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Finite Larmor Radius Stabilization of Ballooning Modes in Tokamaks

K. T. Tsang

MASTER

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Prepared by the
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
operated by
UNION CARBIDE CORPORATION
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CONTENTS

ABSTRACT

A ballooning mode equation that includes full finite Larmor radius effects has been derived from the Vlasov equation for a circular tokamak equilibrium. Numerical solution of this equation shows that finite Larmor radius effects are stabilizing.

I. INTRODUCTION

Recent interest in the stability of high beta tokamak equilibria leads to active investigation of magnetohydrodynamics ballooning modes.[1-4] According to magnetohydrodynamic theory, an unstable mode can develop and balloon in the bad curvature region of a tokamak when beta (= plasma pressure/magnetic field pressure) is higher than a certain critical value that depends on equilibrium. Moreover, this mode is more unstable when the toroidal mode number ℓ is large. However, in the large ℓ limit, the magnetohydrodynamic model fails because finite Larmor radius and kinetic effects become important. Thus, a careful inclusion of these effects in the ballooning eigenmode equation is necessary in order to obtain a more accurate value for the critical beta.

Chu et al.[5] investigated these kinetic effects through a modified energy principle. An approximated eigenmode equation was derived including finite Larmor radius and trapped particle effects. Because the eigenmode equation is no longer self-adjoint, the energy principle approach cannot retain the full finite Larmor radius effect. At the same time, Lee and Van Dam[3] derived a ballooning eigenequation from the Vlasov equation with a model collision operator that simulates finite electron conductivity. However, this equation contains no finite Larmor radius effect and can be reduced to the magnetohydrodynamic ballooning equation only in the low collision frequency limit.

A kinetic derivation of the ballooning equation from the gyro-averaged Vlasov equation is presented here. The equation is no longer self-adjoint when all finite Larmor radius effects are included. In the small finite Larmor radius limit, this equation reduces to the ideal magnetohydrodynamic ballooning equation. The advantage of this derivation is its relatively easy extension to include other kinetic effects properly. Connection to drift waves and other microinstabilities is also easier using this approach. A quadratic form can be constructed if finite Larmor radius terms from the neutrality condition are neglected. Thus it can be shown that the finite Larmor radius terms from Ampere's law are stabilizing. Numerical solution of the ballooning equation shows that the net effect of all finite Larmor radius terms is still stabilizing.

II. BALLOONING MODE EQUATION

For simplicity a simple tokamak equilibrium with circular concentric magnetic flux surfaces is employed. The extension of this derivation to general equilibria will be discussed in a future publication. The variables r, θ, and ζ stand for the radial, poloidal, and toroidal coordinates. Since ζ is the ignorable coordinate, all perturbed quantities X can be expressed as $X(r,\theta,\zeta,t) = X(r,\theta) \exp(-i\ell\zeta - i\omega t)$, where ℓ and ω are the toroidal mode number and frequency, respectively. The gyroaveraged Vlasov equation for the ion can be written as

$$(\omega - k_{\parallel} v_{\parallel} + \omega_{di}) g_i = \frac{e}{T_i} (\omega - \omega_{*i}) F_i J_0 \left(\phi - \frac{v_{\parallel}}{c} A_{\parallel}\right) , \qquad (1)$$

where

$$\omega_{di} = i\left(v_{\parallel}^2 + \frac{v_{\perp}^2}{2}\right)\left(\frac{1}{r} \cos\theta \frac{\partial}{\partial\theta} + \sin\theta \frac{\partial}{\partial r}\right) ,$$

$$J_0 = J_0(k_{\perp} v_{\perp}/\Omega_i) ,$$

$$\vec{k}_{\perp} = \hat{\theta} \frac{1}{ir} \frac{\partial}{\partial\theta} + \frac{\hat{r}}{i} \frac{\partial}{\partial r} , \text{ and}$$

$$k_{\parallel} = (-i\partial/\partial\theta - \ell q)/Rq .$$

Here F_i is the equilibrium distribution function for ions, ϕ is the perturbed electrostatic potential, $A_{\parallel}$ is the parallel component of the perturbed vector potential, g_i is related to the perturbed distribution function f_i by $f_i = -e\phi F_i/T_i + \exp(iL)g_i$, $L = \int^{\phi} d\phi\, \vec{k}_{\perp} \cdot \vec{v}_{\perp}/\Omega_i$, $\Omega_i = eB/M_i c$, $\omega_{*i} = \ell q c T_i/eBrL_n$, $L_n^{-1} = \partial \ln N/\partial r$, q is the safety factor, and the rest of the notation is standard. Decomposing Eq. (1) to its Fourier components, we have

$$\begin{aligned}(\omega - k_{\parallel m}v_{\parallel})g_m + v_{di}[(m - 1)g_{m-1}/r \\ + (m + 1)g_{m+1}/r - \partial(g_{m-1} - g_{m+1})/\partial r] \\ = e(\omega - \omega_{*c})J_0F_i(\phi_m - v_{\parallel}A_{\parallel m}/c)/T_i \ , \qquad (2)\end{aligned}$$

where $k_{\parallel m} = (m - \ell q)/Rq$, $v_{di} = (v_{\parallel}^2 + v_{\perp}^2/2)/\Omega_i R$, $\vec{k}_{\perp} = \hat{\theta}m/r - i\hat{r}\partial/\partial r$, and the Fourier component of a perturbed quantity $X(r,\theta)$ is defined by

$$X_m(r) = \oint (d\theta/2\pi)X(r,\theta)\exp(-im\theta) \ .$$

Equation (2) displays the familiar poloidal mode coupling due to the normal and the geodesic curvature of the magnetic field lines. At this point, we introduce the radial translational invariance[6] assumption in the large m limit:

$$X_{m+j}(x) = X_m(x - j\Delta) \ , \qquad (3)$$

where $x = r - r_m$, $\Delta = 1/\ell q$ is the separation between adjacent mode rational surfaces, and $r = r_m$ is the rational surface on which $k_{\parallel m} = 0$. A general phase factor in Eq. (3) is omitted because this factor equals unity for the most unstable mode. Equation (3) can be shown to be equivalent to the ballooning mode formulism[2-4] used in magnetohydrodynamic calculations. Applying Eq. (3), we can rewrite Eq. (2) as

$$(\omega - k_{\parallel m} v_{\parallel}) g_m(x) + v_{di}\{(m-1) g_m(x+\Delta)/r + (m+1) g_m(x-\Delta)/r$$
$$- \partial[g_m(x+\Delta) - g_m(x-\Delta)]/\partial r\}$$
$$= e(\omega - \omega_{*i}) J_0 F_i [\phi_m(x) - v_{\parallel} A_{\parallel m}(x)/c]/T_i \ . \qquad (4)$$

To avoid the advanced and retarded arguments in g_m, we transform Eq. (4) to the k_x space. Recognizing $k_{\parallel m} = k_{\parallel}' x$, with $k_{\parallel}' = \ell \hat{s}/Rr$ and $\hat{s} = r\hat{q}/r$, we have

$$[\omega - i k_{\parallel}' v_{\parallel} \partial/\partial k_x + v_{di}(k_\theta \cos k_x \Delta + k_x \sin k_x \Delta)]\hat{g}_m$$
$$= e(\omega - \omega_{*i}) J_0 F_i [\hat{\phi}_m - v_{\parallel} \hat{A}_{\parallel m}/c]/T_i \ , \qquad (5)$$

where

$$\hat{X}_m = (2\pi)^{-1/2} \int_{-\infty}^{\infty} dx\, X_m(x) \exp(-i k_x x),$$

$$k_\theta = m/r,$$

$$\vec{k}_\perp = k_\theta \hat{\theta} + k_x \hat{r},$$

and we have ignored one compared with m. Equation (5) is now a simple first-order differential equation which can be solved in principle to obtain $\hat{g}_m$. Rather than solving Eq. (5) now, we take the zero<u>th</u> moment of Eq. (5) and add it to the corresponding moment equation for electrons. The resulting equation is

$$\omega \int d\vec{v}[\hat{g}_i \exp(iL) - \hat{g}_e] - i k_{\parallel}' \frac{\partial}{\partial k_x} \int d\vec{v}\, v_{\parallel} [\hat{g}_i \exp(iL) - \hat{g}_e]$$
$$+ (k_\theta \cos k_x \Delta + k_x \sin k_x \Delta) \int d\vec{v}[v_{di} \hat{g}_i \exp(iL) - v_{de} \hat{g}_e]$$
$$= eN\hat{\phi}[\omega - \omega_{*e} + \tau(\omega - \omega_{*i})\Gamma_0]/T_e \ , \qquad (6)$$

where

$$\Gamma_0 = I_0(b)\exp(-b),$$

$$b = (k_x^2 + k_\theta^2)\rho_i^2 ,$$

$$\rho_i^2 = T_i M_i (c/eB)^2,$$

$$\omega_{*e} = -\tau\omega_{*i},$$

$$\tau = T_e/T_i,$$

and the subscript m has been understood for simplicity. Notice that $\hat{A}_\parallel$ disappears from Eq. (6) because we assume there is no equilibrium parallel ion flow and electron current is ignored. Using the neutrality condition

$$-e\hat{\phi}N/T_i + \int d\vec{v}\hat{g}_i \exp(iL) = e\hat{\phi}N/T_e + \int d\vec{v}\hat{g}_e ,$$

we can rewrite Eq. (6) as

$$\begin{aligned} k_\parallel' \partial/\partial k_x & \left\{ \int d\vec{v}\, v_\parallel [\hat{g}_i \exp(iL) - \hat{g}_e] \right\} \\ & = -i(k_\theta \cos k_x\Delta + k_x \sin k_x\Delta) \int d\vec{v} [v_{di}\hat{g}_i \exp(iL) - v_{de}\hat{g}_e] \\ & + iNe\hat{\phi}(\omega - \omega_{*i})(\Gamma_0 - 1)/T_i . \end{aligned} \quad (7)$$

The left-hand side is related to the perturbed parallel current $\hat{j}_\parallel$, which can be expressed in terms of $\hat{A}_\parallel$ through Ampere's law:

$$(k_x^2 + k_\theta^2)\hat{A}_\parallel = 4\pi\hat{j}_\parallel/c = 4\pi e\left\{ \int d\vec{v}\, v_\parallel [\hat{g}_i \exp(iL) - \hat{g}_e] \right\}/c . \quad (8)$$

Combining Eqs. (7) and (8), we have

$$(\partial/\partial k_x)[(k_x^2 + k_\theta^2)\hat{A}_\parallel]$$
$$= 4\pi ie\{-(k_\theta \cos k_x\Delta + k_x \sin k_x\Delta)\int d\vec{v}[v_{di}\hat{g}_i \exp (iL) - v_{de}\hat{g}_e]$$
$$+ Ne\hat{\phi}(\omega - \omega_{*i})(\Delta_0 - 1)/T_i\}/ck_\parallel' \ . \quad (9)$$

Equation (9) is an equation between $\hat{A}_\parallel$ and $\hat{\phi}$ because $\hat{g}_i$ and $\hat{g}_e$ can be expressed by $\hat{\phi}$ and $\hat{A}_\parallel$. Treating ions in the fluid limit but electron transit frequency as much higher than ω, we have

$$\hat{g}_i = e(1 - \omega_{*i}/\omega)F_iJ_0(\hat{\phi} - v_\parallel\hat{A}_\parallel/c)/T_i \ , \quad (10a)$$

$$ik_\parallel' \ \partial g_e/\partial k_x = -e(\omega - \omega_{*e})F_e\hat{A}_\parallel/cT_e \ . \quad (10b)$$

Equation (10a) can be obtained from Eq. (5) by assuming $\omega >> k_\parallel v_i, kv_{di}$. Equation (10b) is obtained by assuming $k_\parallel v_e >> \omega, kv_{de}$ in the corresponding electron equation.

The perturbed ion density $\tilde{N}_i$ is given by

$$\tilde{N}_i/N = e[-1 + (1 - \omega_{*i}/\omega)\Gamma_0]\hat{\phi}/T_i \ . \quad (11)$$

The perturbed electron density $\tilde{N}_e$ is

$$\tilde{N}_e/N = e\hat{\phi}/T_e - e(\omega - \omega_{*e})\hat{A}_\parallel/k_\parallel cT_e \ . \quad (12)$$

Thus the neutrality condition can be written as

$$\hat{\phi} - \omega A_\parallel/k_\parallel c = \tau(\omega - \omega_{*i})(\Gamma_0 - 1)\hat{\phi}/(\omega - \omega_{*e}) \ . \quad (13)$$

Transformed to k_x space, Eq. (13) becomes

$$\omega\hat{A}_\parallel/c = ik_\parallel'(\partial/\partial k_x)\{[1 - \tau(\omega - \omega_{*i})(\Gamma_0 - 1)\phi/(\omega - \omega_{*e})]\hat{\phi}\} \ . \qquad (14)$$

It is interesting to observe that if finite Larmor radius effects are ignored in Eq. (13) then we have $\phi - \omega A_\parallel/k_\parallel c = 0$, which means the perturbed parallel electric field $\tilde{E}_\parallel = 0$ because $\omega\phi - \omega A_\parallel/c \sim \tilde{E}_\parallel$. Therefore Eq. (13) is the extension of the ideal magnetohydrodynamics assumption $\vec{E} + \vec{v} \times \vec{B}/c = 0$ to include finite gyroradius effects.[7]

Substituting Eqs. (10) in Eq. (9), we get

$$(\partial/\partial k_x)(k_x^2 + k_\theta^2)\hat{A}_\parallel$$
$$= (c/\omega)\{\beta(R/L_n)(q/\hat{s})^2[\cos k_x\Delta + (k_x/k_\theta)\sin k_x\Delta]\lambda$$
$$+ \omega(\omega - \omega_{*i})(\Gamma_0 - 1)/\Omega_0^2\}\hat{\phi} \ , \qquad (15)$$

where

$\lambda = 1 - (\omega/\omega_{*i} - 1)[\Gamma_0 - 1 - b(\Gamma_0 - \Gamma_1)/2]/(1 + 2)$,

$\Omega_0 = k_\theta\lambda_{Di}c\hat{s}/Rq$,

$\lambda_{Di} = (T_i/4\pi Ne^2)^{1/2}$, and

$\beta = 8\pi N(T_i + T_e)/B^2$.

Combining Eqs. (14) and (15), we obtain the following eigenmode equation:

$$(\partial/\partial k_x)(k_x^2 + k_\theta^2)(\partial/\partial k_x)[1 - \tau(\omega - \omega_{*i})(\Gamma_0 - 1)/(\omega - \omega_{*e})]\hat{\phi}$$

$$= \{\beta(R/L_n)(q/\hat{s})^2[\cos k_x\Delta + (k_x/k_\theta) \sin k_x\Delta]\lambda$$

$$+ \omega(\omega - \omega_{*i})(\Gamma_0 - 1)/\Omega_0^2\}\hat{\phi} \ . \tag{16}$$

Aside from more subtle kinetic effects, such as coupling to ion acoustic waves and electron resonance, Eq. (16) includes all finite Larmor radius effects through ω_* and Γ_0. In the limit of small gyroradius, $(k_\theta \rho_i, k_x \rho_i) \ll 1$, Eq. (16) reduces to

$$(\partial/\partial k_x)(k_x^2 + k_\theta^2)(\partial/\partial k_x)\hat{\phi}$$

$$= \{\beta(R/L_n)(q/\hat{s})^2[\cos k_x\Delta + (k_x/k_\theta) \sin k_x\Delta]$$

$$+ \omega(\omega - \omega_{*i})(k_x^2 + k_\theta^2)\rho_i^2/\Omega_0^2\}\hat{\phi} \ . \tag{17}$$

When ω_{*i} is further neglected, Eq. (17) is equivalent to the ideal magnetohydrodynamics ballooning equation. This equivalence is best demonstrated by comparing Eq. (17) with Eq. (24) of Connor et al.[4] and identifying $k_x/\hat{s}k_\theta$ with the variable y employed by Connor et al. In Eq. (16), the term $\omega(\omega - \omega_{*i})$ on the right-hand side agrees with both Chu et al.[5] and Lee and Van Dam.[3] However, Chu et al.[5] could not include all the finite Larmor radius effects because the eigenmode equation is no longer self-adjoint; hence the full finite Larmor radius modification cannot be retained in an energy principle approach. Lee and Van Dam[3] followed a kinetic approach with a model collision operator, but their equation contains no finite Larmor radius effects.

III. QUADRATIC FORMS

A simple conclusion can be drawn from Eq. (17) by construction of a quadratic form. However, trouble arises from the finite Larmor radius term introduced by the neutrality condition, which appears in the magnetic field line bending terms [left-hand side of Eq. (16)]. This finite Larmor radius term changes the differential operator to a non-self-adjoint one. If this finite Larmor radius term is neglected (this is justified a posteriori by numerical solutions), the differential operator in Eq. (16) becomes self-adjoint and the following quadratic form can be constructed:

$$\omega(\omega - \omega_{*i})\Omega_0^{-2}A_0 = A_1 + \beta(R/L_n)(q/\hat{S})^2 A_2 \ , \tag{18}$$

where

$$A_0 = \int_{-\infty}^{\infty} Ak_x(1 - \Gamma_0)|\hat{\phi}|^2 \geqslant 0 \ ,$$

$$A_1 = \int_{-\infty}^{\infty} Ak_x(k_x^2 + k_\theta^2)|\partial\hat{\phi}/\partial k_x|^2 \geqslant 0 \ ,$$

$$A_2 = \int_{-\infty}^{\infty} Ak_x[\cos k_x\Delta + (k_x/k_\theta)\sin k_x\Delta]\lambda \ .$$

Solutions of Eq. (18) are

$$\omega/\Omega_0 = \omega_{*i}/2\Omega_0 \pm \{(\omega_{*i}/2\Omega_0)^2 + [A_1 + \beta(R/L_n)(q/\hat{S})^2 A_2]/A_0\}^{1/2} \ . \tag{19}$$

From Eq. (19) it is clear that the only possible source of instability is the beta term; all other terms inside the square root are positive except L_n. We assume (and verify later) that A_2 is positive. An expression for the critical beta, β_c, above which instability occurs can be obtained from Eq. (19),

$$\beta_c = -[(\omega_{*i}/2\Omega_0)^2 + A_1/A_0](L_n/R)(\hat{S}/q)^2 . \tag{20}$$

This expression for β_c assumes a form similar to a numerical result obtained from magnetohydrodynamic calculation,[8]

$$\beta_c = (\text{form factor})\varepsilon/q^2 ,$$

where ε is the inverse aspect ratio. Equation (20) also tentatively verifies the conjecture that the effect of ω_{*i} is stabilizing. If we take $\omega_{*i} = 0$, then ω^2 is a real number (as in the case of ideal magnetohydrodynamics). If the finite Larmor radius modification to the field line bending term is not negligible, Eq. (16) is not self-adjoint and no useful information can be extracted from the quadratic form. Then we must resort to numerical solutions.

IV. NUMERICAL SOLUTION

Since Eq. (16) is even in k_x, $\hat{\phi}$ can assume either even or odd solutions (in k_x). We solve Eq. (16) by a standard shooting method. Figures 1 and 2 show the most unstable even and odd solutions, respectively, for $k_\theta \rho_i = 0.5$, $q = 2$, $\tau = 1$, $L_n/R = -0.2$, $\hat{s} = 0.5$, and $\beta = 0.2$. For this high beta value, the mode is clearly unstable. Furthermore, these numerical solutions show that the even mode (in k_x) is more unstable than odd modes (in k_x), which agrees with results from fluid calculation.

When beta decreases, the growth rate of the most unstable even mode reduces until finally a critical beta is reached such that the mode becomes marginally stable. This critical beta is plotted in Fig. 3 as a function of $\hat{s}$ for $k_\theta \rho_i = 0.1$, $q = 2$, $\tau = 1$, and $L_n/R = -0.2$. The long-dashed line in Fig. 3 represents the ideal magnetohydrodynamics model in which Eq. (17) is used with ω_{*i} set equal to zero. The short-dashed line is obtained by using Eq. (17) and retaining the effect of ω_{*i}. The result of Eq. (16) is shown by the solid line. Figure 3 shows that the ideal magnetohydrodynamics model is slightly pessimistic. Including a nonzero ω_{*i} in Eq. (17) is indeed more stable, as expected from Eq. (20). This effect is also noted by Chu et al.[5] The full effect of the finite Larmor radius is also stabilizing, and this does not agree with the conclusion of Chu et al., due to their energy principle approach. Figure 3 also shows the stabilizing effect of shear on the critical beta, which also agrees with the fluid result for similar equilibria.

In Fig. 4, we show the dependence of the critical β_i ($= 8\pi NT_i/\beta^2$) on $k_\theta \rho_i$, for $\hat{s} = 0.5$, $L_n/R = -0.2$, $\tau = 1$, and $q = 2$. The critical β_i has a minimum around $k_\theta \rho_i = 0.03$. Any value of $k_\theta \rho_i$ below or above this value is more stable. It is interesting to note that this behavior at small $k_\theta \rho_i$ agrees with magnetohydrodynamics theory, though we should keep in mind that at this low $k_\theta \rho_i$, Eq. (16) is no longer valid, because the ballooning mode formulism holds only in the high m limit.

V. CONCLUSIONS

A collisionless ballooning mode equation, which includes full finite Larmor radius effects and reduces to the corresponding ideal magnetohydrodynamics equation in the small Larmor radius limit, has been derived from the Vlasov equation for a circular concentric tokamak equilibrium. A quadratic form can be constructed if finite Larmor radius terms from the neutrality condition are neglected. This quadratic form shows that the ω_{*i} term is stabilizing. Numerical solution of the ballooning equation verifies this and also verifies that the full finite Larmor radius effect is also stabilizing. The adiabatic electron and fluid ion model is employed here. More subtle kinetic effects, such as electron resonance, ion acoustic effects, and magnetic drifts in the neutrality condition, as well as a self-consistent tokamak equilibrium, must be included before the critical beta values for present or future tokamaks can be determined.

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FIGURE CAPTIONS

FIG. 1. Real part of $\hat{\phi}(k_x)$ for the most unstable even (in k_x) mode. The parameters used are $k_\theta \rho_i = 0.5$, $q = 2$, $\beta = 0.2$, $\tau = 1$, $L_n/R = -0.2$, and $\hat{s} = 0.5$. The eigenfrequency is $\omega/\Omega_0 = -1.17 + i6.59$.

FIG. 2. Real part of $\hat{\phi}(k_x)$ for the most unstable odd (in k_x) mode, with the same parameters as in Fig. 1. The eigenfrequency is $\omega/\Omega_0 = -1.75 + i3.76$.

FIG. 3. Critical $\beta_i = 8\pi NT_i/B^2$ vs $\hat{s}$ for $k_\theta \rho_i = 0.1$, $L_n/R = -0.2$, $q = 2$, and $\tau = 1$, assuming (a) Eq. (17) with $\omega_{*i} = 0$ (long dashes), (b) Eq. (17) with $\omega_{*i} \neq 0$ (short dashes), and (c) Eq. (16) (solid line).

FIG. 4. Critical β_i vs $k_\theta \rho_i$ for $\hat{s} = 0.5$, $L_n/R = -0.2$, $q = 2$, and $\tau = 1$, assuming Eq. (16).

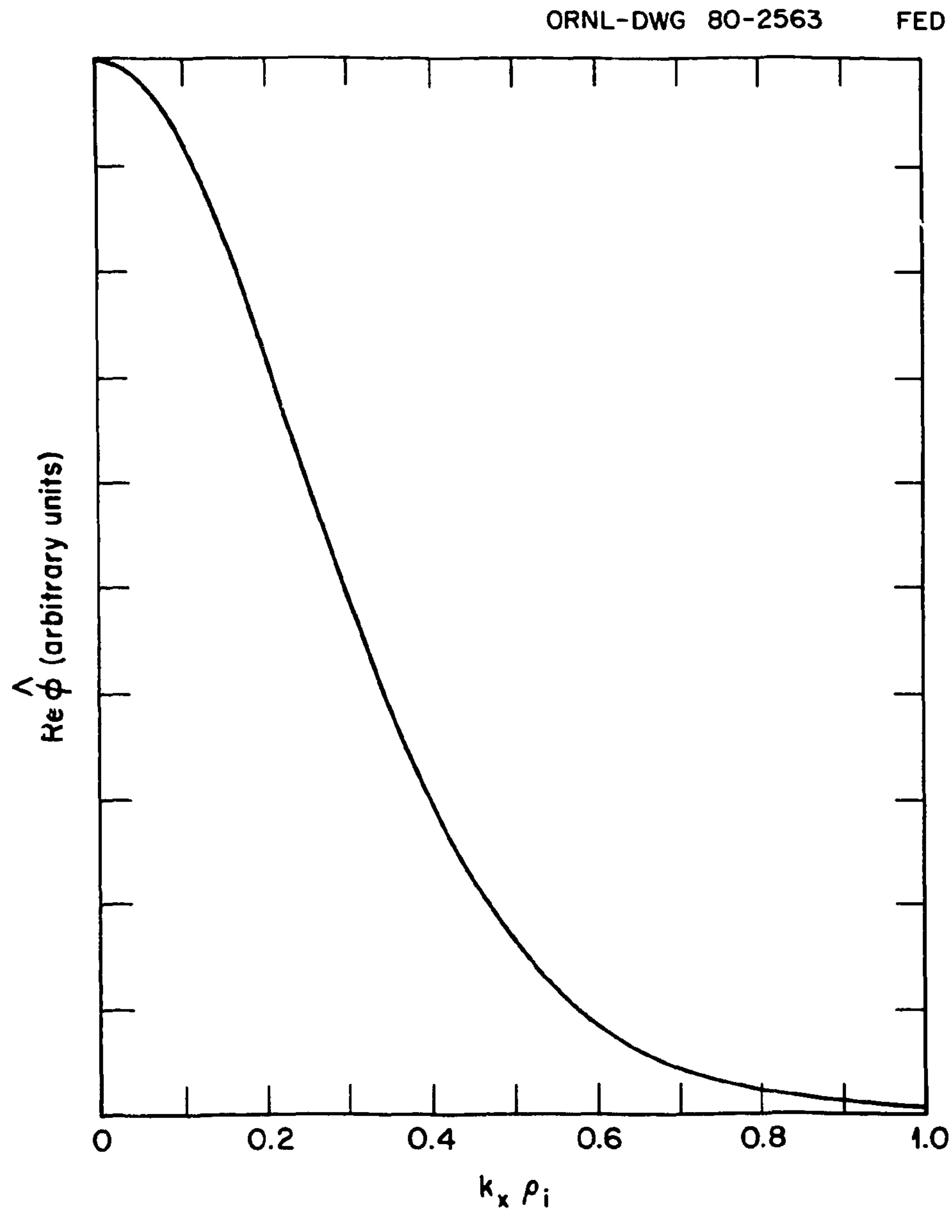
ORNL-DWG 80-2563 FED
Re $\hat{\phi}$ (arbitrary units)
0
0.2
0.4
0.6
0.8
1.0
$k_x \rho_i$

Fig. 1.

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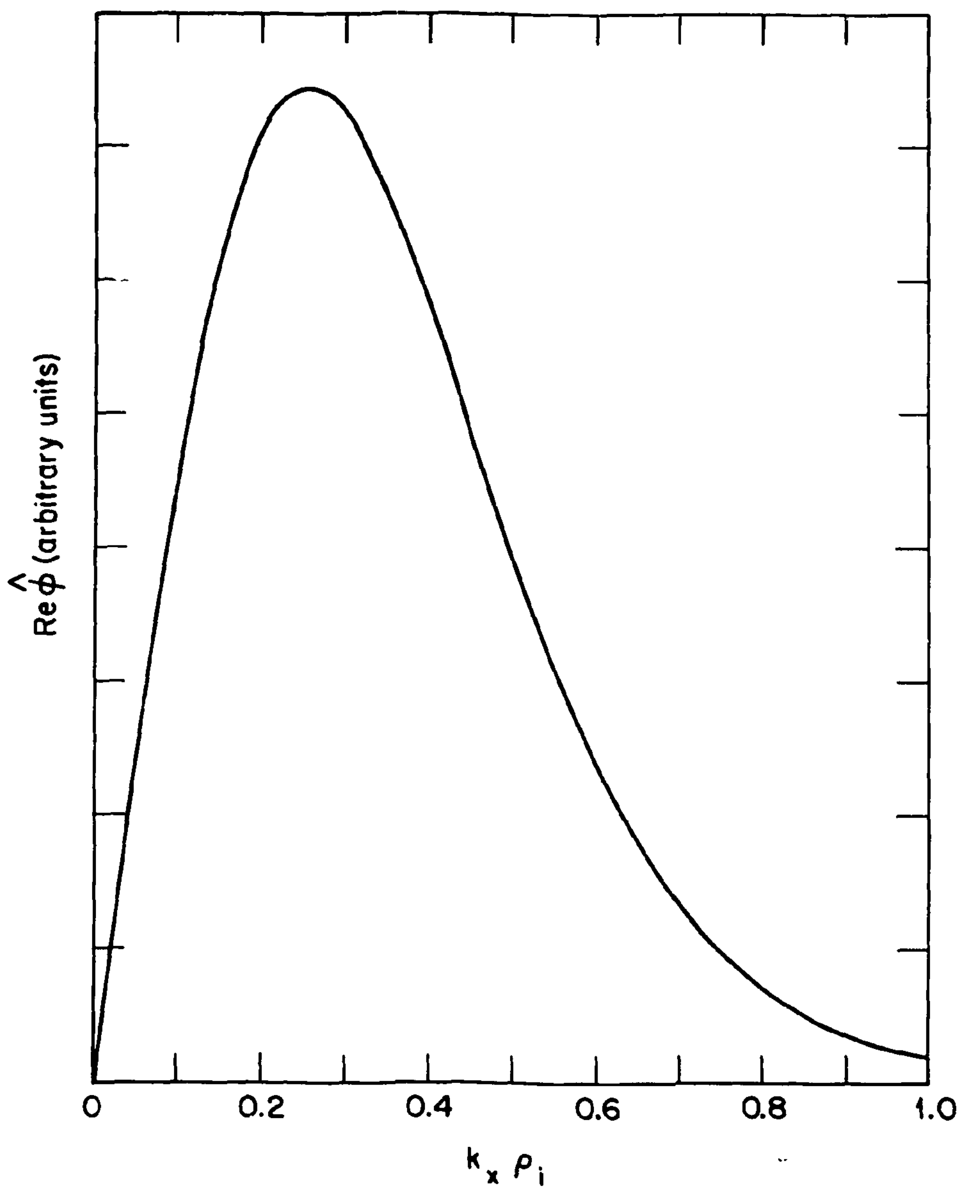
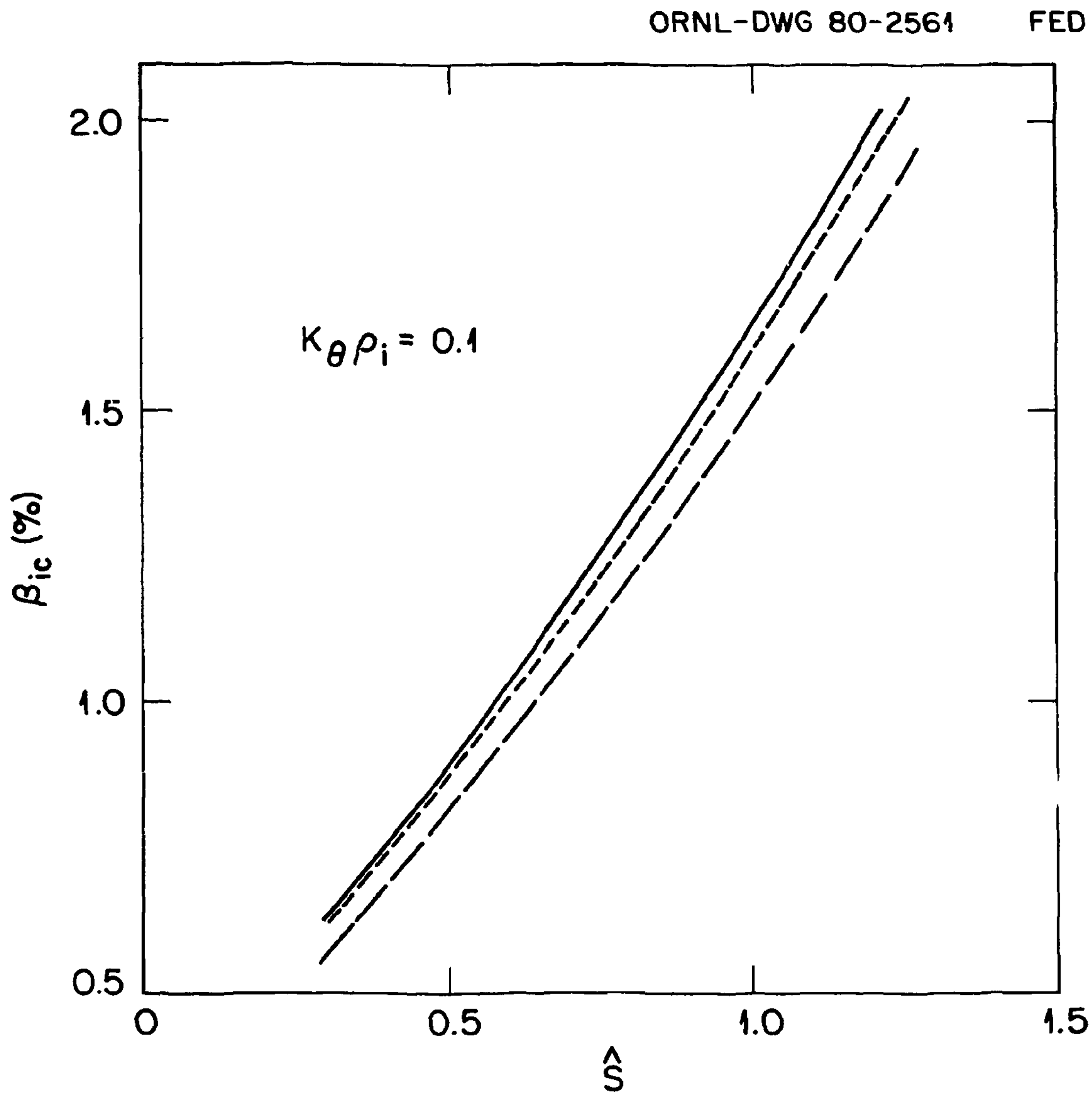


Fig. 2.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2.0
1.5
1.0
0.5
β_{ic} (%)
$K_\theta \rho_i$ = 0.1
0
0.5
1.0
1.5
$\hat{s}$

Fig. 3.

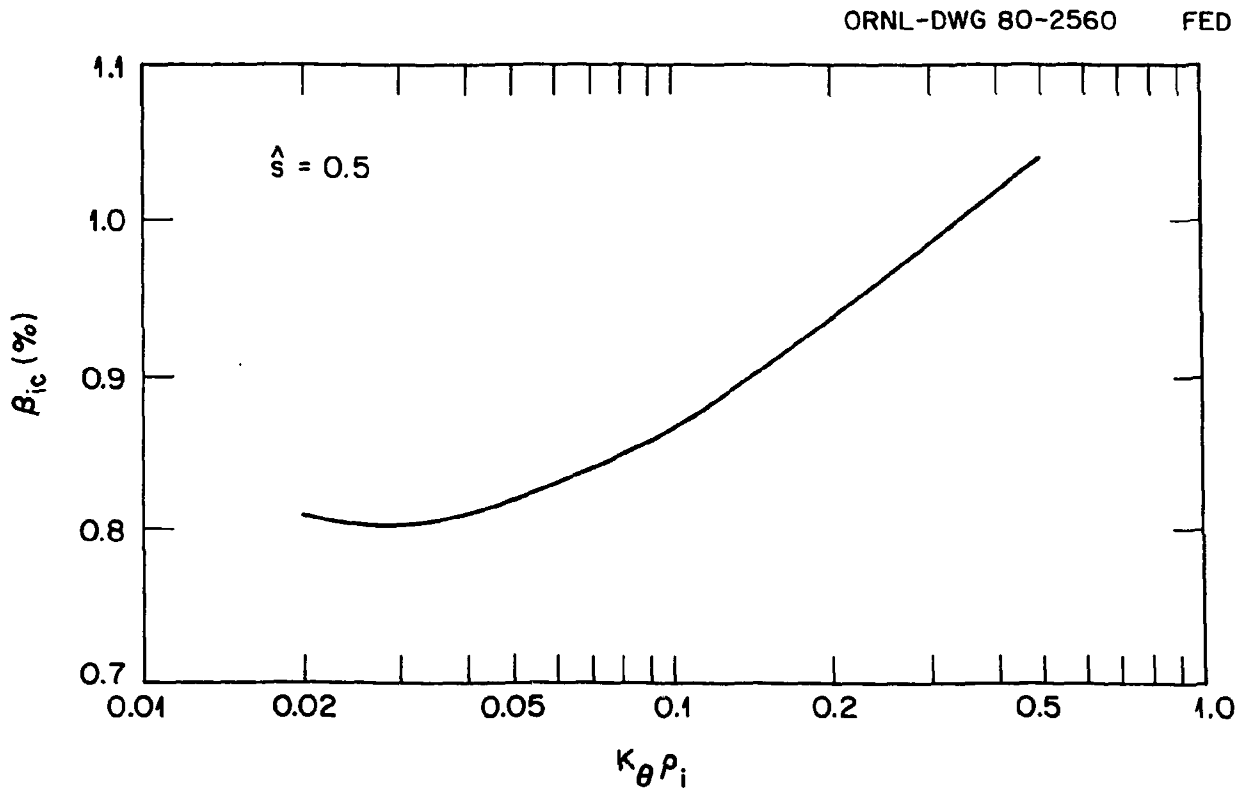
ORNL-DWG 80-2560 FED
ŝ = 0.5
1.1
1.0
0.9
0.8
0.7
β_ic (%)
0.01
0.02
0.05
0.1
0.2
0.5
1.0
K_θ ρ_i

Fig. 4.