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PHASE TRANSITION OVER GAUGE GROUP CENTER AND QUARK CONFINEMENT IN Q.C.D.

Abstract

A lattice gauge model with the phase transition corresponding to spontaneous breakdown of the group center symmetry is considered. It is shown that the phase diagram, obtained in multicolor case, separates phases with confined and non-confined quarks. The possible continuum limit in the phase with the permenently confined quarks is discussed.

A promising approach to the problem of quark confinement in Q.C.D. is based on the symmetry of the gauge group center [1,2]. An adequate tool in attacking this problem are Wilson lattice gauge theories [3]. Although static quarks are confined in the framework of strong coupling expansions, there remains a serious question - Do confinement and asymptotic freedom, observed in perturbation theory, coexist in the same phase of the theory?

1. Recently, there were suggested [4,5] models with a phase transition corresponding to spontaneous breakdown of the symmetry of the gauge group center. In the present paper, we study a model with the same property. The lattice action of our model consists of two terms *)

$$Action(U) = \sum_{p} \left\{ \frac{\beta}{N} \operatorname{Tr} U(\partial p) + \frac{1}{2\lambda} \left| \operatorname{Tr} U(\partial p) \right|^2 \right\}. \quad (1)$$

The first one is a standard trace of the product over four elements, $U[\ell]$, of SU(N) along the boundary ∂p of the plaquette p. The sum over p runs over all plaquettes on the lattice. The second term is invariant under the transformation of $U[\ell] \to Z[\ell]\,U[\ell]$, where $Z[\ell]$ takes values in the group Z_N.

Many properties of the model (1) can be understood in the limit of large N, i.e. $N \to \infty$ for fixed β and λ. In the present paper, we found the phase diagram in the β, λ plane. The obtained phase transition corresponds to sponta-

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*) In the naive local limit (the lattice spacing $a \to 0$ at fixed β, λ), this action reduces to usual continuum Yang-Mills action: $Action(U) \to (\beta/N + N/\lambda)/2 \cdot \int d^4x \operatorname{Tr} F_{\mu\nu}^2(x)$.

neous breakdown of the symmetry of the gauge group center. The phase diagram separates phases with confined and non-confined quarks.

2. Let us first find the phase diagram advertised above. Our method is based on the consideration of equation of motion for the Wilson loop average. In the case of the standard lattice model e corresponding equation was suggested in paper [6]. Here we derive an analogous equation taking into account large fluctuations of variables of the group center. This is a necessary step in considering our model (1).

It is convenient to represent the loop average as follows

$$W[c] = \int d\mu(u) \sum_{z} \frac{1}{N} \operatorname{Tr} U(c)\cdot z(c)\, e^{Action(zu)} \Big/ \int d\mu(u) \sum_{z} e^{Action(zu)}, \quad (2)$$

where the total averaging is divided into the averaging over Z_N and $SU(N)/Z_N$, $d\mu(u)$ being corresponding Haar measure on $SU(N)/Z_N$. $z(c)$ is the product of $z[\ell]$ over $\ell \in C$, and $U(c)$ stands for the ordered product of $U[\ell]$.

In order to derive the equations of motion for $W[c]$, we use a standard trick of shifting variables in the functional integral (2). The derivation of the equation resulting from the infinitesimal variation of U is quite similar to that in the standard model [6]. The derivation of the equetion resulting from the variation of the discrete variable, Z, is somewhat more cumbersome. However, in the case of large N, one can substitute the integral over U(1) for the sum over Z_N in Eq.(2). Now, a standard trick may be used. The obtained infinite set of equations possess to order $O(N^{-1})$ a factorized solution

$$W[C] = \Gamma[C]\cdot\omega[C], \tag{3}$$

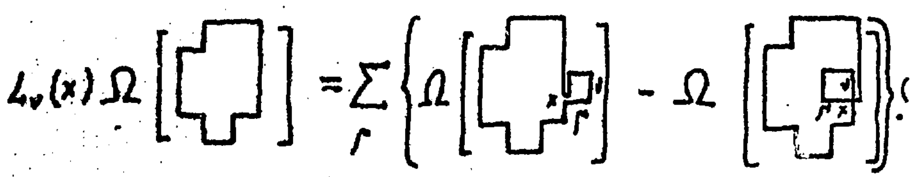
where $\omega[C]$ satisfied the following equation

$$L_\nu(x)\,\omega[C] = \frac{\lambda}{\omega[\partial p]}\sum_{\ell\in C}\tau_\nu(\ell)\,\delta_{xy}\,\omega[C_{xy}]\,\omega[C_{yx}] + O\!\left(\tfrac{1}{N^2}\right), \tag{4}$$

The equation for $\Gamma[C]$ coincides with the equation for the loop average in the compact Abelian gauge theory

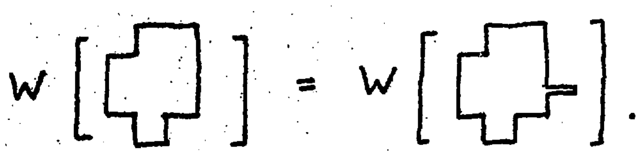
$$L_\nu(x)\,\Gamma[C] = e^2\sum_{\ell\in C}\tau_\nu(\ell)\,\delta_{xy}\,\Gamma[C] + O\!\left(\frac{1}{N^2}\right), \tag{5}$$

where $1/e^2 = \beta\cdot\omega[\partial p]$. The operator $L_\nu(x)$ in the L.H.S. of Eqs.(4),(5) acts as follows


(6)

$\tau_\nu(\ell)$ stands for the unit vector of the link $\ell\in C$. The point Y in the R.H.S. of Eqs.(4),(5) is defined as follows. If the given link ℓ has a positive direction, then Y is the beginning of ℓ. For the case of a negative one, Y is the end of ℓ.

As was noted in paper [6], the equations of motion do not completly determine $W[C]$, and should be accompanied by the Bianchi identity. In the lattice theory, it takes such a simple form


(7)

Eq.(7) for $W[c]$ is satisfied, if both $\Gamma[c]$ and $\omega[c]$ satisfy Eq.(7), separately.

The factorized ansatz (3) for $W[c]$ of Eq.(2) is a consequence, at small λ, from the factorization in the perturbation theory, valid for large N [7]. However, our method gives the possibility to extend the factorization at any λ.

A remarkable fact is that Eq.(4) for $\omega[c]$ depends on the bare action in a very simple form. One should only redefine the coupling constant keeping the equation forminvariant. Thus, $\omega[c]$ and $\Gamma[c]$ in Eq.(3) coincide, after the redefinition of the coupling constants, with the loop averages in the standard Z_N and SU(N) gauge theories, respectively.

This result makes it possible to conclude that there exists a phase transition over β, the critical value being

$$\beta_c(\lambda) = \beta_* / \omega[\partial p], \tag{8}$$

where β_* is a critical point of the Z_N (or U(1)) gauge model [8]. A dependence of $\omega[\partial p]$ on λ is governed by Eq.(4).

Unfortunately, above consideration is not a rigorous proof of the phase transition because the asymptotic behavior of $\omega[c]$ is unknown. However, we were convinced that, for small λ, the asymptotics of the disorder parameter, corresponding to fluctuations of the variables of the gauge group center, is changed at $\beta = \beta_c$. Consequently, Eq.(8) does determine the phase diagram.

3. Now, let us consider a nature of the phase transition described in the previous Sect., and establish its connection with the quark confinement. In dealing with this problem, we use a new criterion (S.B.Khokhlachev, unpublished) for the

quark confinement generalizing the well-known Wilson criterion in the case of the lattice Q.C.D. with light quarks.

Our criterion is based on the fact that quark fields $\psi(x)$ are nontrivial representation of the gauge group center. Under such transformation $\psi(x) \to z(x)\cdot\psi(x)$, where $z(x) \in Z_N$. For this reason one can define an amplitude, $K[c]$, of any quark quantum numbers travelling over the closed loop, C. To define $K[c]$ mathematically, one can do the following. Let us consider the functional

$$\Omega(z[\ell]) = \int d\mu(u)\,\mathcal{D}\psi\,\mathcal{D}\bar\psi\,\exp\{Action(u) + Action(z,\psi)\}, \quad (9)$$

where Action(U) is given by Eq.(1), and

$$Action(z,\psi) = \sum_{\ell} \bar\psi(x)\,\hat\rho(\ell)\,U[\ell]\,z[\ell]\,\psi(x+\ell) + \sum_{x} \bar\psi(x) M \psi(x), \quad (10)$$

X is the beginning of the link ℓ, and $X+\ell$ is its end. $\sum_{\ell}$ runs over all links and their orientations. $\sum_{x}$ runs over all sites on the lattice. Projector $\hat\rho(\ell) = \frac{1+\gamma_\ell}{2}$, with γ_ℓ the Dirac matrices, guarantees the lack of spare quark exitations. Generally speaking, quark fields $\psi(x)$ carry a flavor index.

We see, from Eq.(9,10), that quark travelling is performed by means of subsequent jumps from one site to another neighboring to it. If a quark passed over the contour C, there appears a trace in the form of $z(C)$. States which are singlet representations of the group center, Z_N, produce no traces. Therefore, the amplitude for a quark passed over C is given by

$$K[c] = \frac{1}{\Omega(1)} \sum_{z[\ell]} \Omega(z[\ell]) \prod_{\ell\in C} z^*[\ell]. \quad (11)$$

As in the case of the Wilson criterion, the asymptotics of $K[C] \sim \exp(-\text{Area}/r_c^2)$ corresponds to the confinement phase, but $K[C] \sim \exp(-\text{Longitude}/r_c)$ implies that quaks are non-confined.

For our model (1), $K[C]$ can be found in the multicolor limit. Namely, we show that

$$K[C] = \Gamma[C], \tag{12}$$

where $\Gamma[C]$ is defined in Sect.2. To prove Eq.(12), we will use the equation of motion. For this purpose, change the variable $U[\ell]$ as above. After that the variation of discrete variable, $z[\ell]$, yields an equation for $K[C]$ which is just Eq.(5) for $\Gamma[C]$, and the Bianchi identity follows immediately from the definition of $K[C]$. In deriving this equation, we used the fact that quark loops are negligible for large N [7].

The following conclusion can be drawn on the basis of Eq.(12): <u>the high- ($\beta < \beta_c(\lambda)$) and low- ($\beta > \beta_c(\lambda)$) temperature phases of our model are those with confined and non-confined quarks, respectively.</u>

4. Our above results were based on the lattice gauge model. It is interesting to discuss the possibility of the continuum limit in the confinement phase. A usual way of constructing the local limit of lattice theories is the following. Approach the lattice spacing a to zero and bare charges to their critical values, keeping fixed correlation lengths. If this is the case then the obtained continuum theory will be Lorentz-invariant.

In order to discuss the possibility of such a limiting procedure, let us consider the phase diagram in λ , β^{-1} plane. The phase diagram is determined by Eq.(8). The function

$$X(\lambda) \equiv \omega\,[\partial p] \tag{13}$$

in the R.H.S. of Eq.(8) can be found from the self-consistency condition

$$\omega\left(c;\ \frac{\lambda}{X}\right)\Big|_{c=\partial p} = X \tag{14}$$

which is a consequence of Eq.(4). We gave explicitly a dependence of the coupling constant in Eq.(14).

Eq.(14) has for small λ a solution in the form of perturbation theory expansion. For large λ , strong coupling expansion yields

$$\omega\left(c;\ \frac{\lambda}{X}\right)\Big|_{c=\partial p} = \frac{X}{2\lambda} + O\left(\frac{X^2}{\lambda^2}\right) \tag{15}$$

for the function in the L.H.S. of Eq.(14). In this case, the self-consistency condition (14) has a unique solution, x=0, and consequently $\beta_c^{-1}(\lambda)=0$. We expect this solution to be valid up to some value $\lambda = \lambda_c \sim 1$, as the parameter of the strong coupling expansion is X/λ . Thus, the phase diagram in λ , β^{-1} plane has a shape shown in Fig.1.

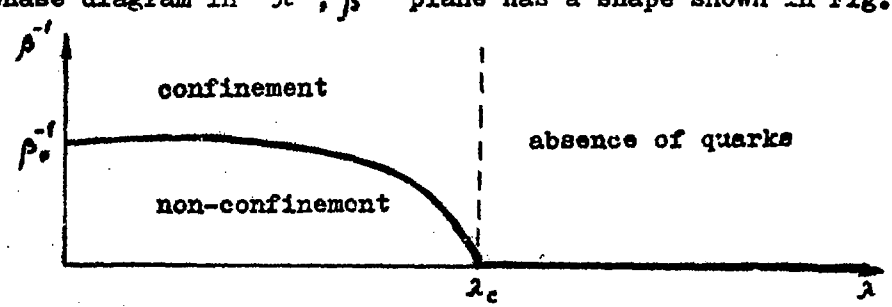


Fig.1. Phase diagramm corresponding to spontaneous breakdown of the center of SU(N) for large N.

All the above results were obtained in the limit N at fixed β, λ. One can show that the correlation length depends on N, at finite N, for $\lambda > \lambda_c$ and vanishes as $N \to \infty$. This implies that there exist only such field configurations when a quark and its antipartner reside in the same site on the lattice, which we call the absence of quarks.

It is interesting to trace how the phase transition discovered for the model (1) in the present paper agrees with that of Yoneya [4] for his model. When $\lambda \ll 1$, $\beta \sim 1$, our results are in fair agreement. An important consequence for the model (1) at $\lambda > \lambda_c$, $\beta \sim N^2$ *), when our approach cannot be used, follows from comparing with the Yoneya model. It is quite natural to expect the critical temperature to be different from zero (β_c^{-1} = const./N^2 for $\lambda > \lambda_c$).

The obtained phase diagram of Fig. 1 allows us to come to some conclusions about a possibility of Lorentz-invariant continuum limit in the confinement phase. One should tend λ to zero which is required by asymptotic freedom over the factor group, and β to its critical value $\beta_* - 0$, keeping fixed the correlation lengths in $\Gamma[c]$ and $\omega[c]$ when $a \to 0$. The local theory obtained in this way has two correlation lengths, their relation being a free parameter.

In principle, another way to obtaine a local theory with asymptotic freedom might have been to tend $\beta^{-1} \to 0$ when $\lambda > \lambda_c$. However, such a theory has no relativistic limit, since as mentioned above the correlation length vanishes for $\lambda > \lambda_c$, $\beta \sim 1$. The correlation length will become

*) The limit $\lambda \gg N^2 \beta^{-1} \sim 1$ is of interest because this is a way arriving at the standard Wilson model.

finite for $N^2\beta^{-1}\sim 1$, but as pointed out above we expect that taking the limit $N^2\beta^{-1}\to 0$, required by asymptotic freedom, one arrives at the phase with non-confined quarks. This question will be considered in more detail in the full publication.

Our local theory with confined quarks guarantees asymptotic freedom over λ . A problem concerning the properties of Z_N gauge theory when $\beta\to\beta_*-0$ has not been studied at present. It is evident that both this problem and the properties of Q.C.D. with two correlation lengths are of great interest and deserve the future investigation.

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