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EFFECTS OF SHORT RANGE CORRELATIONS ON NUCLEAR MASS AND MOMENTUM DISTRIBUTIONS

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Abstract :

Effects of short range correlations on the form factor and the momentum distribution of nuclear systems are investigated. The present analysis, performed in the framework of the Jastrow approach, indicates that an independent-particle wave function (Slater determinant) cannot reproduce simultaneously the form factor and the momentum distribution of a correlated system. It is found that the momentum distribution is strongly affected by correlations beyond $\sim 2\ fm^{-1}$.

Search for experimental evidence of short range correlations (SRC) is an old problem [1] . In theoretical descriptions, SRC play a major role when computing mean values of some two-body operators, for instance when evaluating the ground-state energy. However, it would be desirable to have independent evidence of SRC coming from simpler quantities, i.e. quantities related to one-body operators. Contrarily to what is often believed, there is no guarantee that ground-state one-body quantities can be satisfactorily described in an independent particle model. A certain amount of theoretical work has been devoted in the past to study the effects of SRC on the form factor of some lightnuclei (see, for instance, refs.[2,3]). However it has been noticed that the analysis of elastic electron scattering cannot give a conclusive test on the presence of SRC, since a given form factor can always be reproduced by an independent particle wave function [4] . This can be best illustrated in the case of ^{4}He if one forgets center of mass effects : from the knowledge of the form factor $F(q)$ one can extract the density $\rho(r)$ and the Slater determinant constructed with the single particle wave function $\varphi(r) \sim (\rho(r))^{1/2}$ will give rise to the given form factor $F(q)$. An equivalent remark can be made if one analyzes the momentum distribution $n(p)$: in fact, by working in the momentum space, one can always choose the single particle wave function $\Psi(p) \sim (n(p))^{1/2}$ and consequently reproduce an arbitrary momentum distribution.

The situation may be different if one is interested in reproducing simultaneously several one-body quantities. The purpose of this Letter is to study to what extent the simultaneous knowledge of the form factor $F(q)$ and the momentum distribution $n(p)$ may provide indications on the presence of SRC. In order to carry out the analysis, the knowledge of the functions $F(q)$ and $n(p)$ corresponding to a correlated system is needed. How to compute $F(q)$ for systems with correlated wave functions has been extensively discussed in the past [2,3]. More recently, techniques to compute $n(p)$ are becoming available [5-8]. For the sake of simplicity, instead of using the above-mentioned techniques, we start from a Jastrow wave function and evaluate the one-body density matrix in the single-pair approximation. Let us emphasize that we are not demanding to this approximation to reproduce in general the exact results. Rather we use it as a practical method to generate a correlated density matrix. Of course the method is only justified if the resulting physical quantities -when an adequate choice of the parameters of the Jastrow wave function is made- are in agreement with the corresponding quantities coming from more realistic approaches.

We proceed as follows. We construct a Jastrow wave function

$$\Psi_J(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N) = \frac{1}{\sqrt{C_N}} \prod_{1\le i<j\le N} f(r_{ij})\,\Phi_{SD}(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N) \qquad (1)$$

where the label J stands for Jastrow. In (1) C_N is a normalization constant, Φ_{SD} is an N-particle Slater determinant and $f(r_{ij}) = f(|\vec{r}_i-\vec{r}_j|)$ is the correlation factor. In order to compute the form factor $F_J(q)$ and the momentum distribution $n_J(p)$ deduced from (1) we calculate the one-body density matrix

$$\rho_J(\vec{r}_1,\vec{r}_1') = \int \Psi_J^*(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N)\Psi_J(\vec{r}_1',\vec{r}_2,\ldots,\vec{r}_N)d\vec{r}_2\ldots d\vec{r}_N \qquad (2)$$

To evaluate ρ_J we use the single-pair approximation [2,3] in which one keeps all terms up to second order in $h = f-1$ and first order in $g = f^2-1$ [4]. By choosing for the correlation factor the form

$$f(r) = 1 - e^{-\beta^2 r^2} \qquad (3)$$

the calculation can be performed analytically when single particle harmonic oscillator wave functions are used. For simplicity, only the case of ^{4}He will be discussed.

The resulting normalized one-body density is

$$\begin{aligned}\rho_J(\vec{r}_1,\vec{r}_2) = \frac{\alpha^3}{\pi^{3/2}}\frac{1}{N}\Bigg\{ &\delta\, e^{-\frac{1}{2}\alpha^2(r_1^2+r_2^2)} - \\ &- \frac{3}{(1+y)^{3/2}}\left[e^{-\frac{1}{2}\alpha^2\frac{1+3y}{1+y}r_1^2-r_2^2)} + r_1 \leftrightarrow r_2\right] + \qquad (4)\\ &+ \frac{3}{(1+2y)^{3/2}}\, e^{-\frac{1}{2}\alpha^2\left[(1+2y)(r_1^2+r_2^2)-\frac{2y^2}{1+2y}(\vec{r}_1+\vec{r}_2)^2\right]}\end{aligned} ;$$

in eq.(4) α is the harmonic oscillator parameter, $y^{-1} = \alpha^2/\beta^2$ is a measure of the correlation length of the function $f(r)$ in terms of the size of the system and N and δ are given by $N = 1-12(1+2y)^{-3/2} + 6(1+4y)^{-3/2}$, $\delta = (N+1)/2$. The relation

$$\int \rho(\vec{r}_1,\vec{r}\,')\,\rho(\vec{r}\,',\vec{r}_2)d\vec{r}\,' = \rho(\vec{r}_1,\vec{r}_2) \qquad (5)$$

which is a defining property of Slater determinants, is of course not fulfilled by the one-body density ρ_J of eq.(4).

Generally a violation of eq.(5) will indicate the presence of dynamical correlations in the density matrix. Notice that eq.(5) involves the full one-body density matrix (diagonal and non-diagonal part). Consequently, if one restricts the analysis to one-body properties, one can draw conclusions about ground state correlations only by exploring both the diagonal and non-diagonal part (entering in the expectation value of local and non-local one-body operators respectively). In what follows we study the form factor

$$F(q) = \int \rho(\vec{r},\vec{r})\, e^{i\vec{q}\cdot\vec{r}}\, d\vec{r} \tag{6}$$

and the momentum distribution

$$n(\vec{p}) = \frac{1}{8\pi^3} \int d\vec{r}\, d\vec{R}\; e^{i\vec{p}\cdot\vec{r}}\, \rho(\vec{R},\vec{r}) \tag{7}$$

where relative and center of mass coordinates are introduced $\vec{r} = \vec{r}_1 - \vec{r}_2$, $\vec{R} = (\vec{r}_1 + \vec{r}_2)/2$. The final expressions, when eq.(4) is replaced in (6) and (7) are

$$F_J(q) = \frac{1}{N} \left\{ \delta e^{-\frac{q^2}{4\alpha^2}} - 6(1+2y)^{-3/2}\, e^{-\frac{1+y}{1+2y}\frac{q^2}{4\alpha^2}} + 3(1+4y)^{-3/2}\, e^{-\frac{1+2y}{1+4y}\frac{q^2}{4\alpha^2}} \right\} \tag{8}$$

and

$$n_J(p) = \frac{1}{N}\frac{1}{\alpha^3}\frac{1}{\pi^{3/2}} \left\{ \delta e^{-\frac{p^2}{\alpha^2}} - 6(1+3y)^{-3/2} e^{-\frac{1+2y}{1+3y}\frac{p^2}{\alpha^2}} + 3\left[(1+2y)(1+4y)\right]^{-3/2} e^{-\frac{1}{1+2y}\frac{p^2}{\alpha^2}} \right\} \tag{9}$$

We take $\alpha = 0.82\ \text{fm}^{-1}$, $\beta = 1.69\ \text{fm}^{-1}$ which gives $y = 4.25$;

with these values the r.m.s. radius, when CM and nucleon finite size -corrections are taken into account, is 1.60 fm. In figs. 1a and 1b are plotted the functions $F_J(q)$ and $n_J(p)$ respectively. Both are very close to the ones obtained in more realistic calculations. Notice, in particular, that owing to the smallness of the coefficient $(1+2y)^{-1}$ in the argument of the third gaussian of eq.(9), the momentum distribution $n_J(p)$ has a long queue beyond 2 fm^{-1}. A similar behaviour of n(p) has been recently found by Zabolitky and Ey [7].

The question now is : Is a Slater determinant able to reproduce simultaneously $F_J(q)$ and $n_J(q)$? For that purpose we take a Slater determinant depending on several parameters λ_i and denote by $F_{SD}(q)$ and $n_{SD}(p)$ the corresponding form factor and momentum distribution respectively. In order to perform an analysis based on a realistic situation, we shall consider $F_J(q)$ as known for a finite number of points $q_i (i=1,\dots,n)$ up to some maximum transfer momentum $q_n^2 = q_{max}^2$ and we attach to the points $F_J(q_i)$ error bars $\Delta F_J(q_i)$ which are of the same order of magnitude as the experimental ones [9] (see fig. 1a). We introduce the following quantity

$$\chi^2 = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{|F_J(q_i)| - |F_{SD}(q_i)|}{\Delta F_J(q_i)} \right)^2 \qquad (10)$$

and we retain all the Slater determinants satisfying the condition $\chi^2 < 1$ ($F_{SD}(q)$ compatible with $F_J(q_i) \pm \Delta F_J(q_i)$). We then consider the envelope of the momentum distributions $n_{SD}(p)$ corresponding to these Slater determinants. The compatibility (or incompatibility) of this envelope with the Jastrow momentum distribution $n_J(p)$ will consequently give an answer to the initial question.

In practice we have used Slater determinants built from a radial wave function which is a linear combination of three gaussians :

$$\phi(r) = \sum_{i=1}^{3} \lambda_i \, e^{-\frac{1}{2} \bar{\lambda}_i r^2} \tag{11}$$

This functional form gives rise to a one-body density

$$\rho_{SD}(\vec{r}_1,\vec{r}_2) \propto \sum_{k,\ell=1}^{3} \lambda_k \lambda_\ell \quad e^{-\frac{1}{2}(\bar{\lambda}_k r_1^2 + \bar{\lambda}_\ell r_2^2)} \tag{12}$$

that is well adapted to closely approach the correlated density (4). In the limit of no correlations ($\beta \to \infty$, $y \to \infty$) (12) coïncides with (4) with $\bar{\lambda}_1 = \alpha$ and $\lambda_1 = 1$, $\lambda_2 = \lambda_3 = 0$. Within this class of Slater determinants it is possible to separately reproduce very accurately the Jastrow form factor and momentum distribution. In fig. 1b is plotted $n_J(p)$ and the envelope of the momentum distributions $n_{SD}(p)$ corresponding to Slater determinants giving form factors compatible with $F_J(q)$ up to $q^2 = 20\ \mathrm{fm}^{-2}$. As can be seen, up to 2 fm^{-1} the agreement is very good, but for larger values of p the envelope of $n_{SD}(p)$ and $n_J(p)$ are not any more compatible. Thus we conclude that beyond $p \simeq 2\ \mathrm{fm}^{-1}$ the behaviour of the momentum distribution $n_J(p)$ corresponding to a correlated system cannot be reproduced by a single Slater determinant that fits the form factor $F_J(q)$. However we remark that this discrepancy is significantly smaller than the one found in ref.[7]. The difference between the momentum distributions $n_J(p)$ and $n_{SD}(p)$ is clearly reflected in the total kinetic energy $T = (\hbar^2/2m)\int n(p)p^2 d\vec{p}$ where large values of p may play an important role. In fact, the ratio T_J/T_{SD} is of the order of 1.5.

The same analysis has been performed by evaluating χ^2 for the form factor - eq.(10)- up to a lower value of q_{max} ($q^2_{max} = 7\ fm^{-2}$). The results are similar to the ones corresponding to $q^2_{max} = 20\ fm^{-2}$ (in particular Slater determinants reproducing $n_J(p)$ for low values of p are incompatible for p $\gtrsim 2\ fm^{-1}$). This indicates that the region at high q in the form factor does not play a major role in our analysis. Consequently we expect that additional uncertainties in the form factor at high values of q (due, for example, to the removal of the center of mass motion in the Slater determinant (11), (12), or to exchange current effects) will not introduce essential modifications in the results derived so far.

In summary, we give plausible arguments indicating that the presence of SRC may be inferred by studying simultaneously the form factor and the momentum distribution. Of course, in order to refine quantitatively our conclusions and to perform a realistic analysis of experimental data, the inclusion of center of mass corrections and of meson exchange current effects should be made. Work in these directions is in progress. We finally mention that the determination of the momentum distribution from experiment is by no means a solved problem. However progress in this field may be expected, in particular from experiments on projectile fragmentation with relativistic ^{4}He beams [10].

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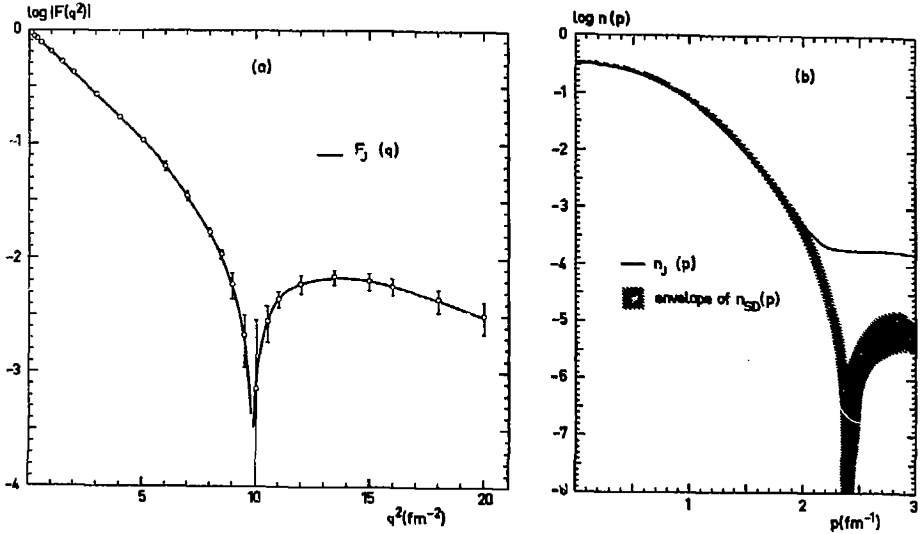
Figure Caption

(a) Continuous line : Jastrow form factor $F_J(q)$; open circles with error bars : values used in computing χ^2 , as explained in the text.

(b) Continuous line : Jastrow momentum distribution $n_J(p)$; dashed area : envelope of $n_{SD}(p)$, as explained in the text.



log |F(q²)|
(a)
— F_J (q)
q²(fm⁻²)
log n (p)
(b)
— n_J (p)
envelope of n_SD(p)
p(fm⁻¹)