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ABSTRACT

The novel method proposed by one of the authors to calculate exactly the response functions of the one-dimensional Tomonaga-model is described in more detail. The method is generalized for the case of a system of coupled chains where both the intrachain and interchain interactions have forward scattering components only. The model does not show real phase transition at any finite temperature indicating that the interchain backward scattering or hopping is needed to have an ordering of the chains at finite temperature.

АННОТАЦИЯ

Излагается более подробно новый метод, предложенный раньше одним из авторов, для точного вычисления функции линейного отклика в одномерной модели Томонаги. Метод обобщается на системы связанных цепей, когда и внутрицепное и межцепное взаимодействия имеют только компоненты рассеяния вперед. Показано, что в данной модели при конечной температуре не появляется фазовый переход. Это указывает на то, что для упорядочения цепей при конечных температурах следует учитывать межцепное рассеяние назад или "hopping".

KIVONAT

Az egyik szerző által az egydimeziós Tomonaga-modell válaszfüggvényének pontos kiszámitására javasolt módszert irjuk le részletesebben. A módszert általánositjuk csatolt láncok rendszerére olyan esetben, amikor a láncon belüli és a láncok közötti kölcsönhatásoknak egyaránt csak előreszórási komponensei vannak. A modell egyetlen véges hőmérsékleten sem mutat valós fázisátalakulást, jelezve, hogy a láncok közötti visszaszórás, vagy "hopping" szükséges ahhoz, hogy a láncok véges hőmérsékleten rendeződjenek.

1. Introduction

Recently quasi-one-dimensional /quasi-1-d/ conductors attracted a great deal of interest because of their unusual properties. For a review of works done in the past few years see |1| - |3|. These systems are built usually of large, flat molecules which are relatively closely stacked in one direction to form chains, the adjacent chains being at a relatively larger distance. Thus the motion of electrons is confined predominantly to motion along the chains with rare hoppings between them.

Various theoretical models have been worked out to study the properties of these systems. One of these models is the Fermi gas model which has been investigated in the strictly 1-d case in great detail. A review of the properties of this model can be found in |4|. Since 1-d system may have specific properties of behaviour /such as absence of phase transition at finite temperature/ which are characteristic for 1-d systems but are not necessarily true in higher dimensions, any realistic model of quasi-1-d conductors should contain some kind of interaction between the chains. Generalizations of the 1-d Fermi gas model in this direction have indeed been attempted |5| - |9|. The Fermi gas model is a model with infrared singularities and it is difficult to treat it in a satisfactory manner at low temperatures and low energies even in its strictly 1-d version already. The treatment becomes increasingly more complicated

when interchain interactions are taken into account. Therefore any simplified model which can be solved exactly is of great value in finding out what the effect of the interaction terms is.

The 1-d Tomonaga model |10| is a well known exactly soluble model of a 1-d interacting electron system. The two important features which make it soluble are t'e linear dispersion relation and the neglect of backward scattering. In the present paper we will consider a generalization of the model to a set of coupled chains but keeping these two features intact, i.e. a one-dimensional linear dispersion relation is taken for every chain which means that no hopping between the chains is allowed and only forward scattering processes are taken into account for both the intrachain and interchain scatterings.

There are various ways to solve the 1-d Tomonaga model. The excitations of the model have been shown to be the boson--like charge- and spin-density excitations |11| and therefore the calculation of the one-particle Green's function and the response functions or two-particle correlation functions in which the fermion operators cannot be easily expressed in terms of the charge- or spin-density operators was a formidable task |12|. It was shown, however, by Luther and Peschel |13| and by Mattis |14| that an exact operator identity can be found which allows to represent the fermion operators in terms of the boson--like density operators. Since the Tomonaga Hamiltonian can be

transformed to a diagonal form of the boson operators, the Green's function and response functions can be calculated by calculating harmonic averages. The operator identity involves, however, a delicate limiting procedure, which is not always easy to perform and therefore other methods are also of interest. Fogedby |15| used functional integrals to solve the 1-d Tomonaga model. Another very elegant method was used by Dzyaloshinsky and Larkin |16| to calculate the Green's function. They could derive a Ward identity between the Green's function and the vertex appearing in the Dyson equation. Making use of this relation, the Dyson equation could be solved exactly. This method has been generalized by one of the authors |17| to calculate the response functions. In this paper we will make a further generalization to study the behaviour of the quasi-1-d system of Tomonaga chains.

First in Sec. 2 the method of Dzyaloshinsky and Larkin |16| is briefly presented since it is the basis of all further considerations. Then the generalization of the method to calculate response functions is described in Sec. 3. Sec. 4 contains the results obtained for the quasi-1-d system of Tomonaga chains. Finally the results are discussed in Sec. 5.

2. The Dzyaloshinsky-Larkin method of summing diagrams in the Tomonaga model

The Tomonaga model was defined originally as a 1-d model of interacting electrons

$$H = \sum_{k,\alpha} \frac{k^2}{2m} c^+_{k\alpha} c_{k\alpha} + \frac{1}{2L} \sum_{\substack{k,k',p \\ \alpha,\beta}} \lambda(k)\, c^+_{k+p\alpha}\, c^+_{k'-p\beta}\, c_{k'\beta}\, c_{k\alpha} \qquad /1/$$

where L is the length of the system and the interaction potential is supposed to be of long range and therefore the Fourier components $\lambda(k)$ are non-zero for small momenta only. If the cutoff on the momentum transfer Λ is much smaller than the Fermi momentum k_F, one can distinguish between particles near the Fermi point $+k_F$ and particles near the other Fermi point $-k_F$, since in the absence of large momentum transfer interaction /backward scattering/ the electrons stay always in the neighbourhood of the same Fermi point after any interaction process. In order to make this distinction more explicit, we will denote the creation /annihilation/ operators of electrons near the right and left Fermi points by a^+_k (a_k) and b^+_k (b_k), respectively.

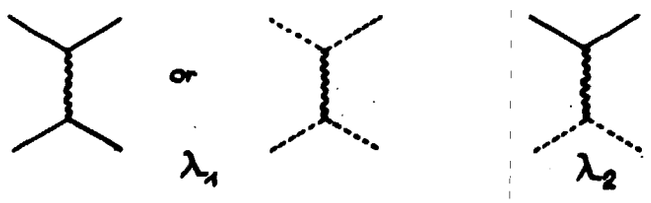
The kinetic energy term can be approximated by a linear relationship near the Fermi points and the free part of the Hamiltonian can be written as

$$H_0 = \sum_{k,\alpha} v_F(k-k_F)\, a^+_{k\alpha} a_{k\alpha} + \sum_{k,\alpha} v_F(-k-k_F)\, b^+_{k,\alpha} b_{k\alpha} \qquad /2/$$

The interaction part of the Hamiltonian can be generalized by allowing for different coupling strengths for the cases when the scattered electrons are from the same or different branches of the spectrum and for parallel or antiparallel orientation of the scattered electrons. In the most general case the Hamiltonian reads:

$$H_{int} = \frac{1}{2L}\sum_{\substack{k,k',p\\ \alpha,\beta}} (\lambda_{4\parallel}\delta_{\alpha\beta} + \lambda_{4\perp}\delta_{\alpha,-\beta})(a^{+}_{k+p\alpha}\, a^{+}_{k'-p\beta}\, a_{k'\beta}\, a_{k\alpha} + b^{+}_{k+p\alpha}\, b^{+}_{k'-p\beta}\, b_{k'\beta}\, b_{k\alpha}) + \frac{1}{L}\sum_{\substack{k,k',p\\ \alpha,\beta}} (\lambda_{2\parallel}\delta_{\alpha\beta} + \lambda_{2\perp}\delta_{\alpha,-\beta})\, a^{+}_{k+p\alpha} b^{+}_{k'-p\beta}\, b_{k'\beta}\, a_{k\alpha} \qquad /3/$$

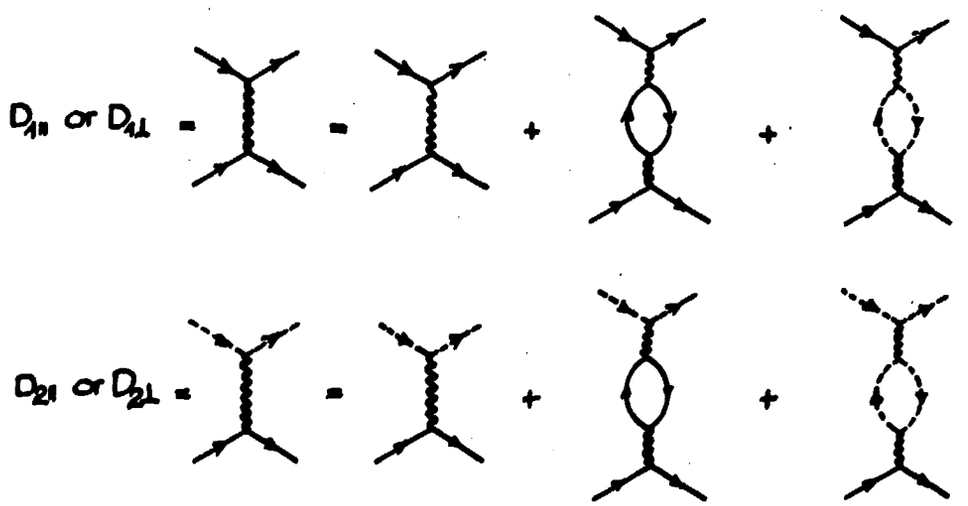
To make contact with other works on the Fermi gas model [4] where backward scattering is also included, the present choice of couplings corresponds in the language of 'g'-ology to $\lambda_1 = g_4$ and $\lambda_2 = g_2$. The scattering processes can be represented diagrammatically as



where the solid and dashed lines correspond to electrons on the right or left branch of the spectrum.

Dzyaloshinsky and Larkin |16| recognized that the two particular features of the Tomonaga model, namely the linearized dispersion relation and the neglect of backward scattering terms lead to an enormous simplification in the contribution of diagrams. They have found that all diagrams which contain electron bubbles with more than two interaction legs are mutually cancelled and only simple bubbles with two interaction legs and series of simple bubbles remain.

Effective interactions can be introduced by summing these series of bubbles, in the same way as the screened interaction is introduced in a RPA calculation, but here working with these interactions will lead to an exact procedure. The diagrammatic equations for the effective interactions denoted by $D_{1/\!/}$, $D_{1\perp}$, $D_{2/\!/}$ and $D_{2\perp}$ are as follows:



The diagrammatic equations are the same for parallel and anti-parallel spin orientations. It is worth mentioning that though the bare λ_1 coupling is the same for electrons on the right or left branches, the effective couplings will be different.

The analytic form of these equations is

$$
\begin{aligned}
D_{1//} &= \lambda_{1//} + \lambda_{1//}\Pi_+ D_{1//} + \lambda_{1\perp}\Pi_+ D_{1\perp} + \lambda_{2//}\Pi_- D_{2//} + \lambda_{2\perp}\Pi_- D_{2\perp} \\
D_{1\perp} &= \lambda_{1\perp} + \lambda_{1//}\Pi_+ D_{1\perp} + \lambda_{1\perp}\Pi_+ D_{1//} + \lambda_{2//}\Pi_- D_{2\perp} + \lambda_{2\perp}\Pi_- D_{2//} \\
D_{2//} &= \lambda_{2//} + \lambda_{2//}\Pi_+ D_{1//} + \lambda_{2\perp}\Pi_+ D_{1\perp} + \lambda_{1//}\Pi_- D_{2//} + \lambda_{1\perp}\Pi_- D_{2\perp} \\
D_{2\perp} &= \lambda_{2\perp} + \lambda_{2//}\Pi_+ D_{1\perp} + \lambda_{2\perp}\Pi_+ D_{1//} + \lambda_{1//}\Pi_- D_{2\perp} + \lambda_{1\perp}\Pi_- D_{2//}
\end{aligned}
\qquad /4/
$$

where Π_+ and Π_- are the polarization bubbles for electrons on the right and left branches, for one spin orientation. These equations can be solved using the expressions

$$
\Pi_+(K,\omega) = \frac{K}{2\pi(\omega - v_F K)}, \quad \Pi_-(K,\omega) = -\frac{K}{2\pi(\omega + v_F K)} \qquad /5/
$$

giving e.g. for $D_{1//}$ which couples two electrons on the right branch

$$
D_{1//}(K,\omega) = (\omega - v_F K)\left[\frac{A+B}{\omega - u_\sigma K + i\delta \operatorname{sign} K} + \frac{A-B}{\omega + u_\sigma K - i\delta \operatorname{sign} K} + \right.
$$

$$
\left. + \frac{C+D}{\omega - u_s K + i\delta \operatorname{sign} K} + \frac{C-D}{\omega + u_s K - i\delta \operatorname{sign} K}\right] \qquad /6/
$$

where

$$u_\sigma^2 = [v_F + \tfrac{1}{2\pi}(\lambda_{4/} - \lambda_{4\perp})]^2 - [\tfrac{1}{2\pi}(\lambda_{2/} - \lambda_{2\perp})]^2$$

$$u_\zeta^2 = [v_F + \tfrac{1}{2\pi}(\lambda_{4/} + \lambda_{4\perp})]^2 - [\tfrac{1}{2\pi}(\lambda_{2/} + \lambda_{2\perp})]^2$$

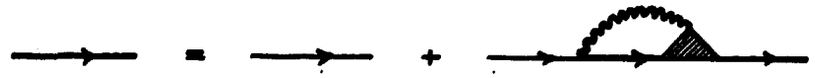
$$A = \tfrac{1}{4}(\lambda_{4/} - \lambda_{4\perp}),\ B = \tfrac{\pi}{2u_\sigma}\{u_\sigma^2 - v_F[v_F + \tfrac{1}{2\pi}(\lambda_{4/} - \lambda_{4\perp})]\} \quad /7/$$

$$C = \tfrac{1}{4}(\lambda_{4/} + \lambda_{4\perp}),\ D = \tfrac{\pi}{2u_\zeta}\{u_\zeta^2 - v_F[v_F + \tfrac{1}{2\pi}(\lambda_{4/} + \lambda_{4\perp})]\}$$

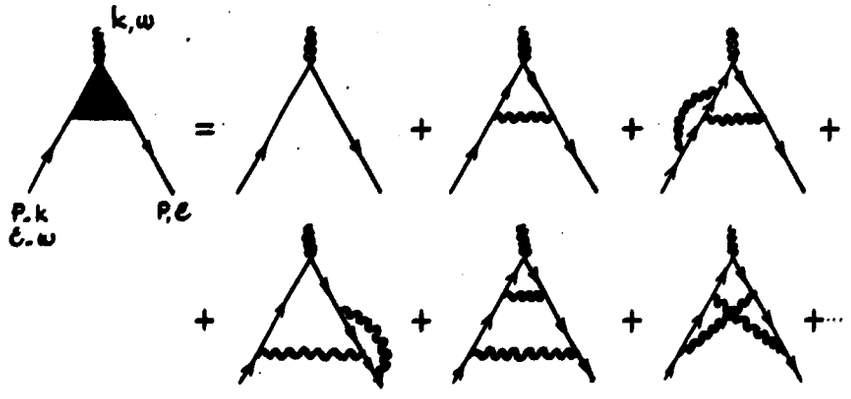
and for $D_{2/}$

$$D_{2/}(k,\omega) = \frac{1}{2}\frac{\omega^2 - v_F^2k^2}{\omega^2 - u_\sigma^2k^2 + i\delta}(\lambda_{2/} - \lambda_{2\perp}) + \frac{1}{2}\frac{\omega^2 - v_F^2k^2}{\omega^2 - u_\zeta^2k^2 + i\delta}(\lambda_{2/} + \lambda_{2\perp}) \quad /8/$$

Once these effective interactions are used, the Dyson equation can be written diagrammatically as

$G_+(p,\varepsilon)$ = 

in which the three-leg vertex has two solid line legs coupled by effective interactions in all possible ways.

$\Gamma_+(p,\varepsilon,k,\omega)$ = 

A further consequence of the particular features of the Tomonaga model is that, as shown by Dzyaloshinsky and Larkin |16|, a Ward identity can be found for general energy and momentum variables by use of which the vertex Γ_+ can be expressed in terms of the Green's function

$$\Gamma_+(p,\varepsilon,K,\omega) = \frac{1}{\omega - v_F K}\left[G_+^{-1}(p,\varepsilon) - G_+^{-1}(p-K,\varepsilon-\omega)\right] \quad /9/$$

and an analogous relation

$$\Gamma_-(p,\varepsilon,K,\omega) = \frac{1}{\omega + v_F K}\left[G_-^{-1}(p,\varepsilon) - G_-^{-1}(p-K,\varepsilon-\omega)\right] \quad /10/$$

holds for the vertex in which the electrons on the external legs belong to the left branch of the spectrum.

By making use of the Ward identity the Dyson equation for the Green's function can be written in a closed integral equation form

$$[\varepsilon - v_F(p-k_F)]G_+(p,\varepsilon) = 1 + \frac{i}{4\pi}\int dK\, d\omega\, \frac{D_{//}(K,\omega)}{\omega - v_F K}\, G_+(p-K,\varepsilon-\omega) \quad /11/$$

where the term leading to Fermi energy renormalization has been neglected. In real space and time representation we get

$$\left(\frac{\partial}{\partial t} + v_F\frac{\partial}{\partial x}\right) G_+(x,t) = K_{//}(x,t)\, G_+(x,t) \quad /12/$$

with

$$K_{1\parallel}(x,t) = \frac{1}{4\pi^2}\int dK\, d\omega\, \frac{D_{\parallel}(K,\omega)}{\omega - v_F K}\, e^{-i(\omega t - kx)} \qquad /13/$$

This equation can be solved in the form

$$G_+(x,t) = G_+^{(0)}(r)\exp\left\{\frac{1}{2v_F}\int_r^s K_{\parallel}(r,s')\,ds'\right\} f_+(r) \qquad /14/$$

where the new variables $r = x - v_F t$ and $s = x + v_F t$ were introduced.

$$G_+^{(0)}(x,t) = \frac{1}{2\pi}\,\frac{1}{x - v_F t + i\delta(t)} \qquad /15/$$

with $\delta(t) = \delta \operatorname{sign} t$ and $f_+(r)$ has to be chosen in a way to ensure the correct analytic properties. Performing the integrations in eqs. /13/ and /14/ with the cutoff factor $\exp(-|K|/\Lambda)$ for the momentum transfer, the final result is

$$G_+(x,t) = \frac{1}{2\pi}\,\frac{1}{x - v_F t + i\delta(t)}\,\frac{x - v_F t + i/\Lambda(t)}{[x - u_\sigma t + i/\Lambda(t)]^{1/2}[x - u_\varsigma t + i/\Lambda(t)]^{1/2}} \times$$

$$\times[\Lambda^2(x - u_\sigma t + i/\Lambda(t))(x + u_\sigma t - i/\Lambda(t))]^{-\alpha_\sigma} \qquad /16/$$

$$\times[\Lambda^2(x - u_\varsigma t + i/\Lambda(t))(x + u_\varsigma t - i/\Lambda(t))]^{-\alpha_\varsigma}$$

with

$$\alpha_\sigma = \frac{1}{4u_\sigma}\left[v_F + \frac{1}{2\pi}(\lambda_{1\parallel} - \lambda_{1\perp}) - u_\sigma\right]$$
$$\alpha_\rho = \frac{1}{4u_\rho}\left[v_F + \frac{1}{2\pi}(\lambda_{1\parallel} + \lambda_{1\perp}) - u_\rho\right] \qquad /17/$$

and $\Lambda(t) = \Lambda \operatorname{sign} t$. The velocities u_σ and u_ρ are given in eq. /7/.

The Green's function of the electrons on the left branch can be calculated similarly leading to the result:

$$G_-(x,t) = G_+(x,-t) \qquad /18/$$

3. Generalized Ward identities and response functions of the 1-d model

One of the best way to get information about the possibility of phase transition to an ordered state is to calculate the response of the system to various external perturbations. A singularity in the response function is an indication that a spontaneous ordering can take place. We will consider the density and pairing responses since they contain logarithmically singular terms in every order of perturbation theory and therefore there is a high chance for them to be singular when an exact summation of all contributions is performed.

These functions can be defined as

$$R(K,\omega) = -i\int dt\, e^{i\omega t} \langle T\{ O(K,t)\, O^{+}(K,0)\}\rangle \quad /19/$$

where

/i/ $$O(K,t) = \frac{1}{L^{1/2}} \sum b^{+}_{p\uparrow}(t)\, a_{p+K\uparrow}(t) \quad /20/$$

for the charge-density response function $N(K,\omega)$ with large momentum $(K \sim 2K_F)$,

/ii/ $$O(K,t) = \frac{1}{L^{1/2}} \sum_{p} b^{+}_{p\uparrow}(t)\, a_{p+K\uparrow}(t) \quad /21/$$

for the spin-density response function $\chi(K,\omega)$ with large momentum $(K \sim 2K_F)$,

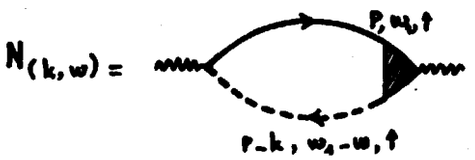
/iii/ $$O(K,t) = \frac{1}{L^{1/2}} \sum_{p} b_{p\downarrow}(t)\, a_{-p+K\uparrow}(t) \quad /22/$$

for the singlet-superconductor response $\Delta_s(K,\omega)$ with small momentum K , and Λ

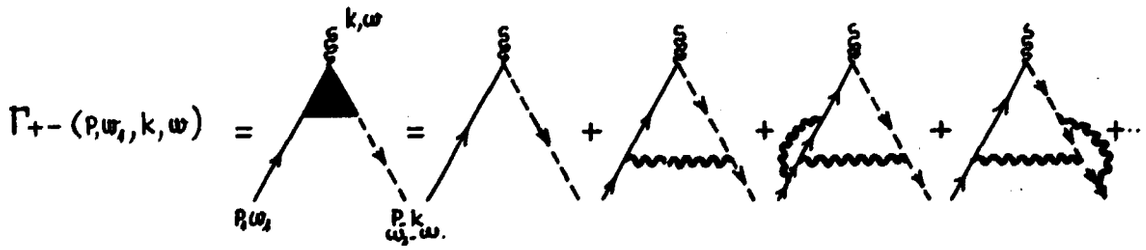
/iv/ $$O(K,t) = \frac{1}{L^{1/2}} \sum_{p} b_{p\downarrow}(t)\, a_{-p+K\uparrow}(t) \quad /23/$$

for the triplet-superconductor response $\Delta_t(K,\omega)$

The diagrams representing these response functions have one solid and one dashed lines running from one external vertex to the other one and these lines are dressed and connected by effective interactions in all possible ways. The charge-density response function can be represented as



where the two lines are renormalized lines and the vertex

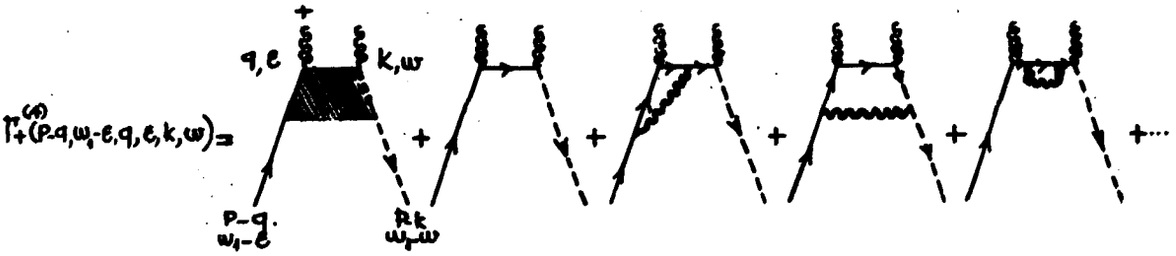


is analogous to the vertices Γ_+ and Γ_- , but have the two legs belong to different branches.

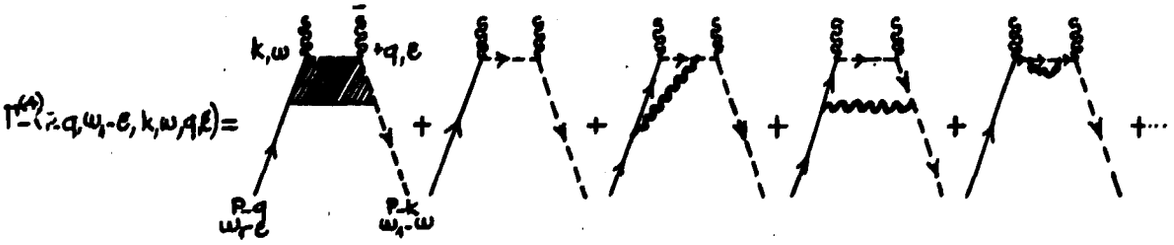
Unfortunately there is no Ward identity which could relate this vertex to the Green's functions. There is, however, an alternative way to write the response functions. All the diagrams can be classified into three classes. Either there is no interaction line coupling to the solid line, or if there is one, the first of them can couple back to the solid line in which case this is an effective $D_{///}$ interaction or it couples to the

dashed line in which case it is an effective $D_{2/\!/}$ interaction. Accordingly

where two new vertices have been introduced:



in which the small momentum transfer interaction couples to the solid line, and



in which the small momentum transfer interaction couples to the dashed line. The two legs and the intermediate line are connected by effective couplings in all possible ways and self energy

corrections on the intermediate line should also be considered. The diagrammatic representation of the spin-density response is similar, but instead of $D_{2\parallel}$ the effective interaction $D_{2\perp}$ should appear. In the pairing responses the vertices have two incoming lines. otherwise the representations are similar.

As was pointed out by Sólyom [17], the particular features of the Tomonaga model allow to derive generalized Ward identities relating the four-leg vertices $\Gamma_{+}^{(4)}$ and $\Gamma_{-}^{(4)}$ to three-leg vertices Γ_{+-} with large momentum transfer. These relations are the consequence of the conservation law for particles in each branch and for each spin orientation and are straightforward generalizations of the Ward identities given in eqs. /9/ and /10/. They can be written analytically as

$$\Gamma_{+}^{(4)}(p-q,\omega_1-\varepsilon,q,\varepsilon,k,\omega) = \frac{1}{\varepsilon-v_F q}\left[\Gamma_{+-}(p-q,\omega_1-\varepsilon,k-q,\omega-\varepsilon) - \right. \qquad /24/$$

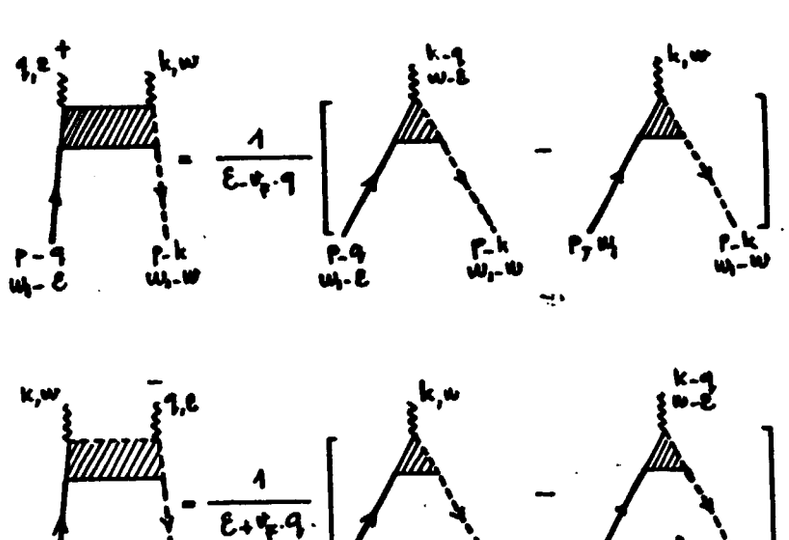
$$\left. - G_{+}^{-1}(p-q,\omega_1-\varepsilon)G_{+}(p,\omega_1)\,\Gamma_{+-}(p,\omega_1,k,\omega)\right]$$

and

$$\Gamma_{-}^{(4)}(p-q,\omega_1-\varepsilon,q,\varepsilon,k,\omega) = \frac{-1}{\varepsilon-v_F q}\left[\Gamma_{+-}(p-q,\omega_1-\varepsilon,k-q,\omega-\varepsilon) - \right. \qquad /25/$$

$$\left. - G_{-}^{-1}(p-k,\omega_1-\varepsilon)G_{-}(p-k-q,\omega_1-\varepsilon-k)\Gamma_{+-}(p-q,\omega_1-\varepsilon,k,q)\right]$$

or diagrammatically as



where the external renormalized Green's function legs should also be taken into account.

The analytic expressions for the charge-density response function $N(k,\omega)$, when the two representations are used, are

$$N(k,\omega) = -i\int\frac{dp}{2\pi}\frac{d\omega_1}{2\pi}\,G_+(p,\omega_1)\Gamma_{+-}(p,\omega_1,k,\omega)\,G_-(p-k,\omega_1-\omega), \qquad /26/$$

and

$$\begin{aligned} N(k,\omega) = {} & -i\int\frac{dp}{2\pi}\frac{d\omega_1}{2\pi}\,G_+^{(0)}(p,\omega_1)G_-(p-k,\omega_1-\omega) + \\ & + \int\frac{dp}{2\pi}\frac{d\omega_1}{2\pi}\,G_+^{(0)}(p,\omega_1)\int\frac{dq}{2\pi}\frac{d\varepsilon}{2\pi}\frac{D_{zz}(q,\varepsilon)}{\varepsilon - v_F q}\cdot \\ & \cdot G_+(p-q,\omega_1-\varepsilon)\Gamma_+^{(4)}(p-q,\omega_1-\varepsilon,q,\varepsilon,k,\omega)G_-(p-q,\omega_1-\omega) + \\ & + \int\frac{dp}{2\pi}\frac{d\omega_1}{2\pi}\,G_+^{(0)}(p,\omega_1)\int\frac{dq}{2\pi}\frac{d\varepsilon}{2\pi}\frac{D_{zz}(q,\varepsilon)}{\varepsilon - v_F q}\cdot \\ & \cdot G_+(p-q,\omega_1-\varepsilon)\Gamma_+^{(4)}(p-q,\omega_1-\varepsilon,k,\omega,q,\varepsilon)G_-(p-k,\omega_1-\omega) \end{aligned} \qquad /27/$$

Introducing the quantity

$$N(p,\omega_1,K,\omega) = G_+(p,\omega_1)\Gamma_{+-}(p,\omega_1,K,\omega)\,G_-(p-K,\omega_1-\omega) \qquad /28/$$

from which $N(K,\omega)$ is obtained by simple integration, making use of the generalized Ward identities is eq. /27/ and comparing eqs. /26/ and /27/ gives

$$N(p,\omega_1,K,\omega)\,G_+^{(0)-1}(p,\omega_1) = G_-(p-K,\omega_1-\omega) +$$
$$+ i\int\frac{dq}{2\pi}\frac{d\varepsilon}{2\pi}\frac{D_{1/\!/}(q,\varepsilon)}{\varepsilon - v_F q}\left[N(p-q,\omega_1-\varepsilon,K-q,\omega-\varepsilon) - N(p,\omega_1,K,\omega)\right] + /29/$$
$$+ i\int\frac{dq}{2\pi}\frac{d\varepsilon}{2\pi}\frac{D_{2/\!/}(q,\varepsilon)}{\varepsilon - v_F q}\left[N(p-q,\omega_1-\varepsilon,K,\omega) - N(p-q,\omega_1-\varepsilon,K-q,\omega-\varepsilon)\right]$$

This equation can be solved by writing it in Fourier transformed form with

$$N(p,\omega_1,K,\omega) = \int dx_1\,dt_1\,dx\,dt\,N(x_1,t_1,x,t)\,e^{i(\omega_1 t_1 - px)}e^{i(\omega t - Kx)} \qquad /30/$$

since then

$$i\left(\frac{\partial}{\partial t_1} + v_F\frac{\partial}{\partial x_1}\right)N(x_1,t_1,x,t) = \delta(x+x_1)\,\delta(t+t_1)\,G_-(-x_1,-t) +$$
$$+ i\,K_{1/\!/}(x+x_1,t+t_1)\,N(x_1,t_1,x,t) - \qquad /31/$$
$$- i\left[K_{2/\!/}(x+x_1,t+t_1) - K_{2/\!/}(x_1,t_1)\right]N(x_1,t_1,x,t)$$

where the term leading to a Fermi energy renormalization has again been neglected and

$$K_{2\parallel}(x,t) = \int \frac{dq}{2\pi}\frac{d\varepsilon}{2\pi} \frac{D_{2\parallel}(q,\varepsilon)}{\varepsilon - v_F q} e^{-i(\omega t - \kappa x)} \qquad /32/$$

The solution can be looked for in the form

$$N(x_1,t_1,x,t) = N_1(x+x_1,t+t_1)\,N_2(x_1,t_1)\,N_3(x,t) \qquad /33/$$

Eq. /31/ can be separated into two equations for N_1 and N_2, and furthermore $N_3(x,t) = G_-(-x,-t)\,N_2^{-1}(-x,-t)$. These equations are solved in the same way as eq. /12/. Putting everything together, taking into account that $N(\kappa,\omega)$ is obtained from $N(p,\omega_1,\kappa,\omega)$ by a simple integration, which means that the Fourier transform $N(\kappa,t)$ of $N(\kappa,\omega)$ is obtained by putting $x_1 = t = 0$ in $N(x_1,t_1,x,t)$, we finally get

$$N(x,t) = -i\,G_+(x,t)\,G_-(-x,-t)\,[\Lambda^2(x-u_\sigma t+i/\Lambda(t))(x+u_\sigma t-i/\Lambda(t))]^{\beta_\sigma} \qquad /34/$$
$$[\Lambda^2(x-u_\rho t+i/\Lambda(t))(x+u_\rho t-i/\Lambda(t))]^{\beta_\rho}$$

with

$$\beta_\sigma = \frac{1}{4\pi u_\sigma}(\lambda_{2\parallel} - \lambda_{2\perp})$$
$$\beta_\rho = \frac{1}{4\pi u_\rho}(\lambda_{2\parallel} + \lambda_{2\perp}) \qquad /35/$$

Since the spin-density response function contains $D_{2\perp}$ instead of $D_{2\parallel}$, a similar expression is obtained for $\chi(x,t)$ with $-\beta_\sigma$ instead of β_σ in the exponent.

The calculation of the pairing responses can be done in a similar way. The generalized Ward identities are also similar, with one overall sign difference in the case when the small momentum transfer interaction leg couples to the dashed line due to the opposite orientation of this line. The result for the singlet-superconductor response is

$$\Delta_s(x,t) = i\,G_+(x,t)\,G_-(x,t)\,[\Lambda^2(x-u_\sigma t+i/\Lambda(t))(x+u_\sigma t-i/\Lambda(t))]^{\beta_\sigma} \cdot [\Lambda^2(x-u_\rho t+i/\Lambda(t))(x+u_\rho t-i/\Lambda(t))]^{\bar{\beta}_\rho} \qquad /36/$$

The triplet-superconductor response has $-\beta_\sigma$ in the exponent instead of β_σ.

Writing in the expressions for the Green's functions from eqs. /16/ and /18/ we finally get

$$R(x,t) = \mp i\,\frac{x-v_F t+i/\Lambda(t)}{x-v_F t+i\delta(t)}\cdot\frac{x+v_F t-i/\Lambda(t)}{x+v_F t-i\delta(t)}\Lambda^{\nu} \cdot [(x-u_\sigma t+i/\Lambda(t))(x+u_\rho t-i/\Lambda(t))]^{\mu_\sigma} \qquad /37/$$
$$\cdot [(x-u_\rho t+i/\Lambda(t))(x+u_\rho t-i/\Lambda(t))]^{\mu_\rho}$$

where

/i/ $$\mu_\sigma = \frac{1}{2}\gamma_\sigma \,, \qquad \mu_\rho = \frac{1}{2}\gamma_\rho \qquad /38/$$

with

$$\gamma_\sigma = \left[\frac{v_F + \frac{1}{2\pi}(\lambda_{11} - \lambda_{11}) - \frac{1}{2\pi}(\lambda_{2\parallel} - \lambda_{21})}{v_F + \frac{1}{2\pi}(\lambda_{11} - \lambda_{11}) + \frac{1}{2\pi}(\lambda_{2\parallel} - \lambda_{21})}\right]^{1/2}$$

$$\gamma_\rho = \left[\frac{v_F + \frac{1}{2\pi}(\lambda_{1\parallel} + \lambda_{11}) - \frac{1}{2\pi}(\lambda_{2\parallel} + \lambda_{21})}{v_F + \frac{1}{2\pi}(\lambda_{1\parallel} + \lambda_{11}) + \frac{1}{2\pi}(\lambda_{2\parallel} + \lambda_{21})}\right]^{1/2} \qquad /39/$$

for the charge-density response function,

/ii/ $$\mu_\sigma = \frac{1}{2\gamma_\sigma} \,, \qquad \mu_\rho = \frac{1}{2}\gamma_\rho \qquad /40/$$

for the spin-density response function,

/iii/ $$\mu_\sigma = \frac{1}{2}\gamma_\sigma \,, \qquad \mu_\rho = \frac{1}{2\gamma_\rho} \qquad /41/$$

for the singlet-superconductor response, and

/iv/ $$\mu_\sigma = \frac{1}{2\gamma_\sigma} \,, \qquad \mu_\rho = \frac{1}{2\gamma_\rho} \qquad /42/$$

for the triplet-superconductor response.

In the case of spin-independent interaction these results agree will those obtained by Fogedby |15|.

4. System of coupled Tomonaga chains

It was shown in the previous sections how the strictly 1-d Tomonaga model can be solved exactly by summing all diagrams by the use of Ward identities. Now this method will be extended to study the properties of a quasi-1-d system, in which 3-d couplings are taken into account.

The model is defined as follows: a set of 1-d Tomonaga chains with linear dispersion relation and intrachain forward scattering is coupled together by interchain coupling which has forward scattering components only. Assigning a chain index i to the electrons on the chain at position R_i, the Hamiltonian of the system can be written as

$$H = H_o + H_{int} \qquad /43/$$

with

$$H_o = \sum_{i,K,\alpha} v_F(K-K_F)\, a^+_{iK\alpha}\, a_{iK\alpha} + \sum_{i,K,\alpha} v_F(-K-K_F)\, b^+_{iK\alpha}\, b_{iK\alpha} \qquad /44/$$

and

$$H_{int} = \frac{1}{2L}\sum_{\substack{i,j\\K,K',p\\\alpha,\beta}} (\lambda_{1\parallel ij}\delta_{\alpha\beta} + \lambda_{1\perp ij}\delta_{\alpha,-\beta})(a^+_{iK+p\alpha}\, a^+_{jK'-p\beta}\, a_{jK'\beta}\, a_{iK\alpha} + b^+_{iK+p\alpha}\, b^+_{jK'-p\beta}\, b_{jK'\beta}\, b_{iK\alpha}) + \qquad /45/$$

$$+ \frac{1}{L}\sum_{\substack{i,j\\K,K',p\\\alpha,\beta}} (\lambda_{2\parallel ij}\delta_{\alpha\beta} + \lambda_{2\perp ij}\delta_{\alpha,-\beta})\, a^+_{iK+p\alpha}\, b^+_{jK'-p\beta}\, b_{jK'\beta}\, a_{iK\alpha}.$$

where λ_{1ij} and λ_{2ij} are the coupling constants between electrons on i^{th} and j^{th} chains. For simplicity equivalent chains are considered, i.e. v_F and k_F are taken to be the same for all chains.

The method of solution is a straightforward generalization of the method used in the strictly 1-d case. Since the quasi-1-d model has the two main characteristic features of the 1-d Tomonaga model, namely the linearity of the dispersion relation and the absence of large momentum transfer /backward scattering/ terms, diagrams with simple bubbles and series of bubbles with only contribute, the effect of more complicated diagrams being cancelled by each other. As a consequence, effective interactions can again be introduced, satisfying the same digrammatic equations as in the 1-d case, but the electrons-hole pair in the intermediate state can be on any chain. The analytic equations for the effective interactions are

$$D_{1\nu ij} = \lambda_{1\nu ij} + \sum_{\ell}[\lambda_{1\nu i\ell}\Pi_{+}D_{1\nu\ell j} + \lambda_{11i\ell}\Pi_{+}D_{11\ell j} + \lambda_{2\nu i\ell}\Pi_{-}D_{2\nu\ell j} + \lambda_{21i\ell}\Pi_{-}D_{21\ell j}]$$

$$D_{11ij} = \lambda_{11ij} + \sum_{\ell}[\lambda_{1\nu i\ell}\Pi_{+}D_{11\ell j} + \lambda_{11i\ell}\Pi_{+}D_{1\nu\ell j} + \lambda_{2\nu i\ell}\Pi_{-}D_{21\ell j} + \lambda_{21i\ell}\Pi_{-}D_{2\nu\ell j}] \qquad /46/$$

$$D_{2\nu ij} = \lambda_{2\nu ij} + \sum_{\ell}[\lambda_{2\nu i\ell}\Pi_{+}D_{1\nu\ell j} + \lambda_{21i\ell}\Pi_{+}D_{11\ell j} + \lambda_{1\nu i\ell}\Pi_{-}D_{2\nu\ell j} + \lambda_{11i\ell}\Pi_{-}D_{21\ell j}]$$

$$D_{21ij} = \lambda_{21ij} + \sum_{\ell}[\lambda_{2\nu i\ell}\Pi_{+}D_{11\ell j} + \lambda_{21i\ell}\Pi_{+}D_{1\nu\ell j} + \lambda_{1\nu i\ell}\Pi_{-}D_{21\ell j} + \lambda_{11i\ell}\Pi_{-}D_{2\nu\ell j}]$$

These effective interactions are of course functions of the momentum component $k_{\parallel}$ which is parallel to the chain direction, and the energy variable ω through the dependence of Π on these variables. Performing a Fourier transformation in the perpendicular direction with

$$D_{\mu ij}(k_{\parallel},\omega) = \frac{1}{N}\sum_{k_\perp} D_{\mu}(k_{\parallel},k_\perp,\omega)\, e^{ik_\perp(R_i - R_j)}, \quad \mu = 1\parallel, 1\perp, 2\parallel, 2\perp \qquad /47/$$

we have

$$D_{1\parallel}(k_\perp) = \lambda_{1\parallel}(k_\perp) + \lambda_{1\parallel}(k_\perp)\Pi_+ D_{1\parallel}(k_\perp) + \lambda_{1\perp}(k_\perp)\Pi_+ D_{1\perp}(k_\perp) + \lambda_{2\parallel}(k_\perp)\Pi_- D_{2\parallel}(k_\perp) + \lambda_{2\perp}(k_\perp)\Pi_- D_{2\perp}(k_\perp) \qquad /48/$$

and three similar equations with

$$\lambda_{\mu}(k_\perp) = \sum_i \lambda_{\mu ij}\, e^{-ik_\perp(R_i - R_j)}, \qquad \mu = 1\parallel, 1\perp, 2\parallel, 2\perp \qquad /49/$$

The $k_\perp$-dependent effective interactions obey exactly the same equations as the effective interactions in the 1-d case, except that everywhere λ should be replaced by $\lambda(k_\perp)$. Keeping this in mind, the solution of these equations is obtained from eqs. /6/ - /8/ if the dependence on $k_\perp$ is taken into account.

The velocities u_σ and u_s will also depend on $k_\perp$, e.g.

$$u_\sigma(k_\perp) = \{[v_F + \frac{1}{2\pi}(\lambda_{1/\!/}(k_\perp) - \lambda_{1\perp}(k_\perp))]^2 - [\frac{1}{2\pi}(\lambda_{2/\!/}(k_\perp) - \lambda_{2\perp}(k_\perp))]^2\}^{1/2} \quad /50/$$

A very important consequence of the neglect of hopping and backward scattering is that perturbations created on any chain can propagate along that chain only. The neighbouring chains can influence this propagation, but there is no response on other chains. The Green's function

$$G_{+ij}(p,\varepsilon) = -i\int dt\, e^{i\varepsilon t} \langle T\{a_{ip}(t)\, a^+_{jp}(0)\}\rangle \quad /51/$$

will be diagonal in the chain index, $G_{+ij} = G_{+ii} \cdot \delta_{ij}$, and similarly all the response functions

$$R_{ij}(k,\omega) = -i\int dt\, e^{i\omega t} \langle T\{\mathcal{O}_i(k,t)\, \mathcal{O}^+_j(k,0)\}\rangle \quad /52/$$

with e.g.

$$\mathcal{O}_i(k,t) = \frac{1}{L^{1/2}} \sum_p b^+_{ip\uparrow}(t)\, a_{ip+k\downarrow}(t) \quad /53/$$

for the charge-density response function, will be diagonal in the chain index.

This is easily seen since in the density responses a large $/\sim 2\ k_F/$ momentum is given to the electrons on one chain and this momentum cannot be transferred to other chains if only small momentum transfer processes are allowed. In the superconducting responses the Cooper pair added to a chain cannot propagate to other chains if hopping is not allowed.

Due to this property, the vertices appearing in the Dyson equation for the Green's function and in the two alternative representations of the response functions have identical chain index on the two electron legs. The Ward identities used in the 1-d case will be valid here as well, because the problem is reduced to a one chain problem with effective interactions

$$D_{\mu ii}(k_{//}, \omega) = \frac{1}{N} \sum_{k_\perp} D_\mu (k_{//}, k_\perp, \omega) , \qquad \mu = W,1\perp, 2W, 2\perp \qquad /54/$$

Using these expressions for the effective intrachain interactions, the Dyson equation /11/ eq. /29/ for the charge-density response and the analogous equations for the other response functions can be solved by taking over the results of the 1-d case to get

$$G_{+ij}(x, t) = \delta_{ij} \prod_{k_\perp} [G_+(x, t, k_\perp)]^{\frac{1}{N}} \qquad /55/$$

and

$$R_{ij}(x, t) = \delta_{ij} \prod_{k_\perp} [R(x, t, k_\perp)]^{\frac{1}{N}} \qquad /56/$$

where N is the number of chains, $G_+(x,t,k_\perp)$ and $R(x,t,k_\perp)$ have the same form as the corresponding expressions for a single chain with $\lambda_\mu(k_\perp)$ instead of λ_μ . E.g.

$$R(x,t,k_\perp) = \mp i \frac{x - v_F t + i/\Lambda(t)}{x - v_F t + i\delta(t)} \frac{x + v_F t - i/\Lambda(t)}{x + v_F t - i\delta(t)} \Lambda^\nu \cdot$$
$$\cdot [(x - u_\sigma(k_\perp)t + i/\Lambda(t))(x + u_\sigma(k_\perp)t - i/\Lambda(t))]^{\mu_\sigma(k_\perp)} \qquad /57/$$
$$\cdot [(x - u_\varrho(k_\perp)t + i/\Lambda(t))(x + u_\varrho(k_\perp)t - i/\Lambda(t))]^{\mu_\varrho(k_\perp)}$$

where for the charge-density response $\mu_\sigma(k_\perp) = \frac{1}{2}\gamma_\sigma(k_\perp)$ and $\mu_\varrho(k_\perp) = \frac{1}{2}\gamma_\varrho(k_\perp)$ with

$$\gamma_\sigma(k_\perp) = \left\{ \frac{v_F + \frac{1}{2\pi}[\lambda_{1\parallel}(k_\perp) - \lambda_{1\perp}(k_\perp)] - \frac{1}{2\pi}[\lambda_{2\parallel}(k_\perp) - \lambda_{2\perp}(k_\perp)]}{v_F + \frac{1}{2\pi}[\lambda_{1\parallel}(k_\perp) - \lambda_{1\perp}(k_\perp)] + \frac{1}{2\pi}[\lambda_{2\parallel}(k_\perp) - \lambda_{2\perp}(k_\perp)]} \right\}^{1/2}$$

/58/

$$\gamma_\varrho(k_\perp) = \left\{ \frac{v_F + \frac{1}{2\pi}[\lambda_{1\parallel}(k_\perp) + \lambda_{1\perp}(k_\perp)] - \frac{1}{2\pi}[\lambda_{2\parallel}(k_\perp) + \lambda_{2\perp}(k_\perp)]}{v_F + \frac{1}{2\pi}[\lambda_{1\parallel}(k_\perp) + \lambda_{1\perp}(k_\perp)] + \frac{1}{2\pi}[\lambda_{2\parallel}(k_\perp) + \lambda_{2\perp}(k_\perp)]} \right\}^{1/2}$$

and the exponents of the other response functions are obtained by the rules given in eqs. /40/ - /42/.

In the limit of weak nearest neighbour interchain interaction this result agrees with that obtained by Klemm and Gutfreund |7|.

5. Discussion

We have given an exact solution of a quasi-1-d model of interacting electrons. Although the model contains explicitely a 3-d interchain coupling, due to the particular choice of this coupling /only forward scattering terms were considered/, this model turns out to be equivalent to a set of effectively decoupled chains. The perturbations can propagate along the chains only, no response is obtained on other chains. Earlier approximate treatments of the backward scattering model |6| - |8| have indicated already that backward scattering terms or interchain hopping are important to have a phase transition in quasi-1-d system. Our exact calculation proves that without these terms no ordering is possible at any finite temperature, since the ordered phase would have a non-decaying response function.

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