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STRUCTURE FUNCTIONS EXTRACTED FROM MUON PAIR PRODUCTION AT THE SPS.

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The data reported have been obtained using the NA3-LEZARD spectrometer at the CERN SPS. The production of massive muon pairs by pions at 200 and 280 GeV/c are measured.

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ABSTRACT

Dimuon data provided by πN interactions were analysed in the framework of the Drell-Yan quark fusion model in order to extract the pion and nucleon structure functions. Our results are compared to the structure functions obtained in other experiments.

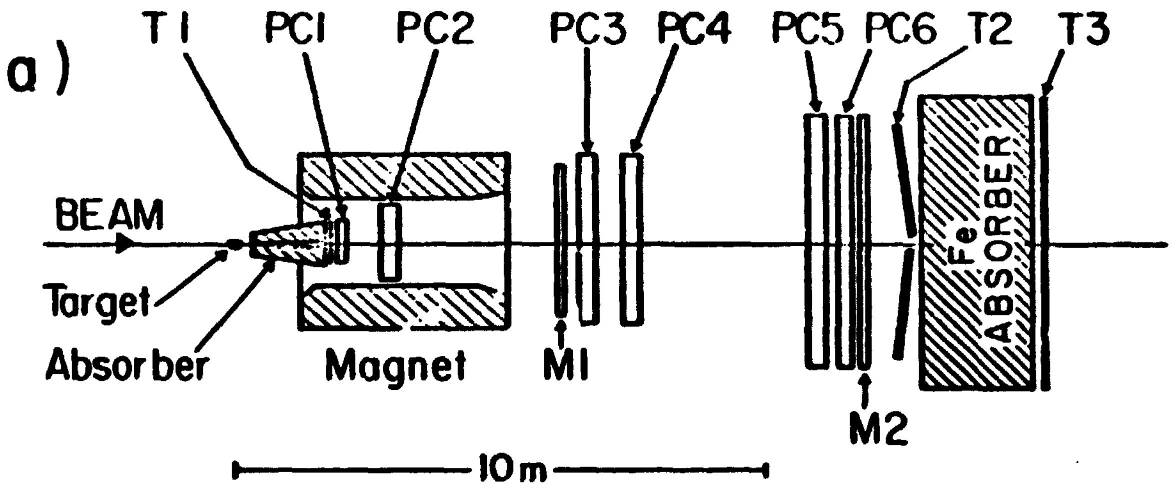
I - INTRODUCTION

Systematic work on the determination of the nucleon structure functions has been done by 3 types of experiments : eN, μN, νN. In these cases, one uses a fundamental probe like e, μ, ν to investigate the structure of the nucleon. The study of the reactions : $hN \rightarrow \mu^+\mu^-X$ (h is an incident hadron) allows to determine in principle the structure functions of the nucleon (target) and of the incident projectile ($\pi^\pm$, $K^\pm$, p, $\bar{p}$). In order to reach this goal, we have performed a series of experiments to measure the production of massive muon pairs in hadron-hadron collisions at CERN SPS. From these data, one of the aims of our analysis was to obtain the structure functions of instable hadrons like π or K, difficult to probe by lepton scattering. The way in which this is possible should be the use of the Drell-Yan mechanism of quark annihilation. In this paper, detailed results on the pion and the nucleon structure functions are presented.

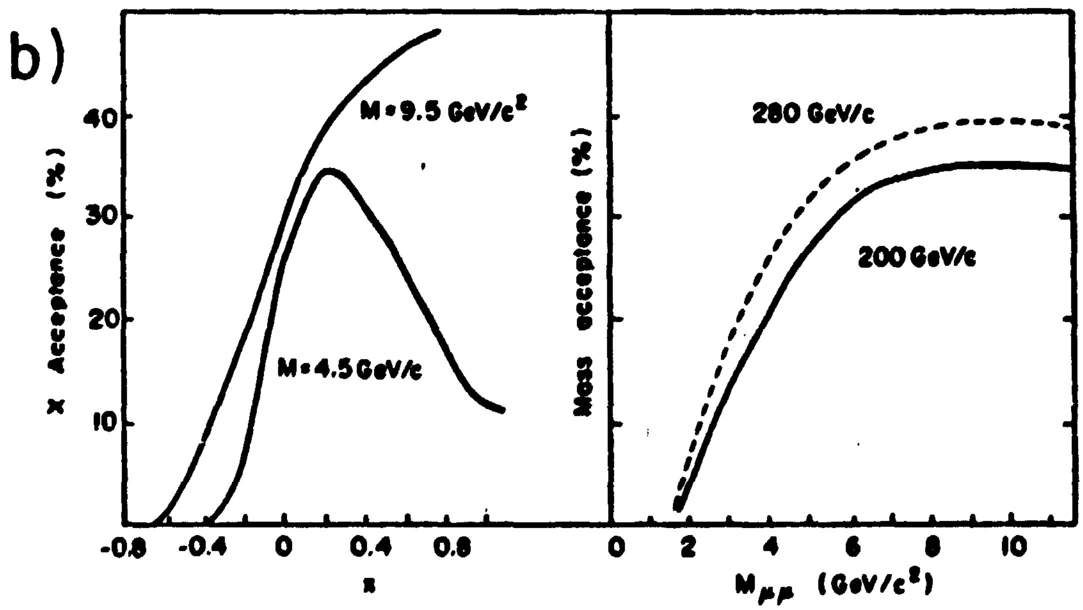
II - BRIEF EXPERIMENTAL DESCRIPTION

The general layout of the experiment is shown in fig. 1a). More details on the apparatus and trigger system, together with others results of the dimuon production by

NA 3 SPECTROMETER



General layout of the NA3 spectrometer for the study of dimuon production in hadronic collision. (Side view).



The acceptance of the apparatus as a function of x and M as calculated by a Monte Carlo method.

FIG. 1

kaons, protons and antiprotons are given elsewhere [1]. Nevertheless I have summarized the main features of the apparatus :

. The "LEZARD" is a large acceptance spectrometer especially designed for the study of particles produced at high transverse momentum and used in a beam dump configuration.

. Two targets (one of platinum, the other of liquid hydrogen) have been used.

. The beam dump and secondary hadron absorber is a 1.50 m long block of stainless steel situated 40 cm downstream of the platinum target.

. The large acceptance superconducting dipole magnet has a vertical field in a cylindrical airgap of horizontal axis of 1.6 m diameter ($\int Bdl = 4.0$ Tm).

: An additional muon filtering is provided by an 1.8 m iron wall placed at the end of the apparatus.

. 31 planes of multiwire proportional chambers are used to reconstruct the trajectory and therefore to get the momentum of the charge particles.

. The identification of the incident particle is made by a set of 4 Cerenkov counters.

. The fast trigger is based on a coincidence of three (T1, T2, T3) hodoscopes divided for two of them (T2, T3) into horizontal strips. The second stage of the trigger is provided by a coincidence between two special checker-board proportional chambers M1 and M2 which allows to select on the transverse momentum of the particles.

The over-all acceptance of the apparatus is shown in fig. 1b) ; for the mass resolution see reference [1].

III - EXPERIMENTAL CONDITIONS

From September 1978 to April 1979, we ran with three different hadrons beams :

i) a negative pion beam of 280 GeV/c without any identification (in fact, only a few percent of incident particles are not π^-). During this run, we had only an 11.1 cm long platinum target.

ii) a negative beam of 200 GeV/c with the following composition : 96.3 % π^-, 3.1 % K^-, 0.62 % $\bar{p}$.

iii) a positive beam of 200 GeV/c with 36 % π^+, 4.6 % K^+, 59.4 % p.

The particle fluxes were in the range $(1-3)10^7$ particle/pulse.

In the 200 GeV/c runs (positive and negative), we have used a 6 cm long platinum target simultaneously with a 30 cm long liquid hydrogen target placed 40 cm upstream of the platinum target. For dimuon masses above 4 GeV/c^2, the distance between the two targets and between the platinum target and the beam dump were large enough to allow an unambiguous determination of the origin of the event [1].

The number of recorded events with a dimuon mass greater than 4 GeV/c^2 is reported in the following table I :

Table I

Recorded events with $M_{\mu^+\mu^-} > 4$ GeV/c^2

P_{inc}		π^-	K^-	$\bar{p}$	π^+	K^+	p
200 GeV/c	Pt	5916	119	54	2195	215	1304
	H_2	138			47		24
280 GeV/c only	Pt	5700	-	-	-	-	-

Two remarks can be made :

i) The large statistic of dimuons induced by π^+ and π^- allows us to measure the structure function of the pion.

ii) Until now the number of dimuons produced by kaons and $\bar{p}$ are not sufficient to obtain the structure functions of these particles. Therefore, we are now taking data in order to perform such an analysis in the near future.

In order to analyse the data in the framework of the Drell-Yan mechanism, we have excluded the resonances mass regions (ψ, ψ', Υ family) to get a clean sample of dimuons events. We then are reduced to the following number of events in the table II :

Table II

Number of events used to measure the structure functions

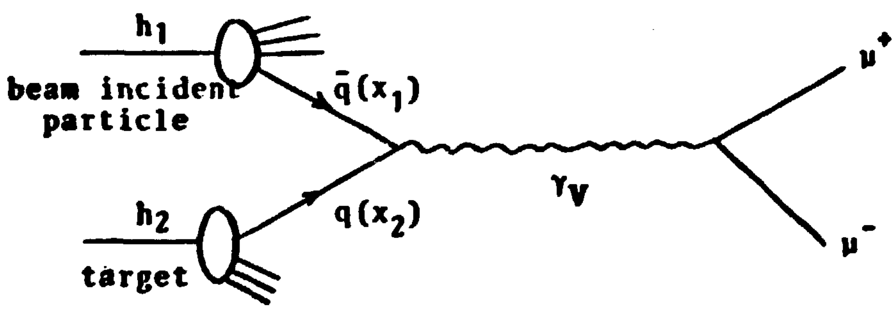
		Number of events	Range in mass cut
200 GeV/c	π^+	2073	$4 < M_{\mu^+\mu^-} < 8.5$ GeV/c^2
	π^-	5607	
280 GeV/c	π^-	3441	$4.5 < M_{\mu^+\mu^-} < 8.5$ GeV/c^2

Because of our worse resolution in mass at 280 GeV/c, due to the fact that we had a platinum target of 11.1 cm instead of 6 cm at 200 GeV/c, we have made a different cut in mass at low masses.

IV - DATA ANALYSIS

A. Drell-Yan formalism

The parton-antiparton annihilation model proposed by Drell and Yan is described by the now well-known following diagram :



In this graph, one assumes that an antiquark $\bar{q}$ (or quark q) of the hadron h_1 (incident particle) annihilates with a quark q (or antiquark $\bar{q}$) of the target h_2 which has the same flavour, into a virtual photon which finally decays into a muon pair.

If transverse momentum of the dimuon is neglected, the kinematical variables of the colliding $\bar{q}q$ pair are defined by the muon pair momentum and the invariant mass M as :

$$\begin{matrix} M^2 = x_1 \, x_2 \, s \\ x = x_1 - x_2 \end{matrix} \longrightarrow x_{\frac{1}{2}} = \frac{1}{2}\left(\pm x + \sqrt{x^2 + 4\frac{M^2}{s}}\right)$$

where x_1 and x_2 are the fractional momenta of the quarks in the beam and target particle respectively ; $x = 2\, p_L^* / \sqrt{s}$, where p_L^* is the longitudinal momentum of the dimuon in the overall c.m. system and $\sqrt{s}$ the total energy.

The Drell-Yan model enables one to express the double differential cross section of the produced dimuons as a factored product of structure functions refering to the beam and the target hadrons in the following way :

$$\frac{d^2\sigma}{dx_1\, dx_2} = \frac{\sigma_0}{3} \sum_i \frac{Q_i^2}{x_1^2\, x_2^2} \left[f_i(x_1)\, f_{\bar{i}}(x_2) + f_{\bar{i}}(x_1)\, f_i(x_2) \right] \quad (1)$$

where $\sigma_0 = 4\pi\alpha^2 (\hbar c)^2 / 3s$. ($\alpha = 1/137$).
and the structure functions f_i for quarks of flavour i and charge Q_i have a valence and a sea contribution
$f_i = f_{iv} + f_{i\,sea}$.

Note that the factor of 3 is due to the color hypothesis.

B. Application to pion-nucleon interaction

In our case, where h_1 is a pion and h_2 is a nucleon (proton or neutron) the definitions and hypothesis for the valence and sea functions are the following :

i) Valence functions

. For the pion, due to isospin invariance and charge conjugaison conservation, we have only one single valence function $V(x_1)$ defined by :

$$V(x_1) = \bar{u}_V^{\pi^-}(x_1) = d_V^{\pi^-}(x_1) = u_V^{\pi^+}(x_1) = \bar{d}_V^{\pi^+}(x_1)$$

. For the <u>nucleon</u>, due to isospin invariance there are only two independent valence functions, that we define for the proton as $u(x_2)$ and $d(x_2)$ for the up and down quarks respectively :

$$u(x_2) = u^p(x_2) = d^n(x_2) \qquad d(x_2) = d^p(x_2) = u^n(x_2)$$

In addition, we have <u>normalized</u> all the valence distribution functions to the corresponding number of valence quarks i.e.

$$\int_0^1 \frac{V(x_1)}{x_1} dx_1 = 1 \qquad \int_0^1 \frac{u(x_2)}{x_2} dx_2 = 2 \qquad \int_0^1 \frac{d(x_2)}{x_2} dx_2 = 1$$

ii) <u>Sea functions</u>

The sea distributions are taken to be SU3 symmetric. For each flavour, we define $S_N(x_2)$ for the nucleon and $S_\pi(x_1)$ for the pion.

The basic idea of the analysis of our data is to compare the experimentally determined cross section to the one calculated by the Drell-Yan formula using :

$$\left(\frac{d^2\sigma}{dx_1\, dx_2}\right)_{exp} = K \left(\frac{d^2\sigma}{dx_1\, dx_2}\right)_{D.Y.} \qquad (2)$$

where K is a scale factor, related either to our experimental normalization error or (and) to a multiplicative correction factor due to QCD effects. [2, 3].

We can now give the general form of the cross section for a pion-nucleon interaction :

$$\frac{d^2\sigma}{dx_1\, dx_2} = K \frac{\sigma_0}{3x_1^2x_2^2} \Big(V(x_1)\, G(x_2) + S_\pi(x_1)\, H(x_2)\Big) \qquad (3)$$

where $G(x_2)$ and $H(x_2)$ are nucleon functions which can be developped into terms of the structure functions $u(x_2)$ $d(x_2)$ and $S_N(x_2)$ of the nucleon as shown in the appendix A.

For a platinum target (where $Z/A = 0.40$) these functions can be written in the following way :

$$G(x_2) = \frac{1}{9}\,(1.6\ u(x_2) + 2.4\ d(x_2) + 5\ S_N(x_2)) \quad \text{for } \pi^-$$

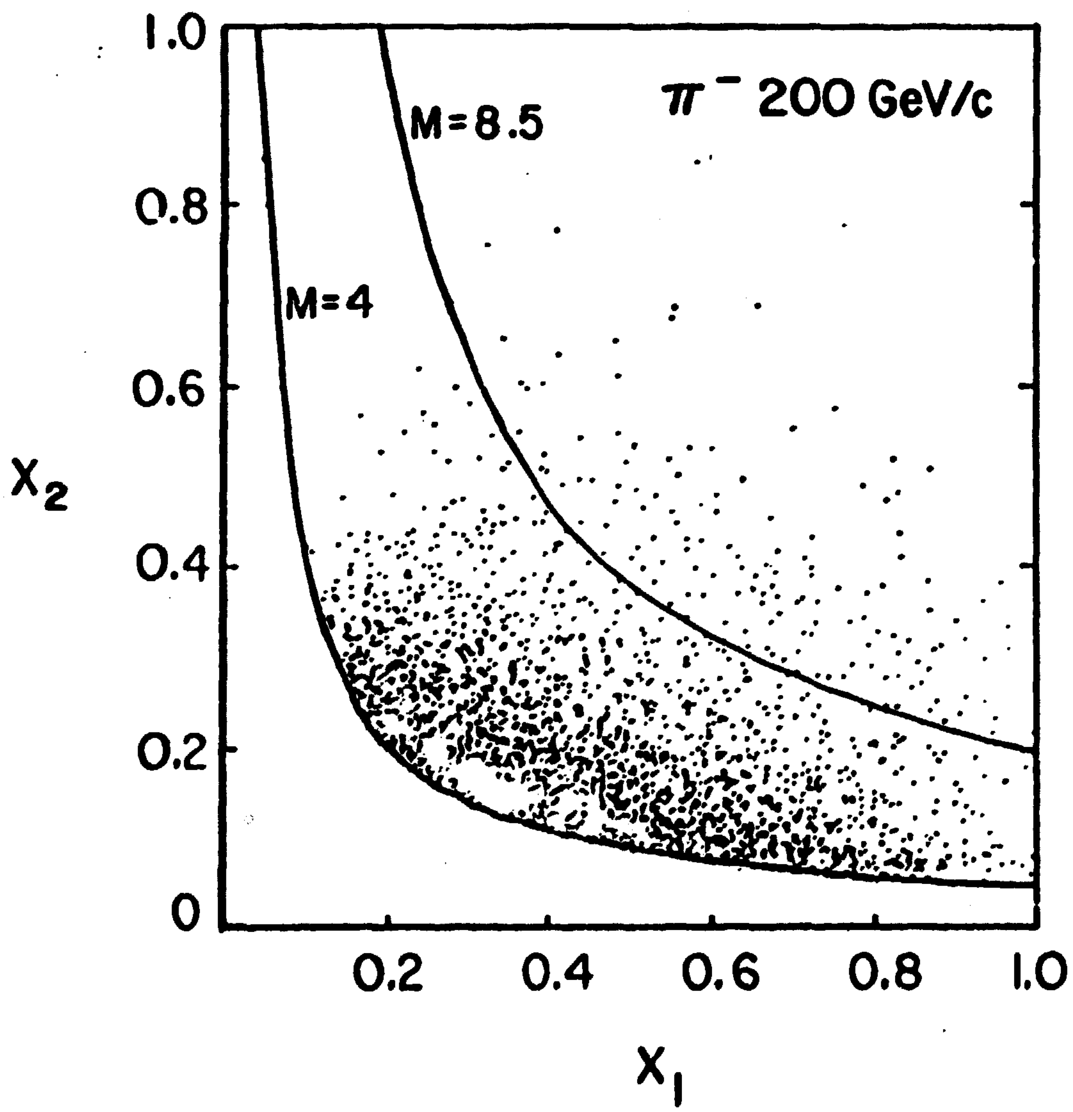
$$G(x_2) = \frac{1}{9}\,(0.6\ u(x_2) + 0.4\ d(x_2) + 5\ S_N(x_2)) \quad \text{for } \pi^+ \qquad (4)$$

$$H(x_2) = \frac{1}{9}\,(2.2\ u(x_2) + 2.8\ d(x_2) + 12\ S_N(x_2)) \quad \text{for } \pi^\pm$$

The raw data of the experiment come in the form of $\mu^+\mu^-$ events for which we know the mass and the longitudinal momentum. From these we extract values of x_1 and x_2 for each event and we obtain a two-dimensional plot (fig. 2).

We have calculated by Monte Carlo method the acceptance at each value of x_1 and x_2, by integrating on the observed P_t distribution and on the $\cos\theta$ and ϕ distribution [1] which was taken to be $P(\theta, \phi) = (1 + \cos^2\theta)$. The experimental errors ($\Delta p/p$, multiple scattering) and the Fermi motion of the nuclear target distort slightly the distribution of events in the $[x_1, x_2]$ array. The main effects are that the $\Delta p/p$ error produces a $\Delta x_1/x_1$ of about 3% at high x_1, while Fermi motion and multiple scattering induces $\Delta x_2/x_2$ of about 10% and 6% respectively. The resulting effects on dN/dx_1 are sizeable only at high x_1 or x_2 and in any case do not exceed 10%.

From the $[x_1, x_2]$ array, corrected for acceptance, we extract the pion and nucleon structure functions by three different methods which are discussed in details in the



Two-dimensional plot in the x_1, x_2 plane of the 200 GeV/c π^- Pt dimuon events. A cut was applied at $M = 4$ GeV/c^2 and $M = 8.5$ GeV/c^2.

FIG. 2

following sections. In the two first methods (called factorization and parametrization methods) we only get results on the shape of the structure functions. The third one (called projection method) combined with results of the parametrization method allows us to make an absolute normalization of our data and therefore to evaluate the scale factor K as defined in equation (2).

V - EXPERIMENTAL RESULTS

Before going into the details of the determination of the structure functions, we have to calculate our A-dependence of the differential cross section. This result is needed in the projection method (see section (V, D)).

A. Measurement of the A-dependence

The analysis of the behaviour of the differential cross section $d\sigma/dM$ with the atomic number A^{α} is a good way to check if the collisions between quarks are hard or not. (A is the atomic mass number of the target material, α is a constant which may depend on the kinematical region under consideration).

A comparison between our hydrogen and platinum data allows us to determine the A-dependence of our Drell-Yan cross section parametrized as A^{α}. Due to the fact that we have π^+ and π^- induced dimuons, we are able to correct the cross sections for the I-spin asymmetry of the platinum nucleus. This is made in computing the difference between the cross sections of dimuons produced by π^- and π^+ incident particles. In addition, this method also eliminates sea quark effects.

We then measure the experimental ratio of $(\pi^- - \pi^+)$ cross sections of dimuon production on hydrogen and platinum, multiplied by the factor A :

$$A \frac{\sigma(\pi^- - \pi^+)\ H_2 \to \mu^+\mu^- X}{\sigma(\pi^- - \pi^+)\ Pt \to \mu^+\mu^- X} = 1.51 \pm 0.28$$

If u and d are the up and down quarks in the proton, we can calculate the expected ratio from equation (3) : (see also appendix B)

$$A \frac{\sigma(\pi^- - \pi^+)\ H_2 \to \mu^+\mu^- X}{\sigma(\pi^- - \pi^+)\ Pt \to \mu^+\mu^- X} = A^{1-\alpha} \frac{\langle 4u - d \rangle}{\langle u + 2d \rangle} \qquad (5)$$

Since u = 2d, we have :

$$C = \frac{\langle 4u - d \rangle}{\langle u + 2d \rangle} = 1.75$$

This expression has been evaluated at the average x_2 of our data because we have to take into account that the ratio of $\frac{d}{u}$ is dependent of x_2. If we use the Field-Feynmann hypothesis [4] (see appendix C) where $d = \frac{1.125\ u}{2} \cdot (1 - x_2)$, and a mean value of $x_2 = 0.15$ corresponding to our data, we get for C the value 1.80. We thus obtain* :

$$A^{1-\alpha} = 0.84 \pm 0.16$$

and for the Drell-Yan process, α becomes :

$$\alpha = 1.03 \pm 0.03$$

This result supports the hypothesis of incoherent proton interactions. Previous experiments have measured this parameter [5, 6].

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* Re-interactions in the 6 cm platinum target was account for $(10 \pm 5)\%$ of the cross section at the J/ψ mass ; we estimate this effect to be ∿ 5% for the Drell-Yan data. We do not correct for this effect.

B. Factorization method

In this method we perform a first analysis of our π^- data by assuming that for the range of x_1 values explored by this experiment, the sea of the pion can be neglected in comparison to the valence. In that case, the observed cross section for incident π^- can be written as (see equation (3))

$$\frac{d^2\sigma}{dx_1\, dx_2} = \frac{1}{x_1^2\, x_2^2}\, V(x_1)\, G(x_2) \qquad (6)$$

The technical way in which we use this method is the following : in the $[x_1, x_2]$ plane, reduced by the mass cuts (see figure 2), the π^- data are binned into rectangular arrays of x_1 and x_2. We divided the range of x_1 ($0.1 < x_1 < 1.$) into N_1 bins ($N_1 = 18$ in our case), the range of x_2 ($0.07 < x_2 < 0.50$) into N_2 bins ($N_2 = 9$). For each bin of given x_1, we have an unknown value of $V(x_1)$, for each bin of given x_2, we have an unknown value of $G(x_2)$. We thus have $N_1 + N_2$ unknowns and $N_1 . N_2$ populated independent cells which are fitted to the form of equation (6). We exclude from the analysis the cells where the acceptance is less than 3%. By minimizing the global χ^2, we obtain the numerical value of the function $V(x_1)$ for N_1 different values of x_1 and the numerical value of $G(x_2)$ for N_2 values of x_2.

The results of this analysis are shown for the 200 GeV π^- run in fig. 3a together with results from a similar analysis by Newman et al. [7] on their 225 GeV π^- data*. The

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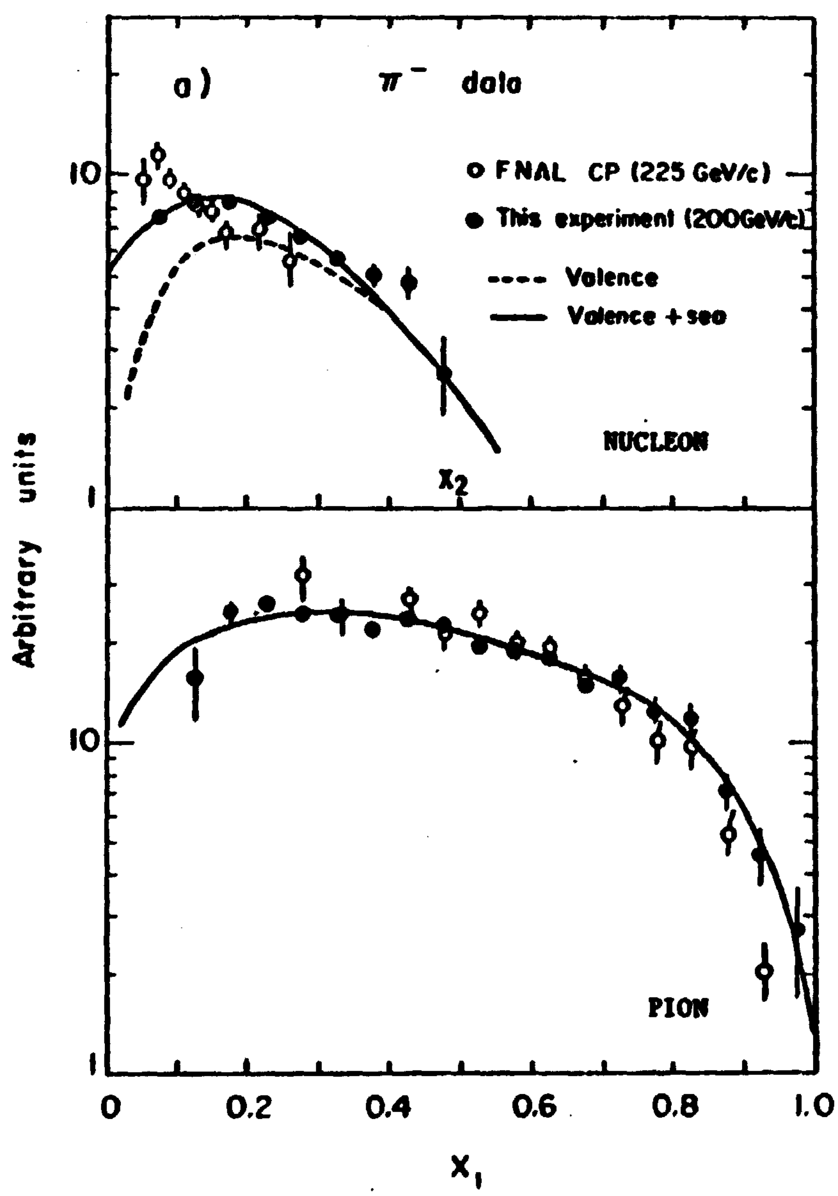
* Notice that in the comparison with the results of ref. [7] the two sets of data are arbitrarily normalized.



The data points are the result of the factorization method (section V,B) applied on the π^- data at 200 GeV/c. Data points from ref. [7] are also plotted with arbitrary relative normalization. The shape of the structure functions obtained by the parametrization method (section V,C) is also shown.

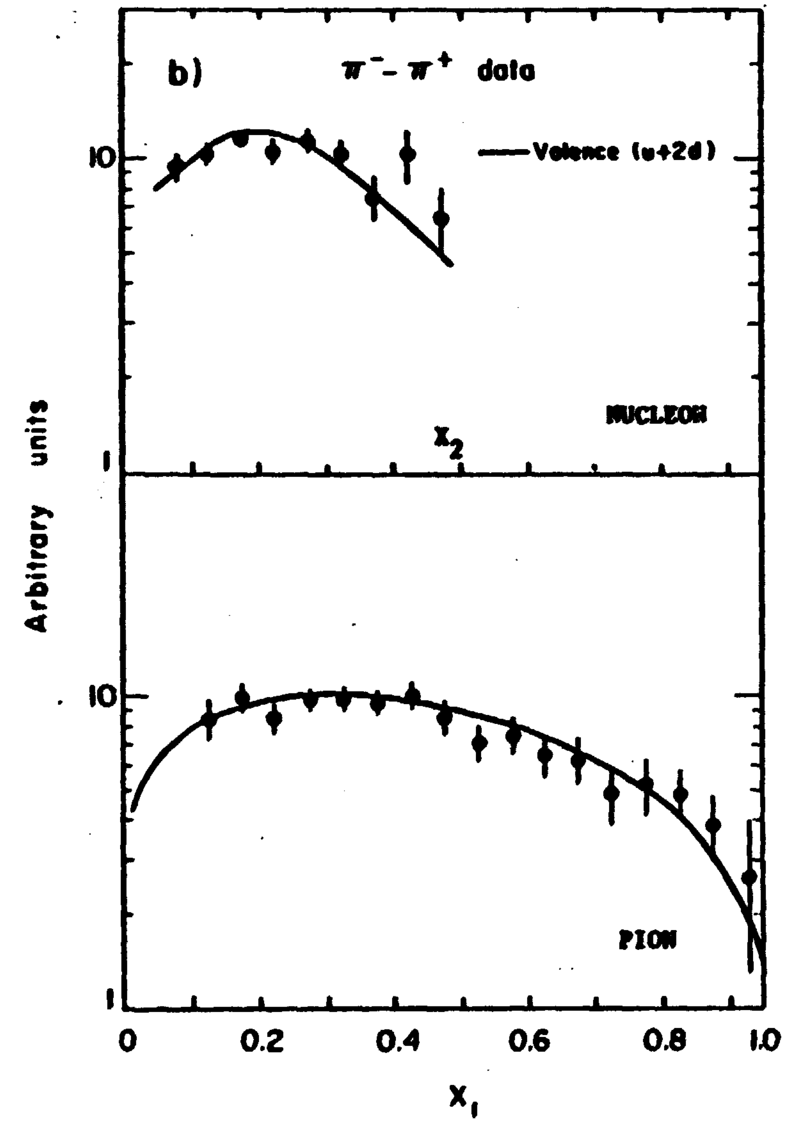
FIG. 3a)

χ^2 of our fit is 95 for 67 degrees of freedom. This value of the χ^2 indicates that in our kinematical range of the variable x_1 and x_2, the factorization hypothesis may not be adequate, whereas it seems to better fit the data of ref. [7] in which the range of x_1 $(0.25 < x_1 < 1.)$ allows clearly to neglect the sea contribution of the pion.

We can also perform a simultaneous analysis of our π^- and π^+ data without any assumptions on the pion sea. In fact, the subtraction of the π^+ induced Drell-Yan cross section from the π^- induced cross section, allows to eliminate the terms involving the sea of the pion and those involving the sea of the nucleon, which are the same for incident π^+ and π^-. For the Pt target $(Z/A = 0.40)$, the combination of up and down valence quark that we obtain is u + 2d (see appendix B)

$$\left(\frac{d^2\sigma}{dx_1\, dx_2}\right)_{(\pi^- Pt) - (\pi^+ Pt)} = \frac{1}{x_1^2\, x_2^2}\, V(x_1)\, \frac{1}{9}\, [u(x_2) + 2d(x_2)]$$

The analysis was done by subtracting in each x_1, x_2 cell the π^+ events from the π^- events, both normalized to the same number of incident pions. This normalization was obtained by using the observed number of J/ψ events and the measured equality (within $\pm$ 2%) of the production cross section for J/ψ on Pt by incident π^+ and π^- [1]. The results are presented in fig. 3b. The χ^2 of this $\pi^- - \pi^+$ difference fit is 43 for 59 degrees of freedom. In the present analysis by the factorization method we have made no attempt of normalization of our data, only the shape of the structure functions are determined.



The data points are the result of the factorization method (section V,B) applied to the $\pi^- - \pi^+$ data at 200 GeV/c. The shape of the structure functions obtained by the parametrization method (section V,C) is also shown.

FIG. 3b)

C. Parametrization method

We assume the following simple x-dependence for the various structure functions [8].

$$V(x) = Ax^{\alpha}(1-x)^{\beta}$$
$$S_{\pi}(x) = B(1-x)^{n}$$
$$u(x) = A'_{u}\, x^{\alpha'}(1-x)^{\beta'}$$
$$d(x) = A'_{d}\, x^{\alpha'}(1-x)^{\beta'+1}$$
$$S_{N}(x) = B'(1-x)^{n'}$$

The choice of $\alpha'_u = \alpha'_d$ and $\beta'_d = \beta'_u + 1$ is the result of theoretical prejudices [4].

As explained in section (IV,B), the parameters A, A'_u, A'_d are fixed in terms of α and β by the normalization condition to the number of valence quarks.

If we use simultaneously the information from our π^- and π^+ data, it is possible to determine the parameters of the sea in addition to the valence. In fact the π^+/π^- ratio gives the relative importance of the sea and the valence contribution, the variation of this ratio as a function of x_1 and x_2 fixes the relative importance of the pion sea and the nucleon sea.

The results of this global fit, done by a maximum likelihood method, on the 200 GeV data are given below

$\alpha = 0.40 \pm 0.06$	$\alpha' = 1.02 \pm 0.15$	
$\beta = 0.90 \pm 0.06$	$\beta' = 4.04 \pm 0.40$	
$B = 0.09 \pm 0.06$	$B' = 0.35 \pm 0.07$	
$n = 4.4 \pm 1.9$	$n' = 6.0 \pm 1.3$	
$A = 0.55$	$A'_u = 10.5$	$A'_d = 6.31$

Only the relative normalisation of the π^+ to the π^- data (known within $\pm$ 2%) is used in this fitting procedure. The absolute normalisation, which is however affected by a larger error, will be exploited in the projection method to evaluate the factor K as defined by equation (2).

Using the parameters obtained from our fit we find :

$$MF_V^\pi = 2\int_0^1 V(x_1)\ dx_1 = 0.34 \pm 0.07$$
$$MF_S^\pi = 6\int_0^1 S_\pi(x_1)\ dx_1 = 0.10 \pm 0.02$$

MF_V^π and MF_S^π are the fractions of the π momentum carried respectively by the valence and the sea quarks ; these fractions agree with general expectation.

In the table III, we compare our results of the pion structure function with previous experiments :

Table III : Comparison of the results of NA3 with other experiments (for π structure function).

Valence quarks	NA3	CIP (ref.[7])	GOLIATH (ref.[10])
α_π	0.40 $\pm$ 0.06	0.5 (fixed)	0.5 (fixed)
β_π	0.90 $\pm$ 0.06	1.27 $\pm$ 0.06	1.56 $\pm$ 0.18
MF_V^π	0.34 $\pm$ 0.07	0.40 $\pm$ 0.07	0.49

For the nucleon, using the parameters from the same fit, we find :

$$\int_0^1 \left(u(x_2) + d(x_2)\right)\ dx_2 = 0.47 \pm 0.09$$
$$6\int_0^1 S_N(x_2)\ dx_2 = 0.30 \pm 0.06$$

Concerning the nucleon structure function (and its integral) we find an average momentum carried by valence quarks bigger than the one obtained in CDHS parametrization (47% instead of 34%) [9]. It should be noticed however that in our fit, we are very sensitive to extrapolation of the structure function to $x_2 = 0$ and hence the values given above, both for the sea and valence quarks, depend on the choice of the analytical representation of $F_N(x_2)$ at small x_2 (see section (V,D)).

D. Projection method

By projecting the content of the x_1, x_2 array on the two axes we get the distribution dN/dx_1 and dN/dx_2. If L is the integrated luminosity calculated from the integrated beam intensity and from the useful number of target nucleons assuming a linear A-dependence of the cross section, as found in section (V,A), we can get from eq. (3) and (4) an expression where only the variable x_1 appears :

$$F_\pi(x_1) \equiv \frac{dN/dx_1}{\frac{\sigma_0}{3}\,\frac{L}{x_1^2}\, I(x_1)} = K\left[V(x_1) + \frac{J(x_1)}{I(x_1)}\, S_\pi(x_1)\right] \qquad (7)$$

The quantities $I(x_1)$ and $J(x_1)$ are integrals involving $G(x_2)$ and $H(x_2)$ and the calculated acceptance of the apparatus $A(x_1, x_2)$

$$I(x_1) = \int \frac{G(x_2)}{x_2^2} A(x_1, x_2)\, dx_2, \quad J(x_1) = \int \frac{H(x_2)}{x_2^2} A(x_1, x_2)\, dx_2$$

These integrals have been evaluated in two different ways :

(i) using for $G(x_2)$ et $H(x_2)$ the results of the fit to our data discussed in section (V,C) ;

(ii) using the results of the CDHS parametrization [9, 11].

The quantity $J(x_1)/I(x_1)$ is nearly constant ($\pm$ 7%) in the relevant x_1 range and is $\sim$ 1.4 for the π^- data and $\sim$ 3.7 for the π^+ data.

The numerical values of K are obtained from the integration of equation (7).

$$K = \frac{\int F_\pi(x_1)\, dx_1}{\int \left(V(x_1) + \frac{J(x_1)}{I(x_1)} S_\pi(x_1) \right) dx_1}$$

where $V(x_1)$ and $S_\pi(x_1)$ are the normalized valence and the sea structure functions as determined in section (V,C).

The results of the pion structure function are displayed in figure 4(a) in the following way :

i) The solid points correspond to our data as determined by the equation :

$$F_\pi^{NA3}(x_1) = \frac{dN/dx_1}{\frac{\sigma_0}{3} \frac{L}{x_1^2} I^{NA3}(x_1)}$$

where $I^{NA3}(x_1)$ is calculated using our data from the parametrization method (section (V,C)).

ii) The fit corresponding to these points is adjusted to the form of equation (7) in which $V(x_1)$ and $S_\pi(x_1)$ are the normalized valence and sea structure functions as determined in section (V,C). We then got for K a value equal to 1.4 - 1.5.

iii) Using now the results of CDHS parametrization we get a second set of data corresponding to the open circle points :

$$F_\pi^{CDHS}(x_1) = \frac{dN/dx_1}{\frac{\sigma_0}{3} \frac{L}{x_1^2} I^{CDHS}(x_1)}$$

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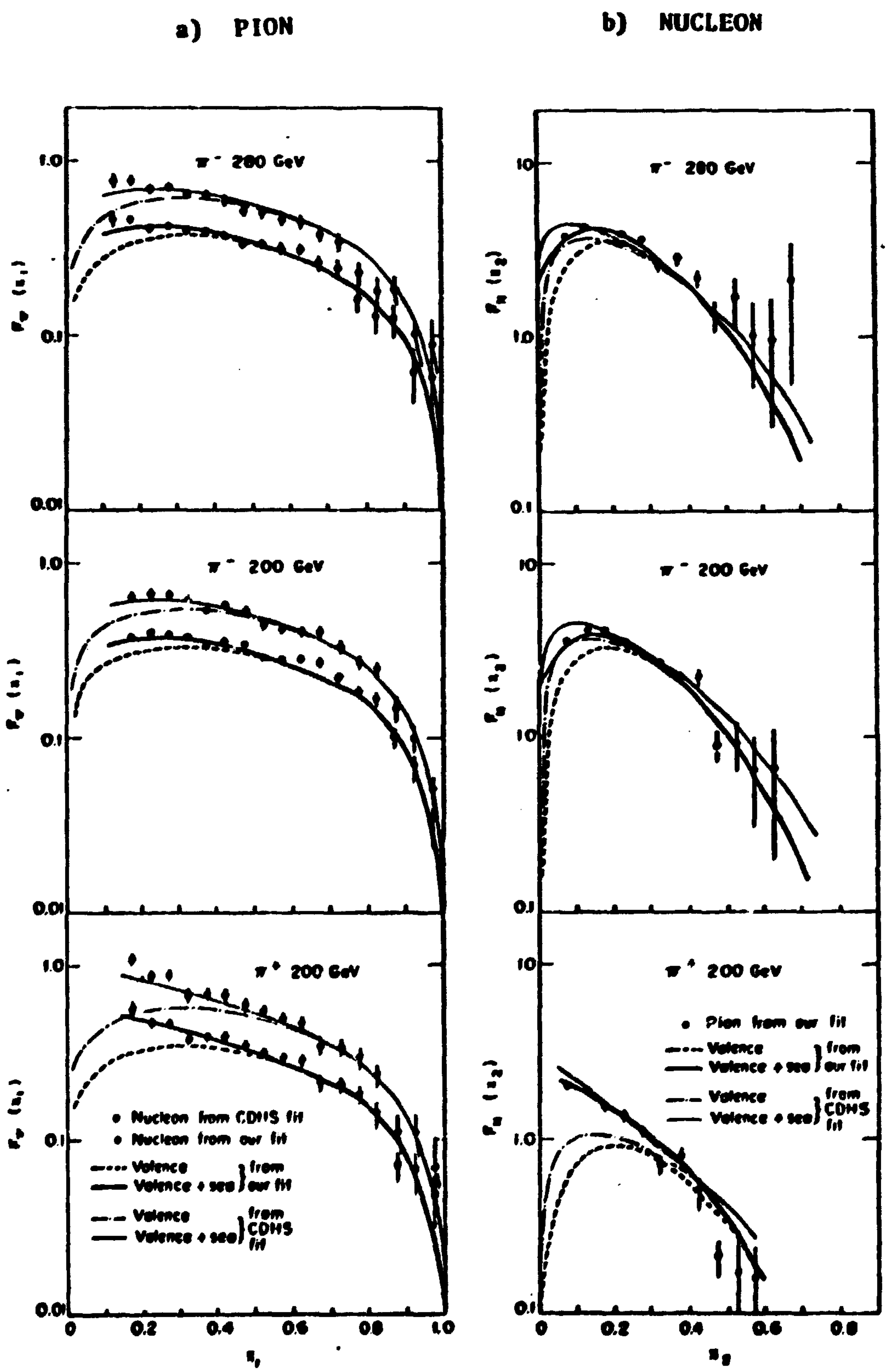

FIG. 4

(a) The data points represent $F_\pi(x_1)$ as defined by eq. (7), using :

- nucleon structure function of our fit (1) (solid points)
- nucleon structure function from CDHS fit (2) (open circle points).

(i) dashed curves represent the valence structure function of the pion obtained from our fit ;

(ii) solid curves represent the (valence + sea) pion structure function as defined by eq. (7).

The curves have been scaled up by a factor K :
($K = 1.4$ for (1), $K = 2.5$ for (2)).

(b) The data points represent $F_N(x_2)$, as defined in section (V,D) using the pion structure function from our fit :

- dashed curves represent the valence part of the nucleon structure function : $1.6u(x_2) + 2.4d(x_2)$ for π^-
$0.4d(x_2) + 0.6u(x_2)$ for π^+ ;

- solid curves represent (valence + sea) nucleon structure function as defined in section (V,D).

The curves have been scaled up by a factor K :
($K = 1.4$ using our fit ; $K = 2.5$ using CDHS fit).

FIG. 4

iv) In order to reproduce this new set of points, we compute another value of K equal to about 2.4 in the same way that ii).

A procedure similar to the one which leads to equation (7) can be used to derive the nucleon structure function using as input the pion structure function from our fit. Only one calculation of $I(x_2)$ and $J(x_2)$ can be made from our pion structure function as determinated in section (V,C). The integrals are :

$$I(x_2) = \int \frac{V(x_1)}{x_1^2} A(x_1, x_2)\, dx_1 \qquad J(x_2) = \int \frac{S_\pi(x_1)}{x_1^2} A(x_1, x_2)\, dx_1$$

In this case for the π^- the valence part is $1.6u + 2.4d$ and $J/I \sim 5.3$, for the π^+ the valence part is $0.6u + 0.4d$ and $J/I \sim 4.5$. The results are given in fig. 4(b).

We now have only one set of data :

$$F_N(x_2) = \frac{(dN/dx_2)}{\frac{\sigma_0}{3} \frac{L}{x_2^2} I(x_2)}$$

Two values of K can be evaluated, using either our structure function for the nucleon (see section (V,C)) or CDHS parametrization :

$$F_N(x_2) = \frac{(dN/dx_2)}{\frac{\sigma_0}{3} \frac{L}{x_2^2} I(x_2)} = \begin{cases} \nearrow K_{NA3} \left(G^{NA3}(x_2) + \frac{J(x_2)}{I(x_2)} H^{NA3}(x_2) \right) \\ \searrow K_{CDHS} \left(G^{CDHS}(x_2) + \frac{J(x_2)}{I(x_2)} H^{CDHS}(x_2) \right) \end{cases}$$

The results for K are summarized in the table IV.

Table IV : K scale factor

	π^+ 200 GeV/c	π^- 200 GeV/c	π^- 280 GeV/c
$G(x_2)$, $H(x_2)$ from this experiment (sect. (V,C))	1.4	1.4	1.5
$G(x_2)$, $H(x_2)$ from CDHS fit	2.4	2.2	2.5

The different possible sources of errors which affected the scale factor K are given in table V.

Table V : Sources of errors on K

	K obtained using for the nucleon $G(x_2)$ and $H(x_2)$ from CDHS	K obtained using for the nucleon $G(x_2)$ and $H(x_2)$ from our fit
Luminosity error of our experiment	± 15%	± 15%
Statistical error	± 10%	± 15%
Systematic error from the acceptance uncertainty	± 10%	± 15%
CDHS normalization error	± 5%	-

We estimate an overall error of ± 30% on K (from CDHS fit) and 35% on K (from our fit)*.

In conclusion, the errors are ± 0.5 for K_{NA3} and ± 0.8 for K_{CDHS}.

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* The large acceptance error in the case of the fit with our data alone is due to the strong dependence of the integral $\int (F_N(x_2)/x_2)dx_2$ on the acceptance at small x_2.

VI - DISCUSSION AND CONCLUSION

First we should note that the shape of the pion structure function is rather insensitive to the choice of the nucleon structure function used in the projection method of section (V,D). Furthermore, the pion and nucleon valence structure function curves obtained from our fit fall nicely on the values obtained from the factorization method (section (V,B)), using the $(\pi^- - \pi^+)$ data (fig. 3(b)) ; this checks the consistency of the two methods. The π^- structure function which we derive from the factorization method agrees in shape with the result of Newman et al. [7]. However the nucleon structure functions, derived by the same methods, are incompatible (fig. 3(a)).

In the parametrization method, the global fit of our π^- and π^+ data fixes the shape of the structure functions. The shape of the pion structure function is insensitive to the choice of the nucleon structure functions. For the nucleon shape, it is found to be low at small x_2 with respect to CDHS fit, whereas the fractional momentum of valence and sea quarks is higher than the results of CDHS, but we have to dwell again on our limited x_2 range.

An absolute normalization was possible in our projection method which allows us to determine the scale factor K. The assumption of K = 1, i.e. "naïve Drell-Yan model" is then excluded by the data if CDHS fit is used for the nucleon structure functions.

Instead of the naïve quark annihilation model, we now consider the introduction of gluon radiative processes in the framework of QCD theory. In this case, the simple Drell-Yan formula of equation (1) must be replaced in order to take into account the contributions (to the cross section) of gluons interactions. Their contribution may be calculated to be proportional to $(\alpha_s \log Q^2)^n$ where α_s is the strong coupling constant [3]. The Drell-Yan formula remains then valid if we replace the structure functions f(x) depending only on x by new ones $f(x, Q^2)$ depending on x and Q^2 (which are called "renormalization group improved structure functions"). This replacement takes into account the contributions of the leading logarithmic terms (i.e. leading order terms in log Q^2). In addition, these new Drell-Yan structure functions $f(x, Q^2)$ should be identical to these measured in deep-inelastic electroproduction and neutrino scattering [3].

In our analysis, we have taken CDHS results at Q^2 = -20 GeV^2/c^2 because most of our data are closed to this value. However the change on α and β in the CDHS structure function in the range of Q^2 from 20 to 70 GeV^2/c^2 is less than 10 %. The log Q^2 dependence of the structure functions on M^2 observed in deep inelastic neutrino scattering (ref. [9]) and predicted by QCD produces only a very small effect in the $Q^2 = -M^2$ range we explored. In conclusion, our comment on the scale factor K made above remains valid even when we introduce in the naïve Drell-Yan formula the leading terms in log Q^2.

We can now do a further step. We take into account the contributions to the cross section of the non leading terms in log Q^2. These QCD corrections to the Drell-Yan prediction are proportional to $1/\log Q^2$ and are expected even when $f(x, Q^2)$ are used. The order of magnitude of the corrections in the case of a reaction $h_1\ h_2 \rightarrow \mu\mu X$ is estimated to introduce effects as large as 100% and of the same shape in the $\tau = M^2/s$ range of the present experiments [2, 3]. The interpretation of our results on the scale factor K might be related to these QCD corrections.

In order to draw conclusions about K and to settle how much of the value of K is due to a multiplicative correction proceeding from QCD effects as discussed above, a more precise knowledge of the nucleon structure function is needed. Actually, the determinations of α_N and β_N (α and β of the nucleon : see section (V,C)) are not precisely known because of the lack of sensitivity of the experiments at small x_2 (for α_N) and x_2 close to 1 (for β_N). We hope that our next data on $\bar{p}$ - Nucleon interactions will be relevant to get a better determination of the nucleon structure function.

In conclusion, whether or not our results on the scale factor K could be explained either by experimental acceptance problems or by the introduction of important QCD corrections is still an open question. The data have to be analyzed in more details in order to shed more light on the normalization problem.

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Appendix A

In order to illustrate how to get the nucleon functions $G(x_2)$ and $H(x_2)$, I give in details the calculation. Due to isospin invariance and charge conjugaison conservation, we have the following set of hypothesis (see section (IV,B)) :

For the pion :

Valence : . $V(x_1) \equiv \bar{u}_V^{\pi^-}(x_1) = d_V^{\pi^-}(x_1) = u_V^{\pi^+}(x_1) = \bar{d}_V^{\pi^+}(x_1)$

Sea : . $S_\pi(x_1) = \bar{S}_\pi(x_1)$

. $S_\pi(x_1) = u_S = d_S = s_S = \bar{u}_S = \bar{d}_S = \bar{s}_S$

(u_S, d_S, s_S, $\bar{u}_S$, $\bar{d}_S$, $\bar{s}_S$ are the sea quarks)

For the nucleon :

Valence : . $u(x_2) = u_V^p(x_2) = d_V^n(x_2)$

. $d(x_2) = d_V^p(x_2) = u_V^n(x_2)$

Sea : . $S_N(x_2) = \bar{S}_N(x_2)$

. $S_N(x_2) = u_S = d_S = s_S = \bar{u}_S = \bar{d}_S = \bar{s}_S$

Notation : p for proton, n for neutron, N for nucleon.

In order to simplify the notation when no confusion is possible, I do not always mention explicityly the x_1 or x_2 dependence of the structure functions in the relations below.

We now can calculate the different terms of equation (1) (section (IV,B)) :

$$F(x_1,x_2) \equiv f_i(x_1)\cdot f_{\bar{i}}(x_2) + f_{\bar{i}}(x_1)\cdot f_i(x_2) =$$

$$\frac{4}{9}\, u^N\cdot\bar{u}^\pi + \frac{1}{9}\, d^N\cdot\bar{d}^\pi + \frac{1}{9}\, S_N\cdot\bar{S}_\pi + \frac{4}{9}\, \bar{u}^N\cdot u^\pi + \frac{1}{9}\, \bar{d}^N\cdot d^\pi + \frac{1}{9}\, \bar{S}_N\cdot S_\pi$$

where :

$$u^N(x_2) = (u^N)_{Valence} + (u^N)_{Sea} = u_V^N + S_N$$

$$d^N(x_2) = (d^N)_{Valence} + (d^N)_{Sea} = d_V^N + S_N$$

u_V^N and d_V^N are the valence structure functions of the nucleon defined as follows :

$$u_V^N = \frac{Z}{A}\, u_V^p + \frac{B}{A}\, u_V^n \qquad \text{and} \qquad d_V^N = \frac{Z}{A}\, d_V^p + \frac{B}{A}\, d_V^n$$

where Z, A, B refer to the composition of the target : Z protons and (A - Z) = B neutrons.

<u>Application : π - Nucleon interaction</u>

As an example, we can calculate these 6 terms for a π^- - nucleon interaction,

$$\frac{4}{9}\, u^N\cdot\bar{u}^{\pi^-} = \frac{4}{9}\,(u_V^N + S_N)(\bar{u}_V^{\pi^-} + S_\pi)$$

$$= \frac{4}{9}\,\bar{u}_V^{\pi^-}(u_V^N + S_N) + S_\pi(u_V^N + S_N)$$

$$= \frac{4}{9}\,\bar{u}^{\pi^-}\left(\frac{Z}{A}\, u_V^p + \frac{B}{A}\, u_V^n + S_N\right) + S_\pi\left(\frac{Z}{A}\, u_V^p + \frac{B}{A}\, u_V^n + S_N\right)$$

From the hypothesis mentioned above, we get :

- $\frac{4}{9}\, u^N\cdot\bar{u}^{\pi^-} = \frac{4}{9}\quad V\left(\frac{Z}{A}\, u + \frac{B}{A}\, d + S_N\right) + S_\pi\left(\frac{Z}{A}\, u + \frac{B}{A}\, d + S_N\right)$

If we play the same game for the other terms, we obtain :

- $\frac{1}{9}\, d^N\, d^{\pi^-} = \frac{1}{9}\, S_\pi\left(\frac{Z}{A}\, d + \frac{B}{A}\, u + S_N\right)$
- $\frac{1}{9}\, S_N\, S_{\pi^-} = \frac{1}{9}\, S_N\, S_\pi$
- $\frac{1}{9}\, d^N\, d^{\pi^-} = \frac{1}{9}\,(V\, S_N + S_N\, S_\pi)$
- $\frac{4}{9}\, u^N\, u^{\pi^-} = \frac{4}{9}\, S_N\, S_\pi$
- $\frac{1}{9}\, S_N\, S_{\pi^-} = \frac{1}{9}\, S_N\, S_\pi$

Assuming that for a platinum target $\frac{Z}{A} = 0.40$ and $\frac{B}{A} = 0.60$, we then obtain :

$$F(x_1,x_2) = V(x_1)\left[\frac{1}{9}\,(1.6u(x_2) + 2.4d(x_2) + 5\,S_N(x_2)\right]$$
$$+ S_\pi(x_1)\left[\frac{1}{9}\,(2.2u(x_2) + 2.8d(x_2) + 12\,S_N(x_2)\right]$$

The same calculation can be made for a π^+ - platinum interaction.

In order to get equation (3) (section IV,B), we define :

$$G(x_2) = \frac{1}{9}\,[1.6u(x_2) + 2.4d(x_2) + 5\,S_N(x_2)] \quad (\text{for } \pi^-)$$
$$G(x_2) = \frac{1}{9}\,[0.6u(x_2) + 0.4d(x_2) + 5\,S_N(x_2)] \quad (\text{for } \pi^+)$$
$$H(x_2) = \frac{1}{9}\,[2.2u(x_2) + 2.8d(x_2) + 12\,S_N(x_2)] \quad (\text{for } \pi^- \text{ or } \pi^+)$$

Appendix B

The calculation made in appendix A allows to determine the cross section ratio of equation (5). We have :

$$\sigma_+^{Pt} = \sigma(\pi^+\ Pt \to \mu^+\mu^- X) \propto [V(x_1)\cdot G^{\pi^+}(x_2) + S_\pi(x_1)\cdot H(x_2)]$$
$$\sigma_-^{Pt} = \sigma(\pi^-\ Pt \to \mu^+\mu^- X) \propto [V(x_1)\cdot G^{\pi^-}(x_2) + S_\pi(x_1)\cdot H(x_2)]$$

Then,
$$\sigma_-^{Pt} - \sigma_+^{Pt} \propto V(x_1)\,[G^{\pi^-}(x_2) - G^{\pi^+}(x_2)]$$
$$\propto V(x_1)\,\frac{1}{9}\,(u + 2d)\ .$$

If we do the same calculation for a hydrogen target, where $Z = A = 1$ and $B = 0$, we obtain : (see Appendix A)

$$\sigma_-^{H_2} - \sigma_+^{H_2} \propto V(x_1)\,\frac{1}{9}\,(4u - d)\ .$$

Appendix C

For the u and d quarks, Field and Feynmann (ref. [4]) assume the following x_2-dependence of the structure functions of the nucleon :

$$\text{for} \quad x_2 \to 1 \qquad u(x_2) \to (1 - x_2)^3$$
$$d(x_2) \to (1 - x_2)^4$$
$$\text{for} \quad x_2 \to 0 \qquad u(x_2) \text{ and } d(x_2) \to \sqrt{x_2}$$

If we introduce the normalization hypothesis

$$\int_0^1 \frac{u(x_2)}{x_2}\, dx_2 = 2 \qquad \text{and} \qquad \int_0^1 \frac{d(x_2)}{x_2}\, dx_2 = 1$$

one can calculate the factor A'_u and A'_d (see section V,C)

$$u(x_2) = A'_u \sqrt{x_2}\,(1 - x_2)^3 \quad \text{and} \quad d(x_2) = A'_d \sqrt{x_2}\,(1 - x_2)^4$$

We get : $A'_u = \frac{35}{16}$ $\quad A'_d = \frac{315}{256}$

With these hypothesis, we then obtain :

$$\frac{d}{u} = \frac{1.125}{2}\,(1 - x_2)\ .$$

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The data of CDHS are analyzed following a methoc loped by Buras and Gaemers [8]. The parametrizatior

their nucleon structure function which is a combination of (u+d) valence quarks is $(u+d) = A_{u+d}\, x^{\alpha_{u+d}} (1-x)^{\beta_{u+d}}$.

In the projection method, we want to determine the normalization factor K ; therefore we have to evaluate the $I(x_1)$ and $J(x_1)$ integrals in which the functions $G(x_2)$ and $H(x_2)$ depending on u and d structure functions of the nucleon are needed (see appendix A).

In order to use CDHS nucleon structure function, we have to extract from their result the u and d structure functions separately. The way in which this is done is the following : u and d are parametrized as $u = A_u\, x^{\alpha_u} (1-x)^{\beta_u}$ and $d = A_d\, x^{\alpha_d} (1-x)^{\beta_d}$. We assume the normalization of the valence distribution functions to the number of valence quarks i.e. $\int_0^1 \frac{(u+d)}{x}\, dx = 3$ and $\int_0^1 \frac{u}{x}\, dx = 2$, $\int_0^1 \frac{d}{x}\, dx = 1$; we now can write [9] :

$$M_{u+d}(2) = M_u(2) + M_d(2)$$

$$M_{u+d}(3) = M_u(3) + M_d(3)$$

where $M_f(2)$ and $M_f(3)$ are the second and third moments of the structure functions f. The moment of order n of the function f is defined as : $M_f(n) = \int_0^1 x^{n-2} f\, dx$.

Besides we assume that $\alpha_u = \alpha_d$ and $\beta_d = \beta_u + 1$ [4]. The resolution of the above system of two equations allows to determine α_u, α_d, β_u and β_d from α_{u+d} and β_{u+d}.

In our case, the choice of the results of CDHS for $Q^2 = 20\ \text{GeV}^2/c^2$ (justified in section VI), leads to the following results : for $\alpha_{u+d} = 0.51$ and $\beta_{u+d} = 3.03$ [9], we obtain $\alpha_u = \alpha_d = 0.51$ and $\beta_u = 2.8$, $\beta_d = 3.8$. From these values and from the normalization of the valence distributions to the number of valence quarks, we get A_u and A_d. The u and d structure functions determined from CDHS results on (u+d) combination of valence quarks are then used in our fit to get K_{CDHS}.