TS}(R) \otimes {}^{R}\overset{*}{B}_{[2K+\ell]}]_J | V(\xi,\Omega) | [\Gamma_{T'S'}(R') \otimes {}^{R'}\overset{*}{B}_{[2K'+\ell']}]_J \rangle \qquad (B.1)$$

where

$$V(\xi,\Omega) = \sum_{i>j} V(\vec{x}_{i,j}) = \sum_{i>j} \pi^{\frac{1}{2}} \sum_{K''\ell''m''\alpha} (-1)^{K''} {}^{\alpha}a^{m''}_{\ell''(ij)} {}^{\alpha}V^{(6,\ell'')}_{2K''}(\xi) \cdot A^{\ell'',m''}_{2K''}(\varphi^{(i,j)},\Omega) . \qquad (B.2)$$

The states $|[\Gamma_{TS}(R) \otimes {}^{R}\overset{*}{B}_{[2K+\ell]}]_J\rangle$ are completely antisymmetric and so

$$\langle [\Gamma_{TS}(R) \otimes {}^{R}\overset{*}{B}_{[2K+\ell]}]_J | V(\xi,\Omega) | [\Gamma^{(R')}_{T'S'} \otimes {}^{R'}\overset{*}{B}_{[2K'+\ell']}]_J \rangle =$$

$$= 3\langle [\Gamma_{TS}(R) \otimes {}^{R}\overset{*}{B}_{[2K+\ell]}]_J | V(r_{ij}) | [\Gamma^{(R')}_{T'S'} \otimes {}^{R'}\overset{*}{B}_{[2K'+\ell']}]_J \rangle \qquad (B.3)$$

Introducing (B.2) into (B.3) one must calculate

$$\langle [\Gamma_{TS}(R) \otimes {}^{R}\overset{*}{B}_{[2K+\ell]}]_J | V(\xi,\Omega) | [\Gamma^{(R')}_{T'S'} \otimes {}^{R'}\overset{*}{B}_{[2K'+\ell']}]_J \rangle =$$

$$= 3\,\pi^{\frac{1}{2}} \sum_{K''\ell''m''\alpha} (-1)^{K''} \langle [\ ]_J | {}^{\alpha}a^{m''}_{\ell''(ij)} \cdot {}^{\alpha}V^{(6,\ell'')}_{2K''}(\xi) \cdot A^{\ell'',m''}_{2K''}(\varphi^{(i,j)},\Omega) | [\ ]_J \rangle , \qquad (B.4)$$

where

$$A^{\ell",m"}_{2K"}(\varphi^{(i,j)},\Omega)=\frac{\Gamma(\ell"+\frac{1}{2})}{\Gamma(\frac{\ell"}{2}+3)}\sum_{\lambda_1\lambda_2}(-1)^{\lambda_1+\lambda_2}\frac{\Delta(\ell"\lambda_1\lambda_2)}{\sqrt{4\pi(2\ell"+1)}}\,{}^{(2)}P^{\lambda_2\lambda_1}_{2K"+\ell"}(\varphi^{ij})\cdot$$

$$\cdot\,{}^{(2)}P^{\lambda_2\lambda_1}_{2K"+\ell"}(\phi)\;Y^{m"}_{(\lambda_1\lambda_2)\ell"}(\tilde{\xi}_1\tilde{\xi}_2) \qquad (B.5)$$

Choosing for (i,j) the pair (1,2) eq. (B.5) becomes :

$$A^{\ell",m"}_{2K"}(\varphi^{1,2},\Omega)=\frac{\Gamma(\ell"+\frac{3}{2})}{\Gamma(\frac{\ell"}{2}+3)}\sum_{\lambda}\frac{\Delta(\ell"\lambda\lambda)}{\sqrt{4\pi(2\ell"+1)}}\,{}^{(2)}P^{\lambda\lambda}_{(\pi/2)2K"+\ell"}\,{}^{(2)}P^{\lambda\lambda}_{(\phi)2K"+\ell"}\,Y^{m"}_{(\lambda\lambda)\ell"}(\tilde{\xi}_1,\tilde{\xi}_2)\cdot \qquad (B.6)$$

Performing the operations proceeding from the tensorial product contained in the vectors $|[\Gamma_{TS}(R)\otimes {}^{R^*}B_{2K+\ell}]^{M_J}_J>$, we have :

$$<[\Gamma_{TS}(R)\otimes {}^{R^*}B_{2K+\ell}]^{M_J}_J|V(\xi,\Omega)|[\Gamma_{T'S'}(R')\otimes {}^{R'^*}B_{2K'+\ell'}]^{M_J}_J>=$$

$$3\sum_{K"\ell"\alpha ss'tt'}(-1)^{K"+\ell+\ell'}\pi^{\frac{3}{2}}\,{}^{R^*}N_{2K+\ell}\,{}^{R'^*}N_{2K'+\ell'}\sum_{m"\ell_1\ell_2\ell_1'\ell_2'}\hat{\ell}_1\hat{\ell}_2\hat{\ell}_1'\hat{\ell}_2'\cdot$$

$${}^{R^*}F^{\ell_2\ell_1}_{2K+\ell}\cdot{}^{R'^*}F^{\ell_2'\ell_1'}_{2K'+\ell'}\cdot\begin{pmatrix}\ell & \ell_1 & \ell_2\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\ell' & \ell_1' & \ell_2'\\ 0 & 0 & 0\end{pmatrix}b^t_s(R)b^{t'}_{s'}(R)\cdot$$

$$<(t\tfrac{1}{2})Tm_T|{}^{\alpha}V^{(6,\ell")}_{(\xi)2K"}|(t'\tfrac{1}{2})Tm'_T>\cdot<{}^{(2)}P^{\ell_2\ell_1}_{2K+\ell}(\ell_1\ell_2)\ell(s\tfrac{1}{2})SJM_J|{}^{\alpha}a^{m"}_{\ell"(ij)}A^{\ell"m"}_{2K"}(\varphi^{ij},\Omega)$$

$$|(\ell_1'\ell_2')\ell'(s'\tfrac{1}{2})S'JM_J\,{}^{(2)}P^{\ell_2'\ell_1'}_{2K'+\ell'}>\cdot \qquad (B.7)$$

Let us define the coupling coefficients $C^{(2K'+\ell')R's'S'J}_{(K",\alpha,\ell")(2K+\ell)RsSJ}$ appearing in the previous formula by :

$$C^{(2K'+\ell')R's'S'J}_{(2K+\ell)RsSJ}{}^{(K'',\alpha,\ell'')} = \pi^{\frac{1}{2}}\ {}^{R}\mathcal{N}^{*}_{2K+\ell}\ {}^{R'}\mathcal{N}^{*}_{2K'+\ell'}\ (-1)^{\ell+\ell'} \sum_{m''\ell_1\ell_2\ell_1'\ell_2'} \hat{\ell}_1\hat{\ell}_2\hat{\ell}_1'\hat{\ell}_2'$$

$$\begin{pmatrix} \ell & \ell_1 & \ell_2 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \ell' & \ell_1' & \ell_2' \\ 0 & 0 & 0 \end{pmatrix} b^{t}_{s}(R)b^{t'}_{s'}(R')\cdot\ {}^{R}\mathcal{F}^{*}{}^{\ell_2\ell_1}_{2K+\ell}\ {}^{R'}\mathcal{F}^{*}{}^{\ell_2\ell_1}_{2K'+\ell'} <|\ {}^{\alpha}Q^{m''}_{\ell''}(ij)A^{\ell''m''}_{2K''}(\varphi^{ij},\Omega)|> \qquad \text{(B.8)}$$

- For $\ell''=0$, $m''=0$ $\alpha=(C,LS,LL,L2,Q)$

$${}^{\alpha}Q^{0}_{0}(ij) = \sqrt{4\pi}\ \Xi^{\alpha}$$

$$\Xi^{\alpha} = (1,\ \vec{\ell}_{ij}.\vec{s}_{ij}, LL_{ij},\ Q_{ij},\ \vec{\ell}^{2}_{ij}).$$

We deal with the coupling coefficients

$$C^{(2K'+\ell')R's'S'J}_{(2K+\ell)RsSJ}{}^{(K'',\alpha,0)} = 2\pi^{2}\ {}^{R}\mathcal{N}^{*}_{2K+\ell}\ {}^{R'}\mathcal{N}^{*}_{2K'+\ell'}\ \frac{(-1)^{\ell+\ell'}}{\hat{\ell}\hat{\ell}'} \sum_{\ell_1\ell_2\ell_1'\ell_2'} {}^{R}\mathcal{F}^{*}{}^{\ell_2\ell_1}_{2K+\ell}\cdot$$

$${}^{R'}\mathcal{F}^{*}{}^{\ell_2'\ell_1'}_{2K'+\ell'}\ \Delta(\ell_1\ell_2\ell)\ \Delta(\ell_1'\ell_2'\ell)\ b^{t}_{s}(R)b^{t'}_{s'}(R') <|\Xi^{\alpha}A^{0,0}_{2K''}(\varphi^{ij}\Omega)|> . \qquad \text{(B.9)}$$

The calculation of the matrix elements $<|\Xi^{\alpha}\ A|>$ leads to the determination of the two following matrix elements :

$$<(\ell_2\ell_1)\ell(s\tfrac{1}{2})SJM_J|\ A^{\ell''m''}_{2K''}(\varphi^{ij}\Omega)\ |(\ell_2'\ell_1')\ell'(s'\tfrac{1}{2})S'J'M_J'>=(4\pi)^{-\frac{1}{2}}\ \frac{\Gamma(\ell''+\frac{1}{2})}{\Gamma(\frac{\ell''}{2}+3)}\cdot$$

$$\delta_{ss'}\delta_{SS'}\hat{J}\hat{J}'\sum_{\lambda_1\lambda_2}(-1)^{\ell_1'+\ell_2'+\ell'+S+J+J'-M_J}\Delta(\lambda_1\lambda_2\ell'')\Delta(\ell_1\lambda_1\ell_1')\Delta(\ell_2\lambda_2\ell_2')\hat{\ell}\hat{\ell}'$$

$$\begin{Bmatrix} J' & \ell'' & J \\ \ell & S & \ell' \end{Bmatrix}\begin{Bmatrix} \ell_1 & \lambda_1 & \ell_1' \\ \ell_2 & \lambda_2 & \ell_2 \\ \ell & \ell'' & \ell' \end{Bmatrix}\cdot\begin{pmatrix} J' & \ell'' & J \\ -M_J' & -m'' & M_J \end{pmatrix}{}^{(2)}P^{\lambda_2\lambda_1}_{2K''+\ell''}(\varphi^{(i,j)})\cdot\ {}^{(2)}P^{\lambda_2\lambda_1}_{2K''+\ell'}(\phi)\ , \qquad \text{(B.10)}$$

and

$$\langle(\ell_2\ell_1)\ell;(s\tfrac{1}{2})S;JM_J|\Xi^\alpha|(\ell_2''\ell_1'')\ell'';(s''\tfrac{1}{2})S'';J''M_J''\rangle =$$

$$\delta_{ss''}\delta_{SS''}\delta_{\ell\ell''}\delta_{JJ''}\delta_{M_JM_J''}\delta_{\ell_1\ell_1''}\delta_{\ell_1\ell_2''}\sum_{j_1j_2}\hat{j}_1^2\hat{j}_2^2\hat{S}^2\hat{\ell}^2\begin{Bmatrix}\ell_1 & \ell_2 & \ell\\ s & \frac{1}{2} & S\\ j_1 & j_2 & J\end{Bmatrix}\Xi^\alpha(\ell_1sj_1) \qquad (B.11)$$

where $\Xi^\alpha(\ell_1sj_1) = \langle(\ell_1s)j_1|\Xi^\alpha|(\ell_1s)j_1\rangle$. (B.12)

$\hat{j}^2 = (2j+1)$.

Using B.10 for $\ell''=m''=0$ one finds :

$$\langle(\ell_1\ell_2)\ell;(s\tfrac{1}{2})S;JM_J|A^{0,0}_{2K''}(\varphi^{ij})|(\ell_2'\ell_1')\ell';(s'\tfrac{1}{2})S';JM_J\rangle=\frac{1}{32\pi}\ \delta_{ss'}\,\delta_{\ell\ell'}\delta_{SS'}$$

$$\sum_\lambda(-1)^{\lambda+\ell}\hat{\lambda}^{-2}\Delta(\ell_1\lambda\ell_1')\Delta(\ell_2\lambda\ell_2')\ \begin{Bmatrix}\ell_1 & \ell_1' & \lambda\\ \ell_2' & \ell_2 & \ell\end{Bmatrix}\ {}^{(2)}P^{\lambda\lambda}_{2K''}(\varphi^{(i,j)})\ {}^{(2)}P^{\lambda\lambda}_{2K''}(\phi) \qquad (B.13)$$

Then

$$C^{(2K'+\ell')R's'S'J}_{(2K+\ell)RsSJ}{}^{(K'',\gamma,0)} = \frac{\pi}{16}\delta_{ss'}\delta_{SS'}\delta_{\ell\ell'}\ {}^{R}N^{*}_{2K+\ell}\ {}^{R'}N^{*}_{2K'+\ell}\sum_{\substack{\ell_1\ell_2\ell_1'\ell_2'\\ j_1j_2\lambda}}$$

$$(-1)^{\lambda+\ell}\Delta(\ell_1\lambda\ell_1')\Delta(\ell_2\lambda\ell_2')\Delta(\ell_1\ell_2\ell)\Delta(\ell_1'\ell_2'\ell)\hat{j}_1^2\hat{j}_2^2\hat{S}^2\hat{\lambda}^{-2}\ {}^{(2)}P^{\lambda\lambda}_{(\varphi^{(i,j)})2K''}$$

$$\Xi^\gamma(\ell_1sj_1)\ \begin{Bmatrix}\ell_1 & \ell_2 & \ell\\ s & \frac{1}{2} & S\\ j_1 & j_2 & J\end{Bmatrix}^2 b^t_S(R)b^{t'}_{S'}(R')\ \begin{Bmatrix}\ell_1 & \ell_1' & \lambda\\ \ell_2' & \ell_2 & \ell\end{Bmatrix}\ {}^{R}F^{*}_{2K+\ell}{}_{z\ell_1}\ {}^{R'*}F^{\ell_2'\ell_1'}_{2K'+\ell}$$

$$\langle\ {}^{(2)}P^{\ell_2\ell_1}_{2K+\ell}|\ {}^{(2)}P^{\lambda\lambda}_{2K''}|\ {}^{(2)}P^{\ell_2'\ell_1'}_{2K'+\ell}\ \rangle\ . \qquad (B.14)$$

If we choose the pair (1,2) as a reference, $\varphi^{(1,2)}=\frac{\pi}{2}$

$${}^{(2)}P^{\lambda,\lambda}_{(\pi/2)2K''} = \delta_{\lambda0}\ P^{0,0}_{(\pi/2)2K''}$$

$$C^{(2K'+\ell')R's'S'J}_{(K'',\gamma,0)\,(2K+\ell)RsSJ} = \frac{\pi}{16}\,\delta_{ss'}\delta_{SS'}\delta_{\ell\ell'}\;{}^{R}\!N^{*}_{2K+\ell}\;{}^{R'}\!N^{*}_{2K'+\ell}\;P^{0,0}_{2K''(\pi/2)}$$

$$\sum_{\ell_1\ell_2 j_1 j_2}(2\ell+1)\begin{pmatrix}\ell_1 & \ell_2 & \ell\\ 0 & 0 & 0\end{pmatrix}^2 b^{t}_{s}(R)\,b^{t'}_{s'}(R')\,\hat{\ell}_1^2\hat{\ell}_2^2\hat{s}^2\hat{j}_1^2\hat{j}_2^2\,\Xi^{\gamma}(\ell_1 s j_1)\begin{Bmatrix}\ell_1 & \ell_2 & \ell\\ s & j & S\\ j_1 & j_2 & J\end{Bmatrix}^2$$

$${}^{R}\!F^{*}_{2K+\ell}{}^{\ell_2\ell_1}\;{}^{R'}\!F^{*}_{2K'+\ell}{}^{\ell_2\ell_1} < {}^{(2)}P^{\ell_2\ell_1}_{2K+\ell}\,|\,{}^{(2)}P^{0,0}_{2K''}\,|\,{}^{(2)}P^{\ell_2\ell_1}_{2K'+\ell}> \,. \qquad (B.15)$$

β- For tensor forces $\ell''=2$

We must consider the coefficients

$$C^{(2K'+\ell')R's'S'J}_{(K'',T,2)\,(2K+\ell)RsSJ} = \pi^{\frac{3}{2}}\;{}^{R}\!N^{*}_{2K+\ell}\;{}^{R'}\!N^{*}_{2K'+\ell'}\,(-1)^{\ell+\ell'}\sum_{\substack{m''\ell_1\ell_2\ell_1'\ell_2'\\ mm'm_S m_S'}}$$

$${}^{R}\!F^{*}_{2K+\ell}{}^{\ell_2\ell_1}\;{}^{R'}\!F^{*}_{2K'+\ell'}{}^{\ell_2\ell_1}\;\hat{\ell}_1\hat{\ell}_2\hat{\ell}_1'\hat{\ell}_2'\,\hat{J}^2(-1)^{S-\ell-M_J+S'-\ell'-M_J}$$

$$\begin{pmatrix}\ell_1 & \ell_2 & \ell\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\ell_1' & \ell_2' & \ell'\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\ell & S & J\\ m & m_S & -M_J\end{pmatrix}\begin{pmatrix}\ell' & S' & J\\ m' & m_S' & -M_J\end{pmatrix}<(s\tfrac{1}{2})Sm_S|^{T\mathcal{C}m''}_{2}|(s'\tfrac{1}{2})S'm_S'>$$

$$b^{t}_{s}(R)b^{t'}_{s'}(R') < {}^{(2)}P_{2K+\ell}{}^{<(\ell_1\ell_2)\ell m|}\,A^{2,m''}_{2K''}(\varphi^{(i,j)})\,|(\ell_2'\ell_1')\ell'm'>\,{}^{(2)}P^{\ell_2\ell_1}_{2K'+\ell'}> \qquad (B.16)$$

We have

$$<(\ell_1\ell_2)\ell m|\,A^{2,m''}_{2K''}(\varphi^{(i,j)})\,|(\ell_1'\ell_2')\ell'm'>= \frac{5}{128\pi}\sum_{\ell_1''\ell_2''}(-1)^{\ell+m+\ell_1'+\ell_2'}$$

$$\Delta(\ell_1''\ell_2''2)\,\Delta(\ell_1\ell_1''\ell_1')\,\Delta(\ell_2\ell_2''\ell_2')\,\hat{\ell}\hat{\ell}'\begin{pmatrix}\ell' & \ell & 2\\ -m' & m & -m''\end{pmatrix}\begin{Bmatrix}\ell_1 & \ell_1'' & \ell_1'\\ \ell_2 & \ell_2'' & \ell_2'\\ \ell & 2 & \ell'\end{Bmatrix}\;{}^{(2)}P^{\ell_2''\ell_1''}_{2K''+2}(\varphi^{(i,j)})\,.$$

$$\cdot\,{}^{(2)}P^{\ell_2''\ell_1''}_{2K''+2}(\phi) \qquad (B.17)$$

and

$$\langle (s\tfrac{1}{2})Sm_S | {}^{T}Q_2^{m''}(ij) | (s'\tfrac{1}{2})S'M'_S \rangle = 24\sqrt{2\pi}\,\hat{s}\hat{s}'\hat{S}\hat{S}' \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s \\ \frac{1}{2} & \frac{1}{2} & 1 \end{Bmatrix} \begin{Bmatrix} s' & 2 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} S & 2 & S' \\ s' & \frac{1}{2} & s \end{Bmatrix}$$
$$\cdot (-1)^{2S-\frac{1}{2}-m'_S} \begin{pmatrix} S & 2 & S' \\ m_S & m'' & -m'_S \end{pmatrix} \qquad (B.18)$$

Inserting these expressions in eq. B.16, one finds

$$C^{(2K'+\ell')R's'S'J}_{(2K+\ell)RsSJ} = \frac{15\sqrt{2}}{16}\pi\, {}^{R}\mathcal{N}_{2K+\ell}\, {}^{R'}\mathcal{N}_{2K'+\ell'}\, \hat{J}^2\hat{S}\hat{S}'\hat{S}\hat{S}'\hat{\ell}'^{-2} \sum_{\substack{\ell_1\ell_1'\ell_2\ell_2' \\ \ell_1''\ell_2''}}$$
$$ {}^{R}F^{\ell_2\ell_1}_{2K+\ell}\, {}^{R'}F^{\ell_2'\ell_1'}_{2K'+\ell'}\, b^{t}_{s}(R)b^{t'}_{s'}(R')\, {}^{(2)}P^{\ell_2''\ell_1''}_{2K''+2}(\varphi^{i,j})\Delta(\ell_1\ell_2\ell)\Delta(\ell_1'\ell_2'\ell')\Delta(\ell_1''\ell_1'')$$
$$\Delta(\ell_2\ell_2''\ell_2')(-1)^{S-S'-\frac{1}{2}+J} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s' \\ \frac{1}{2} & \frac{1}{2} & 1 \end{Bmatrix} \begin{Bmatrix} s' & 2 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} S & 2 & S' \\ s' & \frac{1}{2} & s \end{Bmatrix} \begin{Bmatrix} S' & J & \ell' \\ \ell & 2 & S \end{Bmatrix} \begin{Bmatrix} \ell_1 & \ell_1'' & \ell_1' \\ \ell_2 & \ell_2'' & \ell_2' \\ \ell & 2 & \ell' \end{Bmatrix}$$
$$\langle {}^{(2)}P^{\ell_2\ell_1}_{2K+\ell} | {}^{(2)}P^{\ell_2''\ell_1''}_{2K''+2} | {}^{(2)}P^{\ell_2'\ell_1'}_{2K'+\ell'} \rangle \qquad (B.19)$$

Appendix C

Multipole matrix elements

The multipoles of the two body potential appearing in the coupled equations (III.53) are defined by :

$$V^{\gamma,\ell''}_{2K''}(\xi,R,R') = 6\,\frac{\Gamma(\frac{\ell''}{2}+3)\,\Gamma(K''+\ell''+\frac{1}{2})}{\Gamma(\frac{\ell''+3}{2})\,\Gamma(K''+\frac{1}{2})\,\Gamma(\ell''+\frac{1}{2})}\; b^{t}_{s}(R)\; b^{t'}_{s'}(R') \int_0^\infty du\,(1-u)^{\frac{1}{2}}$$

$$u^{\ell''+2}\; {}_2F_1(-K'',\ell'+K''+5,\ell''+\tfrac{3}{2};u^2).\langle (s\tfrac{1}{2})S m_S,(t\tfrac{1}{2})T m_T | V^{\ell''}_{\gamma}(\xi u) | (s'\tfrac{1}{2})s' m'_S,(t'\tfrac{1}{2})T' m'_T\rangle \qquad (C.1)$$

For $\ell''=0$, $V^{0}_{\gamma}(r_{ij})$ which contains all the exchange terms may be written as follow

$$V^{0}_{\gamma}(r_{ij}) = \frac{1}{2}(1-P^{\tau}_{ij}\,P^{\sigma}_{ij})V^{+}_{\gamma}(r_{ij}) + \frac{1}{2}(1+P^{\tau}_{ij}\,P^{\sigma}_{ij})V^{-}_{\gamma}(r_{ij}) - \frac{1}{2}(P^{\tau}_{ij}-P^{\sigma}_{ij})U^{+}_{\gamma}(r_{ij})$$
$$+ \frac{1}{2}(P^{\tau}_{ij}+P^{\sigma}_{ij})U^{-}_{\gamma}(r_{ij}) \quad , \qquad (C.2)$$

where P^{σ} and P^{τ} are the usual Bartlett and Heisenberg exchange operators while

$$V^{\pm}_{\gamma}(r_{ij}) = \frac{1}{2}\,(V^{3\pm}_{\gamma}(r_{ij}) + V^{1\pm}_{\gamma}(r_{ij}))$$
$$U^{\pm}_{\gamma}(r_{ij}) = \frac{1}{2}\,(V^{3\pm}_{\gamma}(r_{ij}) - V^{1\pm}_{\gamma}(r_{ij})) \qquad (C.3)$$

where $V^{3\pm}_{\gamma}(r_{ij})$ and $V^{1\pm}_{\gamma}(r_{ij})$ refers respectively to the triplet and singlet even or odd potentials.

The matrix elements of the exchange operators are

$$\langle (s\tfrac{1}{2})Sm_S|P^{\sigma}_{12}|(s'\tfrac{1}{2})S'm'_S\rangle = \delta_{ss'}\delta_{m_S m'_S}\delta_{SS'}(-1)^{s+1}$$

$$\langle (s\tfrac{1}{2})Sm_S|P^{\sigma}_{23}|(s'\tfrac{1}{2})S'm'_S\rangle = \delta_{ss'}\delta_{m_S m'_S}\delta_{SS'}\begin{Bmatrix} s' & \frac{1}{2} & S' \\ s & \frac{1}{2} & \frac{1}{2} \end{Bmatrix}(-1)^{s'} \qquad \text{(C.4)}$$

$$\langle (s\tfrac{1}{2})Sm_S|P^{\sigma}_{13}|(s'\tfrac{1}{2})S'm'_S\rangle = \delta_{ss'}\delta_{m_S m'_S}\delta_{SS'}\begin{Bmatrix} s & \frac{1}{2} & S \\ s' & \frac{1}{2} & \frac{1}{2} \end{Bmatrix},$$

and analogous expressions for the isospin.

Since we have chosen the reference pair (1,2) for the calculations, the matrix elements of the interaction become :

$$\langle (s\tfrac{1}{2})Sm_S(t\tfrac{1}{2})Tm_T|V^0_{\gamma}(\xi u)|(s'\tfrac{1}{2})S'm'_S(t'\tfrac{1}{2})T'm'_T\rangle =$$

$$\frac{1}{2}\delta_{ss'}\delta_{tt'}\delta_{SS'}\delta_{TT'}\delta_{m_S m'_S}\delta_{m_T m'_T}[(1+(-1)^{s+t+1})V^+_{\gamma}(\xi u)+(1+(-1)^{s+t})V^-_{\gamma}(\xi u)+$$

$$((-1)^t+(-1)^{s+1})\,U^+_{\gamma}(\xi u)+((-1)^{t+1}+(-1)^{s+1})\,U^-_{\gamma}(\xi u)]\,, \qquad \text{(C.5)}$$

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Table Captions

Table 1 — Values of the coefficients occuring in eq. B12 (appendix).

Table 2 — Parameters of the shell δ even central potentials fitted to the $^1S_0$ and $^3S_1$ phase shifts according to fig .

Table 3 (4) — Convergence of the solutions obtained in integrating the coupled equations for the shell δ potential up to a maximum K value for ^3H (^3He). Binding energies, percentage of the S and S' symmetry states and matter radiu are given.

Table 5 — a) Binding energies c ained in solving the coupled equations up to L=2K=24, extrapolated binding energies, ^3H-^3He energy differences, percentage of the S, S' and D components of the wave functions and matter radius for the realistic potentials.
b) Charge radii, position of the zeros and amplitude of the maxima of the charge form factors for ^3H and ^3He.

Table 6 — Convergence of the solutions in terms of the maximum grand orbital L=2K used :
a) For G2 only one S' term is used, the extrapolated energy is given
b) For GPDT and SSCB potentials, together with the matter radius R_m.

Table 7 — Comparison between our results and those obtained by Demin-Efros (15,16) and by Laverne-Gignoux (34) in solving the Faddeev equation for the same potentials.

	s=0	s=1			
V	$\ell=\ell'=j$	$\ell=\ell'=j$	$\ell=\ell'=j-1$	$\ell=j-1,\ell'=j+1$	$\ell=\ell'=j+1$
S_{12}	0	2	$-\frac{2(j-1)}{2j+1}$	$6\frac{\sqrt{j(j+1)}}{2j+1}$	$-2\frac{(j+2)}{2j+1}$
L²	$j(j+1)$	$j(j+1)$	$j(j+1)$	0	$(j+1)(j+2)$
L.S	0	-1	$j-1$	0	$-(j+2)$
TL	$-2j(j+1)$	$2j(j+1)-1$	$j-1$	0	$-(j+2)$
Q	0	$3-4j(j+1)$	$(j-1)(2j-3)$	0	$(j+2)(2j=5)$

Table 1

Potential	Triplet or Singlet	r	α	$-E_D$(MeV)	a	r_e
Shell 6	t	1.508	1.3875	2.202	5.40	1.73
	s	1.508	0.93		-21.54	2.24

Table 2

K	NS	NS'	$-E_{^3H}$ (MeV)	P(S)	P(S')	R_m
3	3	3	8.231	99.49	.507	1.522
6	6	6	9.219	99.41	.584	1.497
9	9	9	9.708	99.37	.628	1.488
12	12	10	10.010	99.36	.637	1.48
14	14	10	10.141	99.36	.636	1.478

Table 3

K	NS	NS'	$-E_{^3He}$ (MeV)	P(S)	P(S')	R_m
3	3	3	7.435	99.4	.596	1.54
6	6	6	8.416	99.32	.68	1.52
9	9	9	8.902	99.27	.73	1.51
12	12	10	9.202	99.26	.73	1.5
14	14	10	9.332	99.26	.735	1.5

Table 4

Potential	$-E(^3H)$ Mev calculed	$-E(^3H)$ Mev extrapolated	E_c	P(S)	P(S')	P(D)	$R_M(^3H)$
GPDT	8.58	8.58	.66	94.3	.97	4.72	1.77
SSC.A	7.44	7.51	.645	93.5	.76	5.7	1.76
SSC.B	7.34	7.41	.65	93.8	.81	5.4	1.78
SSC.C	7.01	7.13	.68	92.2	.85	6.98	1.81
Experiment	8.48						1.7

Table 5.a

Potential		R_{CH}(fm)	First dip $\lvert F_{CH}\rvert$ $q^2(fm^{-2})$	First max $\lvert F_{CH}(q^2)\rvert$ $q^2(fm^{-2})$	$\lvert F_{CH}(q^2)\rvert$ x10^3	2^{nd}dip $\lvert F_{CH}\rvert$ $q^2(fm^{-2})$	2^{nd} max $\lvert F_{CH}(q^2)\rvert$ $q^2(fm^{-2})$	$\lvert F_{CH}(q^2)\rvert^2$ x10^3	$F_{CH}(1)$
GPDT	^{3}He	1.93	15.5	20	0.78	58	72.	0.0075	.538
	^{3}H	1.77	15.4	20	1.10				.574
SSC.A	^{3}He	1.95	15.5	20.	1.18	64.8	84	0.016	.541
	^{3}H	1.78	15	20	1.48	63.5	78	0.025	.565
SSC.B	^{3}He	1.95	14.7	20.	1.15	61.4	82	0.015	.535
	^{3}H	1.78	15.2	20	1.38	65.7	82	0.018	.565
SSC.C	^{3}He	1.96	15.	20.	1.15	62.	80	0.015	.52
	^{3}H	1.78	14.7	20	1.38	64.7	78	0.065	.551
Exp	^{3}He	1.87±.05	11.6	18.	6.	68	78	0.03	.567±.004
	^{3}H	1.7 ±.05							.622±.007

Table 5.b

G2 (^{3}H)				
L=2K	N(s)	N(s')	-E (MeV)	ΔE
4	2	1	1.752	
6	3	1	3.610	1.858
8	4	1	5.236	1.626
10	5	1	6.564	1.328
12	6	1	7.525	.961
14	7	1	7.829	.304
16	8	1	8.046	.217
18	9	1	8.218	.172
20	10	1	8.284	.066
22	11	1	8.334	.050
24	12	1	8.370	.036
Ext :			8.41	
Exp :			8.48	

Table 6.a

	GPDT ( )			SSC.B		
L=2K	-E	ΔE	R_m	-E	ΔE	R_m
4	6.614		1.73			
6	7.509	.894	1.70	4.111		1.784
8	7.977	.469	1.70			
10	8.248	.271	1.717		2.553	
12	8.395	.146	1.727	6.664		1.688
14	8.468	.074	1.737			
16	8.515	.047	1.745		1.417	
18	8.543	.028	1.752	7.181		1.73
20	8.559	.016	1.757			
22	8.569	.010	1.761		0.134	
24	8.575	.006	1.764	7.315		1.762
26	8.579	.004	1.766	7.332		1.769
28	8.58	.003	1.767	7.345		1.774

Table 6.b

Potentials	Authors	$-E(^3H)$ Mev	P(S)	P(S')	P(D)	$R_{CH}(^3He)$
GPDT	Laverne-Gignoux	8.28	94.65	1.28	4.07	1.87
	Demin et al.	8.50	94.6	.6	4.7	-
	Ballot-Fabre	8.58	94.3	.97	4.72	1.93
SSC.A	Laverne-Gignoux	7.58	92.	1.5	6.5	1.85
	Demin et al.	7.38	93.2	.58	6.25	1.94
	Ballot-Fabre	7.51	93.5	.76	5.7	1.95
SSC.B	Laverne-Gignoux	7.60	92.3	1.4	6.3	1.82
	Demin et al.	7.53	93.25	0.79	5.96	1.96
	Ballot-Fabre	7.41	93.8	0.81	5.4	1.95
SSC.C	Laverne-Gignoux	7.34	90.6	1.4	8.	1.83
	Ballot-Fabre	7.13	92.2	.85	6.98	1.96

Table 7

Figure Captions

Figure 1 Radial shape of local two-nucleon interaction combination $\frac{1}{2}(V^{3+}+V^{1+})$ for G2 and V^x potentials.

Figure 2 The ^3H charge form factor for the two body interactions G2 (continuous line) and V^x (dotted line). The experimental data are from Ref. 64.

Figure 3 The ^3He charge form factor for the two body interactions G2 (continuous line) and V^x (dotted line). The experimental data are from Ref. 65.

Figure 4 First two S and S' partial waves $u_{2K}(\xi)$, K=0,1 calculated with G2 (continuous lines) and V^x (dotted line) potentials.

Figure 5 First three S partial waves $u_{2K}(\xi)$, K=0,2,3 calculated with V^x potential.

Figure 6 The N-N $^3S_1$ phase shifts of the shell δ potential. The points correspond to the single energy phase shift of Arndt-Mac Gregor-Wright Ref. 60.

Figure 7 The N-N $^1S_0$ phase shifts of the shell δ potential.

Figure 8 The ^3He charge form factor using the shell δ potential.

Figure 9 The ^3He charge form factor calculated with realistic potentials, GPDT (1 dot-dashed line), SSCA (dashed line), SSCB (continuous line), SSCC (4 dot-dashed line). The experimental data are from Ref. 65.

Figure 10 One body density and two body correlation functions using the two first partial waves (K=0,2) calculated for G2 (continuous line), V^x (dashed line), GPDT (dotted line).

Figure 11 One body density (continuous line) and two body correlation function (dashed line) calculated with V^x potential.

Figure 12 One body density (continuous line) and two body correlation function (dashed line) calculated with G2 potential.

Figure 13 One body density $R_1(\xi)$ and two body correlation function $R_2(\xi)$ of ^3H and ^3He calculated with GPDT potential.

Figure 14 ^3He proton densities calculated with G2 (continuous line) and V^x (dashed line) potentials. The points correspond to the Sick analysis given in Ref. 59.

Figure 15 ^3He proton density calculated with GPDT potential.

Figure 16 The overlapping functions $R_i^\alpha(\xi)$ between the deuteron wave function from Ref. 40 and ^3He wave function calculated for G2 (continuous line), V^x (dashed line) and GPDT (dot-dashed line) potentials.

Figure 17 Total cross section of the reaction ^3He $(\gamma,p)d$. The theoretical curves are obtained with G2 (continuous line) and V^x (dashed line) potentials. The experimental data are from Ref. 62.

Figure 18 Total cross section of the reaction ^3He $(\gamma,p)d$ calculated using the ^3He wave function corresponding to the GPDT interaction. The continuous line is the cross section calculated with the complete wave function. The two dot-dashed curve is the cross section calculated with the total symmetric S wave function. The dashed line is the cross section calculated with the total and mixed symmetric S' wave functions.

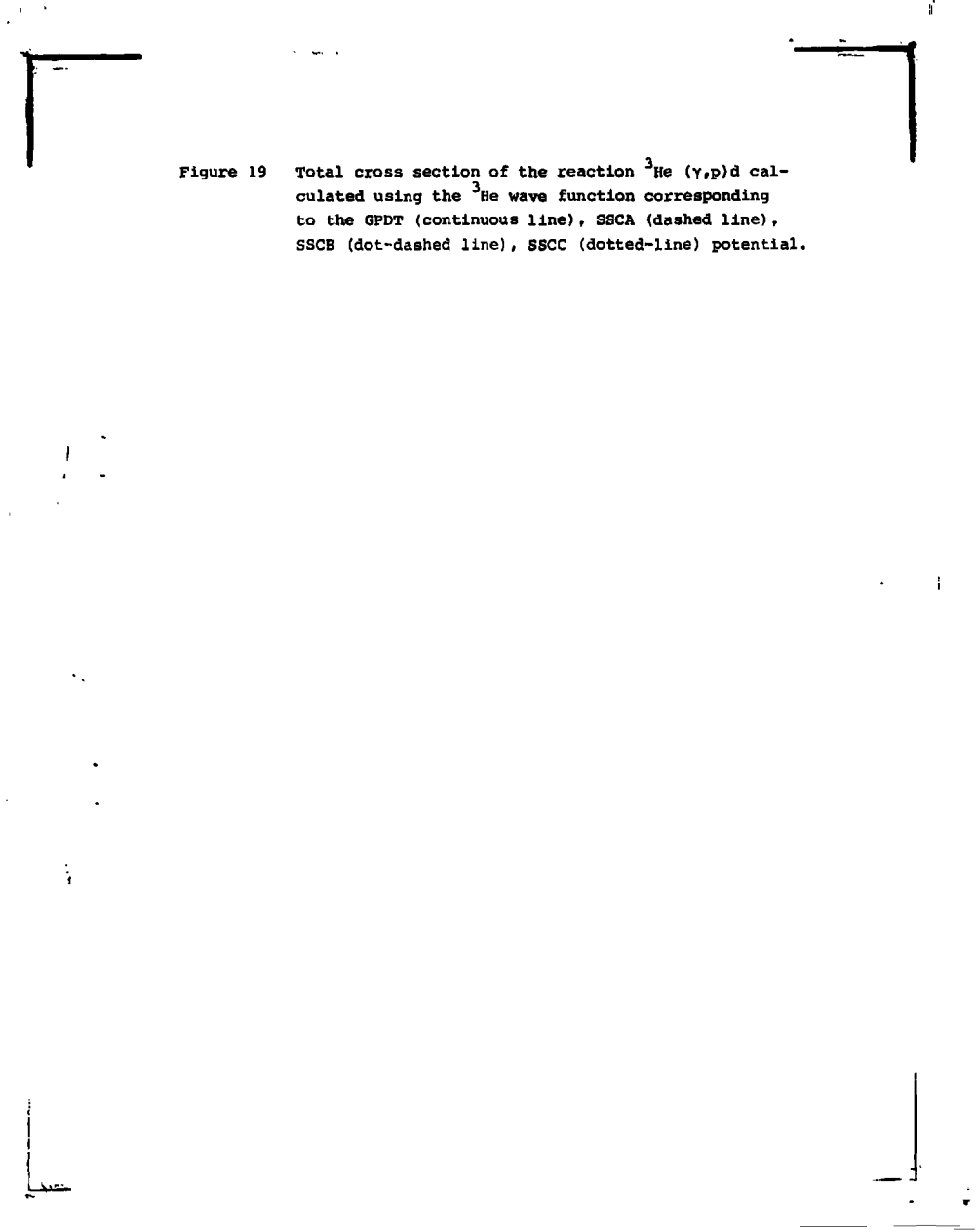


Figure 19 Total cross section of the reaction ^{3}He $(\gamma,p)d$ calculated using the ^{3}He wave function corresponding to the GPDT (continuous line), SSCA (dashed line), SSCB (dot-dashed line), SSCC (dotted-line) potential.

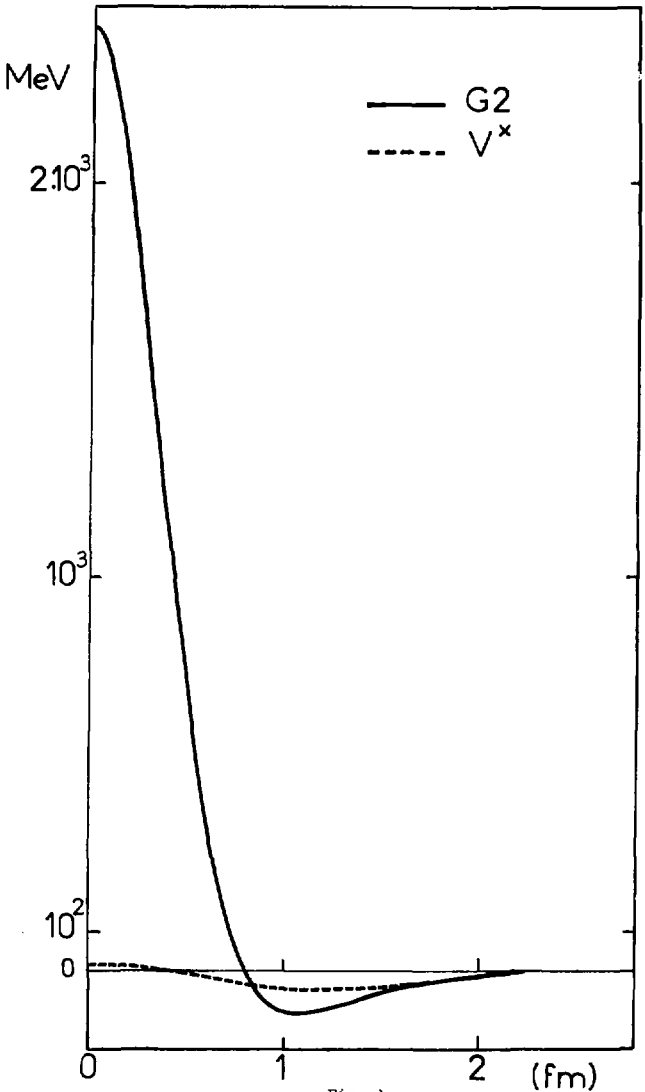


Fig. 1

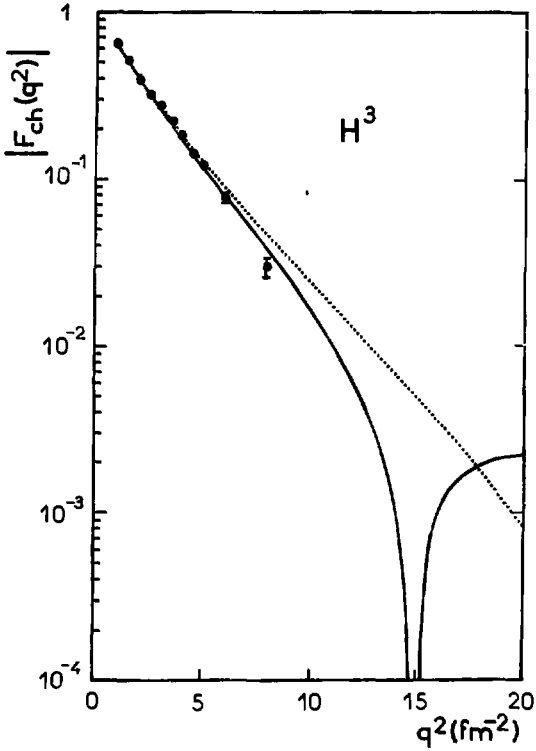


Fig. 2

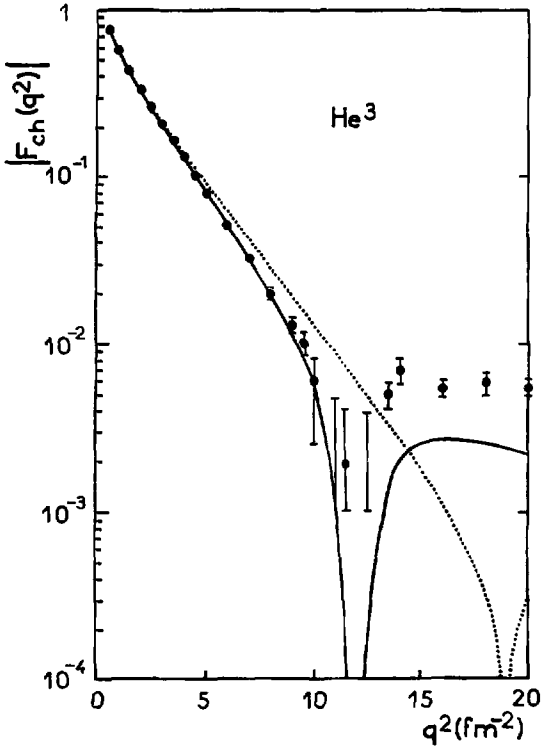


Fig. 3

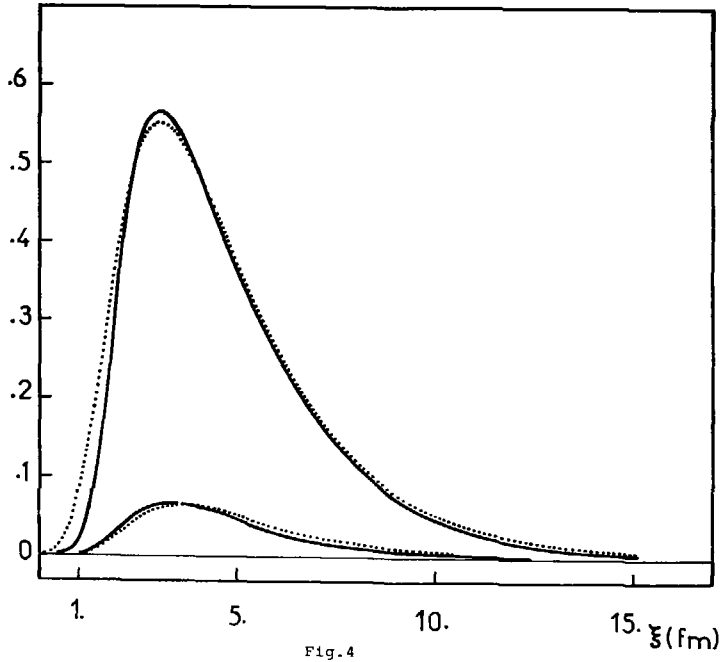


Fig.4

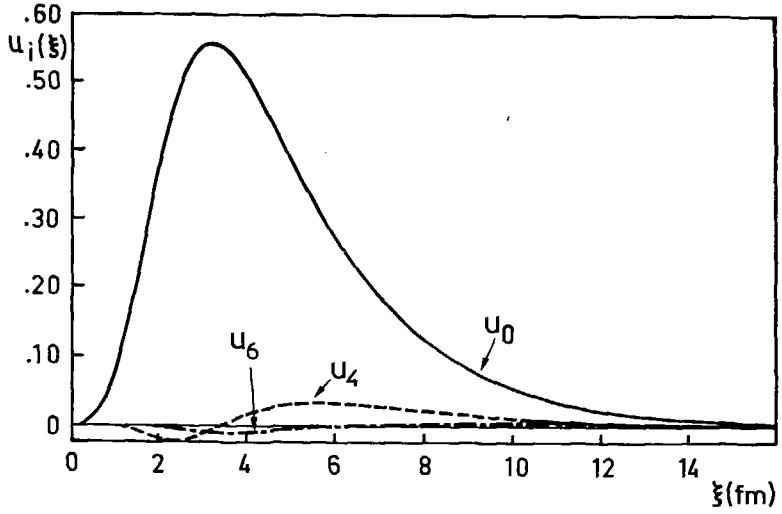


Fig. 5

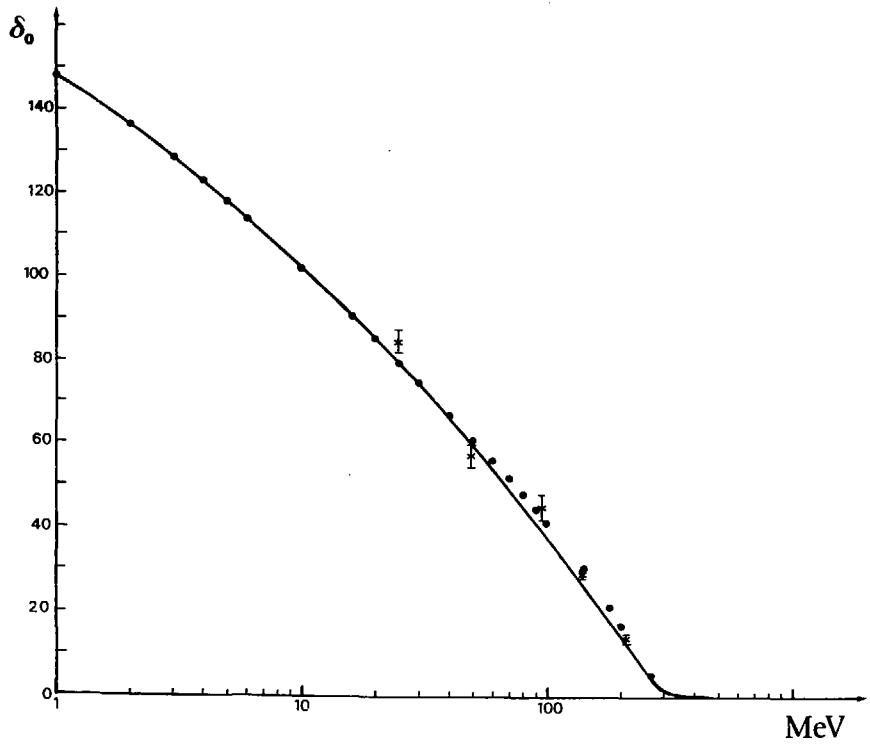


Fig. 6

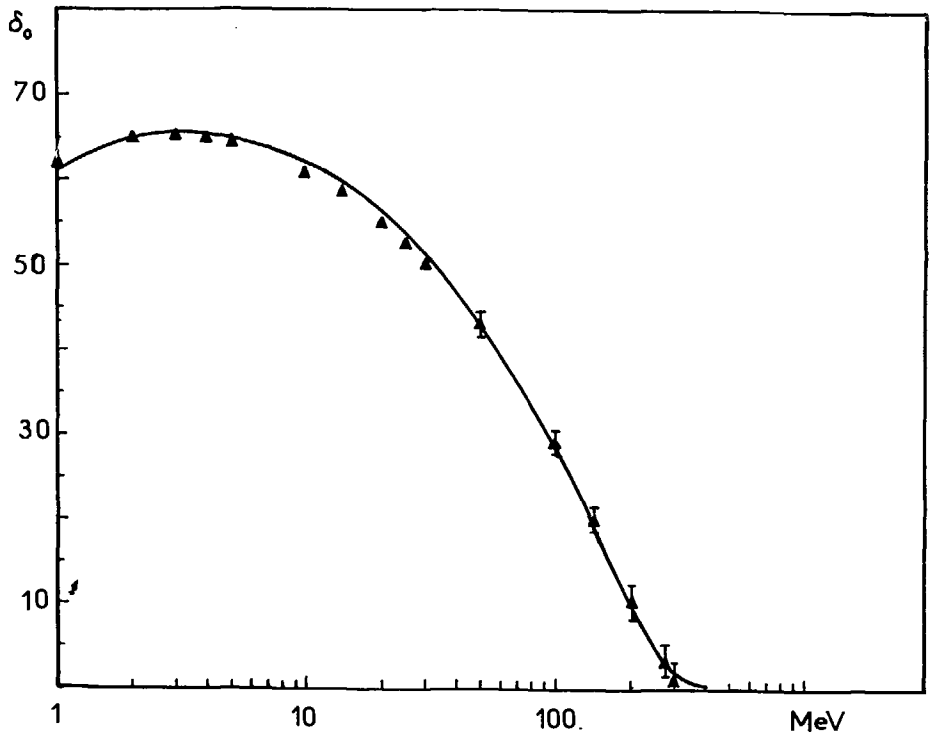


Fig. 7

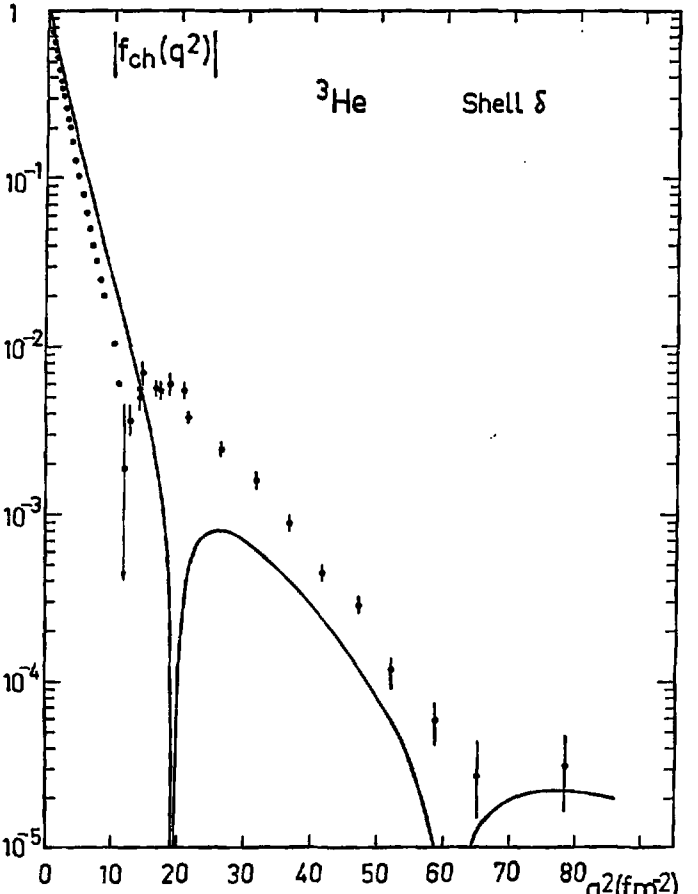

1
$|f_{ch}(q^2)|$
^{3}He
Shell δ
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
10
20
30
40
50
60
70
80
$q^2(fm^2)$

Fig. 8

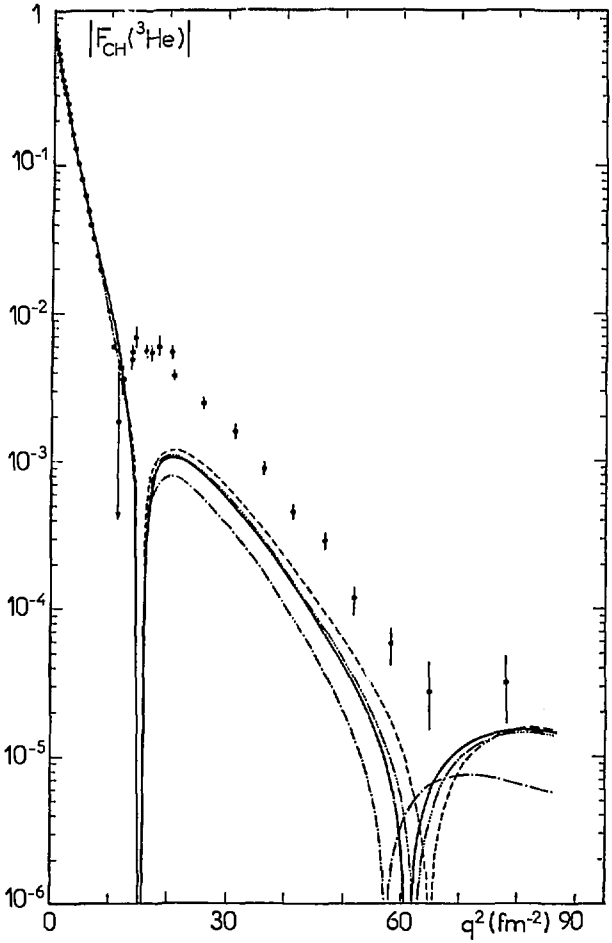


Fig. 9

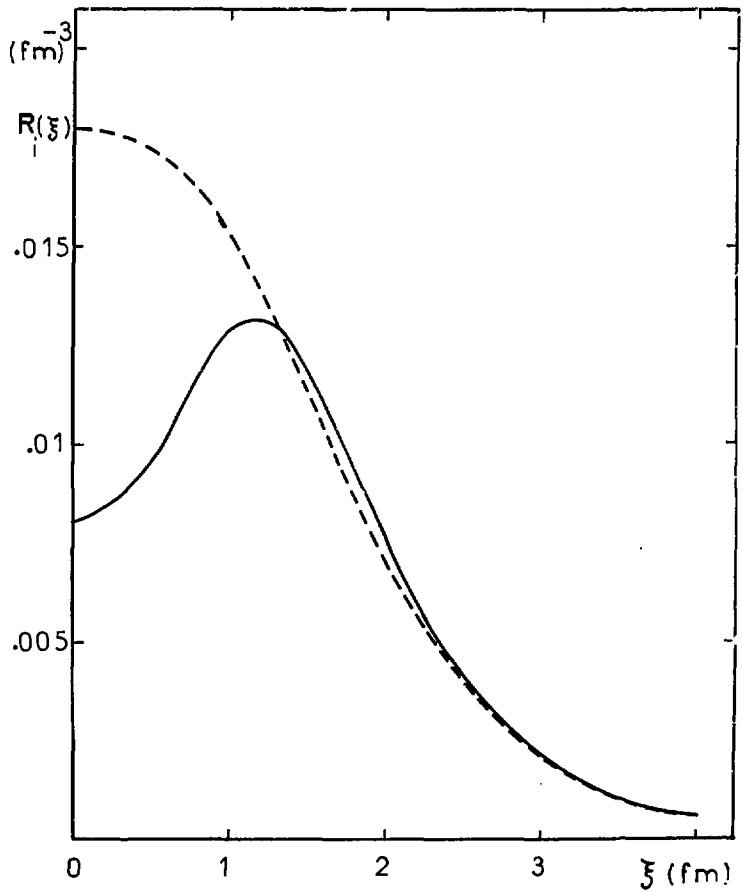


Fig. 10

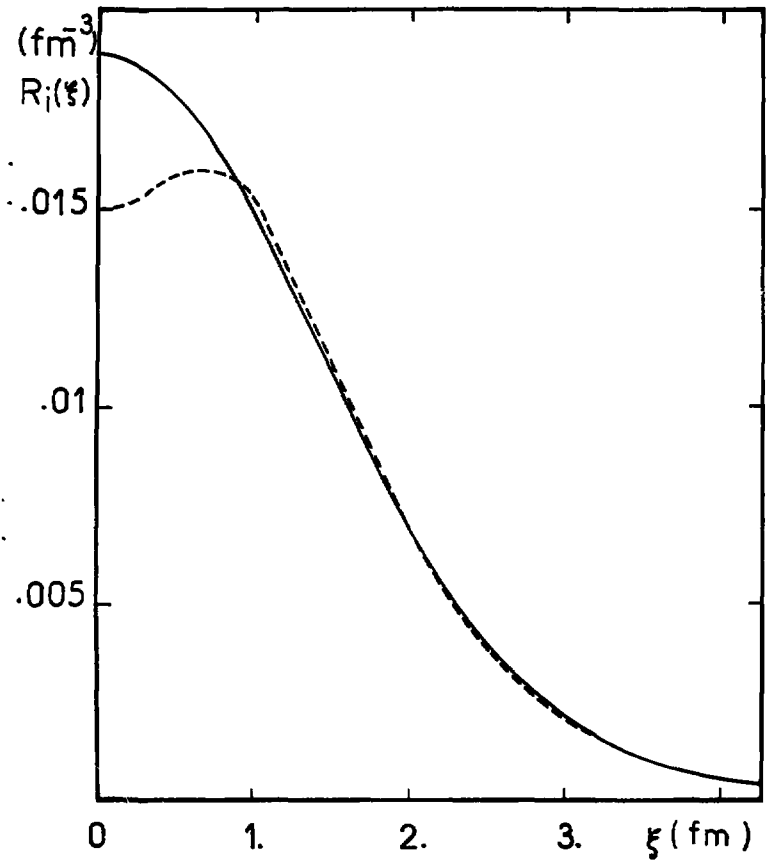
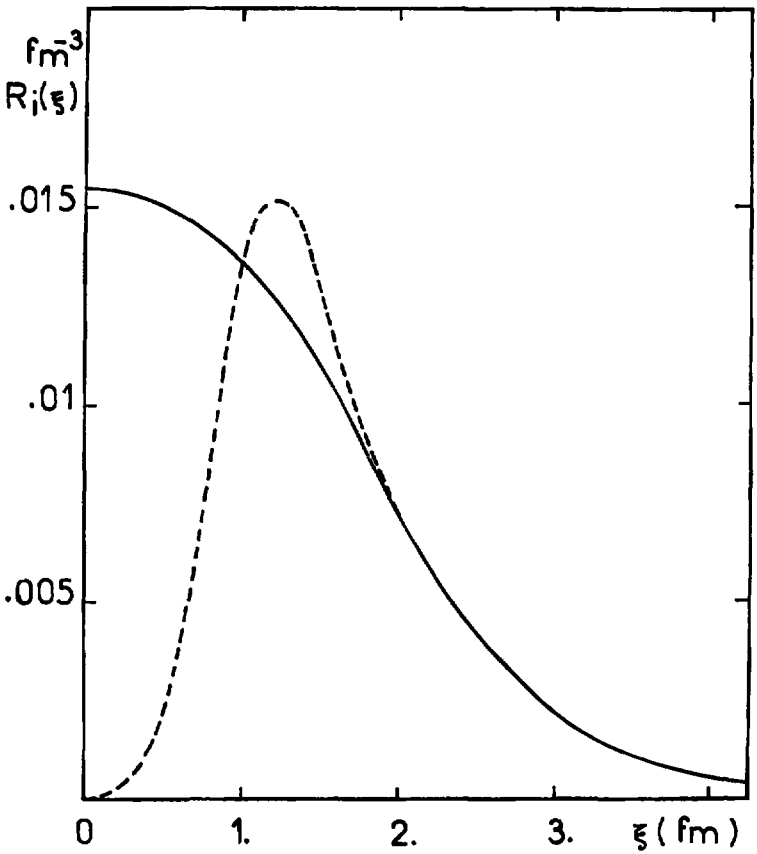


Fig. 11

Fig. 12



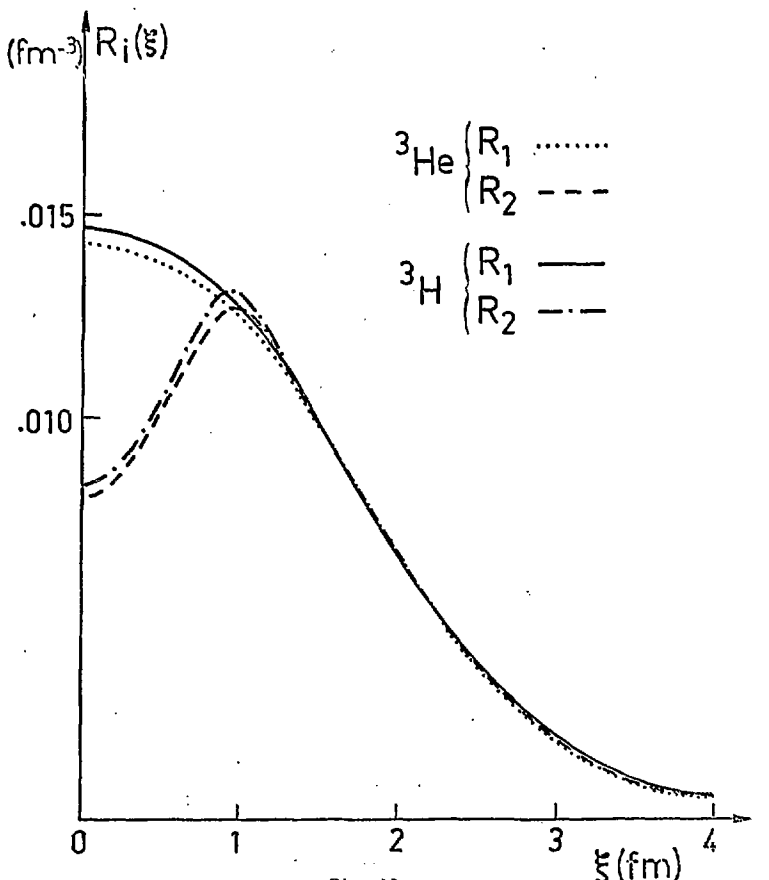


Fig. 13

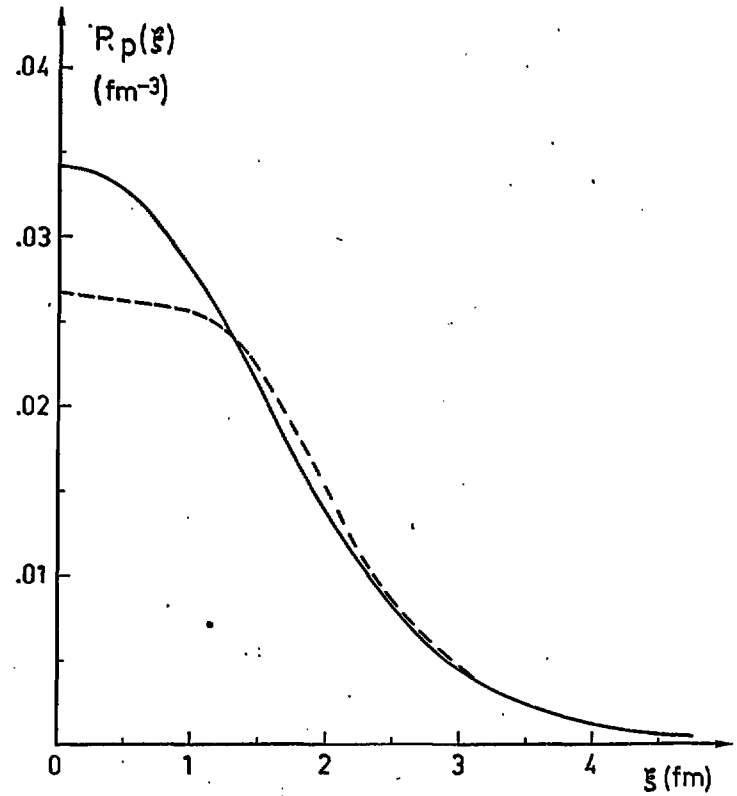


Fig. 14a

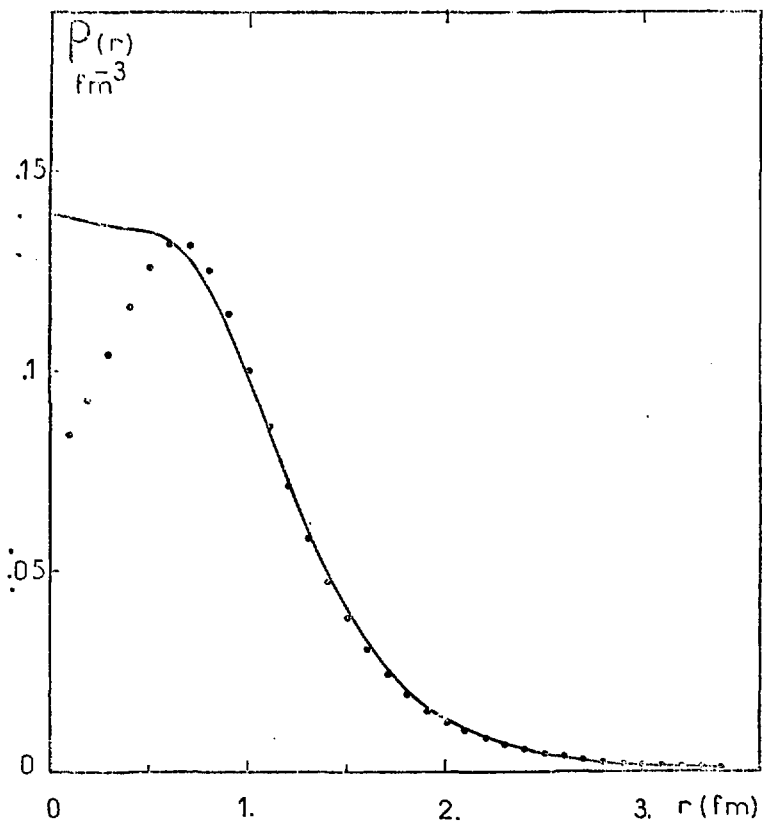


Fig. 14b

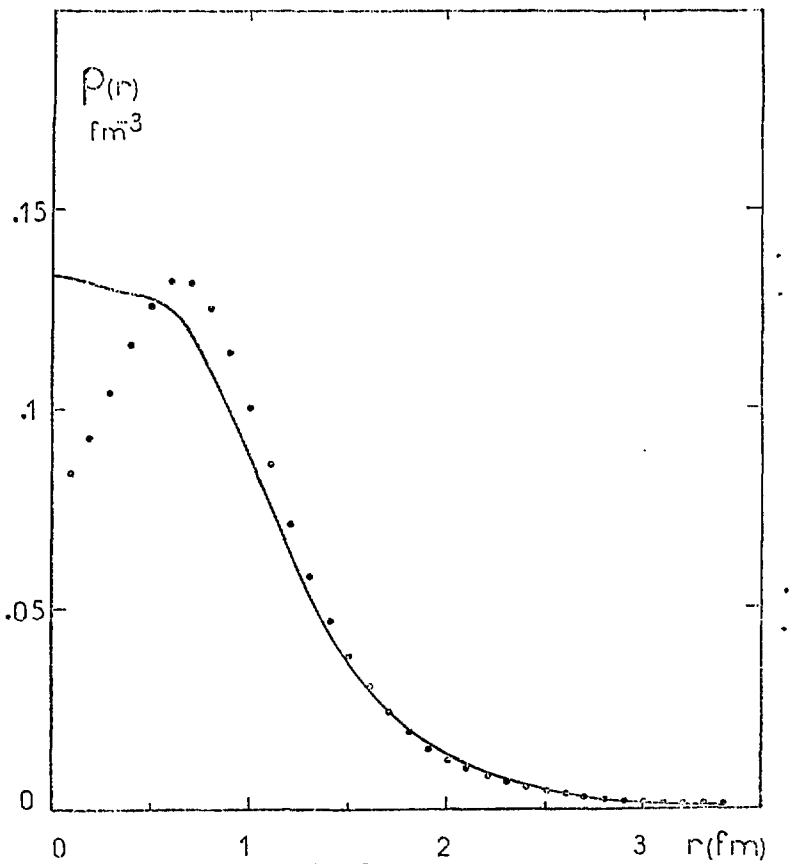


Fig. 15

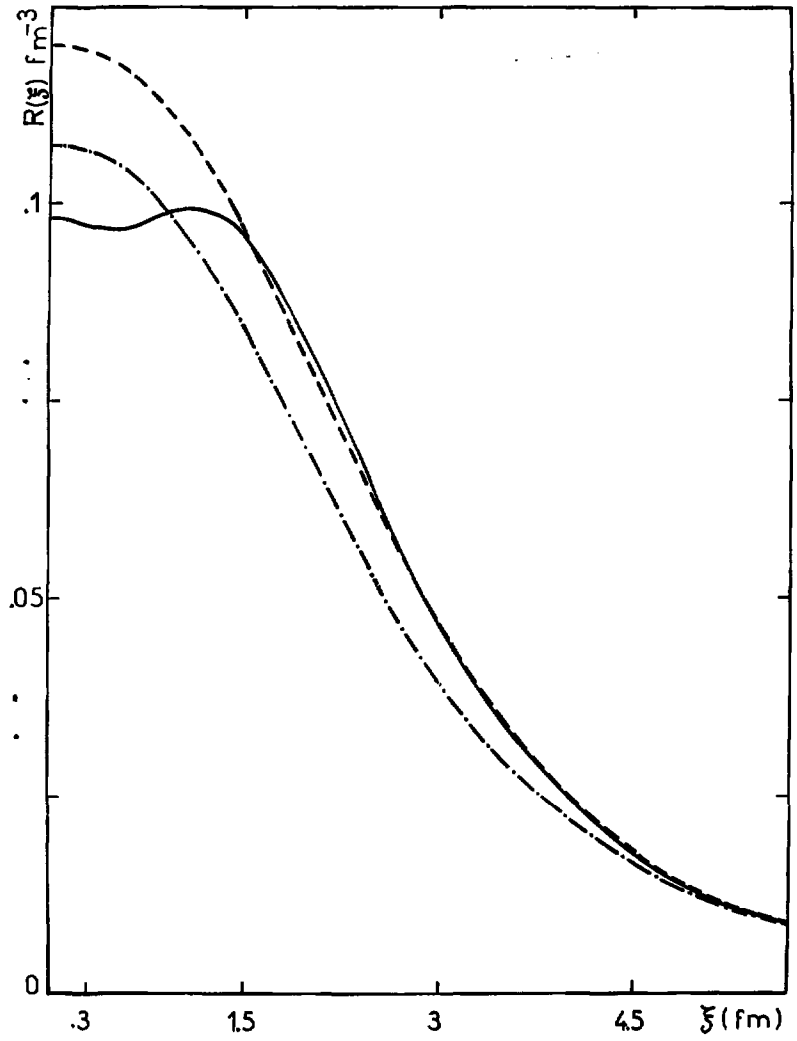


Fig. 16

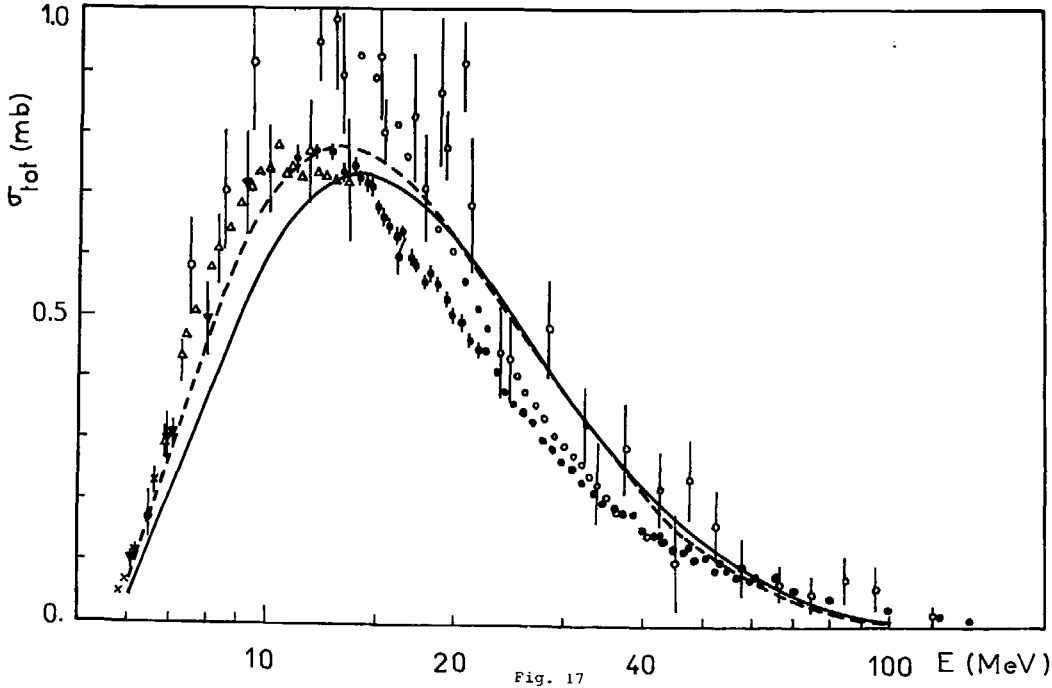


Fig. 17

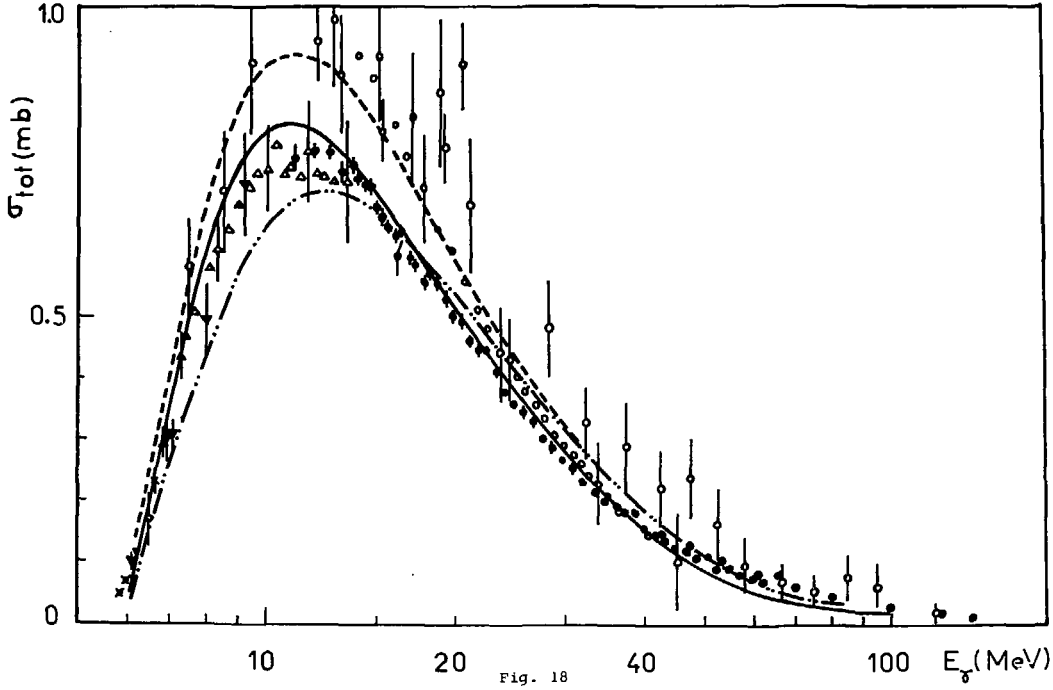


Fig. 18

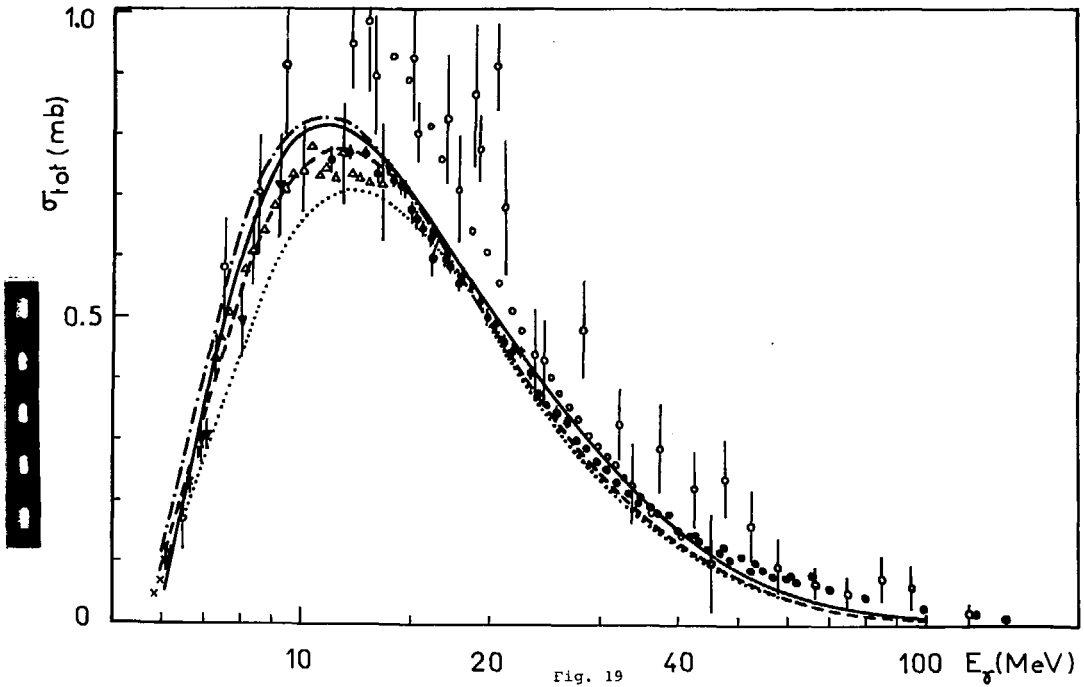


Fig. 19