CONF-800724--45

DIFFICULT STATES IN THE QUARK MODEL:

GLUEBALLS AND THE PION*

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*Invited talk at the XX International Conference on High Energy Physics, Madison, Wisconsin, July 1980

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†This work is supported in part through funds provided by the U.S. DEPARTMENT OF ENERGY (DOE) under contract DE-AC02-76ER03069

August 1980
CTP #874

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ABSTRACT

Work on the spectroscopy of glueballs and on the pion is reviewed.

INTRODUCTION

Quark models are now mature enough to start confronting the really hard questions. In this talk, I will give mini-reviews of some efforts on two such topics-glueballs and the pion. Due to space limitations my comments will tend to be compact; hopefully the references cited will satisfy further curiosity.

GLUEBALLS

It is widely believed that the spectrum of QCD includes bound states of gluons-glueballs. However, there is less unanimity as to their properties. There are some common expectations for glueball spectroscopy,[1-8] and I will review these first. Then I go through a series of comments, elaborations and other points of view in order to illuminate the difficult aspects of glueballs. The hope is to convince some theorists that there are interesting and worthwhile areas for further study in this system. I have elsewhere reviewed the more phenomenological aspects of glueballs.[7]

There are two common ways to treat gluons in bound states. One is to consider color electric and magnetic fields in a region of space,[1, 3] essentially as a boundary condition problem in E and M. In this method the various modes can be classified as transverse electric (TE) or transverse magnetic (TM). With the confining boundary conditions of the bag model, the lowest mode is TE, $\ell=1$. The other method envisions what can be called "lumps of glue", i.e., the gluon as a massive spin one particle in a spin independent potential.[2, 4] The problem is then one of combining together what could be viewed as colored ω or ϕ mesons in various orbital and spin states.

Both methods agree on the two gluon sector. All states here will have positive charge conjugation. The ground states will consist of 0^{++} and 2^{++}. In the first excited state one can form 0^{-+}, 1^{-+}, 2^{-+}. (Note, however, that later on I will argue that the 1^{-+} does not belong here.) Many more excited states can be formed through radial and orbital excitations, although, until the low lying states are found, these are of only academic interest.

The three gluon sector is more subtle, and much of the literature is confused on this subject.[8] Color singlets can be formed with the f_{ABC} or d_{ABC} coefficients. Because of the intrinsic odd charge conjugation, states formed with f_{ABC} will have C=+1 while those with d_{ABC} will have C=-1. In the bag model the lowest mode is TE, which is dominantly a magnetic field, a 1^+ mode. The ground state of these gluons are 0^{++}, 1^{+-} and 3^{+-}. In contrast, the potential models combine up three 1^- particles in an S wave. Their spectrum has the opposite parity: 0^{-+}, 1^{--} and 3^{--}. The reason for this difference is that for a massless gluon the $\ell=0$ mode (which would be 1^-) is the Coulomb mode which does not exist in the absence of sources. The first transverse mode is then either the $\ell=1$ TE mode (1^+) or $\ell=1$ TM (1^-). The requirement that no flux leave the surface ($\eta_\mu F^{\mu\nu}=0$) favors the TE mode. It is exciting that experiment can then provide a clear distinction between the bag model picture and the potential models.

A rough guide to the mass spectrum can be found by using the bag model without intergluon interactions.[1] Not surprisingly the spectrum then starts at 1 GeV. In particular the predictions are:

$$
\begin{aligned}
M\left[(TE)^2 = 0^{++},\ 2^{++}\right] &= 960\ \text{MeV} \\
M\left[(TE)(TM) = 0^{-+},\ 1^{-+},\ 2^{-+}\right] &= 1290\ \text{MeV} \\
M\left[(TE)^3 = 0^{++},\ 1^{+-},\ 3^{+-}\right] &= 1460\ \text{MeV}
\end{aligned}
\tag{1}
$$

One interesting feature of the standard picture is the existence of a light exotic 1^{-+} state in the first excited multiplet of two gluons. Since it can not be formed by $q\bar{q}$ its presence would be good evidence for glueballs. However, I feel that this state is spurious, for the reasons to be discussed below.

There are good reasons to be cautious about naive models for glueballs. Potential models describe a massive field with 3 spin degrees of freedom instead of two for massless gluons. Bag models deal with a fixed bag which acts as an extra body in the problem. These limitations can lead to spurious states in the spectrum.

An alternative trial definition of a two gluon glueball is as a state that is a strong resonance in gluon-gluon scattering. Years ago Yang analyzed this situation for photons,[10] and found that the families (0^{++}, 2^{++}, 4^{++}...), (0^{-+}, 2^{-+}, 4^{-+}...) and (3^{++}, 5^{++}, 7^{++}...) were the only ones allowed by gauge and Lorentz invariance. In particular 1^{-+} cannot be formed by two real gluons, while its partners 0^{-+} and 2^{-+} can.

Yang's results require real gluons; however, there are other ways to obtain these results even for bound gluons. A useful technique in quark model physics to describe a quark state with a given set of quantum numbers is to form quark bilinears and then project out the appropriate state. For example, the fields

$$\bar{\psi}\gamma_5\psi \text{ or } p_\mu\bar{\psi}\gamma^\mu\gamma_5\psi$$
$$\varepsilon^\mu\bar{\psi}\gamma_\mu\psi \text{ or } V_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi \tag{2}$$

with $V_{\mu\nu} = (p_\mu\varepsilon_\mu - p_\nu\varepsilon_\nu)$ project out the ground state pseudoscalar and vector states of two quarks. Similar methods can be applied to glueballs. Gauge invariance requires that we work with the field tensors $F^A_{\mu\nu}$: The only nonvanishing combinations with no derivatives are

$$\begin{aligned} 0^{++} &\sim F^A_{\mu\nu}F^{A\mu\nu} \\ 2^{++} &\sim \varepsilon^{\mu\nu}F^A_{\mu\lambda}F^{A\lambda}{}_\nu \\ 0^{-+} &\sim F^A_{\mu\nu}\tilde{F}^{A\mu\nu} \\ 2^{-+} &\sim \varepsilon^{\mu\nu}F^A_{\mu\lambda}\tilde{F}^{A\lambda}{}_\nu \end{aligned} \tag{3}$$

where $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$. The 1^{-+} candidate $(V_{\mu\nu}F^{A\mu\lambda}F^{A\nu}_\lambda)$ vanished by the symmetry of the Lorentz indices. Even with one derivative the 1^{-+} candidate $(\varepsilon_\mu F^A_{\nu\lambda}\partial^\nu F^{A\lambda\mu})$ can be shown to vanish. Since my talk, Johnson has informed me of a third method of removing the 1^{-+} state in the bag model by use of the transversality of the gluon field and a bag model method for removing spurious states introduced by the fixed cavity.[11] All three methods use gauge and Lorentz invariance to say that the 1^{-+} does not belong with the 0^{-+} and 2^{-+} in the first excited state. It is possible to form 1^{-+} states with three gluons (or perhaps at high excitation with two gluons). In the bag model this state is in the first excited state of three gluons, and lies near 1.8 GeV. While such a state should be looked for, it will probably be much more difficult to find than a light two gluon state.

In $q\bar{q}$ states the spin-spin interaction, mediated by one gluon exchange, are responsible for splitting the ρ and π. Similar forces for gluons will split the multiplets of the above models. Because the gluon-gluon force is 9/4 of the $q\bar{q}$ force, we expect the splittings to be considerably larger in glueballs. Common expectation is that low spin states are pushed down in mass while high spin states move up. No systematic investigation has been done, but Thorn has studied the two gluon ground state.[12] He finds that the 2^{++} goes up to 1290 MeV, while the 0^{++} drops to 110 MeV! (With a center of mass correction[18] this would have $m^2<0$). This would appear to be disastrous; however, there is a reasonable way out. Another scalar state exists: the vacuum. When one finds such a low mass state one is suspicious that one has found a component of the vacuum. The two 0^{++} states must be orthogonalized, pushing the nonvacuum state up in mass. This program has not yet been carried out.

Giles has found a potential instability in the bag model glueballs.[13] The bag shape is usually assumed to be spherical. However, Giles found that a single gluon wavefunction in a cavity can also satisfy the bag boundary condition for a family of nonspherical surfaces, which could have lower total energy. This result does not always hold for two superimposed gluons so its consequences for glueballs is not yet understood.

Alternative descriptions of glueballs exist. For example Suura[6] uses a gauge invariant wavefunction of E and B fields at different space time points with links connecting them. He then derives a wave equation with a potential, similar to the Breit equation, for the spin zero sector. He finds 0^{++} and 0^{-+} degenerate, and when using a linear potential from light quark states bounds the mass to be less than 2 GeV. Adding a coulomb piece lowers this considerably and no lower bound can be obtained.

Another viewpoint is to study glueballs on a lattice.[5] A glueball can be formed by creating a closed path of flux linking lattice sites. The lightest configuration involves four sites-"a boxiton". Depending on orientations for the boxes, 0^{++}, 2^{++} and 1^{++} states may be formed. Kogut, Sinclair, and Susskind have calculated mass ratios to be

$$\frac{M(2^{++})}{M(0^{++})} = 1.003 \qquad \frac{M(1^{++})}{M(0^{++})} = 1.575 \tag{4}$$

Six link states (including 0^{-+} and 1^{-+}) are heavier. In principle the absolute glueball mass can be computed in lattice theories (by comparison to the string tension, for example), but this apparently is difficult and has not been done.

Finally, it seems appropriate to comment on the widths of glueballs. Some authors have favored widths which are the mean between standard quark widths and Zweig suppressed (a "square root of Zweig" suppressed) on the assumption that the OZI rule comes half from converting quarks into glue and half from turning the glue back into quarks. However I have never seen this argument made convincing, and it is particularly suspect for light glueballs. It is quite plausible that glueball widths are more or less typical of hadronic widths. In the limit of a large number of colors, N_C, glueballs are only narrower than quark states by a factor $1/N_C$. In the limit of a large number of light flavors, N_F, glueballs are broader by a factor of N_F. I have recently done a calculation which obtains the remaining space time factors by doing a bag model calculation using the P-matrix,[14] averaging over 16 levels of glueballs and 15 levels of quark states.[15] The resulting factor is of order two:

$$\frac{\Gamma(gq)}{\Gamma(q\bar{q})} = \frac{N_C}{N_C^2-1} \; N_F \quad .48 \tag{5}$$

Since the widths of quark states fluctuate more than a factor of three, all these numbers fall in a "typical" hadronic range.

To sum up, glueballs should exist and we think that we know some of their features. However, the subject needs much more theoretical work. We need more models for the spectrum, and a better understanding of the dynamics of mixing with $q\bar{q}$, decays, gauge invariance, and good phenomenology. In addition the role of glueballs in the known meson spectrum is an interesting subject which I don't have time to treat. Most importantly, experimenters need to find glueballs!

THE PION

It is well known that the pion is difficult to account for in the quark model. In the first approximation the mass of $q\bar{q}$ states ($\rho+\pi$) is roughly 2/3 of the nucleon's mass. Spin splittings from transverse gluon exchange make the pion the lightest state, but generally not light enough. In particular there is no natural limit where $M_\pi=0$. Worse than simply the problem of the light mass is the connection with chiral symmetry. QCD is nearly invariant under the chiral transformation (exact in the limit $M_u=M_d=0$), but the vacuum presumably is not. In this limit, a massless pion is required, with well defined chiral couplings. For M_q small we get a small pion mass

$$M_\pi^2 = (M_u+M_d)\langle\pi|\bar{q}q|\pi\rangle$$

Standard quark models do not contain the chiral symmetry and generally are not compatibles with it.

Perhaps the best paper on what a theory of a pion is all about is the 1961 article by Nambu and Jona Lasinio.[16] They considered a massless quark with a local four fermion interaction (with a cutoff). By summing bubble graphs they found that it became the theory of a <u>massive</u> quark and a <u>massless</u> bound state. They calculated the chiral couplings, and found the vacuum as a condensate of $q\bar{q}$ pairs. What is needed is a similar model in QCD which also ties in with other quark model calculations.

Pagels and Stokar[17] have attempted to do something like this using a Bethe-Salpeter formalism. They sum ladder graphs and use a dynamical quark mass $M(p^2) = 4M_D^3/p^2$. This allows them to calculate f_π and $F_\pi(Q^2)$ in terms of M_D

$$f_\pi = \left[\frac{2^{1/3}}{2\pi\sqrt{3}}\right]^{1/2} M_D \sim 83 \text{ MeV (vs 93 MeV)} \tag{6}$$

$$Q^2F_\pi(Q^2) \underset{Q^2\to\infty}{\to} \frac{4\ln^2}{3\pi(2)^{1/3}}\,\frac{M_D^4}{f_\pi^2} \sim .17 \text{ GeV}^2 \text{ (vs .38 GeV}^2)$$

The absolute numbers come from an estimate of M_D=244 MeV by Hagiwara and Sanda. The agreement is reasonably good. The drawbacks of this framework are that it doesn't explain the pion's relation to other states, and it disagrees with the perturbative QCD results on $F_\pi(Q^2)$. However its advantage is that it does incorporate the chiral properties of the pion.

From the other side of the fence the pion has been reconsidered in the bag model by Johnson and myself.[18] We argue that while the bag model does not contain the chiral symmetry, it is consistent with it. The point is that the bag model is really a guess at the vacuum structure of QCD, and the vacuum should not be chirally invariant. The bag has two forms of vacuum; where fields are strong (inside hadrons) one has the perturbative vacuum while outside of hadrons is the true vacuum. The order parameter is $\bar{q}q$, as in chiral theories, leading to the Lagrangian

$$L_{Bag} = (L_{QCD}-B)\theta(\bar{q}q) \tag{7}$$

where B is the Energy/Volume difference between the two vacuums. Since the bag builds the vacuum into L_{Bag} it isn't chirally invariant but then again it shouldn't be. A massless pion will not be automatic; however if this is the vacuum structure of QCD, a massless pion should be <u>possible</u>. With old techniques[19] one had M_π=280 MeV. However standard quark model techniques do not apply to light states. We developed the appropriate techniques for handling light static states, through use of wave packets. Using these, one can easily obtain M_π=0. We feel that this should be imposed in the chiral limit to be consistent with QCD. Expanding about this limit, we can obtain the chiral perturbation formula

$$M_\pi^2 = (M_u+M_d)\langle\pi|\bar{q}q|\pi\rangle \tag{8}$$

and evaluate it to obtain $1/2(M_u+M_d)$=33 MeV, M_s=330 MeV. Most chiral estimates[20] calculate a different mass $M_q^*=M_qZ$ where $Z=\langle H|\bar{q}q|H\rangle$. In the bag model $Z\simeq 1/2$, so that $1/2(M_u^*+M_d^*)$ = 17 MeV and M^*=1[illegible] MeV, [illegible] be compared with the chiral SU(3) estimate of 7 MeV and 1[illegible] MeV. [illegible] that even though we obtain the expansion Eq. 8, we don't obtain the chiral SU(3) mass ratios. This is because the expansion fails for the kaon; M_K is larger than other scales in the problem and the expression for its mass is not the simple linear expansion. This may be correct and chiral SU(3) could fail. There is some indication of this from the sigma term.[15] We also calculate $f_\pi = .5/R_\pi \simeq .15$ GeV, which is closer than previous bag estimates and, most importantly, finite as $M_\pi \to 0$. The conclusion is that some hints of chiral symmetry seem to emerge from the bag model, although much more lies uncovered.

In a separate talk at this conference, R. Haymaker has described an approach which is somewhere between the two above viewpoints, done in collaboration with T. Goldman.

In summary, the pion is being brought slowly into line. In particular the mass turns out <u>not</u> to be a major problem, but the connection between chiral symmetry and quark models is not yet understood.

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