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EFFECT OF THE THREE BODY POTENTIAL ON THE TRINUCLEON BOUND STATE

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The effect of the three body force, generated by two pions exchange with an intermediate Δ, on the trinucleon system is investigated.

A fundamental question which arises in the description of nuclei is to determine whether a nucleus is bound by mutually interacting nucleons or whether many-body interactions contribute significantly to the nuclear states. Let us begin with the three body force : the best nucleus to test a three body effect is obviously the bound state of the trinucleon system. It is for this reason that an accurate method for solving the ^{3}H-^{3}He isodoublet was actually needed.

Accurate numerical method enabling to solve the three body bound states have been developed in the 70's [1-6]. They all lead to the conclusion that the binding energy of the trinucleon system obtained in using realistic local two body interactions (including the new energy dependent Paris potential [14]) is too small by an amount of 1.3 ± 0.2 MeV according to the investigated realistic potential.

To solve the Schrödinger equation we used an hyperspherical expansion of the wave function and we checked that our solution has nearly converged by using known extrapolation rules.

TABLE 1

Potentials	Authors	$-E(^3H)$MeV	P(S)	P(S')	P(D)	$R_{CH}(^3He)$
GPDT	Laverne-Gignoux	8.28	94.65	1.28	4.07	1.87
	Ballot-Fabre	8.58	94.3	.97	4.72	-
SSC.A	Laverne-Gignoux	7.58	92	1.5	6.5	1.85
	Ballot-Fabre	7.51	93.5	.76	5.7	1.95

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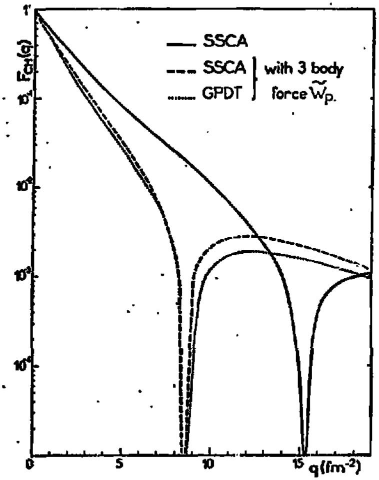
In table 1 is shown a comparison between results obtained with our method and with the solution of the Fadeyev equation obtained by Gignoux and Laverne (Ref.2) for the Gogny-Pirès-de Tourreil[7] and the super soft core Sprung-de Tourreil SSCA[8] potentials (for other potential see ref.9)

The binding energies and percentage of the various symmetry components (S fully space symmetry, S' mixed symmetry, S state and D mixed symmetry state) and the radii of ^{3}H and ^{3}He are in fair agreement in spite of a significant discrepancy in the percentage of the S' state. In all cases an energy of $1 \sim 1.5$ MeV is missing with respect to the triton experimental binding energy (-8.48 MeV).

The most accurate probe for testing the quality of the wave function is to compare the charge form factor obtained with the experimental data. In fig.1 are plotted the charge form factors $F_{CH}(q)$ using the solution of the Schrödinger equation for the investigated potential.

FIGURE 1



It appears immediately that the minimum of $|F_{CH}(q)|$ is not at the right position and that the amplitude of the maximum which appear experimentally around 16 fm^{-2} is too small by a factor 5

TABLE 2

Potential		R_{CH}(fm)	1st dip q^2 (fm^{-2})	$\lvert F_{CH}(q^2)\rvert$ 1st max q^2(fm^{-2})	 $\lvert F_{CH}\rvert \times 10^3$	$F_{CH}(1)$
GPDT	^{3}He	1.93	15.5	20	0.78	.538
	^{3}H	1.77	15.4	20	1.10	.574
SSCA	^{3}He	1.95	15.5	20	1.18	.541
	^{3}H	1.78	15	20	1.48	565
Exp	^{3}He	1.87 ±.05	11.6	18	6	.567 ±.004
	^{3}H	1.7 ±.05				.622 ±.007

In table 2 are summarized the main features of the charge form factor obtained for the potential GPDT and super soft core SSCA. There are small discrepancies between the number given by various potentials [9,10], therefore general conclusions can be drawn :

1°) the value of $F_{CH}(1)$ which corresponds to a small momentum transfer is too small by 5% for ^{3}He and the differences between $F_{CH}(1)$ for the two nuclei ^{3}He and ^{3}H is also too small implying that the charge radii are systematically too large and that the percentage of the mixed symmetry S' into the ground state, which is responsible for the difference between the radii of the isodoublet ^{3}H-^{3}He, is too small.

2°) the position of the first minimum is too far by about 5fm^{-2}.

3°) the amplitude of the first maximum is too small by a factor 5. The position of the second minimum and of the second maximum are nearly correct.

The fact that the experimental data cannot be obtained when the nucleons are assumed to interact by two-body realistic interaction constitutes what is called by the gordian knot [11] of the trinucleon system.

One of the most striking feature of the disagreement with the experimental data is the large amplitude of the first maximum of the charge form factor which cannot be reached, according to the Sick[12] analysis, without the occurence of a hole at the center of the charge density. On the other hand it seems impossible to obtain this hole with a solution of the Schrödinger equation obtained with two body interactions[9]. We have then to inquire what is the missing piece which brings trouble in our description of the trinucleon system. When constructing a two body potential with meson exchange it has been found that the two meson exchange graph (fig.2a) contributes significantly to the potential but on the other hand when dealing with a three nucleon system the two meson exchange graph (b) which corresponds to a 3-body force has been completly ignored. This omition is clearly inconsistent with the claim that the two meson exchange contributes largely to the N-N potential. We intend to show that the gordian knot could be cut by the introduction of the 3-body force and that the large amplitude of the first maximum may be the signature of the effect of this force. The contribution of the 3-body force derived by Fujita and Miyazawa on the trinucleon has been studied by Loiseau and Nogami[13].

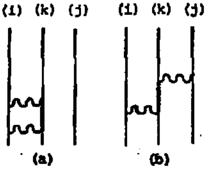


FIGURE 2

We intend to investigate the effect on the predominant trinucleon S state of the component $W_p(k)$ of this force (in the Loiseau-Nogami notation) in which a pion in the p state is exchanged between the lines (i)-(k) and (k)-(j) (see fig.2b). The graph of fig.2b generates a force

$$W_p(k) = C_p\,(3\cos^2\theta_k - 1)\,F(x_{jk})\,F(x_{ki}) \qquad (1)$$

where $F(x) = \left(1 + \frac{3}{\mu x} + \frac{3}{(\mu x)^2}\right)\frac{e^{-\mu x}}{\mu x}$ and where $C_p = 0.46$ MeV is the strength. The total interaction is the cyclic permutation of (1) with respect to $k = 1,2,3$.

THREE BODY FORCE EFFECT ON THE TRINUCLEON

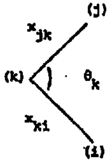


FIGURE 3

Performing an hyperspherical harmonic expansion of $F(x_{jk})\ F(x_{ki})$ and retaining only the first term $V_0(r)$ which contain the predominant part of the interaction one obtains

$$\tilde{W}_p = \sum_C \tilde{W}_p(k) = 6\, C_p \cos\theta_1 \cos\theta_2 \cos\theta_3\, V_0(r)$$

with $r^2 = \frac{2}{3} \sum_C x_{ij}^2$

This three body force is respulsive when one of the θ is obtuse and has a maximum attraction when the three nucleon form an equilateral triangle. On the other hand the potential is 8 times stronger for alined nucleons than for the stable equilateral shape. This force should create the needed hole in the center of the density. The sum of the two other components W_s and W_σ of the 3 body interaction [13] are attractive everywhere but weaker when the nucleons are alined strengthening the effect of $\tilde{W}_p$. As a preliminary step only the $\tilde{W}_p$ has been studied.

The two first components of the h.h. expansion of W_p have been used only and to investigate the effect of the strong repulsion at short distances we truncated $F(x)$ at distances $x_0 = 0.5$ fm and $x_0 = 0.7$ fm taking $F(x) = F(x_0)$ for $x < x_0$. The result obtained in introducing this simplified version of W_p into the Schrödinger equation with the GPDT and SSCA soft core potentials is shown on

TABLE 3

Pot.	x_0	$-E(^3H)$	P(S)	P(S')	P(D)	R_m fm	1st min of $\|F_{CH}\|$	1st max of $\|F_{CH}\|$	
							q^2 fm^{-2}	q^2	$\|F_{CH}\| \times 10^3$
	0	4.20	91.5	4.14	4.37	2.59	8	11	2.0
GPDT	0.5	4.36	91.7	3.85	4.42	2.53	9	12	1.9
	0.7	4.54	91.9	3.57	4.46	2.47	9	12	1.9
SSCA	0.7	4.51	91.3	2.92	5.76	2.29	9	12	2.8

A binding energy of 3 ~ 4 MeV has been lost increasing the range of the tail and consequently the size of ^{3}H. The percentage of the mixed symmetry S' state has been largely enhanced. The effect of the truncation is rather small. The change on the charge form factor is shown in fig.1. The too small position of the minimum around 9 fm^{-2} is due to the too large radius, but the amplitude of the maximum has been more than twice increased. On the other hand it seems that the effect of the three body force is sensitive to the choice of the two-body potential because 4 MeV has been lost with the GPDT potential instead of 3 MeV only with the SSCA potential.

From this preliminary calculation it comes out that the three body force $\tilde{W}_p$ has an important effect on the structure of the trinucleon system and cannot be ignored. On the other hand it seems that the two-body and three-body forces cannot be chosen independently of each other because the comulative effect of these forces in the trinucleon system is sensitive to the used combination. Finally we expect an increase of binding proceeding from the attractive $W_s + W_\sigma$ component[13] of the three body force which may balance the destructive effect of W_p.

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