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REMARK ON CHARGE QUANTIZATION AND LEFT-HANDED NEUTRINOS*

Nilendra G. Deshpande
Institute of Theoretical Science
University of Oregon
Eugene, Oregon 97403

ABSTRACT

Renormalizability of the standard SU(2) X U(1) unified gauge model requires charge quantization provided the right-handed neutrinos are absent from the theory.

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The standard SU(2) X U(1) model of Weinberg[1] and Salam[2] with the Glashow-Iliopoulos-Maiani[3] (GIM) mechanism for quarks has recently emerged as the successful model of weak and electromagnetic interactions. It has also been known for some time that the model is free of Adler-Bell-Jackiw triangle anomalies[4] with the conventional charge assignment for quarks and leptons and the assumption that the quarks are color triplets, as required by Quantum Chromodynamics (QCD). This cancellation further takes place generation by generation.

Once the charge assignment is assumed, charge quantization, i.e., the equality of proton and positron charges and absence of neutron charge, follows. What is however not realized is that the charge assignment normally assumed is the only one that leads to a renormalizable theory, provided however that neutrinos occur only in the left-handed state.

Let us first restrict ourselves to the first generation of particles. These are:

$$\text{doublets:} \quad \begin{bmatrix} u^\alpha \\ d^\alpha \end{bmatrix}_L \qquad \begin{bmatrix} \nu \\ e^- \end{bmatrix}_L \tag{1}$$

$$\text{singlets} \quad u_R^\alpha \,,\; d_R^\alpha \,,\; e_R^- \tag{2}$$

where α is the color index. In the SU(2) X U(1) model we have the generator $\vec{T}$ and Y and the gauge fields $\vec{W}$ and B. The only possible anomalies are in W_3W_3B and BBB couplings, and conditions for their removal are

$$\sum_{\text{doublets}} Y(i) = 0 \tag{2}$$

$$\sum_{\text{left-handed}} Y(i) - \sum_{\text{right-handed}} Y(i) = 0 \tag{3}$$

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We can always define charge as

$$Q = T_3 + Y \tag{4}$$

where a unit of charge is the charge of W^+ boson. Then Eq. (2) and Eq. (3) reduce to

$$\sum_{\text{left-handed}} Q_i = \sum_{\text{right-handed}} Q_i = 0 \tag{5}$$

Now we also require that QCD together with weak and electromagnetic theory be anomaly-free. The only possible anomaly is in the BGG vertex, and this requires

$$\sum_{\substack{\text{left-handed}\\ \text{hadrons}}} Q_i = \sum_{\substack{\text{right-handed}\\ \text{hadrons}}} Q_i \tag{6}$$

From Eq. (1) and (4) we further have the relation

$$Q_{u_L} = Q_{d_L} + 1 \tag{7}$$

$$Q_{\nu_L} = Q_{e_L} + 1 \tag{8}$$

We write Eqs. (5) and (6) explicitly

$$3(Q_{u_L} + Q_{d_L}) + Q_{\nu_L} + Q_{e_L} = 0 \tag{9}$$

$$3(Q_{u_R} + Q_{d_R}) + Q_{e_R} = 0 \tag{10}$$

$$Q_{u_L} + Q_{d_L} - Q_{u_R} - Q_{d_R} = 0 \tag{11}$$

Now the weak requirement that electron and d quark have mass leads to $Q_{e_L} = Q_{e_R}$ and $Q_{d_L} = Q_{d_R}$. Then the only solution to the above equations is

$$Q_\nu = 0 \quad Q_e = -1, \quad Q_u = \frac{2}{3}, \quad Q_d = -\frac{1}{3} \tag{12}$$

To see that this result is special to the W-S model, note that in $SU(2)_L \times SU(2)_R \times U(1)$ model[5], since both helicities of ν are present we would obtain

$$3(Q_u + Q_d) + Q_\nu + Q_e = 0 \tag{13}$$

and

$$Q_u - Q_d = Q_\nu - Q_e = 1 \tag{14}$$

This is sufficient to establish

$$Q_{\text{proton}} = -Q_e \tag{15}$$

$$Q_{\text{neutron}} = -Q_\nu \tag{16}$$

but not $Q_\nu = 0$. Thus we see that the left-handed neutrino plays a crucial role in charge quantization.

We have restricted ourselves to only the first generation in proving the above results. Clearly, if we add more quarks and leptons, the equation would remain the same in number, but the unknowns would increase. This suggests that the additional generation of quarks and leptons occurs through a higher group like SU(2) X U(1) X SU(2) suggested by Wilczek and Zee[6]. Then charge quantization holds generation by generation.

We note that charge quantization is also obtainable in a grand unification scheme, like the SU(5) scheme of Georgi and Glashow[7] or $SU(8)_L \times SU(8)_R$ model[8]. These schemes are suggestive but not established because they require the instability of the proton. It is therefore nice to be able to understand the quantization of charge within the conventional picture.

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