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Abstract

By computing the spin as well as orbit motion of each particle, it is shown that acceleration of polarized protons is feasible if a rapid change is made in ν_z at four medium strong resonances out of eleven in total. The remaining seven resonances are so strong that at each of them, the polarization only changes the sign without a serious loss. The final polarization at 12 GeV will be higher than 83 % of the initial one at the injection energy of 0.5 GeV, if the vertical beam radius is 25 mm at the injection. This estimation includes a polarization loss of 7 % due to the influence of synchrotron oscillation.

1. Introduction

In previous papers[1,2] we have examined a feasibility of accelerating polarized protons in the KEK 500 MeV booster Proton Synchrotron (PS). Loss of polarization can be kept small with or without a ν_z - jump technique.

A feasibility of accelerating polarized protons in the main 12 GeV PS was studied before by Khoe[3], who showed the following: The main resonance of $\gamma G = \nu_z$ is so strong that ν_z has to be changed by 0.6 in 1 μsec in order to significantly reduce the depolarization. Here γ is the conventional Lorentz factor of protons, $G = g/2 - 1 = 1.79285$ the anomalous magnetic moment of protons in nuclear magnetons and ν_z the vertical betatron wave number per revolution. Abandoning the rapid change in ν_z, he suggested a possibility to avoid the strong resonances by choosing 2.25 for ν_z instead of the design value of 7.25. With the new value for ν_z, the total depolarization during acceleration up to the final energy may be as small as 10 %.

The reduction of ν_z to 2.25, however, reduces the acceptance by a factor of three. It is not necessarily a favourable solution.

In this paper, we examine a possibility of accelerating polarized protons with ν_z kept at the designed value of 7.25[4]. For this aim, the spin as well as orbit motion of each particle is traced step by step during many revolutions around the resonance. We show that four medium strong resonances out of eleven in total can be crossed with only a small depolarization by employing a ν_z - jump technique[5] and that the remaining seven strong resonances can be crossed with only a small loss in the polarization and with a change of sign at each resonance. Influence of synchrotron oscillation is also discussed to give a rough estimate for the polarization loss.

2. Depolarizing Resonances

Only intrinsic resonances are considered because imperfection resonances are less important. Eleven resonances listed in Table 1 have to be crossed from the injection at 0.5 GeV to the extraction at 12 GeV. They occur when the frequency of spin precession falls on the frequencies of rotating magnetic fields felt by protons in the particle rest frame:

$$\gamma G = kP \pm \nu_z \qquad (k = 0, \pm 1, \pm 2, \ldots)$$

where P = 4 is the number of superperiods.

3. Computation

Taking the guiding magnetic field vertical, let us adopt the turning rest frame (xyz) of particles (see Fig. 1) for convenience in numerical computation. This frame[2] moves with particles with the z - axis always vertical and with its origin on the equilibrium orbit. The y - axis is along the equilibrium orbit and the x - axis outward radially.

The computational method is the same as described in ref. 2. By assuming the energy as well as transverse coordinates constant within each element whose length is much shorter than the betatron wave - length, the spin equation of motion is solved analytically[2]. The spin as well as orbit motion of each particle is traced step by step along the ring.

Machine parameters are given in ref. 6. The synchrotron ring is of a separate function type and has four superperiods. The cell structures are presented in Fig. 2. In computations edge effects of magnets are treated in a linear approximation for simplicity. The flux density gradient of quadrupole magnets is chosen to be $G_Q = 1.36536 \times p$ Wb/m^3 (p : the particle momentum in GeV/c) to give $\nu_z = 7.25$. The present value of G_Q gives $\nu_x = 7.245$ (the design value ~ 7.25). The acceleration time from 0.5 GeV to 12 GeV is taken 0.6 sec.

The polarization after each resonance is presented in Fig. 3 as a function of the amplitude of vertical betatron oscillation z_{max} at the resonance. The polarization before the resonance is taken +1.0 in the vertical direction. The spin almost completely flips at each resonance if z_{max} is as large as 25 mm. Fortunately, the resonances can be divided into two groups; seven very strong and four medium strong resonances. To save the computing time, computations are mostly carried out in the vicinity (±0.15 GeV) of each resonance. The magnitude of depolarization does not change even if computation is extended to much wider energy regions. It hardly changes either even if the amplitude of radial betatron oscillation is widely changed.

The beam polarization after each resonance can be calculated from Fig. 3 by assuming a uniform distribution of particles in the phase space. The result is presented in Fig. 4 as a function of the vertical beam radius at the resonance. The beam radius is referred by its maximum along the ring. For the vertical beam radius as large as 25 mm at the injection as for unpolarized beams, the polarization loss at strong resonances is small. The polarization changes the sign each time a strong resonance is crossed.

On the other hand, the medium strong resonances are dangerous. A variation of beam polarization with respect to energy is given in Fig. 5 by a dotted line. The vertical beam radius is taken 25 mm at 0.5 GeV and adiabatically damped to 7 mm at 12 GeV in proportion to (momentum)$^{-1/2}$. The polarization is almost completely lost at 3.64 GeV.

Reducing the Depolarization

The depolarization essentially comes from the four medium strong resonances and may be greatly reduced by employing a ν_z - jump technique. We take for simplicity a variation of $kP \pm \nu_z$ as shown in Fig. 6. Around the resonance, $kP \pm \nu_z$ quickly crosses γG and then slowly restores the starting value in parallel with the γG line. The total change of ν_z is taken $\delta\nu_z$ = 0.2 with the rise time of 25 μsec (∿20 revolutions) and the fall time of 5 msec. By employing the ν_z - jump technique, depolarization is significantly reduced as seen in Figs. 3 and 4 by dotted curves. The total depolarization during acceleration from 0.5 to 12 GeV is as small as 10 % if the vertical beam radius is taken 25 mm at the injection. Variation of the beam polarization as a function of energy is given by a solid curve in Fig. 5 (and also in Table 2) after taking into account the influence of synchrotron oscillation discussed in the next section.

A change of 0.20 in ν_z corresponds to a change in the quadrupole fields by 2.8 %, and can be realized by adding a 15 cm thick (effective thickness) quadrupole magnet made of ferrite core to each of existing quadrupoles. The peak excitation required is 0.1 kGauss/cm at the highest energy (5.73 GeV) of the medium strong resonances.

4. Influence of Synchrotron Oscillation

Synchrotron oscillation has been neglected throughout the computation presented above. Synchrotron oscillation has two effects[1.2): modulation of the crossing speed across the resonance and multiple crossing across the same resonance energy. The magnitude of these effects strongly depend on actual machine parameters. Though tracing of spin motion for each particle is easy with the present computation programme by including the synchrotron oscillation, the result will not be transparent, because two additional parameters, i. e., the amplitude and phase of oscillation, come in. Therefore, instead of computation, let us make a rough analytical estimation.

Parameters of synchrotron oscillation in the present PS are roughly as follows: the synchronous phase $\sim 20^\circ$ (or 160°) below (or above) the transition energy of 5.86 GeV and the frequency $\lesssim$ a few kHz depending on the energy. The total phase width of longitudinal phase space decreases from 130° at 0.5 GeV to 25° at the transition energy, above which the width does not change much.

Medium Strong Resonances

If a ν_z - jump technique is employed for the four medium strong resonances, modulation of the crossing speed across the resonance has only a negligibly small effect, because the crossing speed is predominantly determined by the quick change of ν_z. Multiple crossing does not occur either because of the similar reason as above, that is, the variation of $kP \pm \nu_z$ is very much faster than the oscillation in γG. As a result, no sizable effect will exist for these resonances.

Strong Resonances

Among the seven strong resonances, the influence is largest for 2.86 GeV resonance as seen in Table 2. Let us first consider this resonance. As the longitudinal phase space is extended between -10° and 50°, approximately 5/9 of particles may experience deceleration. From the ratio of peak decelerating voltage to the average accelerating one, $-\sin 10^\circ/\sin 20^\circ$, and from the period during which decelerating field is experienced, we may estimate that less than 15 % of particles may cross the resonance three times, if all the particles have the maximum synchrotron oscillation. Multiplying the above two numbers ($0.15 \times 5/9$) and reducing the product by half because the synchrotron oscillation amplitude is distributed between zero and the maximum, only less than 4 % of particles may cross the resonance three times. Even if we assume a worst case that the multiple crossing might completely destroy the polarization, the polarization loss is as small as 4 %.

Let us then estimate an effect of modulation of the crossing speed. The maximum crossing speed at 2.86 GeV is twice that for synchronous particles. Twice large crossing speed is equivalent to a decrease in the vertical beam radius by a factor of $1/\sqrt{2}$ with the crossing speed as before[2]. If the maximum crossing speed is taken for half amount of particles, the polarization loss due to this effect is read from Fig. 5 to be 0.6 %.

From the above two effects, the influence of synchrotron oscillation at the 2.86 GeV resonance is a polarization loss less than 5 %. The actual loss may be much smaller than this, because the estimation is conservative.

Similar estimation is made for the other six strong resonances. For them, however, multiple crossing does not occur because particles do not experience decelerating r. f. phases. Only the modulation in the crossing speed is considered, and the polarization loss is given in Table 2 in the parenthesis after P_f/P_i.

5. Conclusion

Summarizing the above and extending them further, we have the following conclusions:

(1) Acceleration of polarized protons in the KEK 12 GeV PS is feasible with the design value of $\nu_z = 7.25$ as it is by employing a ν_z - jump technique for the four medium strong resonances (0.76 GeV, 1.55 GeV, 3.64 GeV and 5.73 GeV) out of eleven in total. A change of 0.2 in ν_z within 25 μsec is more than sufficient to suppress the depolarization small. The other seven resonances are so strong that polarization only changes the sign without a significant loss. If the vertical beam radius at injection is 25 mm at the maximum along the ring, the final polarization at 12 GeV is higher than 83 % of the initial one, after a polarization loss of 7 % due to synchrotron oscillation effects is taken into account.

(2) Influence of synchrotron oscillation will be negligibly small for the four medium strong resonances if a ν_z - jump technique is employed. The influence may be largest for the 2.86 GeV resonance, because here both effects, i. e. modulation of the crossing speed across the resonance and multiple crossing, are allowed. A resultant polarization loss is expected to be less than 5 %.

(3) At the remaining six strong resonances, synchrotron oscillation affects the spin flip only through the modulation of crossing speed. The polarization loss will be as small as or much smaller than 1 % at each of the resonances. Multiple crossing does not occur because particles do not experience decelerating r. f. phases.

(4) The strongest resonance at 7.83 GeV is entirely difficult to overcome by employing a ν_z - jump technique, if the following limits are tentatively put for the parameters: the rise time of quadrupole magnet excitation $\gtrsim$ 3 μsec and a quick change in $\nu_z \lesssim 0.2$.

(5) As the polarization loss due to synchrotron oscillation effects

amounts several percents at the 2.86 GeV resonance, it is happy if a ν_z - jump technique may be employed here. This resonance lies on the boundary about a possibility of a ν_z - jump technique. If the above limits are put on the fast crossing parameters and if the vertical beam radius at this resonance is as large as 13 mm (25 mm at 0.5 GeV), depolarization as large as a few tens of percent may occur. If the beam radius is reduced typically to half (for example by a charge exchange injection of polarized H^- ions directly from a linac, etc.) or if a quick change in ν_z larger than 0.25 can be handled without making serious harm on the beam dynamics, a ν_z - jump technique may become useful.

(6) A further experimental study is necessary on the influence of synchrotron oscillation, while at ZGS a polarization loss was observed[7] which is consistent with a computation including synchrotron oscillation. In order to make such studies and in order to realize the acceleration of polarized proton beam in the KEK accelerator complex, it will be quite necessary to develope internal polarimeters[8] which can measure the beam polarization in the course of acceleration.

Acknowledgements

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Figure Captions

Fig. 1 The turning rest frame (xyz) of particle (see text).

Fig. 2 Structures of a superperiod. F and D indicate horizontally focusing and defocusing quadrupole magnets respectively. B indicates bending magnets.

Fig. 3 The polarization after each resonance as a function of z_{max}, the amplitude of vertical betatron oscillation at the resonance. The initial polarization is +1.0 in the vertical direction. T_{res} is the resonance kinetic energy. Four curves indicated by ν_z - jump are obtained if a ν_z - jump technique is employed (see text).

Fig. 4 The beam polarization after each resonance as a function of the vertical beam radius at the resonance. Particles are assumed to be uniformly distributed in the phase space. The initial polarization is +1.0 in the vertical direction. T_{res} indicates the resonance kinetic energy. Four curves indicated by ν_z - jump are obtained if a ν_z - jump technique is employed (see text).

Fig. 5 Typical variations of beam polarization with respect to energy. The injected beam is 100 % polarized in the vertical direction. The vertical beam radius decreases from 25 mm at 0.5 GeV to 7 mm at 12 GeV according to adiabatic damping. The dotted (solid) curve corresponds to without (with) a rapid change in ν_z (see text). The solid curve is obtained from Table 2 and includes the influence of synchrotron oscillation.

Fig. 6 A rapid change in ν_z assumed in computations. An example presented in the present paper corresponds to $\delta\nu_z = 0.2$, $\delta\gamma_r = 0.00055$ and $\delta\gamma_f = 200\ \delta\gamma_r = 0.11$. The increment in γ per revolution is $\Delta\gamma = (2.4 \sim 3.3) \times 10^{-5}$ depending on energy.

Table 1 Intrinsic depolarizing resonances between 0.5 and 12 GeV

γG	γ	Kinetic Energy (GeV)	Notes
$-4 + \nu_z$	1.813	0.763	
$0 + \nu_z$	4.044	2.856	* (strong)
$4 + \nu_z$	6.275	4.949	*
$8 + \nu_z$	8.506	7.043	*
$12 + \nu_z$	10.737	9.136	*
$12 - \nu_z$	2.649	1.548	
$16 + \nu_z$	12.968	11.230	*
$16 - \nu_z$	4.881	3.641	
$20 - \nu_z$	7.112	5.734	
$24 - \nu_z$	9.343	7.828	*
$28 - \nu_z$	11.574	9.921	*

Table 2 Variation of the beam polarization as a funciton of energy+

Kinetic Energy (GeV)	Vertical Beam Radius (mm)	$\frac{P_f}{P_i}$ **	Beam Polarization
0.500	25.0	1.0	1.0
0.763*	21.9	0.990 (~ 0)***	0.990
1.548*	17.2	0.998 (~ 0)	0.988
2.856	13.6	-0.950 (< 0.05)	-0.939
3.641*	12.3	0.998 (~ 0)	-0.938
4.949	10.8	-0.960 (< 0.01)	0.899
5.734*	10.2	1.0 (~ 0)	0.899
7.043	9.3	-0.970 (< 0.005)	-0.872
7.828	8.8	-0.990 (< 0.005)	0.864
9.136	8.2	-0.982 (< 0.003)	-0.848
9.921	7.9	-0.993 (< 0.002)	0.842
11.230	7.5	-0.991 (< 0.002)	-0.835

\+ Vertical beam radius is assumed to decrease in proportion to $(\text{momentum})^{-1/2}$ from 25 mm at the injection.

* A ν_z - jump technique of Fig. 6 is employed for these medium strong resonances.

** P_f(or P_i) is the beam polarization after (or before) the resonance.

*** The number in the parenthesis is a rough estimate for the polarization due to synchrotron oscillation effects. This is already included in P_f/P_i.

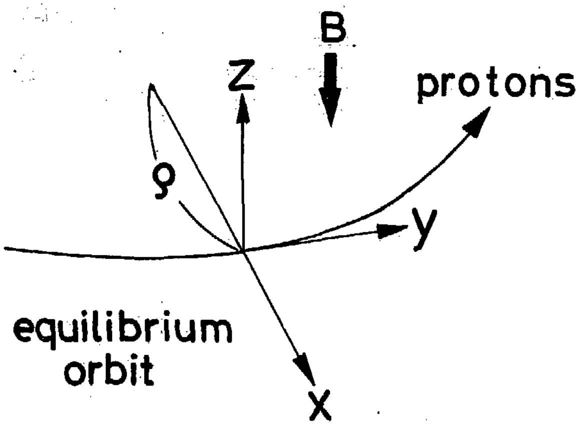
B
z
protons
ϱ
y
equilibrium
orbit
x

Fig. 1

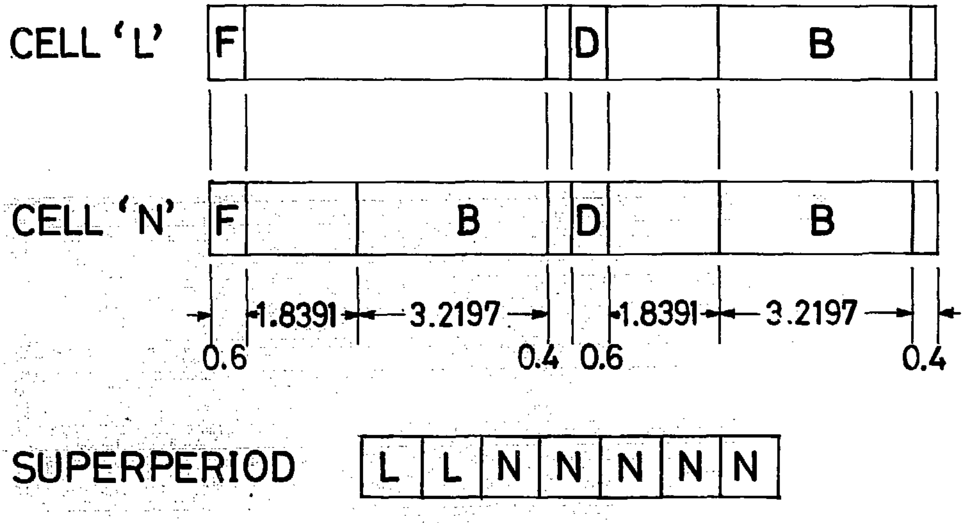
CELL 'L'
F
D
B
CELL 'N'
F
B
D
B
1.8391
3.2197
1.8391
3.2197
0.6
0.4
0.6
0.4
SUPERPERIOD
L
L
N
N
N
N
N

Fig. 2

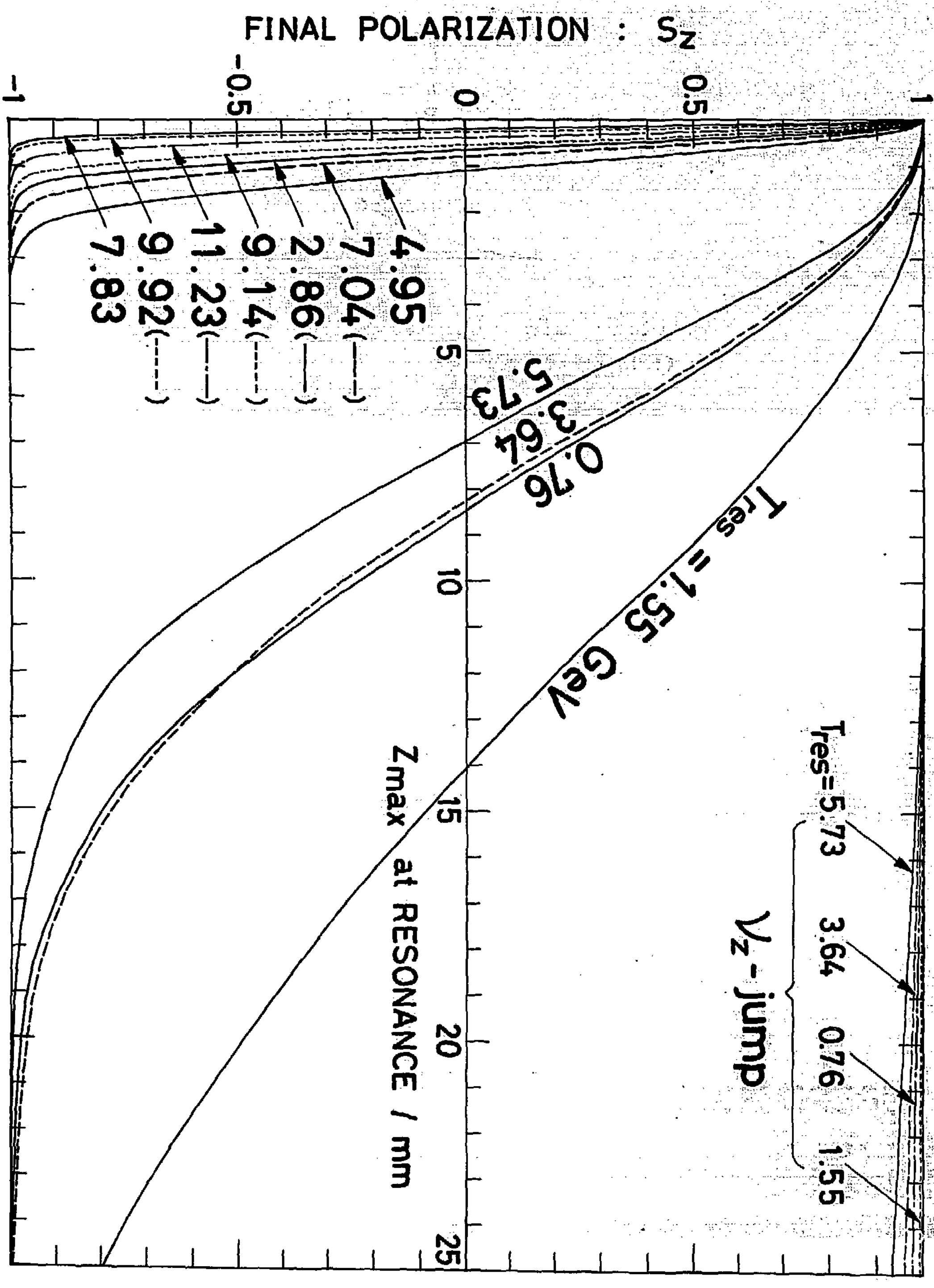
FINAL POLARIZATION : Sz
Tres = 1.55 GeV
5.73
3.64
0.76
4.95
7.04(——)
2.86(——)
9.14(----)
11.23(——)
9.92(----)
7.83
Tres=5.73 3.64 0.76 1.55
νz - jump
Zmax at RESONANCE / mm

FIG. 3

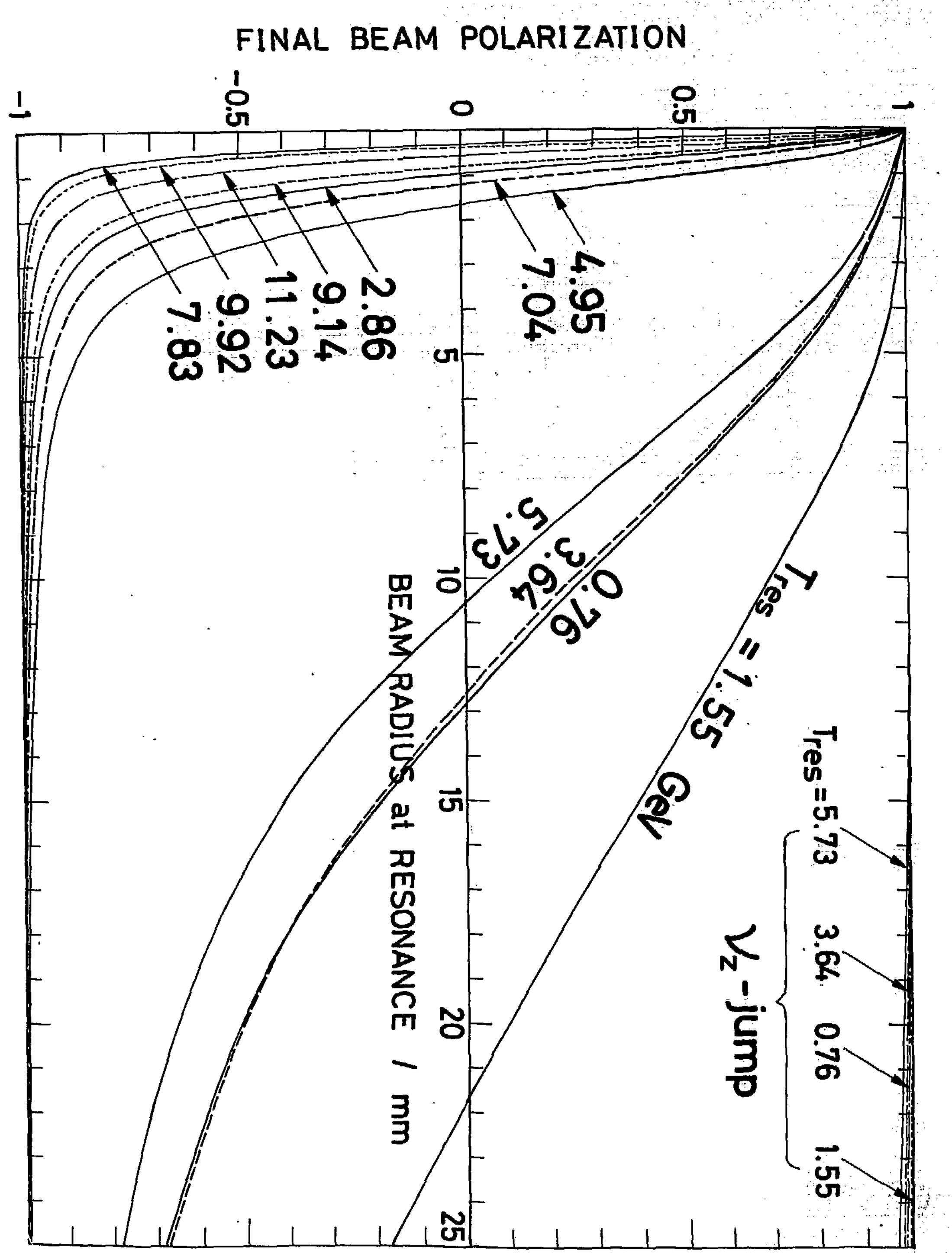
FINAL BEAM POLARIZATION
-1
-0.5
0
0.5
1
4.95
7.04
2.86
9.14
11.23
9.92
7.83
0.76
3.64
5.73
T_{res} = 1.55 GeV
T_{res}=5.73 3.64 0.76 1.55
ν_z-jump
0
5
10
15
20
25
BEAM RADIUS at RESONANCE / mm

FIG. 4

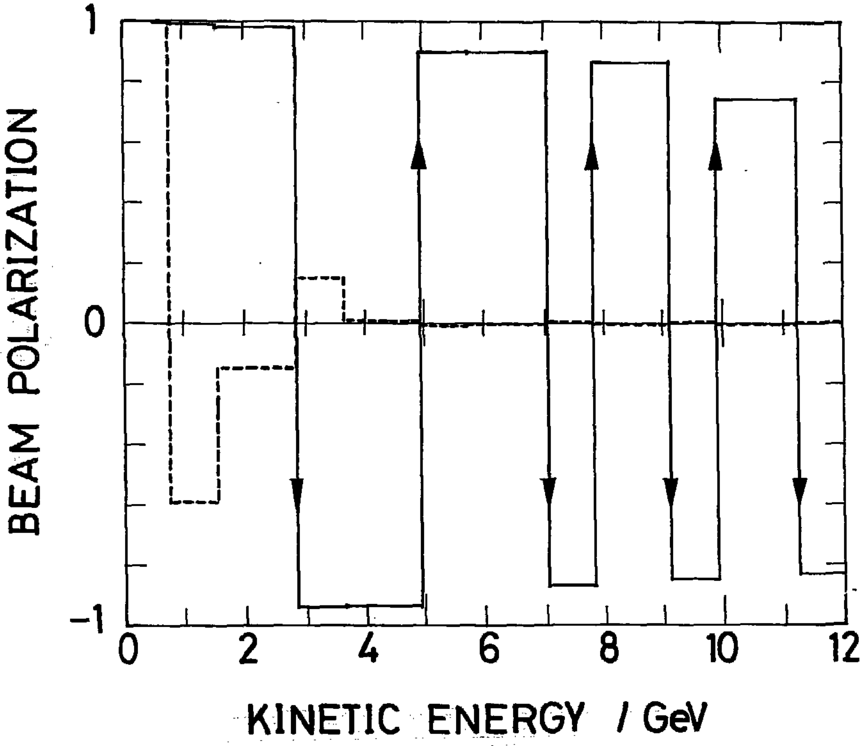

Fig. 5

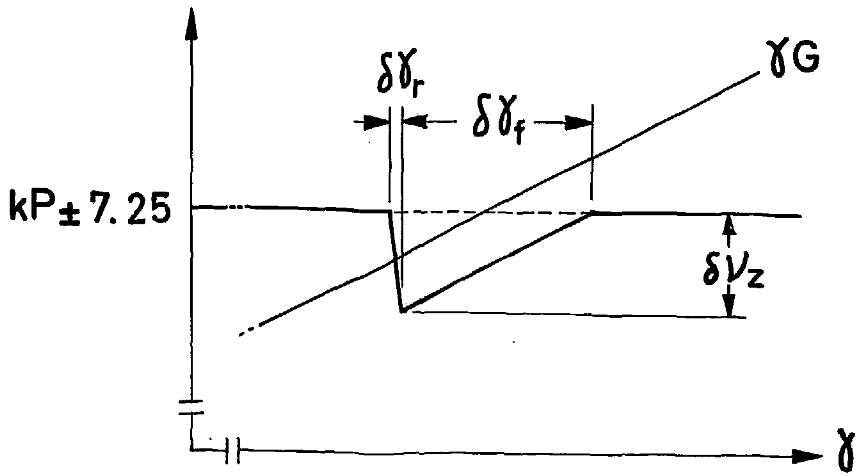

Fig. 6