CONSTRAINTS FROM JET CALCULUS ON QUARK RECOMBINATION

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Constraints from Jet Calculus on Quark Recombination

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With the QCD jet calculus formalism, we deduce an equation describing recombination of quarks and antiquarks into mesons within a quark or gluon jet. This equation relates the recombination function $R(x_1, x_2, x)$ used in current literature to the fragmentation function for producing that same meson out of the parton initiating the jet. We submit currently used recombination functions to our consistency test, taking as input mainly the u-quark fragmentation "data" into π^+ mesons, but also s-quark fragmentation into K^- mesons. The constraint is well satisfied at large Q^2 for large moments. Our results depend on one parameter, Q_o^2, the constraint equation being satisfied for small values of this parameter.

I. INTRODUCTION

Over the past few years, the success of parton-model calculations in current-initiated reactions has led to general acceptance of quarks and gluons as useful calculational descriptions of hadronic structure functions. Clearly the next step is to use these same quark-content functions to predict purely hadronic reactions. Use in high transverse momentum (p_t) reactions for prediction of, e.g., jet cross sections, is well advanced; however, the application of these ideas to other regions of multiparticle phase space is still in its infancy.

A very intuitive approach applying parton-model concepts to low p_t inclusive reactions was suggested by Das and Hwa[1] to explain the empirical observation of Ochs[2] that inclusive pion production at large x and small p_t in proton-proton collisions reflected the proton structure function. Various versions and applications of this recombination model, as it has become called, have appeared[3-8]; most of these resemble the original Das-Hwa paper in essential concept although there are differences in implementation. The basic idea is: Forward non-diffractive meson production can be calculated by assuming that most of the momentum in the meson came from a valence quark in the beam particle. This quark recombined with a sea quark of low momentum, also from the beam, to create a meson.

To compute the x distribution of the mesons from this process two hitherto unstudied functions were introduced: $F(x_1,x_2)$, describing the probability that the incident beam has a quark of the correct flavor at x_1 and an antiquark of the correct flavor at x_2, and a

recombination function $R(x_1, x_2, x)$ which tells how the two constituents

join to create a meson of momentum $x_1 + x_2 = x$. Originally, both

functions were created from various physical arguments; a fair number

of the improvements of subsequent papers are directed towards refining

these arguments.

One should be careful about specifying parton model functions

entering in such a calculation, however, since the functions are not

completely arbitrary. The Q^2 dependence of hadron structure functions,

and presumably of other quantities related to hadronic wave functions,

is known. According to quantum chromo-dynamics, which predicts the

various couplings among quarks(q), antiquarks($\bar{q}$), and gluons(g), a

very energetic quark generates a cloud of partons (q, $\bar{q}$, and g) that

accompany it. The development of this cloud (with the Q^2 of the

probing current) is described by the Altarelli-Parisi[9] evolution

equations. The application of the physics in these equations to

the description of jet production from quarks was formalized by

Konishi, Ukawa, and Veneziano[10,11] (KUV). With their jet calculus,

it is possible to calculate quickly various properties of a jet

including the distribution of multiple constituents in longitudinal

momentum fraction.

We adopt the basic ideas of jet calculus to get an expression

for the probability of finding a quark at x_1 and an antiquark at

x_2 and then require that they recombine to make the final state

meson with momentum fraction x. The resulting function must be a major

contribution to the fragmentation function for the quark that produced

the jet, that is, the probability that a given quark will materialize
into a specific hadron with momentum fraction x. Such quark frag-
mentation functions have been phenomenologically determined by
Field and Feynman from electron-positron jet production and hadron
lepto-production data.[12]

Hence, as a matter of principle, the recombination function R is
not completely independent of other functions used in parton model
phenomenology. In fact, it is severely constrained by the fragment-
ation function D. This relationship will be derived in Section II,
and although the general solution for R in terms of D is difficult, with
present popular forms for R the moments of D can be written in terms of
the moments of R. We believe that the relation between R and D is non-
trivial because, as pointed out by Owens,[13] D should satisfy an evolution
equation of the Altarelli-Parisi type. Results of our calculations
are presented in Section III where a detailed comparison with the
fragmentation function of the u-quark into positive pions is made.
Comparison with fragmentation of s-quarks into K^- mesons also is
satisfactory. Section IV contains our conclusions and discussion of
implications of our results.

II. DERIVATION OF THE RELATION BETWEEN FRAGMENTATION
AND RECOMBINATION FUNCTIONS

We briefly sketch the logic behind our relation: R measures
the probability that two quarks will stick together to form a

meson and D measures the probability that a meson will be found in

a particular quark jet. If we can compute from first principles

the probability that the quark jet in question contains the two

partons necessary to form the meson, we can multiply this probability

by R to obtain D. (This fragmentation function will be denoted $D_{M,i}$.)

The jet calculus[10,11] leads to the probability for finding two

partons a_1, a_2 with momentum between x_1 and $x_1 + dx_1$, x_2 and $x_2 + dx_2$,

respectively, when they originated from parton i,

$$\frac{1}{\sigma_{i \to jet}} \frac{d\sigma(i \to a_1 a_2 + X)}{dx_1 \, dx_2} \equiv D_{a_1 a_2, \, i}(x_1, x_2, Q^2) =$$

$$\sum_{b_1, b_2, j} \int_0^Y dy \int_0^1 dx \, dz dw_1 dw_2 \, D_{a_1 b_1}(w_1, y) \, D_{a_2 b_2}(w_2, y) \quad X$$

$$\hat{P}_{j \to b_1 b_2}(z) \, D_{ji}(x, Y-y) \, \delta(x_1 - xzw_1) \, \delta(x_2 - x(1-z)w_2) \quad . \quad (2.1)$$

The cross section for parton i to produce a jet is $\sigma_{i \to jet}$, with

$D_{a_1 a_2, i}(x_1, x_2, Q^2)$ the joint probability distribution for parton i

with 4-momentum squared Q^2 to produce in its jet partons a_1 and

a_2 at longitudinal momenta x_1 and x_2. The functions D_{ab} play the role

of parton "propagators" in the variable $Y \sim \ln(\ln Q^2)$ of the Altarelli-

Parisi equations, the vertices P are the fundamental QCD vertices

(quark → quark + gluon, etc.) as used in those equations. The partons

a_1 and a_2 can then be dressed into hadrons of interest by use of

phenomenological functions; the resulting di-hadron distributions

possess reasonable size and properties.[14] The δ-functions impose

momentum conservation upon the momentum fraction variables x, z, w_1, and w_2. The KUV variable[11] is

$$Y = (2\pi b)^{-1} \ln[1 + \alpha_o b \ln (Q^2/\Lambda^2)] \quad , \qquad (2.2)$$

where $12\pi b = 11N_c - 2N_f$ for N_c colors and N_f flavors and α_o and Λ^2 are constants determining the strength and scale of the QCD coupling.

If we then require that partons a_1 and a_2 recombine to make a meson, we obtain an expression for the fragmentation function $D_{M,i}$ for quark i making meson M,

$$D_{M,i}(x,Y) = \sum_{a_i,b_i,j} \int_{Y_o(Q_o^2)}^{Y} dy \int_0^1 d\xi \, dz \, dw_1 \, dw_2 \, dx_1 \, dx_2 \; X$$

$$R^M_{a_1 a_2}(x_1,x_2,x) \, D_{a_1 b_1}(w_1,y-Y_o) \, D_{a_2 b_2}(w_2,y-Y_o) \, P_{j \to b_1 b_2}(z) \; X$$

$$D_{ji}(\xi,Y-y) \, \delta(x_1-\xi z w_1) \, \delta(x_2-\xi(1-z)w_2) \quad . \qquad (2.3)$$

The integral over y in (2.3) has a lower limit Y_o determined by $Q^2 = Q_o^2$ in Eq. (2.2), consistent with the requirement that some minimum energy is needed to make meson M. A pictorial representation of Eq. (2.3) is given in Fig. (1).

We now need a form for R^M. In this paper we concentrate our attention on the form used by previous workers in the field, although it is a very special one and perhaps should be generalized or altered as we discuss later. The recombination function of Das and Hwa,[1] and the more general form of Van Hove,[7] can be written as

$$R(x_1,x_2,x) = x\int_0^1 d\eta \; \tilde{P}_{a_1a_2\to M}(\eta) \; \delta(x_1-\eta x) \; \delta(x_2-[1-\eta]x) \quad . \qquad (2.4)$$

When this is substituted into Eq. (2.3) and the delta functions removed, we find

$$D_{M,i}(x,Y) = \sum_j \int_{Y_o}^Y dy \int_x^1 \frac{dt}{t} D_{ji} (\frac{x}{t},Y-y) \; f_j(t,y) \quad , \qquad (2.5)$$

where

$$f_j(t,y) = t\int_0^1 d\eta \; \tilde{P}_{a_1a_2\to M}(\eta) \int_{\eta t}^{1-(1-\eta)t} \frac{dz}{z(1-z)} D_{a_1b_1} (\frac{\eta t}{z},y-Y_o) \; X$$

$$D_{a_2b_2} (\frac{(1-\eta)t}{(1-z)},y-Y_o) \; P_{j\to b_1b_2} (z) \quad . \qquad (2.6)$$

We find it most convenient to work with moments of these functions rather than with the functions themselves, since for the simple R under study this removes all integrations over the x variables. If we define the moments as

$$D_{M,i}^n (Y) = \int_0^1 x^n \; D_{M,i}(x,Y) \; dx \quad , \qquad (2.7)$$

then (2.5) becomes

$$D_{M,i}^n(Y) = \sum_j \int_{Y_o}^Y dy \; D_{ji}^n(Y-y) \; f_j^n(y) \quad , \qquad (2.8)$$

with

$$f_j^n(y) = \int_0^1 \int_0^1 d\xi_1 \, d\xi_2 \, \theta(1-\xi_1-\xi_2) \, (\xi_1+\xi_2)^n \quad X$$

$$\tilde{P}_{a_1 a_2 M} \left(\frac{\xi_1}{\xi_1+\xi_2}\right) \psi_{a_1 a_2 j}(\xi_1,\xi_2,y) \quad , \tag{2.9a}$$

and

$$\psi_{a_1 a_2 j}(\xi_1,\xi_2,y) = \sum_{b_1 b_2} \int_{\xi_1}^{1-\xi_2} \frac{dz}{z(1-z)} \, D_{a_1 b_1}\left(\frac{\xi_1}{z},y-Y_0\right) D_{a_2 b_2}\left(\frac{\xi_2}{1-z},y-Y_0\right)$$

$$X \quad P_{j \to b_1 b_2}(z) \quad . \tag{2.9b}$$

The special case proposed by Das and Hwa and used by most other workers in this field has

$$\tilde{P}_{a_1 a_2 \to M}\left(\frac{\xi_1}{\xi_1+\xi_2}\right) = \frac{\xi_1 \xi_2}{(\xi_1+\xi_2)^2} \, C_{a_1 a_2 \to M} \quad . \tag{2.10}$$

This has the virtue, for our purposes, that the moments f^n reduce to double moments of ψ,

$$f_j^n(y) = \sum_{a_1 a_2} \binom{n-2}{m} \int_0^1 d\xi_1 \int_0^1 d\xi_2 \, \theta(1-\xi_1-\xi_2)$$

$$X \quad \xi_1^{n-1-m} \, \xi_2^{m+1} \, \psi_{a_1 a_2 j}(\xi_1,\xi_2,y) \, C_{a_1 a_2 \to M} \quad , \tag{2.11}$$

which can be calculated explicitly,

$$f_j^n(y) = \sum_{\substack{m \\ a_1 a_2}} \binom{n-2}{m} C_{a_1 a_2 \to M} \, D_{a_1 b_1}^{n-1-m}(y-Y_o) \, D_{a_2 b_2}^{m+1}(y-Y_o) \, P_{j \to b_1 b_2}^{n-1-m,m+1} \quad . \tag{2.12}$$

Thus the moments of the fragmentation functions can be calculated from Eq. (2.8) using only the known moments of basic D_{ij} parton "propagators" and the known vertices, P, plus knowledge of which quarks combine to make which mesons.

The coefficients C are typically determined by recombination modelists as follows: Only the valence quarks in the produce' meson are considered as it is believed that the sea develops later from gluons spawned by these valence quarks. Then, the constraint

$$\int_0^1 d\xi_1 \int_0^1 d\xi_2 \; R(\xi_1, \xi_2) = 1 \quad , \tag{2.13}$$

is either imposed or approximated. This forces all such pairs to make pseudoscalars rather than the other possible mesons which could be formed, e.g., vector or tensor mesons. Hence, the size is fixed. Typically, therefore,

$$C_{u\bar{d} \to \pi^+} = C_{d\bar{u} \to \pi^+} \leq 6 \quad . \tag{2.14}$$

Most applications choose 6 for this, to satisfy Eq. (2.13), although Van Hove[7] chooses 4.0.

III. RESULTS FOR QUARK FRAGMENTATION

Before comparing the fragmentation functions calculated from Eq. (2.8) with those extracted from the e^+e^- jet data, we must face the fact that (2.8) produces moments with different Q^2 dependence than that normally expected of fragmentation functions. Typically, one would compute the Q^2 behavior of the $D_{M,i}$ functions using one of the "quark propagators" D_{ij}^n (y). Our formula is more complicated than this and can *a priori* produce quite different behavior.

However, both the jet calculus and the ordinary evolution equations for the fragmentation functions are expected to be valid for Q^2 very large. Furthermore, if Q^2 is large, and if both m and n in Eq. (2.12) are large, then the Q^2 evolution of the (p + n) $\underline{th}$ moment of Eq. (2.8) is the same as that derived from the transposed Altarelli-Parisi equations given by Owens[13] for the evolution of the fragmentation $D_{m,i}$'s. We can see this by going to a basis in which the D_{ij} functions are diagonal and of the form $e^{y\lambda_m}$, with λ_m an eigenvalue of the Altarelli-Parisi set. In Eq. (2.8),

$$D^{n+p}(Y) \sim \int_{Y_0}^{Y} dy \, e^{(Y-y)(\lambda_{n+p})} e^{(y-Y_0)\lambda_p} e^{(y-Y_0)\lambda_n}$$

$$\sim e^{(\lambda_p+\lambda_n)(Y-Y_0)} - e^{\lambda_{n+p}(Y-Y_0)} \quad . \tag{3.1}$$

Examination of the leading eigenvalues of the Altarelli-Parisi equations shows that if both n and p are large, then $|\lambda_p+\lambda_n| > |\lambda_{n+p}|$

so the Y dependence is given by $\sim e^{Y\lambda_{n+p}}$, exactly as in Owens'

equations. (The eigenvalues are negative. Also, Y_o and Y are the

values of y at opposite ends of the diagram, Fig. 1.)

We therefore will concentrate our comparison at large Q^2 and

large moment, with the idea that the fragmentation functions must

match here if the recombination model is to satisfy the constraint.

The parameters Λ^2 and α_o in Eq. (2.2) must be specified; $\Lambda^2 = 0.25$,

a relatively standard value, and α_o is taken as 10.0. It is desirable

that our basic results be insensitive to α_o for large α_o; the

dependence on α_o roughly indicated by $Y-Y_o$ in Eq. (3.1) assures this.

Numerical calculation confirms this insensitivity as very little

(few percent) change in the double moments of Eqs. (2.8) – (2.11)

occurred when α_o was varied up to 10^6.

The only quantity which might be termed a free parameter is

Q_o^2, which determines the lower limit of the y integration in Eqs.

(2.3) – (2.8). It seems reasonable to assume that Q_o^2 might be small,

on the basis of transverse momentum for produced pions relative to the

jet axis. In fig. 2a and 2b we show two large moments, n = 20 and n = 27,

of the fragmentation function $D_{\pi^+,u}$, as a function of Q^2. The dashed

curve labeled OFF is calculated from the QCD evolution equation of

Ovens with the Field–Feynman empirical fit for $D^n_{\pi^+,u}$ as the initial

"value". The solid curves are calculated from Eqs. (2.8), (2.9a),

and (2.9b) with the somewhat special, popular choice for R, Eqs. (2.4)

and (2.10). The Q_o^2 values, as indicated on the curves, are 0.3, 0.5,

and 1.5; clearly, the smaller Q_o^2 values reproduce the shape of the O-FF

fragmentation function best in the range Q^2 plotted.

In the calculation of these solid curves, the coefficient $C_{\overline{ud} \to \pi^+}$

in Eq. (2.12) is set equal to unity. Comparing the solid and dashed

curves, therefore, we note that the actual value of C is quite sensitive

to the value of Q_o^2 chosen.

It is of interest to examine these same fragmentation functions

for fixed Q^2 as a function of n or moment. In Fig. 3 we show values

of $D_{M,i}^n$ at fixed $Q^2 = 10^6$ GeV^2 as a function of n for the fragmentation

of the u-quark into π^+ and the s-quark into K$^-$. For this comparison

we have made the non-zero $C_{a_1, a_2 \to M}$ coefficients in Eq. (2.12) equal

to unity. Calculations of curves as in Figs. 2 and 3 were performed for

several other Q_o^2 values in addition to 0.3, 0.5, and 1.5 GeV^2. On the basis

of Fig. 2, comparison of curve shapes with O-FF suggests $Q_o^2 = 0.3 - 0.5$

is preferred. In Fig. 3a, it appears that the curves labeled $Q_o^2 = 0.3$

and $Q_o^2 = 1.5$ are least like O-FF. From this moment dependence we

clearly prefer $Q_o^2 \cong 0.5$. The two cases $u \to \pi^+$ and $s \to K^-$ in Figs. 3a

and 3b from our jet-calculus recombination calculations are identical

because of the assumed flavor independence of the propagators and

vertices. The Owens-Feynman-Field fragmentation functions become very

similar also at this large Q^2. (We use the "effective" fragmentation

functions of Ref. 12, which include the consequences of vector meson

creation and decay.) Notice that for large moments the two curves are

nearly parallel; they differ by a factor of about 5.5 in the case of

$u \to \pi^+$ and a factor of 4-5 in the case of $s \to K^-$. These obey the

constraint $C \leq 6$ following from Eq. (2.13) and used by Das and Hwa;
these factors are slightly higher than the value $C = 4$ adopted by
Van Hove.[7]

Thus we see that in spite of the apparently ad hoc methods used
by Das and Hwa to guess their recombination function, it essentially
passes our test of being consistent with the measured fragmentation
functions given the following caveats:

i) The two cases shown in Fig. 3 involve recombination of the main
    quark in the jet with a 'created' quark; this is the case which
    recombination modelists would term "valence-sea" recombination.
    This is the major case addressed in their works and there is some
    uneasiness on their part about extending the model to other cases.
    We will discuss "sea-sea" recombination briefly in the next section.

ii) We have demonstrated agreement in the region where the two
    approaches should have similar Q^2 dependence, namely very large
    Q^2 and large moment. The formula, Eq. (2.8), with the naive
    form for R in fact generates fragmentation functions which
    have rather different Q^2 dependence from that normally assumed,
    at low Q^2. This behavior needs to be explored also, and the
    actual Q^2 dependence of R needs to be deduced.

The true R function must correct the difficulty illustrated
in Fig. 4. Moments of our calculated u quark fragmentation function
into π^+ are shown versus n for two different Q^2; these are the solid
lines labelled by $Q^2 = 100$ and 10^5 GeV^2. For comparison, the dashed

lines are the moments at the same values of Q^2 given by the Owens[13]
evolution equation with the Field-Feynman[12] initialization. The
dependence on Q^2 at the largest n shown is similar but not identical.
The dashed curve falls by a factor of four between $Q^2 = 10^2$ GeV^2
and $Q^2 = 10^5$ GeV^2, whereas the solid curve falls only a factor of
three. However at small n, there is substantial disagreement; the
solid lines rise in this same Q^2 interval while the dashed lines
fall. Two possible modifications of R which might solve this problem
are: (1) R may actually have an ntrinsic Q^2 dependence, or (2)
R may have a different x_1 and x_2 dependence from that assumed and
explored here. (This will produce a different combination of the
y dependent moments than that given in Eq. (2.11)).

Examination of the moments in Figs. 2 and 3 suggests that
the reconstruction of the fragmentation functions from the moments
will not lead to the same shape in x as the Owens-Feynman-Field
curves. For example, the dashed and solid curves in Fig. 3 diverge
most for small n, so the inverted curves should tend to diverge
most at small x. The curves for $xD_{\pi^+,u}(x)$ obtained by use of Yndurain's
method[15] for inverting the moments are shown in Fig. 5. For the
reader's benefit, a complete set of curves for (a) $Q^2 = 10^6$ GeV^2,
(b) $Q^2 = 10^5$ GeV^2, ..., (f) $Q^2 = 10^1$ GeV^2 is shown. The dashed lines,
consistent with the code of the other figures, resulted from inverting
the moments calculated with Owens'[13] evolution equation using Field-
Feynman[12] for initialization. The solid curves are the results of
inverting our recombination theory moments from Eqs. (2.8) – (2.12).

These curves are labeled by the values used for $C_{\bar{u}d \to \pi^+}$, either 4 or 6 being the popular values[1,7] in the recent literature. At the highest Q^2 of 10^6 or 10^5 and somewhat lower the solid curves calculated for $C_{\bar{u}d \to \pi^+} = 4$ are in surprisingly "reasonable" agreement with the dashed (Owens-Feynman-Field). In fact, the curves for $C_{\bar{u}d \to \pi^+} = 6$ are "good", considering the stringent nature of our constraint equation. The entire recombination model approach was constructed for x "reasonably" large; the best agreement between solid and dashed curves is at $x \gtrsim 0.5$. In the mid $- Q^2$ range, $10^2 - 10^4$ GeV^2, agreement exists between the dashed curve and solid curve labeled "$C_{\bar{u}d \to \pi^+} = 6$". When the smallest $Q^2 = 10$ GeV^2 is reached, Fig. 5f, the wrong Q^2 dependence produced by the popular R function becomes clearly evident; $C_{\bar{u}d \to \pi^+}$ must be 9 to have the dashed and solid curves match at $x \gtrsim 0.8$.

V. DISCUSSION AND CONCLUSIONS

We have derived an equation relating the quark fragmentation function D to the recombination function R used in the recombination model. The basis for this derivation is the jet calculus, which allows us to find the probability distribution in momentum fraction variables for two partons in a parton jet. If these two partons are quark and antiquark, the recombination function folds them into a meson. The result is an expression for D, the probability that an initial parton i will yield a meson with momentum fraction x of the momentum of i.

Our relation might be viewed as an integral equation which specifies R in terms of $D_{M,i}$. We could not solve this in general; instead we inserted popular Q^2 independent R functions and computed $D_{M,i}$. At large Q^2, for large moments, the computed $D_{M,i}$ function is much like the ones derived from other sources. This tends to support the belief of recombination modelists that their functions describe meson production at large x.

With the choice $Q_0^2 = 0.5$ GeV^2, the values $4 \leq \alpha \leq 6$, chosen by recombination model advocates to normalize R, give numerical agreement with $D_{M,i}$ at large x over a wide range of large Q^2. The Q^2 dependence of $D_{M,i}$ obtained from our equation with a Q^2 independent R is not, however, quite correct. Presumably the correct Q^2 and x_i dependence of R is specified by the "known" Q^2 dependence of $D_{M,i}$ and our relation between them. Applications of the recombination model to date have not attempted to include any Q^2 dependence in R nor to vary its x_1 dependence from the simple form studied here. We believe that investigation of this point is necessary before the recombination model can be regarded as having a solid foundation.

The results presented here have focused on valence-sea recombination in which the sea-quark ($\bar{u}$ or $\bar{d}$) is light, since that is where the recombination model has been most successful. We are currently studying two other situations which present additional complications, (a) valence-sea recombination in which the sea quark is heavy, and (b) sea-sea recombination with either

light or heavy quarks. We shall briefly sketch the new physics

involved to contrast with the case of light sea-quark recombination.

Field and Feynman[12] find a suppression of valence-sea frag-

mentation functions for cases such as $u \rightarrow K^+$, due presumably to

problems of extracting the heavy mass of the $\bar{s}$ quark from the

sea. Our calculation of $D_{M,i}$ from R does not at present allow

for such suppression, since no quark mass effects are included.

The same R would be used for $u \rightarrow K^+$ and for $\bar{s} \rightarrow K^+$, so the $D_{K^+,i}$

functions calculated from Eq. (2.3) would be the same. Manipulation

of Q_o^2 can, of course, yield different values but it is not clear

at present whether this is an appropriate solution to the problem.

Sea-sea recombination has presented some difficulties for

advocates of the recombination model. Some applications have led

to claims of a dramatically enhanced sea,[3] whereas others have

found a not-so-enhanced sea.[7] Our calculation of the O-FF

curve yields much smaller values for $D_{K^-,u}$ than $D_{\pi^-,u}$. On the

other hand our recombined jet calculus result for $D_{K^-,u}$ is over

an order of magnitude smaller yet. We suspect that this small

value may be related to the necessity of sea enhancement in some

recombination calculations.

Proper calculation of $D_{K^-,u}$ should take account of the mass

effects discussed above. In addition, because the number of K's

produced is small, contributions from other mechanisms which would

normally be neglected must be included. One of these is recombination

from gluons. Another, more important, effect is the recombination production of heavy mesons which decay to produce K's. For instance, there is a contribution to $u \to K^-$ from $u \to (\bar{s}s$ mesons, e.g., $\phi) \to K^+K^-$.

Recombination into heavy mesons and their subsequent decay is also expected to produce additional numbers of pseudoscalars, mainly at small x. This will certainly serve to increase agreement between the dashed and solid curves in Figs. 3 and 5. However, explicit calculation showed this contribution to have small effects in the region of agreement apparent between these curves in Fig. 5.

In conclusion, we have deduced an integral equation for the recombination function R in terms of the quark fragmentation function $D_{M,i}$. This equation suggests that the recombination function may have Q^2 dependence, contrary to popular assumptions in the literature. However, this intuitive, popular R function is shown by explicit calculation to have a wide range of reliability at large Q^2 for large moments of $D_{M,i}$.

REFERENCES

*   Permanent address.

1. K. P. Das and R. C. Hwa, Phys. Lett. <u>68B</u>, 459 (1977). M. E. Vislay and C. E. Carlson, Phys. Rev. <u>D12</u>, 2670 (1975), in an electroproduction application gave identical reasoning to Das and Hwa in arguing that large x mesons reflect the parton momentum distribution in the proton.

2. W. Ochs, Nucl. Phys. <u>B118</u>, 397 (1977).

3. D. W. Duke and F. E. Taylor, Phys. Rev. <u>D17</u>, 1788 (1978).

4. T. A. DeGrand and H. I. Miettinen, Phys. Rev. Lett <u>40</u>, 612 (1978).

5. H. I. Miettinen, T. DeGrand, D. Duke, T. Inami, and H. Thacker, to be submitted to Phys. Rev. (Private communication).

6. Van Chang and K. E. Lassila, Ames Laboratory preprint IS-4434 (unpublished).

7. S. Pokorski and L. Van Hove, CERN preprint Th-?'27, 1978 (unpublished); L. Van Hove, CERN preprint TH-2628, 1979.

8. Van Chang and R. C. Hwa, University of Oregon preprint, 1979.

9. G. Altarelli and G. Parisi, Nucl. Phys. <u>B126</u>, 298 (1977).

10. K. Konishi, A. Ukawa and G. Veneziano, Phys. Lett. <u>78B</u>, 243 (1978).

11. K. Konishi, A. Ukawa and G. Veneiano, Rutherford Laboratory Preprint RL-79-026, February, 1979. We use the abbreviation KUV to refer to this paper and the letter listed in Ref. 10.

12. R. D. Field and R. P. Feynman, Phys. Rev. <u>D15</u>, 2590 (1977), Nucl. Phys. <u>B136</u>, 1 (1978), and Phys. Rev. <u>D18</u>, 3320 (1979).

13. J. F. Owens, Phys. Lett. 76B, 85 (1978). The abbreviation O-FF

    in the text is used for the result of calculation with the

    Owens evolution equation from initial conditions specified

    by the fragmentation functions of Field and Feynman, Ref. 12.

    See also T. Uematsu, ibid. 79B, 97 (1978).

14. D. Willen, University of Illinois preprint, "Two Particle

    Distributions in e^+e^- Jets".

15. F. J. Yndurain, Phys. Lett. 74B, 68 (1978).

FIGURE LEGENDS

Fig. 1.   Schematic representation in the jet calculus for the

fragmentation function of parton i into meson M as

given by Eq. (2.3) of the text.   The circular blobs

on the parton lines i, b_1, b_2 stand for the complete

QCD evolution of a parton cloud out of which partons

j, a_1, a_2, respectively, are selected.   The partons

a_1 and a_2 are recombined to make the di-parton state

M through use of R.

Fig. 2.   Moments (a) n = 20 and (b) n = 27 of the fragmentation

function for a u quark into π^+ mesons.   The dashed

curves are calculated from Owens' evolution equations

with the Field-Feynman fit to e^+e^- annihilation and hadron

lepto-production data as the initializing condition.   The

solid curves show our results for the moments of the

fragmentation function for u into π^+ calculated from Eqs.

(2.8) – (2.12) of the text for Q_0^2 = 0.3, 0.5, and 1.5 GeV^2.

Fig. 3.   Moments of fragmentation functions for (a) $u \rightarrow \pi^+$ and

(b) $s \rightarrow K^-$ versus n, the moment number, for our calculation

(solid line) compared with that (dashed line) of Owens-

Feynman and Field.   The dashed lines are calculated from

Owens' evolution equations with the Field-Feynman fit of

Ref. 12 for the initial (low Q^2) condition.   In (a) we show

our calculation for Q_o^2 = 0.3, 0.5, and 1.5 to illustrate

that the smaller Q_o^2 value produces too fast a fall off

of D^n with moment number. For completeness, we note

that the dashed curve rises at n = 2 to 0.037 for

$u \rightarrow \pi^+$ and 0.022 for $s \rightarrow K^-$.

Fig. 4. Curves showing the moments of the (u-quark into π^+)

fragmentation function versus n for $Q^2 = 10^2$ and 10^5 GeV^2.

The dashed curves are calculated with the Owens evolution

equation for D given the Field-Feynman initialization. The

solid curves are given by Eqs. (2.8) – (2.12) with the recombination

function of Das and Hwa, Ref. 1, C = 1, and $Q_o^2 = 0.5$ GeV^2.

Fig. 5. Fragmentation functions times momentum fraction x vs x

for $u \rightarrow \pi^+$. The dashed curves labelled O-FF are calculated

for the various $Q^2 = (a)10^6$, (b)10^5, (c)10^4, (d)10^3, (e)10^2,

and (f)10 GeV^2 from Owens' evolution equations with Field-

Feynman initialization. The solid curves are calculated

from Eqs. (2.8) – (2.12) for our jet calculus recombination

model with normalization of the recombination function

fixed by the values $C_{u\bar{d} \rightarrow \pi^+} = 4$ or 6 as given in the

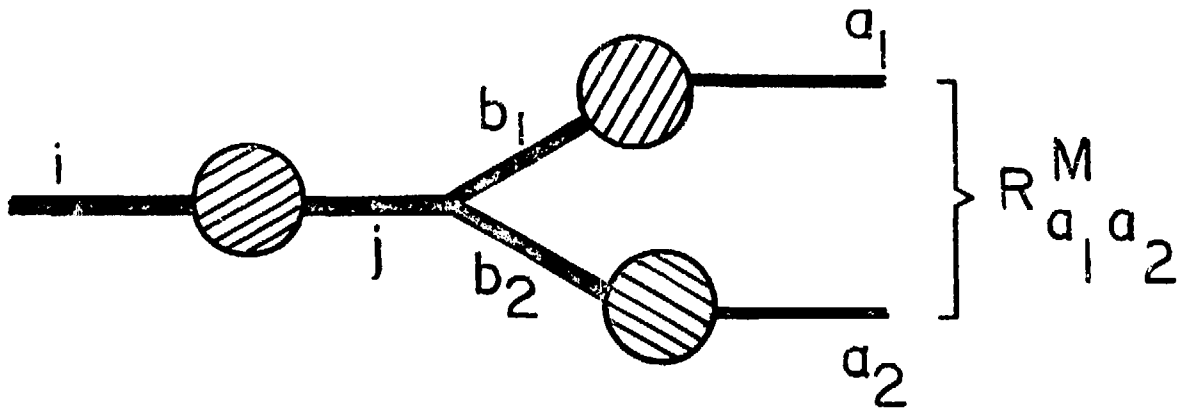literature. For a discussion of (f), see text.

Fig. 1. Schematic representation in the jet calculus for the fragmentation function of parton i into meson M as given by Eq. (2.3) of the text.
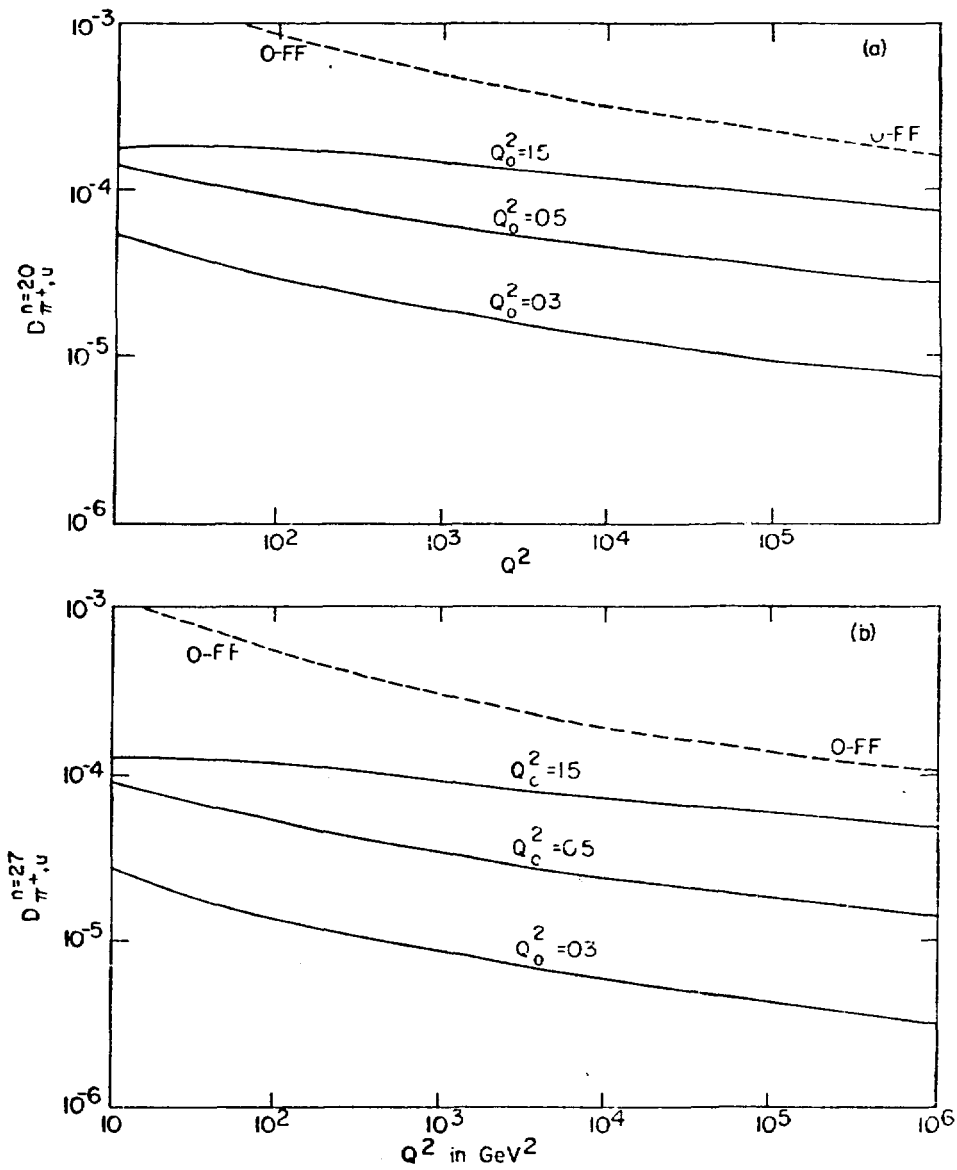
Fig. 2. Moments (a) $n = 20$ and (b) $n = 27$ of the fragmentation function for a u quark into π^+ mesons.
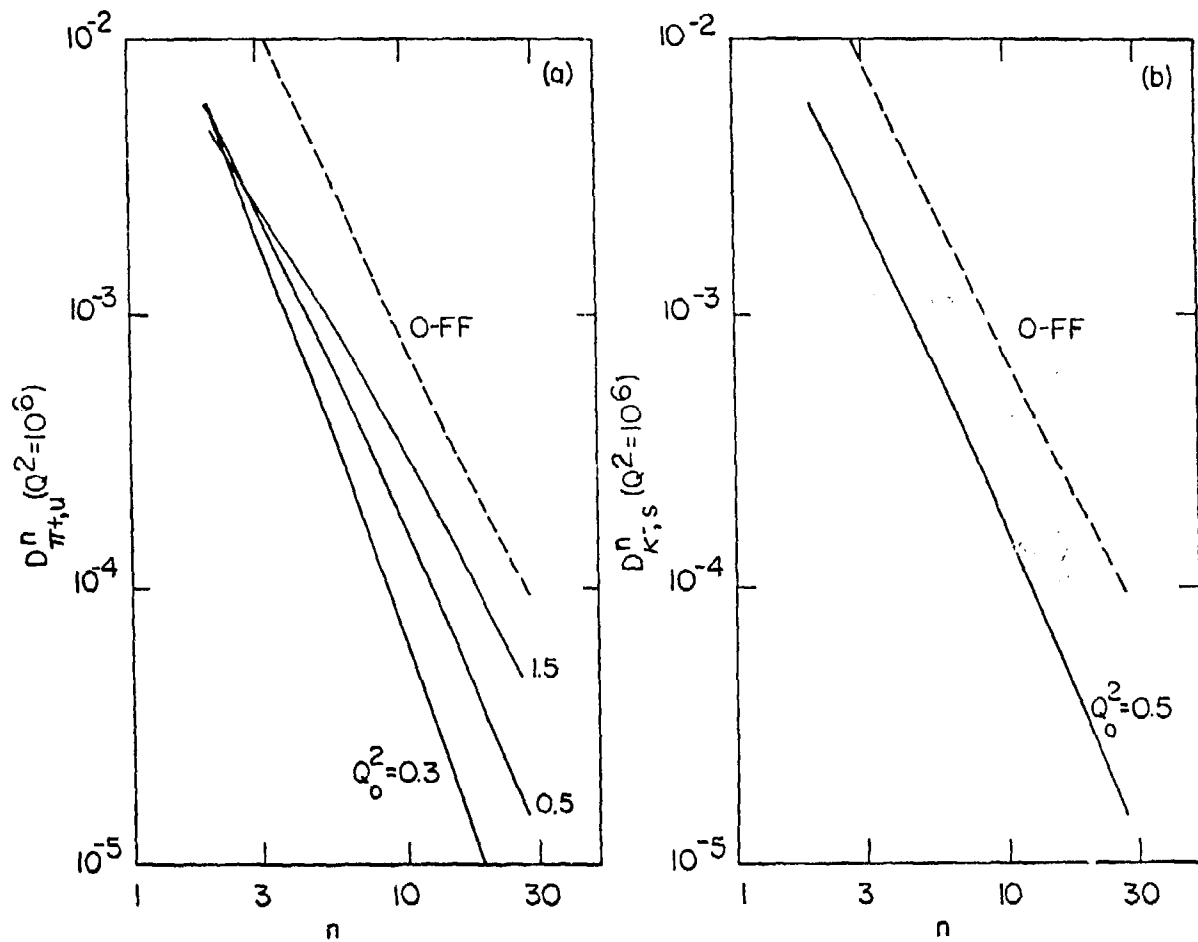
Fig. 3. Moments of fragmentation functions for (a) $u \to \pi^+$ and (b) $s \to K^-$ versus n, the moment number, for our calculation (solid line) compared with that (dashed line) of Owens-Feynman and Field.

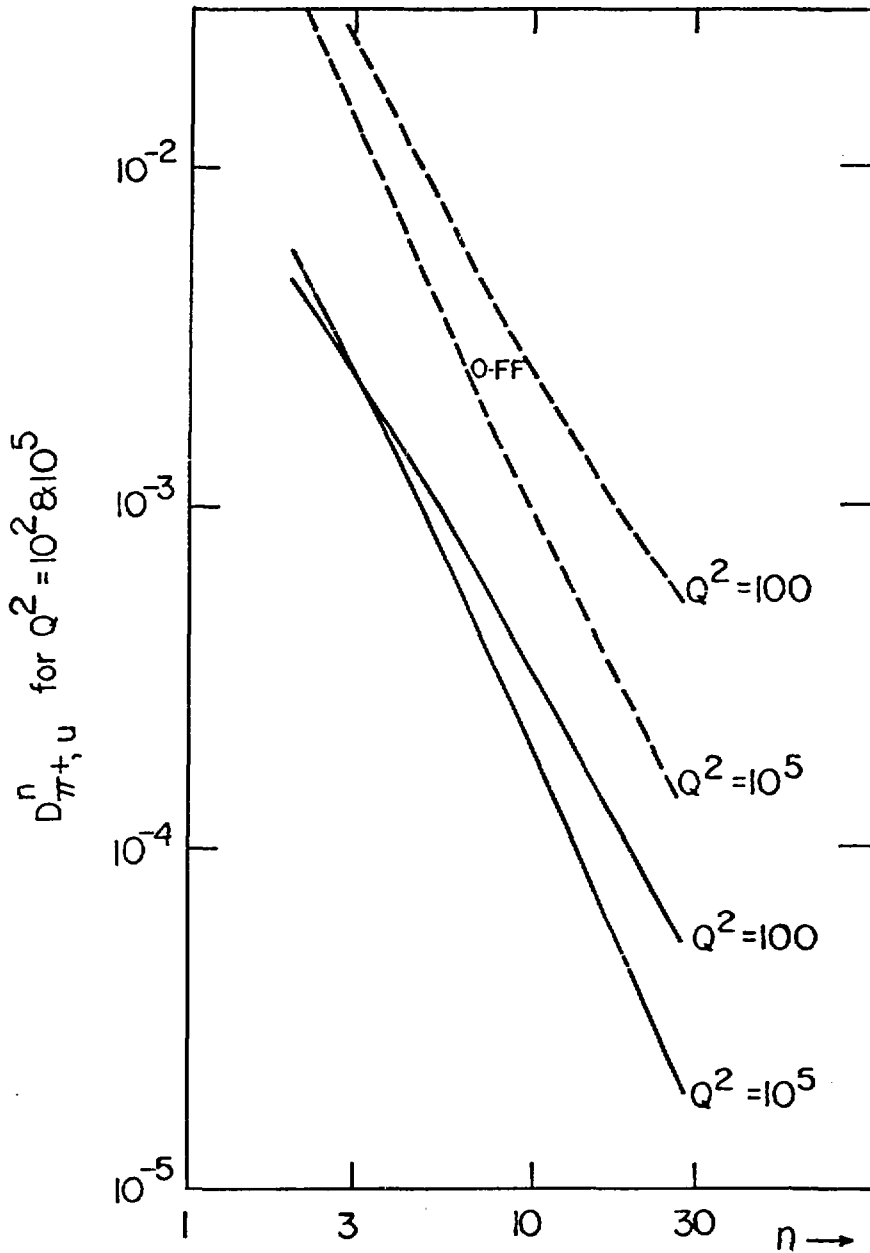Fig. 4. Curves showing the moments of the (u-quark into π^+) fragmentation function versus n for $Q^2 = 10^2$ and 10^5 GeV^2.
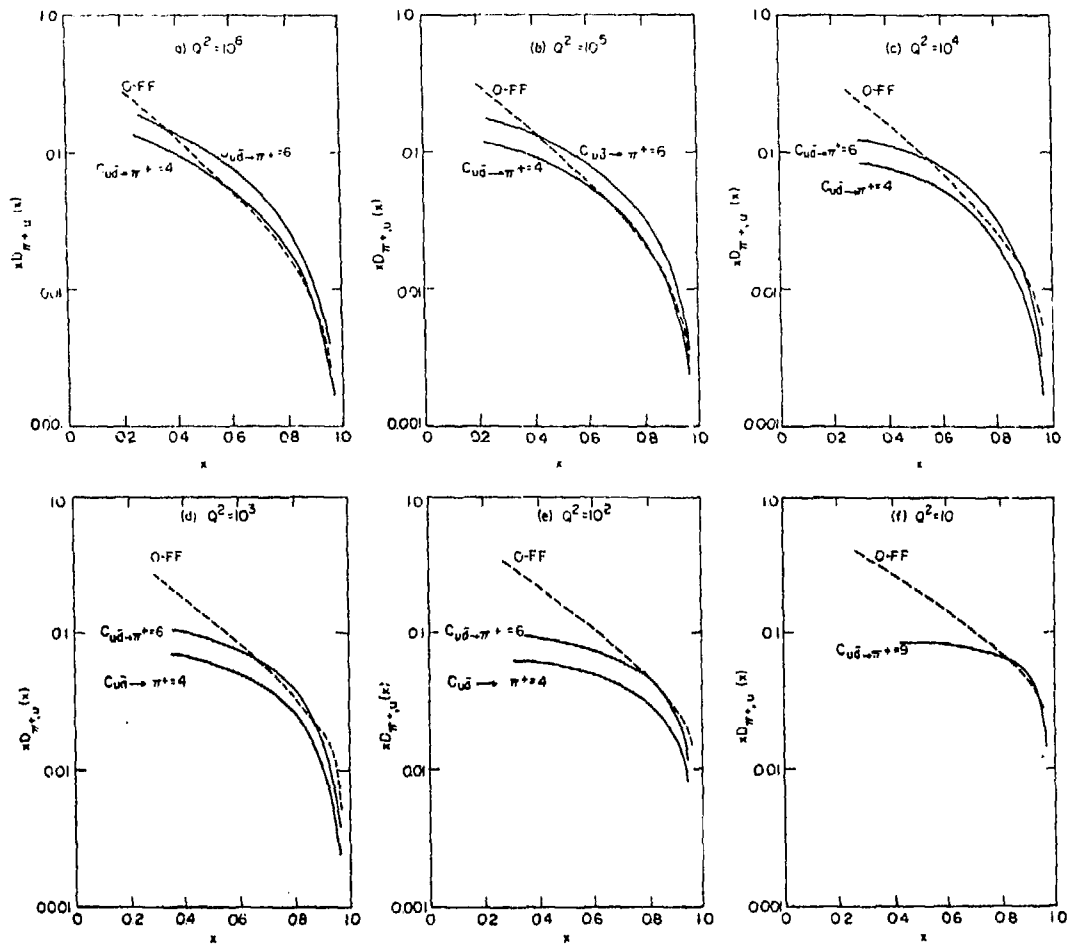
Fig. 5. Fragmentation functions times momentum fraction x vs x for $u \to \pi^+$.