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A GEOMETRICAL APPROACH OF THE LEPTONS

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Abstract :

Starting from the schematic models of Harari [1] and Shupe [2] we propose a geometrical approach of the leptons. The model is extended to consideration of the force mediating bosons.

Recently H. Harari [1] and M.A. Shupe [2] have published in the same journal a paper proposing a schematic model of leptons and quarks. They proposed to consider that quarks and leptons are composites of only two types of fundamental spin $\frac{1}{2}$ objects with electric charge $\frac{1}{3}$ and 0 . These objects have been denominated " rishons " by Harari and " quips " by Shupe. The colour is then linked to a distinguishability assumption. These models have tried to be extended to the force-mediating bosons but many questions have been remained opened. In this letter we use the notation and hypothesis of Harari's paper and try to go further. Let us first recall the basis of the above model.

i) Two $J = \frac{1}{2}$ objects : the rishons (and their antiparticles) are introduced

one charged rishon T ($Q = \frac{1}{3}$)

one neutral rishon V ($Q = 0$)

ii) T T T is identified as e^+

iii) T V T, V T T, T T V are the three colours of the u - quark

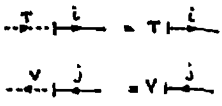
iv) T V V, V T V, V V T are the three colours of the d - quark

v) V V V is the ν_e .

The mediating boson W^+ ($Q = 1$, $B-L = 0$) corresponds to a state of the form (T T T V V V) .

This paper will only deal with the leptons and the mediating bosons of the interactions (Z^0 γ and $W^{\pm}$) .

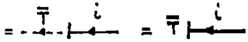
We first consider that the rishons T and V are in fact normed vectors of a quantum space $\mathcal{T}$ for the rishon T and $\mathcal{V}$ for the rishon V . For simplicity sake, we can call " charged space " the $\mathcal{T}$ - space and " neutral space " the $\mathcal{V}$ - space. We give a graphical representation [3] of these colourless vectors of spin $\frac{1}{2}$:

$T_i = \langle T \mid i \rangle =$  (1)

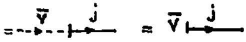
$V_j = \langle V \mid j \rangle =$

We define their antivectors in the dual space :

$$\bar{T}_i = \langle i \,|\, \bar{T} \rangle =$$



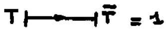
$$\bar{V}_j = \langle j \,|\, \bar{V} \rangle =$$



(2)

The norm of these vectors is set equal to one

$$T.\bar{T} = \sum_i \langle T \,|\, i \rangle \langle i \,|\, \bar{T} \rangle =$$

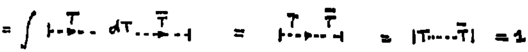


$$V.\bar{V} = \sum_i \langle V \,|\, j \rangle \langle j \,|\, \bar{V} \rangle =$$

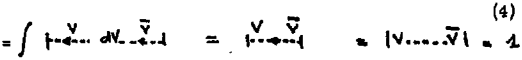
(3)

The completeness of these $\mathcal{T}$ and $\mathcal{V}$ spaces is given with the decomposition of unity

$$\int |T\rangle \, dT \, \langle \bar{T}| =$$

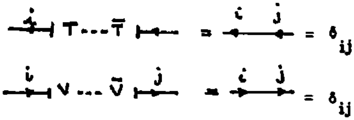


$$\int |V\rangle \, dV \, \langle \bar{V}| =$$



(4)

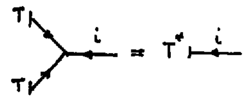
and it comes immediately that



(5)

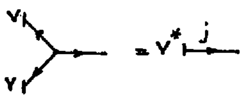
Our fundamental hypothesis will be set after introduction of the pseudo-vectors and the antipseudo-vectors [4] of spin 0 :

$$(T \wedge T)_i = T_i^* =$$



and its dual

$$(V \wedge V)_j = V_j^* =$$



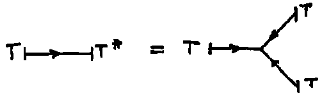
and its dual

(6)

We note that the T^* pseudo-vector has an electric charge $Q = \frac{2}{3}$ while the V^* pseudo-vector is neutral.

As previously done with quarks and hadrons [4] we suppose that the only obervables are the scalars of the $\mathcal{T}$ and $\mathcal{V}$ spaces. It appears immediately with (3) and (5) that only one scalar is possible in the $\mathcal{T}$ - space :

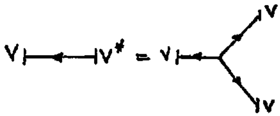
$$T.(T\wedge T) = T.T^* = \quad (7)$$



$$= e^+$$

and following Harari's hypothesis it represents the e^+ $(Q = 1, L = -1)$; in the same way, ν_e $(Q = 0, L = 1)$ is the scalar of the $\mathcal{V}$ - space :

$$V.(V\wedge V) = V.V^* = \quad (8)$$



$$= \nu_e$$

These particles have a spin $\frac{1}{2}$ and this explain why we have set that pseudo-vectors had a zero value spin number. If the coupling of the $\frac{1}{2}$ spin of the rishon could give a pseudo-vector of spin 1 the observables e^+ and ν_e should have had a spin $\frac{1}{2}$ or $\frac{3}{2}$.

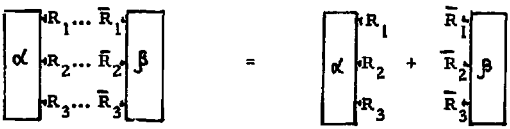
If we call R the rishon T or V, the leptons are then the scalars

$$R.R^* = R.(R\wedge R) \quad (9)$$

We shall use now two fundamental rules which allow the transformation of the diagrams [5].

i) <u>Separation rule</u> :

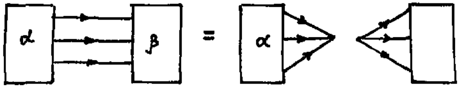
Three R-lines (dotted lines) may be separated (or recombined) altogether



(10)

ii) Pinching rule :

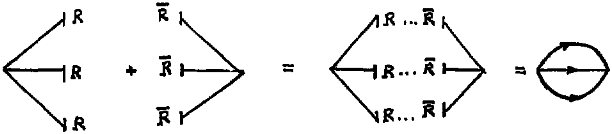
Three vector-lines (main lines) may be separated (or recombined) by a pinch



(11)

Let us now consider the mediating bosons of the interaction in the γ - space (electromagnetic interaction) and in the ν - space (weak interaction).

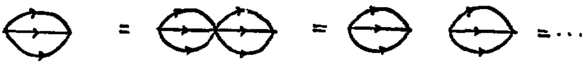
If we recombine a particle-antiparticle pair with (11) and (10) one gets



(12)

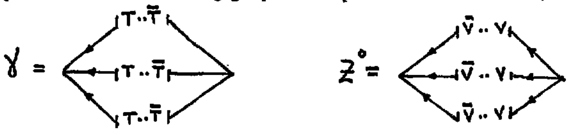
We have thus obtained the graphical representation of the mediating boson (particle of spin 1 with our hypothesis and neutral electrically).

The pinching rule allows the production of several mediating bosons since

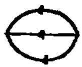


(13)

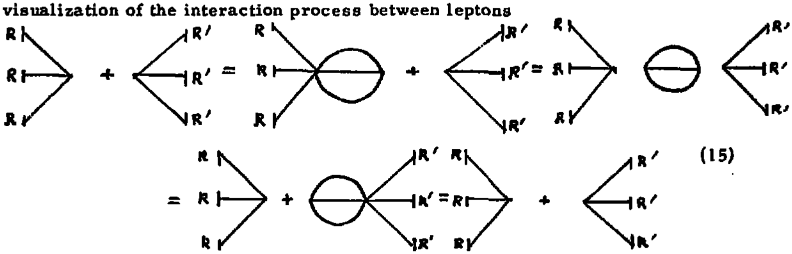
It appears then the following graphical representations of the γ and the Z^o



(14)

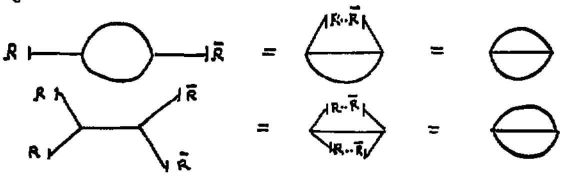
Both diagrams are contained in  which is the mediating boson of the electroweak interactions. This confirms M.A. Shupe's remark [2] concerning the possibility that the photon is a six-rishon state and the photon and Z^o a linear combinations of the same entity.

It is interesting to note that our graphical approach gives a quasi-visualization of the interaction process between leptons



(15)

The separation and pinching rules explain too why one can get the only e^+ and ν_e leptons. Let us take an example with a diagram built with two poles



(16)

The leptonic number L is thus linked to the fact that one must have with the leptons three rishons linked to the same pole.

We introduce

$$n = n(\bar{T}) - n(T) + n(V) - n(\bar{V}) \qquad (17)$$

and p the number of poles (with three outgoing or three ingoing lines) and one easily obtains the leptonic number

$$L = \frac{n}{3} \; \delta_{p \, \frac{|n|}{3}} \qquad (18)$$

the electric charge

$$Q = -\frac{1}{3} \left(n(\bar{T}) - n(T) \right) \qquad (19)$$

and the baryonic number B :

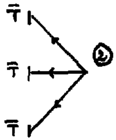
$$B = \frac{n}{3}\left(\delta_{p\,\frac{|n|}{3}} - 1\right) \qquad (20)$$

The last equation has been obtained with the B-L value as given by H. Harari [1] and our expression (18) of L . One easily thus get

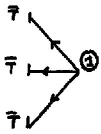
$$\begin{array}{llll} e^+ & (Q = 1 & L = -1 & B = 0) \\ \nu_e & (Q = 0 & L = 1 & B = 0) \\ \gamma & (Q = 0 & L = 0 & B = 0) \\ Z^0 & (Q = 0 & L = 0 & B = 0) \end{array} \qquad (21)$$

The leptons ν_e and e^- having the same leptonic number form a family $\binom{\nu_e}{e^-}$ called the first generation of leptons. The second generation $\binom{\nu_\mu}{\mu^-}$ and the third generation $\binom{\nu_\tau}{\tau^-}$ are constructed in an analogous way with the same set of states at higher energy values [1] .

One can distinguish if necessary the different generations of leptons by marking the pole. For instance

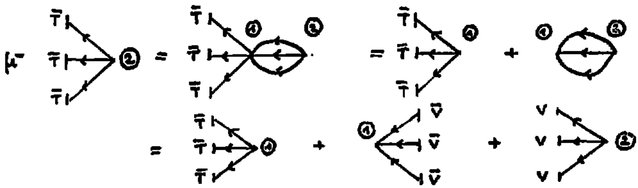


$= \mu^-$ while



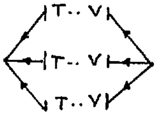
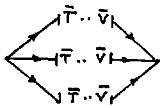
$= e^-$

and one can imagine the desexcitation of an element of the second generation into an element of the first generation



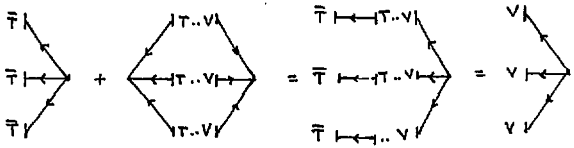
$$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$$

One can finally consider the two electrically charged mediating bosons

$W^+ =$  (22) and $W^- =$  (23)

Their quantum numbers are W^+ ($Q = +1$ $L = 0$ $B = 0$) and W^- ($Q = -1$ $L = 0$ $B = 0$).

The W^+ may transform an e^- into a neutrino

$e^- + W^+ =$  $= \nu_e$ (24)

with tbe use of normalization relation (3) .

One have to note that as soon as a pseudo-vector has been created and combines with a dual vector to make an observable (a lepton) it is impossible to break a junction. The rishon states are thus not observable and there is no hope to ever observe them. The only way to eliminate a junction (a lepton) is in an annihilation of a particle-antiparticle and creation of photons or Z^0 mediating bosons or in a recombination of an electron e^+ (or e^-) with a neutrino ν_e (or antineutrino ν_e) thus creating a W^+ (or W^-) intermediate boson.

Our topological bootstrap approach of the leptons has not any dynamical implication. It gives however a clear understanding of well-known facts and perhaps opens a new approach for the comprehension of the elementary particles.

References

[1] H. Harari, Phys. Lett. B86 (1979) 83.

[2] M.A. Shupe, Phys. Lett. B86 (1979) 87.

[3] E. Elbaz, Traitement graphique de l'algèbre de Racah et de l'analyse vectorielle, Lyon (1979).

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