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Neutrino Mixing and Oscillation in a Grand Unified Field Theory SO(10)*

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1. Introduction

In view of the empirical possibility that neutrinos appear to be massive,[1] it is timely to investigate neutrino mixing and oscillation[2] in the framework of a grand unified field theory, analogously to fermion mixing. There is at present, interest in obtaining relations among the masses of fermions and fermion mixing angles in the framework of grand unified field theories. In particular, one can obtain ratios between the charged-lepton masses and down-quark masses[3] $9m_e/m_d = m_\mu/m_s = 3m_\tau/m_b$, which are roughly consistent with observation at a mass scale of 10^2 GeV. The quark mixing angles given in the Kobayashi-Maskawa (K-M) form[4] are predicted to be[5] $\sin\theta_1 = \sin\theta_c = (m_d/m_s)^{\frac{1}{2}}$, $\sin\theta_2 = -(m_c/m_t)^{\frac{1}{2}}$, and $\sin\theta_3 = -(m_u/m_t)^{\frac{1}{2}}(\sin\theta_c)^{-1}$.

In the standard $SU(2)\times U(1)$ gauge model the fermions are grouped into three families

$$\begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix}\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}, \tag{1}$$

where the t quark is assumed to have a mass larger than 17 GeV. The d and s quarks are mixed and the linear combinations

$$\begin{aligned} d_\theta &= d\cos\theta_c + s\sin\theta_c \\ s_\theta &= -d\sin\theta_c + s\cos\theta_c \end{aligned} \tag{2}$$

are formed, where θ_c is the Cabibbo angle. If the neutrinos are massive, they may mix in a similar manner. Before we proceed to the calculation of the neutrino mixing in SO(10) gauge model[6] in which the aforementioned fermion mass ratios and mixing angles hold, we call your attention to the excellent review articles on ν mixing and oscillation by Bilenky and Pontecorvo[7] and De Rujula et al.[8] For completeness, we introduce some concepts and notations.

Suppose the neutrinos $\alpha = \nu_e, \nu_\mu, \nu_\tau$ in the weak isospin states are orthogonal superposition of fields ν_i, $i = 1,2,3$, of definite and finite different masses m_i

$$\nu_\alpha = U_{\alpha i} \nu_i , \tag{3}$$

where $U_{\alpha i}$ can be parametrized in the K-M form. If a beam of ν_α is produced in a weak process, at a time t the beam will be a coherent superposition of ν_β, that is, there arises an oscillation, $\nu_\alpha \rightleftarrows \nu_\beta$.

Let $V_{\alpha\beta}$ be the probability amplitude for ν_α to oscillate into ν_β at time t or path length R,

$$V_{\alpha\beta} = \sum_i U_{\alpha i} U^\dagger_{\beta i} e^{-iE_i t/\hbar} . \tag{4}$$

At $t = 0$

$$V_{\alpha\beta} = \sum_i U_{\alpha i} U^\dagger_{\beta i} = \delta_{\alpha\beta} . \tag{5}$$

Note from Eq. (4) that ν_α changes to ν_i and propagates with definite mass m_i and then changes back to ν_β. The probability of oscillation is

$$P_{\alpha\beta} = |V_{\alpha\beta}|^2 . \tag{6}$$

Let ν_j have a mass very much different from the other ν_i. Then we may rewrite Eq. (4) with the aid of Eq. (5) as

$$\begin{aligned} V_{\alpha\beta} &= \sum_{i \neq j} U_{\alpha i} U^\dagger_{\beta i} e^{-iE_i t/\hbar} + U_{\alpha j} U^\dagger_{\beta j} e^{-iE_j t/\hbar} \\ &= e^{-iE_j t/\hbar} \Big[\delta_{\alpha\beta} - \sum_{i \neq j} U_{\alpha i} U^\dagger_{\beta i} (1 - e^{-i\Delta_{ij}}) \Big] , \end{aligned} \tag{7}$$

where

$$|E_i - E_j| t/\hbar = |m_i^2 - m_j^2| R/2E\hbar c \equiv \Delta_{ij} \equiv 2\pi R/L \tag{8}$$

Here $L = 4\pi E \hbar c / |m_i^2 - m_j^2|$ is the oscillation length and oscillation effect is observable only if $L \leq R$.

2. $\nu_e - \nu_\tau$ Mixing

As an illustration, consider the toy model in which ν_e and ν_τ mix,

$$\begin{aligned} \nu_e &= \nu_1 \cos\theta + \nu_3 \sin\theta , \\ \nu_\tau &= -\nu_1 \sin\theta + \nu_3 \cos\theta , \end{aligned} \tag{9}$$

where θ is the mixing angle. Put $\alpha = \nu_e$ and $\beta = \nu_\tau$ in Eqs.(6) and (7)

$$P_{\nu_e \nu_\tau} = |V_{\nu_e \nu_\tau}|^2 = |U_{\nu_e 1} U^{+}_{\nu_\tau 1} (1 - e^{-i\Delta_{13}})|^2 \tag{10}$$

We obtain from (9) and (10)

$$P_{\nu_e \nu_\tau} = \sin^2 2\theta \sin^2(\Delta_{13}/2) \quad . \tag{11}$$

In a similar way from (7)-(11), one obtains

$$\begin{aligned} P_{\nu_e \nu_e} = \left| 1 - (1 - e^{-i\Delta_{13}}) |U_{\nu_e 1}|^2 \right|^2 &= 1 - \sin^2 2\theta \sin^2(\Delta_{13}/2) \\ &= 1 - P_{\nu_e \nu_\tau} \quad . \end{aligned} \tag{12}$$

The measured result of Ref. 1 for the Avignone spectrum is

$$\begin{aligned} R &= (\bar{\sigma}_{ccd} / \bar{\sigma}_{ncd})_{exp} / (\bar{\sigma}_{ccd} / \bar{\sigma}_{ncd})_{theory} \\ &= 0.43 \pm 0.17 , \end{aligned} \tag{13}$$

where $\bar{\sigma}_{ccd}$ and $\bar{\sigma}_{ncd}$ are the spectrum averaged cross sections of $\bar{\nu}_e + d \to n + n + e^+$ and $\bar{\nu}_e + d \to n + p + \bar{\nu}_e$, respectively. We may regard the deviation of the value of R in (13) from 1 as due to the second term on the right-hand-side of (12). We find from Eq.(8)

$$\Delta_{13} = 2.54\,|m_1^2 - m_3^2|\,R/E \tag{14}$$

where in case of Ref. 1, $R = 11.2$ meters, $E = 4.6 - 7.6$ MeV, and $|m_1^2 - m_3^2|$ is in $(\mathrm{eV})^2$. In order to get a rough idea of the size of $|m_1^2 - m_3^2|$ let us put in (12) and (14) $\sin^2 2\theta = 0.5$, and $E = 6.1$ MeV and allow one standard deviation from the measured value of (13) $0.26 < 1 - \frac{1}{2}\sin^2 2.33\,|m_1^2 - m_3^2| < 0.6$ which leads to $0.47(\mathrm{eV})^2 < |m_1^2 - m_3^2| < 0.67(\mathrm{eV})^2$. An accurate evaluation by Barger et al.[9] for $\sin^2 2\theta = 0.5$ yields $0.5(\mathrm{eV})^2 < |m_1^2 - m_3^2| < 0.85(\mathrm{eV})^2$.

3. Calculation of Mixing

We briefly review the scenario of calculating the fermion mixing in gauge models. Choose a gauge group G and write a Yukawa interaction between fermions and Higgs scalars invariant under G. Impose a discrete symmetry to reduce the coupling parameters and to assure that higher order corrections do not change the results from the tree approximation. Assume spontaneous symmetry breakdown by requiring a non-zero vacuum expectation value of the Higgs scalars. It is usually required that the number of parameters in the resulting mass term is then equal to the number of masses in the mass matrix. The resultant mass terms for up- and down-quarks M^U and M^D, respectively, are written as

$$\bar{\psi}_1^0 M^U \psi_1^0 + \bar{\psi}_2^0 M^D \psi_2^0 + \mathrm{H.c.}, \tag{15}$$

where $\psi_1^0 = (u,c,t)$ and $\psi_2^0 = (d,s,b)$. The mass matrices M^U and M^D are diagonalized by a biunitary transformation,

$$V_i^{\dagger} M U_i = M^{\text{diagonal}} \tag{16}$$

The left-handed charged current is given by

$$J_{\mu L} = \bar{\Psi}_{1L}^{0} \gamma_\mu \Psi_{2L}^{0} = \bar{\Psi}_{1L} V_1^{\dagger} U_2 \gamma_\mu \Psi_{2L} = \bar{\Psi}_{1L} \Gamma \gamma_\mu \Psi_{2L} . \tag{17}$$

The mixing angles appear in $\Gamma = V_1^{\dagger} U_2$ that is expressed in the K-M form. The quark mixing angles and fermion mass ratios mentioned in the Introduction has been obtained in SU(5)[3,10] and SO(10).[5]

If the neutrinos are massive, then it is natural to extend the previous calculations to up-quarks and neutrinos. A straightforward extension leads to $m_{\nu_e} \sim m_u$, $m_{\nu_\mu} \sim m_c$, and $m_{\nu_\tau} \sim m_t$ which is absurd. It was noted, however, by Gell-Mann et al.[11] that these relations can be modified by introducing a heavy Majorana mass for the right-handed neutrino ν_R. See further discussions by Barbieri et al. and Witten.[12]

In SU(5) gauge model,[13] the fermions of each generation are in $\bar{\underset{\sim}{5}}_L \sim d^c_{iL}, e_L, \nu_L$ and $\underset{\sim}{10}_L \sim u^c_{iL}, u_{iL}, d_{iL}, e^c_L$, where $i = R,G,B$. There is no ν_R so there is no Dirac mass term $\bar{\nu}_R \nu_L$. In the simple version, baryon minus lepton number is conserved so there is no $\nu_L \nu_L$ term. One can introduce a ν_R in a SU(5) singlet but it appears more attractive to adopt SO(10) in which the fermions are in $\underset{\sim}{16}_L$. The SU(5) decomposition is $\underset{\sim}{16} = \bar{\underset{\sim}{5}} + \underset{\sim}{10} + \underset{\sim}{1}$ so that the $\nu_R = \overline{\nu_L^c}$ appears in a natural way. We assign ν_R a large mass, then let us see what happens. The mass matrix for each generation of neutrinos is

	ν_L	ν_L^c
ν_L	0	m
ν_L^c	m	A

where m is the Dirac mass related to the up-quark mass and A is the Majorana mass and c means charge conjugate. The characteristic equation is $\lambda^2 - \lambda A - m^2 = 0$, and the eigenvalues are A, $-m^2/A$. There is a large mass A and another mass m^2/A (the minus sign may be transformed away) that may be of the order of the neutrino masses. For example, $m^2/A \sim 0.6$ eV for $m = m_t = 25$ GeV and $A = 10^{12}$ GeV. We suppose the neutrinos acquire masses via this Gell-Mann-Ramond-Slansky mechanism.[11]

Let us consider the following Yukawa interaction between quarks and Higgs scalars:[14]

$$\mathcal{L} = (A\,16_1 16_2 + B\,16_3 16_3)\overline{126}_1 + (a\,16_1 16_2 + b\,16_3 16_3)10 + c\,16_2 16_2 \overline{126}_2 + d\,16_2 16_3 \overline{126}_3 + \text{H.c.} \tag{18}$$

Here 16_i $(i = 1,2,3)$ represents the left-handed fermions of the ith generation, 10, 126_1, 126_2, and 126_3 represent different Higgs scalars, A, B, a, b, c, and d are real coupling constants, and it is assumed that 126_1, 126_2, and 126_3, have vacuum expectation values along $\underset{\sim}{1}$, $\underset{\sim}{45}$, and $\bar{\underset{\sim}{5}}$ of SU(5). The up-quark and Dirac and Majorana neutrino mass matrices, M^U, ν, and M, respectively, that follow from (18) are

$$M^U = \begin{pmatrix} 0 & a & 0 \\ a & 0 & d \\ 0 & d & b \end{pmatrix}, \quad \nu = \begin{pmatrix} 0 & a & 0 \\ a & 0 & m \\ 0 & m & b \end{pmatrix}, \quad M = \begin{pmatrix} 0 & A & 0 \\ A & 0 & 0 \\ 0 & 0 & B \end{pmatrix}, \tag{19}$$

where $m = -3d$. The vacuum expectation values of the Higgs scalars are absorbed into the coupling constants, and A and B are of the order of $10^{12} - 10^{15}$ GeV where SO(10) breaks down to SU(5). One finds from the characteristic equation for M^U that

$$a \simeq (m_u m_c)^{\frac{1}{2}}, \quad b \simeq m_t, \quad \text{and} \quad d \simeq (m_c m_t)^{\frac{1}{2}}. \tag{20}$$

The a and b terms of (18) contribute to all the mass matrices and c term of (18) contributes only to the M^D and M^ℓ mass matrices, whereas A and B terms contribute to M, d to M^U and ν so that one may vary these terms A, B, and d by a suitable discrete symmetry to obtain different models.

The 6×6 neutrino mass matrix is expressible in block form

$$M^\nu = \begin{pmatrix} 0 & \nu \\ \nu & M \end{pmatrix}, \tag{21}$$

which refers to the neutrino fields written as $\psi = (\nu_e, \nu_\mu, \nu_\tau, \nu_e^c, \nu_\mu^c, \nu_\tau^c)$ where c means charge conjugate. The eigenvalues of M^ν, with the assumption $A \sim B$, are approximately $\lambda_1 = -a^3 b/2m^2 A$, $\lambda_2 = 2abm^2/(m^2 + b^2)A$, $\lambda_3 = -(m^2 + b^2)/B$, $\lambda_4 = -A$, $\lambda_5 = A$, and $\lambda_6 = B$. The corresponding eigenvectors yield the unitary matrix $U_{\alpha i}$, that relates the weak isospin states $\alpha = e, \mu, \tau$ to the mass eigenstates $i = 1, 2, 3$, according to $\nu_\alpha = U_{\alpha i} \nu_i$. The K-M form of $U_{\alpha i}$ is

$$U_{\alpha i} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 & c_1 c_2 s_3 + s_2 c_3 \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 & -c_1 s_2 s_3 + c_2 c_3 \end{pmatrix}, \tag{22}$$

where $c_1 = \cos\theta_1$, $s_1 = \sin\theta_1$, etc., θ_1, θ_2, and θ_3 are the weak mixing angles, and a CP violating phase δ is put equal to zero.

We obtain[15] from Eqs.(19)-(21),

$$U_{\alpha i} = \begin{pmatrix} 1 & \theta_1 & 0 \\ -c_2\theta_1 & c_2 & s_2 \\ \theta_1 s_2 & -s_2 & c_2 \end{pmatrix}, \tag{23}$$

where

$$\theta_1 = -a\,(m^2+b^2)^{\frac{1}{2}}/2m^2\,,$$
$$s_2 = m/(m^2+b^2)^{\frac{1}{2}}\,,$$
$$\theta_3 = 0. \tag{24}$$

There is a large $\nu_\mu - \nu_\tau$ mixing, the mixing angle being given by $|\tan\theta_2| \simeq 3(m_c/m_t)^{\frac{1}{2}}$, whereas the other mixing angle is small, $|\theta_1| \simeq [m_u(m_t + 9m_c)/m_c m_t]^{\frac{1}{2}}/18$.

While this may be a priori reasonable, there is some evidence[16] that if neutrino oscillations are significant, they do not involve ν_μ. Therefore, we have searched for models in which there is large mixing between other neutrino types, by varying the Majorana term, and by varying the d-term that contributes to the up-quark and neutrino masses. These variations do not alter the down-quark and charged-lepton relations. The forms of ν that are considered by varying the d-term of (18) are

$$\nu_1 = \begin{pmatrix} 0 & a & 0 \\ a & n & 0 \\ 0 & 0 & b \end{pmatrix}, \qquad \nu_2 = \begin{pmatrix} 0 & a & m \\ a & 0 & 0 \\ m & 0 & b \end{pmatrix}. \tag{25}$$

The ν_1 case leads to a stable b quark and the ν_2 case leads to $m_u \geqslant m_c$, so both are unacceptable. Nevertheless it is instructive to note that the ν_2 case yields large $\nu_e - \nu_\tau$ mixing while the ν_1 case yields very little neutrino mixing.

4. Comments

We can summarize the results of our investigation by stating that it is very difficult to achieve neutrino mixing of other than the $\nu_\mu - \nu_\tau$ type in any minimal SO(10) model in which neutrino masses are generated by the Gell-Mann-Ramond-Slansky mechanism, because of the severe constraints placed on the mass matrix by quark phenomenology. In order to search for new models it is not necessary to diagonalize a 6×6 matrix (21) with $\begin{pmatrix} U & W \\ W^+ & V \end{pmatrix}$ but rather $-\nu M^{-1} \nu$ with U, where ν and M are the block diagonal elements of (21) and U satisfies (3), because of the huge disparity between the Majorana mass scale and the quark mass scale.[15] In a recent article, neutrino oscillation effects are reviewed by Barger et al.[17]

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