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A SIMPLE MONOTONIC INTERPOLATION SCHEME*

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This paper presents an alternative procedure for presenting tabular data, such as are contained in the ENDF/B files[1], that is simpler, more general and potentially much more compact than the present schemes used with ENDF/B.

It is well known that ENDF/B allows the form of a function to vary from panel to panel, so as to take advantage of the fact that the local variation of a function may be most efficiently represented by one of the four monotonic interpolation schemes it provides. These schemes include linear x-linear y, log x-linear y, linear x-log y and log x-log y variation. The monotonic nature of the approach recognizes that the important details in a function should be included in the data tabulation and should not be introduced by the interpolational procedure, in order to avoid violating the intentions of the data evaluator. In other words, the evaluator will supply all maxima, minima, and inflection points that describe the shape of a cross section curve.

There are two major shortcomings with the ENDF/B approach. First, there are only four allowed forms of variation, causing the use of excess points. Secondly, the use of some of the forms gives rise to inconvenient analytic expressions. For reasons of accuracy and efficiency, many computer codes integrate ENDF/B data analytically; in some cases, even the products of two functions in ENDF/B form are analytically integrated[2]. For some of the combinations, the resulting analytic expressions do not lend themselves to evaluation on a digital computer, thereby forcing the use of numerical integration.

Consider the possible benefits of a scheme which could allow more generality in the variation within a data panel than ENDF/B and which would be in a form very convenient for the evaluation of its integrals by computer.

Consider any function such as a cross section. If one isolates the maxima, minima and inflection points, the intervening regions are monotonic regions of like (concave, convex, or no) curvature. Each of these regions is a candidate

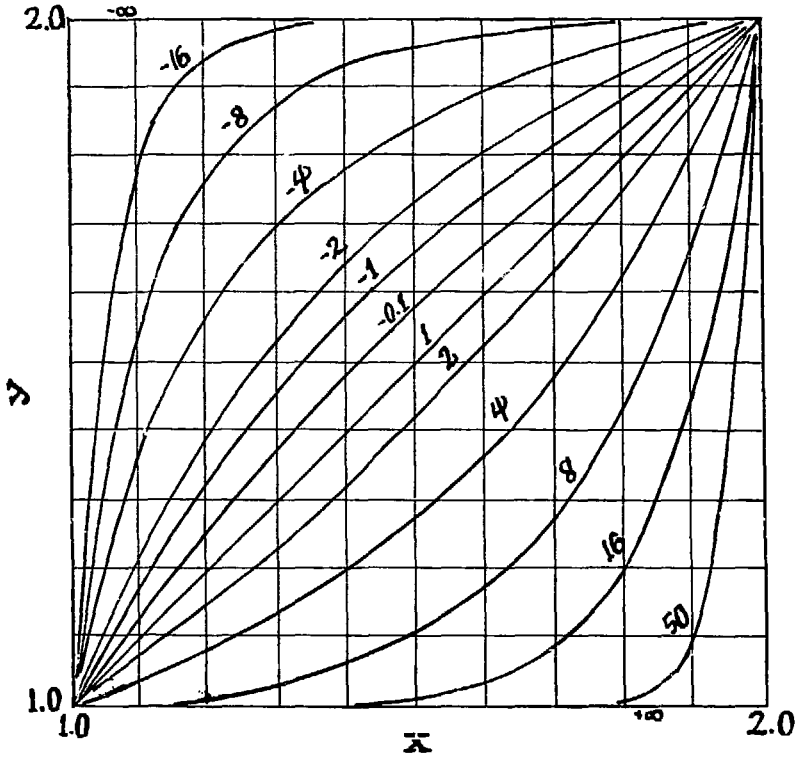

Figure 1: Demonstration of Generality of New Scheme

as a region of "pure" monotonic variation. In the event that a single "pure" function cannot be found, the regions can be subdivided until the function is represented to the desired accuracy, as is the practice now.

There are a variety of forms which will provide monotonic variation in a simple manner, including expressions involving e^{px} and $(\log x)^p$, but one of the more attractive is a simple expression involving x^p. Consider the following equation for interpolating a value for y at an x between the points (x_a,y_a) and (x_b,y_b):

$$y = y_a + \frac{x^p - x_a^p}{x_b^p - x_a^p}(y_b - y_a) \qquad (1)$$

When p=1, this expression defines ordinary linear interpolation. Figure 1 illustrates the degree of variation possible, if one allows p to take on values ranging between $-\infty$ and $+\infty$. In this example, the end points are (1,1) and (2,2). Obviously, the expression provides a very large number of monotonic paths for traversing the panel, suggesting that the presentation of a function in triplets of x, y and p would be very compact, provided, of course, that regions of "pure" monotonic variation can be found.

Consider other advantages that this scheme offers for cross section processing. Setting p=-0.5 accomodates a 1/v variation, while a p=-1.0 would be used for a 1/E flux variation. The integral of Eq. (1) involves nothing more complicated than integrating x^p; in fact, the integral of the product of two or more functions in the form involves terms of the x^p form.

A simple scheme for determining p illustrates why this method is efficient in terms of the number of panels required to represent a function. Consider a three point monotonic region consisting of three (x,y) pairs. If one uses Eq. (1) to determine the interior point:

$$y_2 = y_1 + \frac{x_2^p - x_1^p}{x_3^p - x_1^p}(y_3 - y_1) \qquad (2)$$

a value for p can be determined (using, for example, the Newton root-solving technique). Given this p, the interior point can be discarded, since it is

readily determined by Eq. (1). This eliminates two points, while requiring one additional point to store p. More elaborate schemes based on least-squares would yield even greater storage savings.

The compact fit is many times better than the original, because the p is determined by fitting the local variation of the function. This is illustrated in Table 1, which is concerned with the resonance-like function, $1/(1 + x^2)$. Values of x were determined such that the integral from assuming a linear variation in the panels was 5% different from the analytic integral. In the table, the second and third points are included because there is an inflection point at $x = \sqrt{1/3}$, and the second point is needed to determine a p for this region of convex shape. As evidenced, the power function integrals are everywhere superior to the linear integrals, especially when p approaches its asymptotic limit of 2.

This method has been successfully used for Bondarenko factor interpolation[3] in a module of the AMPX system[4], called BONAMI-2. In this case, it proved itself vastly superior to the Lagrangian schemes used in BONAMI-1, as will be noted in the oral presentation.

In conclusion, the new scheme offers an attractive method for representing tabular data which is compact, simple and general. The ability to retrofit and improve existing files, as demonstrated in Table 1, is also important, as it eliminates the requirement for a re-evaluation.

Table 1. Results of Applying Proposed Interpolation Scheme to Produce Integrals for the $1/(1 + x^2)$ Function

No.	x	y	p	Integrals between points			Error (%)	
				Analytic	Linear	Power	Linear	Power
1*	1.0E-5	1.000	1.000	0.2809	0.2774	0.2803	-1.23	-0.207
2	0.289	0.932	--	0.2425	0.2414	0.2433	-0.44	-0.364
3*	0.577	0.750	-0.350	0.4320	0.4536	0.4189	5.00	-3.03
4	1.414	0.333	--	0.1393	0.1987	0.1924	5.00	1.64
5*	2.203	0.171	-1.613	0.1222	0.1283	0.1219	5.00	-0.236
6	3.133	0.0896	--	0.0846	0.0888	0.0848	5.00	0.219
7*	4.486	0.0473	-1.893	0.0602	0.0632	0.0601	5.00	-0.059
8	6.230	0.0251	--	0.0434	0.0455	0.0434	5.00	0.054
9*	8.597	0.0136						

*These points were retained in the final "power" fit.

References

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