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# PRE-COULOMB BEHAVIOR
# OF A HEAVY QUARKONIUM LEVELS

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## Abstract

Mass splittings and $e^+e^-$ widths of heavy quarkonium states are considered for such values of the quark mass that the properties of the quarkonium levels are determined dominantly by the Coulomb-like short-distance gluon exchange and deviations from the "Coulomb" behavior can be considered as corrections. The corrections discussed are due to interaction of heavy quarks with nonperturbative fluctuations of gluonic field in the true vacuum of QCD. Expressions for corrections to mass spacing of 2S and 1S levels are obtained as well as for 2S - 1P splitting and for $e^+e^-$ widths of $1^3S_1$ and $2^3S_1$ states. The results obtained suggest a very natural interpolation of the $e^+e^-$ widths between the mass region of applicability of our approach and the data on $\Upsilon$ resonances. With this interpolation numerical estimates of the widths for arbitrary quark mass above $m_\ell$ are given incorporating the $Z$ boson contribution.



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The discovery of the $\Upsilon$ family of resonances [1] made from $b$-quarks with mass around 4.7 GeV and widely discussed expectations of new still heavier (top?) quarks convince oneself that a consideration of quarkonium system made from very heavy (tens of GeV mass) quarks is not an entirely empty exercise and can have a direct bearing to future experiments. From theoretical point of view the dynamics of lowest states of superheavy quarkonium is greatly simplified since a dominant part of it is described by an essentially Coulomb-like interaction potential

$$V(r) = -\frac{4}{3}\,\frac{\alpha_s(r^{-1})}{r} \tag{1}$$

where $\alpha_s(k)$ is the QCD effective coupling, obeying the famous asymptotic freedom relation [2]

$$\alpha_s(k) = \frac{2\pi}{b\ln(k/\Lambda)} \tag{2}$$

That lowest energy levels of superheavy quarkonium are described by the potential (1) is a trivial consequence of the nonrelativistic potential model [3] (in fact this behavior is built in it from the very begining), but it can be also traced back to the first principales of QCD. To verify this one needs only to recall that if a quark of mass $m$ and the corresponding antiquark are interacting via the potential (1), the wave function of the $n$-th eigenstate is localized at distances $r \lesssim k_n^{-1}$, with $k_n$ being the solution of the equation

$$k_n = \frac{1}{n}\,\frac{2}{3}\,m\,\alpha_s(k_n)\ ;\ n = 1, 2, 3, \ldots \qquad (3)$$

Therefore if the mass $m$ is large enough, for few first values of $n$ (the heavier the quark is - the larger is the critical value of $n$ ) the wave function localizes the quarks at sufficiently short distances belonging to asymptotic freedom, which in turn is reduced in the nonrelativistic (static) limit to the interaction potential (1) (see e.g. Ref. 4 ).

A more interesting problem, however, is not the Coulomb-like behavior but deviations from it emerging for each specific level as the quark mass goes down. In particular these deviations place limits on validity of "asymptotically free" description in terms of the potential (1). Recently it has been demonstrated [5,6]. that deviations from asymptotic freedom at larger distances arise from nonperturbative QCD effects rather than from growth of the effective coupling $\alpha_s$ . These nonperturbative phenomena come into play when $\alpha_s$ is still small.

In dynamics of heavy quarks these effects are due to interaction of quarks with nonperturbative fluctuations of gluonic field in the true vacuum of QCD. The very existence of such fluctuations became obvious after the discovery of the instanton solutions [7] . It turns out, however, that instantons are not the whole story and that the main role is played [8] by large scale fluctuations for which the instanton (quasiclassical) approximation [9] fails to be helpful.

In this paper I shall consider a nonperturbative contribution to the properties (mass splittings, $e^+e^-$ widths) of superheavy quarkonium levels for such quark mass that this contribution can be considered as a small perturbation on the "Coulomb" background. As a result estimates will be obtained for mass spacing of 2S and 1S levels, 2S - 1P splitting and the $e^+e^-$ widths of the $1\,^3S_1$ and $2\,^3S_1$ states of superheavy quarkonium. A comparison of the results obtained for the widths $\Gamma(1\,^3S_1 \to e^+e^-)$ and $\Gamma(2\,^3S_1 \to e^+e^-)$ with experimental data on $\Upsilon$ and $\Upsilon'$ resonances (which are far bellow the mass region in which the approach considered can be trusted) suggests a very natural interpolation of the widths between the $\Upsilon$ region and superheavy masses. With this interpolation it is possible to estimate the $e^+e^-$ widths of lowest quarkonium levels for arbitrary quark mass larger than $m_b$ which is of particular importance for experimental search for new quarkonium.

Here I shall use a general approach to nonperturbative effects in dynamics of heavy quarkonium developed in Ref. 10, which is more or less analogous to the standard Wilson operator product expansion. In particular, it has been shown [10] that the leading corrections to the "asymptotically free" behavior of quarkonium levels is due to nonvanishing vacuum expectation value of the square of the gluonic field tensor $\langle 0 | \pi \alpha_s G^a_{\mu\nu}(0) G^a_{\mu\nu}(0) | 0 \rangle$. This vacuum expectation value (v.e.v.) had been estimated by Shifman et al [5,6] from experimental data on hidden charm production in the $e^+e^-$ annihilation

$$\varkappa^4 \equiv \frac{1}{144} \langle 0 | \pi \alpha_s G^a_{\mu\nu}(0) G^a_{\mu\nu}(0) | 0 \rangle \simeq \simeq (0.17\ \mathrm{Gev})^4, \tag{4}$$

with a possible (20–40%) uncertainty.

To avicd possible confusions it should be noted that along with the nonperturbative corrections there are surely usual corrections, say, of the Breit-Fermi type whose relative magnitude is $\sim \alpha_s^2$. They are not considered here and most of them can be adapted from positronium results. The nonperturbative corrections are parametrically different form these and rapidly become the most important ones with diminishing the quark mass.

Specific effects in quarkonium due to the v.e.v. (4) can be evaluated from the expansion of the nonrelativistic Green's function of relative motion in quarkonium [10]

$$G(\vec{x},\vec{y},\varepsilon) = G_{(0)}(\vec{x},\vec{y},\varepsilon) - \\ - 8\varkappa^4 \int G_{(0)}(\vec{x},\vec{z},\varepsilon)\, z_i\, G_{(8)}(\vec{z},\vec{z}',\varepsilon)\, z'_i\, G_{(0)}(\vec{z}',\vec{y},\varepsilon)\, d^3z\, d^3z' + \dots \tag{5}$$

where $\varepsilon$ is the total nonrelativistic energy and $G_{(0)}(\vec{x},\vec{y},\varepsilon)$ is the Green's function for the potential (1):

$$\left[ -m^{-1}\left(\frac{\partial}{\partial \vec{x}}\right)^2 - \frac{4}{3}\frac{\alpha_s}{|\vec{x}|} - \varepsilon \right] G_{(0)}(\vec{x},\vec{y},\varepsilon) = \delta(\vec{x}-\vec{y}) \tag{6}$$

and $G_{(8)}$ is the same for the potential in the color-octet state

$$\left[ -m^{-1}\left(\frac{\partial}{\partial \vec{x}}\right)^2 + \frac{2}{3}\frac{\alpha_s}{|\vec{x}|} - \varepsilon \right] G_{(8)}(\vec{x},\vec{y},\varepsilon) = \delta(\vec{x}-\vec{y}). \tag{7}$$

Here $\vec{x}$ $(\vec{y})$ is the relative coordinate between the quark and the antiquark : $\vec{x} = \vec{x}_Q - \vec{x}_{\bar{Q}}$

The term $G_{(0)}$ in eq. (7) describes the Coulomb-like dynamics in the potential (1), while the second term in the right-hand-side generates the corrections we are interested in. The dots in eq. (5) refer to contribution of vacuum expectation values of operators of dimension $d>4$. It should be noted also that the Green's function $G_{(8)}$ describes dynamics of color-octet quark-antiquark pair (which appears in the intermediate state). Since we need only short distance behavior a consideration of colored states seems quite legitimate.

The terms in the expansion (5) beyond the two explicitly written out can be neglected as far as relevance of only short distances is asumed; moreover the second term can be considered as a small correction to the first one. This corresponds to negative energy

$$\varepsilon = -k^2/m \quad , \qquad k^2 > 0 \tag{8}$$

and $k$ being sufficiently large. In this case the Green's functions fade as exp. ( $-k\tau$ ) and a contribution of distances $\tau \gg k^{-1}$ in the integral in eq. (5) is exponentially suppressed if one considers the function $G(\vec{x},\vec{y},-k^2/m)$ for $|\vec{x}|, |\vec{y}| \leq k^{-1}$.

For large enough mass, such that the momentum $k_n$ in eq. (2) belongs to asymptotic freedom, one can consider eq. (5) at energy $\varepsilon$ around the value

$$\varepsilon_n = -k_n^2/m \tag{9}$$

which corresponds to the Coulomb pole of the function $G_{(0)}(\vec{x},\vec{y},\varepsilon)$ :

$$G_{(0)}(\vec{x},\vec{y},\varepsilon) = \frac{\psi_n(\vec{x})\,\psi_n(\vec{y})}{\varepsilon_n - \varepsilon} + O\left[(\varepsilon-\varepsilon_n)^0\right]. \qquad (10)$$

Here $\psi_n(\vec{z})$ is the wave function of the n-th Coulomb level (problems with degeneracy of states with different $L$ can be readily resolved by expanding (5) in partial waves). Correspondingly to eq. (10) the Laurent series for the integral in eq. (5) starts with a term $\propto(\varepsilon-\varepsilon_n)^{-2}$ which can be considered as the effect of a shift $\delta\varepsilon_n$ of the pole position

$$\frac{1}{\varepsilon_n+\delta\varepsilon_n-\varepsilon} = \frac{1}{\varepsilon_n-\varepsilon} - \frac{\delta\varepsilon_n}{(\varepsilon_n-\varepsilon)^2} + O(\delta^2)$$

Using eq. (10) one obtains [10]

$$\delta\varepsilon_n = 8\varkappa^4\int \psi_n^*(\vec{z})\,z_i\,G_{(8)}(\vec{z},\vec{z}',\varepsilon_n)\,z_i'\,\psi_n(\vec{z}')\,d^3z\,d^3z'. \qquad (11)$$

Note that the shift $\delta\varepsilon_n$ is strictly positive.

Eq. (11) can be rewritten in a form convenient for computation of the energy shift of a level with quantum numbers ( $n, L$ ) using the partial wave expansion of $G_{(8)}(\vec{z},\vec{z}',-k_n^2/m)$:

$$G_{(8)}(\vec{z},\vec{z}',-\frac{k_n^2}{m}) = \frac{m}{2\pi}\int\frac{dp}{p^2+k_n^2}\sum_{L=0}^{\infty}\frac{2L+1}{4\pi}R_L^{(8)}(p,z)R_L^{(8)*}(p,z)P_L\left(\frac{\vec{z}\vec{z}'}{zz'}\right)$$

$R_L^{(8)}(p,z)$ are the radial eigenfunctions of the continuum spectrum of the Schrödinger operator involved in eq. (7).

(This operator corresponds to the repulsive "Coulomb" potential and thus it has only continuum spectrum labeled by the momentum $p$ ). The functions $R_L^{(8)}(p,r)$ are normalized by the condition

$$\int_0^\infty r^2 R_L^{(8)}(p,r) R_L^{(8)}(p',r)\, dr = 2\pi\, \delta(p-p')$$

and they have the following form [11]

$$R_L^{(8)}(p,r) = \frac{2}{3} m\alpha_s \left(e^{-\frac{\pi\nu}{2}}\right) \frac{|\Gamma(L+1+i\nu)|}{(2L+1)!} (2pr)^L e^{ipr} \,. \tag{12}$$
$$\cdot\ \Phi(L+1+i\nu,\ 2L+2;\ -2ipr),$$

where $\nu = m\alpha_s/3p$ and $\Phi(a,c;z) \equiv {}_1F_1(a,c;z)$ is the standard confluent hypergeometric function.

Thus, using the wave function of a level with given ( $n, L$ ) in the form

$$\psi_{n,L,m} = Y_{Lm}\left(\frac{\vec{r}}{r}\right) R_{nL}(r)$$

and juggling a little with spherical harmonics we arrive at the final expression

$$\delta\varepsilon_{nL} = \frac{4\varkappa^4 m}{\pi(2L+1)} \int_0^\infty \frac{dp}{p^2+k_n^2} \left\{ (L+1) \left| \int_0^\infty r^3 R_{nL}^*(r) R_{L+1}^{(8)}(p,r)\, dr \right|^2 + \right.$$
$$\left. + L \left| \int_0^\infty r^3 R_{nL}^*(r) R_{L-1}^{(8)}(p,r)\, dr \right|^2 \right\} \tag{13}$$

In each specific case the integrals over $r$ in this expression are readily calculable and the last integration over $p$ can be performed numerically.

For the energy shifts of 1S, 2S and 1P levels the results are:

$$\frac{\delta \varepsilon_{1S}}{|\varepsilon_{1S}|} = \frac{\varkappa^4 m^2}{k_1^6} \frac{[\Gamma(6)]^2}{9} \int_0^\infty \frac{dx\, x^9 (4+x^2) \exp(2x \arctan x^{-1})}{(1+x^2)^7 (e^{\pi x}-1)} \tag{14}$$

$$\simeq 11,5 \frac{\varkappa^4 m^2}{k_1^6},$$

$$\frac{\delta \varepsilon_{2S}}{|\varepsilon_{2S}|} = \frac{\varkappa^4 m^2}{k_2^6} \frac{2^5 [\Gamma(5)]^2}{9} \int_0^\infty dx\, (4+x^2)(19x^2-5)^2 \varphi(x) \simeq \tag{15}$$

$$\simeq 45 \frac{\varkappa^4 m^2}{k_2^6},$$

$$\frac{\delta \varepsilon_{1P}}{|\varepsilon_{1P}|} = \frac{\varkappa^4 m^2}{k_2^6} \frac{2^7 [\Gamma(5)]^2}{3^4} \int_0^\infty dx\, x^2 \varphi(x) \left[(19x^2-5)^2 + \right. \tag{16}$$

$$\left. + 32(1+x^2)(4+x^2)\right] \simeq 26 \frac{\varkappa^4 m^2}{k_2^6};$$

where in the last two integrands

$$\varphi(x) = \frac{x^9}{(1+x^2)^9} \frac{\exp(4x \arctan x^{-1})}{e^{2\pi x}-1}.$$

Since quarks are confined objects it is more appropriate to discuss energy spacings of levels rather than the energies measured from the threshold $2m$. Thus for the 2S - 1S spasing one obtains

$$M(2S) - M(1S) \simeq \frac{k_1^2}{m} - \frac{k_2^2}{m}\left(1 - 45 \frac{\varkappa^4 m^2}{k_2^6}\right) \tag{17}$$

This expression can be trusted as long as the correction (15) can be considered as small. (Note, that the shift of 1S level is negligibly small in comparison with that of the 2S since $k_1 \simeq 2k_2$ ). The splitting of the levels 2S and 1P (which are degenerate in a pure Coulomb potential) is given by

$$M(2S) - M(1P) \simeq 19 \frac{x^4 m}{k_2^4} \tag{18}$$

Now we proceed to consideration of corrections to $e^+e^-$ widths of ${}^3S_1$ states due to v.e.v. (4). The width of a pure "Coulomb" level is given by the well known expression:

$$\Gamma(n\,{}^3S_1 \to e^+e^-) = \frac{4\pi}{m^2}\alpha^2 Q^2 |\psi_n(0)|^2 \left(1 - \frac{16}{3\pi}\alpha_s(m)\right) = \tag{19}$$
$$= 4\alpha^2 Q^2 \frac{k_n^3}{m^2}\left(1 - \frac{16}{3\pi}\alpha_s(m)\right)$$

where $Q$ is the electric charge of the quark. In this expression the first "radiative" correction ( $1 - \frac{16}{3\pi}\alpha_s(m)$ ) is included, which can be found from QED results (see e.g. Ref. [12] ). For a large quarkonium mass the $e^+e^-$ width is also contributed by the $Z$ exchange. The modification of eq. (19) which accounts for this contribution is quite straght-forward and will be given below.

To evaluate corrections to the width due to v.e.v. (4) one should consider the vacuum polarization $P(q^2)$ by the electromagnetic current of the heavy quarks $j_\mu(x) = \bar{Q}(x)\gamma_\mu Q(x)$ in the near-threshold region of the $q^2$ variable, $q^2 = 4m^2 - 4k^2$ . In the nonrelativistic limit ( $k^2 \ll m^2$ ) the amplitude $P(4m^2 - 4k^2)$ is proportional [10, 13] to the $|\vec{x}|, |\vec{y}| \to 0$ limit of the Green's function (5) at energy $E = -k^2/m$ :

$$P(4m^2 - 4k^2) = C_1 \cdot G(0, 0, -k^2/m) + C_2 \tag{20}$$

The $|\vec{x}|, |\vec{y}| \to 0$ limit of $G(\vec{x}, \vec{y}, -k^2/m)$ is singular, but the singularity is independent of $k$ and can be absorbed into the nonphysical subtraction constant $C_2$. The constant $C_1$ depends on normalization of $P(q^2)$ and can be reconstructed [13] say using eq. (19).

The amplitude $P(4m^2 - 4k^2)$ developes poles corresponding to $^3S_1$ quarkonium states, whose residues are proportional to the $e^+e^-$ widths ( $^3D_1$ poles do not contribute to the order in v/c considered). Therefore, the relative magnitude of the corrections to the widths is given by the relative magnitude of the corrections to the residues of the poles of the function $G(0, 0, -k^2/m)$ . (One should also remember that the positions of the poles are shifted by $\delta\varepsilon_n$ ).

To find the correction it is convenient to rewrite eq. (5) in the following form

$$G(0, 0, -k^2/m) = G_{(0)}(0, k^2) + G_2(k^2) \tag{21}$$

where

$$G_2(k^2) = -8\varkappa^4 \int G_{(0)}(r, k^2)\, r_i\, G_{(1)}(\vec{r}, \vec{r}\,', -k^2/m)\, r_i' \cdot G_{(0)}(r', k^2)\, d^3r\, d^3r' \tag{22}$$

and

$$G_{(0)}(r, k^2) \equiv G_{(0)}(0, \vec{r}, -k^2/m) \tag{23}$$

The poles of $G_{(0)}(0, k^2)$ correspond to Coulomb S levels

$$\frac{\rho_n}{\varepsilon_n - \varepsilon} = \frac{|\psi_n(0)|^2}{\varepsilon_n + k^2/m} = \frac{m\,|\psi_n(0)|^2}{k^2 - k_n^2} \tag{24}$$

and the residues $\propto |\psi_n(0)|^2$ reproduce eq. (19). With the correction included the pole term takes the form $(\rho_n + \delta\rho_n)(\varepsilon_n + \delta\varepsilon_n - \varepsilon)^{-1}$ which is perpoduced by eqs. (21) and (22) in a form of the expansion

$$G(0,0,-k^2/m) = \frac{m\rho_n}{2k_n(k-k_n)} - \frac{m\rho_n\,\delta\varepsilon_n}{4|\varepsilon_n|(k-k_n)^2} + \tag{25}$$

$$+ \frac{m\rho_n\,\delta\varepsilon_n}{4k_n|\varepsilon_n|(k-k_n)} + \frac{m\,\delta\rho_n}{2k_n(k-k_n)} + O[(k-k_n)^0] + O(\delta^2)$$

The double pole term $\propto (k-k_n)^{-2}$ corresponds to eq. (11) while among those with the simple pole there is one with the correction $\delta\rho_n$. Therefore extracting a piece proportional to $(k-k_n)^{-1}$ from $G_2(k^2)$ and using $\rho_n = k_n^3/\pi$ (eq. (24)) one finds from eq. (25)

$$\frac{\delta\Gamma_n}{\Gamma_n} = \frac{\delta\rho_n}{\rho_n} = \frac{2\pi}{mk_n^2}\frac{\partial}{\partial k}\left[G_2(k^2)(k-k_n)^2\right]\Big|_{k=k_n} - \frac{1}{2}\frac{\delta\varepsilon_n}{|\varepsilon_n|} \tag{26}$$

The Laurent expansion of $G_2(k^2)$ around $k = k_n$ can be performed using an explicit expression[13] for $G_{(0)}(r,k^2)$:

$$G_{(0)}(r,k^2) = \frac{mk}{2\pi}\Gamma\left(1-\frac{2m\alpha_s}{3k}\right)e^{-kr}\cdot$$

$$\cdot\,\Psi\left(1-\frac{2m\alpha_s}{3k}, 2; 2kr\right) \tag{27}$$

where the singular at $r = 0$ confluent hypergeometric function $\Psi(a, c; z)$ has the following representation (see e.g. [14])

$$\Psi(a,c;z) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-zt} t^{a-1} (1+t)^{c-a-1} dt \qquad (28)$$

With the help of this representation the expansion of $G_{(0)}(r, k^2)$ is directly found in the following form

$$G_{(0)}(r,k^2) = \frac{mk}{2\pi} e^{-kr} \left[ \frac{\varphi_n(r)}{1-k_n/k} + \eta_n(r) \right] + O(k-k_n) \qquad (29)$$

Here the function $\varphi_n(r)$ is related simply to the n-th S-wave eigenfunction

$$e^{-k_n r} \varphi_n(r) = \psi_n(r)/\psi_n(0) \qquad (30)$$

while the functions $\eta_n(r)$ are somewhat more complicated. Substituting expression (29) into eq. (22) and using eq. (26) one finally obtains

$$\begin{aligned} \frac{\delta\Gamma_n}{\Gamma_n} = -\,\mathrm{Re}\, \frac{16\varkappa^4 m_c^2}{\pi} k_n \int_0^\infty \frac{dp}{p^2+k_n^2} \Big( \int_0^\infty r^3 \varphi_n^*(r) e^{-k_n r} \cdot \\ \cdot R_1^{(1)}(p,r)\, dr \Big) \cdot \Big\{ \int_0^\infty r^3 e^{-k_n r} \big[(2-k_n r)\varphi_n(r) + \\ + \eta_n(r)\big] R_1^{(1)*}(p,r)\, dr - \frac{k_n^2}{p^2+k_n^2} \int_0^\infty r^3 \varphi_n(r) e^{-k_n r} \cdot \\ \cdot R_1^{(1)*}(p,r)\, dr \Big\} - \frac{1}{2} \frac{\delta\varepsilon_n}{|\varepsilon_n|} \end{aligned} \qquad (31)$$

The explicit form of the functions $\varphi_n(r)$ and $\eta_n(r)$ for n = 1 and 2 is given by

$$\varphi_1(r) = 1 \; ; \quad \eta_1(r) = \frac{1}{2k_1 r} - \ln 2k_1 r - C \; ;$$

$$\varphi_2(r) = (1 - k_2 r) \; ;$$

$$\eta_2(r) = \frac{1}{2k_2 r} + 2k_2 r \left[ \ln 2k_2 r - \frac{1}{2} + C \right] - 2\ln 2k_2 r - C .$$

where C = 0,5772.. is the Bernouilli constant. A computation of integrals entering eq. (31) with these functions gives the following results

$$\frac{\delta \Gamma_1}{\Gamma_1} \simeq 30,6 \; \frac{\varkappa^4 m^2}{k_1^6} \; ; \quad \frac{\delta \Gamma_2}{\Gamma_2} \simeq 290 \; \frac{\varkappa^4 m^2}{k_2^6} \qquad (32)$$

In Fig. 1 plots of the widths $\Gamma(1\,^3S_1 \to e^+e^-)$ and $\Gamma(2\,^3S_1 \to e^+e^-)$ vs. the quarkonium mass $M = 2m$ are displayed. These plots are calculated from eqs. (19) and (32) for $|Q| = 1/3$. These expressions do not include the $Z$ contribution and we refer to these widths as to normalization ones ( $\Gamma^{(norm)}$). In numerical estimates we take in eq. (2) $\Lambda = 0.1$ GeV [4,5,6,13] and calculate $k_1$ and $k_2$ from eq. (3) by iterations. The solid lines in Fig. 1 are the predictions and they refer to such masses $M$ that the corrections (32) contribute less than 30%. For $\Upsilon$ and $\Upsilon'$ resonances the normalization widths coincide with the experimental ones [1] and the latter are also shown in Fig. 1 with the error bars. A remarkable feature of the mass dependence of the widths shown in Fig.1 is that when the mass $M$ goes down and the corrections (32) come into play they stabilize the widths in a wide range of mass at levels practically equal to those of $\Upsilon$ and $\Upsilon'$. This behavior suggests a tempting and quite natural interpolation of the curves between the superheavy mass region and the $\Upsilon$ resonances. The interpolation shown in Fig. 1 by dashed lines corresponds to constant widths $\Gamma^{(norm)}(1^3S_1 \to e^+e^-) \simeq$ 1,0 KeV and $\Gamma^{(norm)}(2^3S_1 \to e^+e^-) \simeq 0.39$ KeV. An experimental test of this suggestion seems very interesting and will surely be possible with discovery of new quarkonia in $e^+e^-$ annihilation.

In view of the possibility of experimental verification I display in Figs. 2 a) and b) the plots of predicted experimental $e^+e^-$ widths of 1S and 2S quarkonium levels for both cases: $Q = 2/3$ and $Q = -1/3$, with $Z$ -boson contribution incorporated in the framework of the standard Weinberg--Salam theory. The conversion from plots of Fig.1 is quite straightforward and is performed according to the formulas

$$\Gamma(Q=2/3) = 4\Gamma^{(norm)}\left\{\left[1 + \frac{3}{32}\frac{(1-\frac{8}{3}\xi)(4\xi-1)}{\xi(1-\xi)}\frac{M^2}{M_Z^2-M^2}\right]^2 + \left[\frac{3}{32}\frac{(1-\frac{8}{3}\xi)}{\xi(1-\xi)}\frac{M^2}{M_Z^2-M^2}\right]^2\right\}, \tag{33}$$

$$\Gamma(Q=-1/3) = \Gamma^{(norm)}\left\{\left[1 + \frac{3}{16}\frac{(1-\frac{4}{3}\xi)(4\xi-1)M^2}{\xi(1-\xi)(M_Z^2-M^2)}\right]^2 + \left[\frac{3}{16}\frac{(1-\frac{4}{3}\xi)}{\xi(1-\xi)}\frac{M^2}{M_Z^2-M^2}\right]^2\right\}, \tag{34}$$

with $\xi \equiv \sin^2\theta_W$. In calculating the plots of Fig. 2 the numerical value $\sin^2\theta_W = 0.25$ is used which also corresponds to $M_Z$ = 86 GeV.

In conclusion it is worth noting that the consideration presented above suggests a more rapid approach of characteristics of superheavy quarkonium levels to the "Coulomb" behavior than in the potential model [3] with the potential

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + gr \tag{35}$$

which predicts the relative magnitude of the pre-Coulomb corrections $O(gmk^{-3})$ . It is not excluded that these two predictions can be distinguished experimentally.

I am thankful to M.I.Vysotsky for valuable help in numerical calculations.

## FIGURE CAPTIONS

Fig. 1. Plots of the normalization widths ( $|Q| = 1/3$ and only the photon contribution is accounted for) of the $1^3S_1$, and $2^3S_1$ quarkonium states *vs.* the mass $M$. The solid curves are predictions from eqs. (19) and (32). The dashed lines are an interpolation between the data on $\Upsilon$ and $\Upsilon'$ $e^+e^-$ widths and the mass region where eqs. (19) and (32) are applicable.

Fig. 2. The behavior of the $e^+e^-$ widths with the $Z$ contribution accounted for:

a) the widths $\Gamma(1^3S_1 \to e^+e^-)$,

b) the widths $\Gamma(2^3S_1 \to e^+e^-)$.

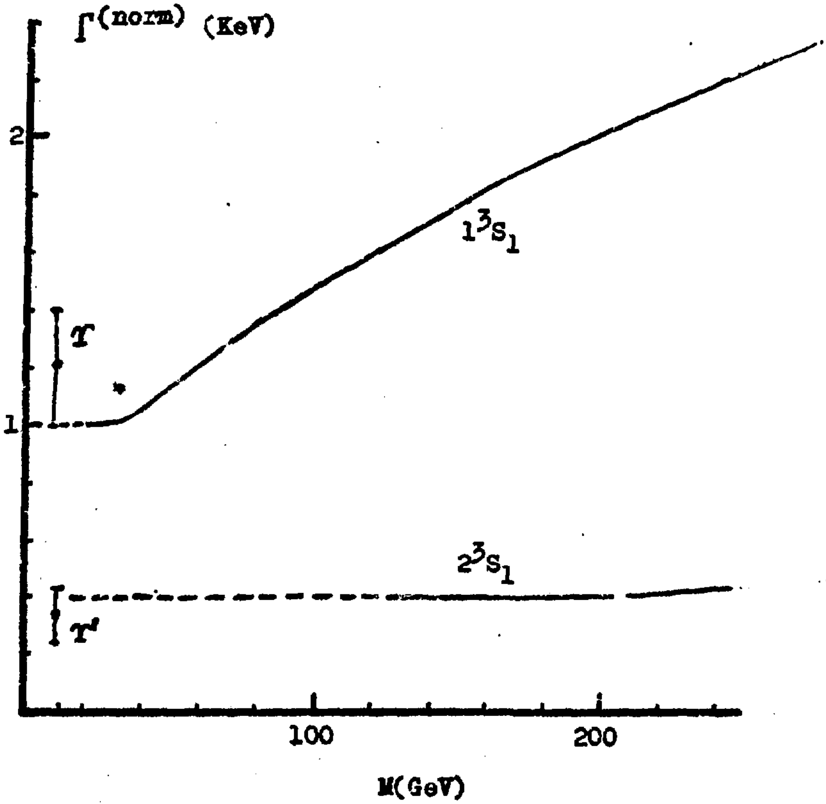


Fig.1

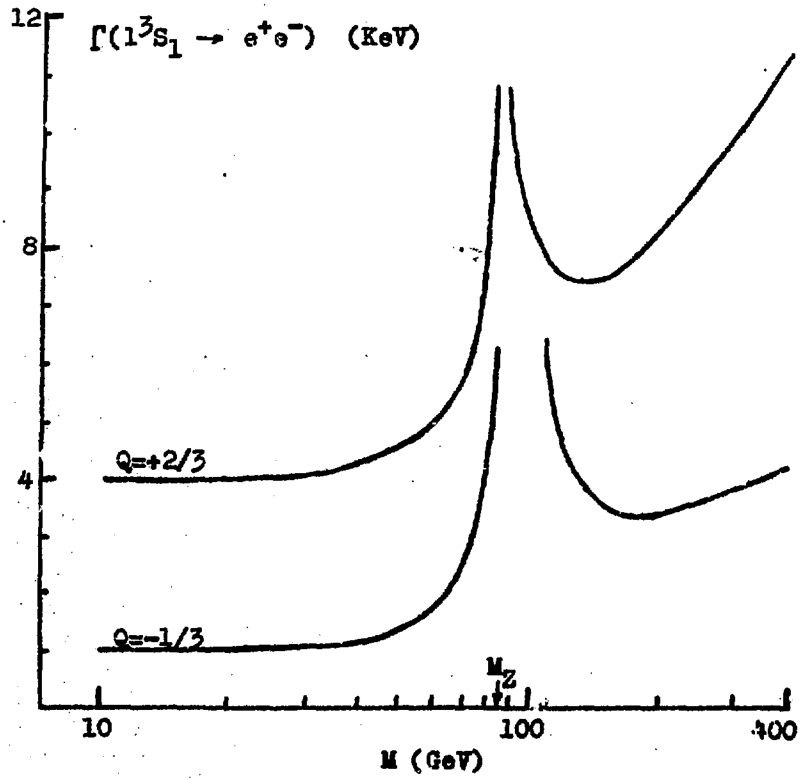


Fig.2a

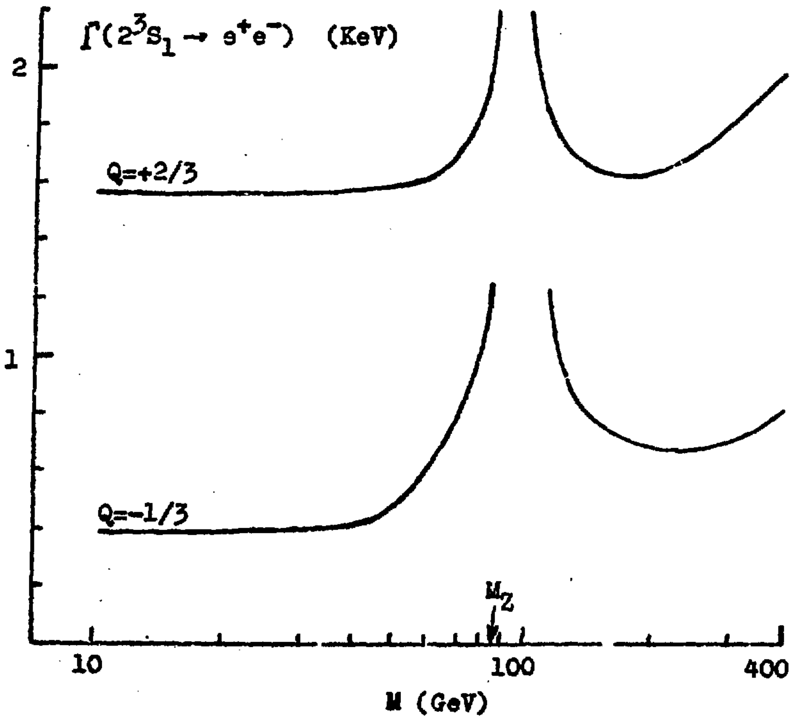


Fig.2b

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