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The Electroweak Anomaly and Current Algebra for $K \to \gamma \ell \nu$

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Abstract

The process $K \to \gamma \ell \nu$ is calculated using the electroweak axial-vector anomaly with the quark color factor of 3, together with standard current-algebra techniques. The result, which generalizes that of Das, Mathur, and Okubo for the axial-vector part, is in good agreement with experiment.

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In the context of the development of what presumably is the theory of strong interactions, quantum chromodynamics, current algebra and associated questions are receiving something of a renaissance. One of the key advances in that earlier development was the understanding of the two-photon decay of the pion. It was the axial-vector anomaly[1] which precluded the zero predicted in the soft-pion limit by Sutherland's theorem,[2] and which in fact predicts a rate for $\pi^0 \to \gamma\gamma$ completely consistent with experiment.[1]

The synthesis of electromagnetic and weak interactions[3] provides an arena for testing extensions of this anomaly-mediated process. The electroweak anomaly, for example, should underly the vector form factor of the decay $K \to \gamma \ell \nu$. This and the attendant axial-vector form factor resulting from current algebra are particularly ripe for study on both theoretical and experimental grounds. For although there is an extensive literature[4] on the theory of these and related amplitudes, the models are principally of a phenomenological character. Beyond the inclusion of the anomaly, what we attempt here is a generalization of the treatment by Das, Mathur, and Okubo[5] of the axial-vector form factor for the analogous decay $\pi \to \gamma e \nu$. Thus, we are testing the sufficiency of anomaly-mediation, PCAC for the kaon, current algebra, and the concomitant soft kaon limit. Experimentally, $K \to \gamma e \nu$, which proceeds at almost the same rate as $K \to e\nu$, has now been measured over a significant portion of its phase space,[6] so that one knows more than simply the magnitude of the amplitude. Perhaps surprisingly, agreement with experiment is excellent.

The essential matrix element of the weak <u>vector</u> current governing the structure-radiation[2] amplitude for the decay $K^+(p) \to \gamma(k) + \ell^+ + \nu$ in the soft-kaon limit $(p \to 0)$ is given by

$$\langle \gamma(k,\epsilon)|V_\mu^+(0)|K^+(p)\rangle = i\int d^4x \frac{1}{(2\pi)^{\frac{3}{2}}}\frac{1}{\sqrt{2\omega_k}} e^{-ipx}$$

$$\times \frac{1}{F_K}\langle \gamma(k,\epsilon)|T(V_\mu^+(0)[\partial^\alpha A_\alpha^-(x) - a\, F_{\alpha\beta}(x)\, {}^*W^{-\alpha\beta}(x)])|0\rangle \quad , \tag{1}$$

where $V_\mu^+ (= V_\mu^4 - i\, V_\mu^5)$ is the weak vector current coupled to the W vector boson. The PCAC relation with the anomaly term for the interpolating field of the K^- has been incorporated in (1). It is given by

$$\partial^\alpha A_\alpha^- = m_K^2 F_K \phi_{K^-} + a\, F_{\mu\nu}\, {}^*W^{-\mu\nu} \quad , \tag{2}$$

where

$$A_\alpha^- = A_\alpha^4 + i\, A_\alpha^5 \quad , \tag{3}$$

$${}^*W^{-\alpha\beta} = \tfrac{1}{2}\,\epsilon^{\alpha\beta\rho\sigma}(\partial_\rho W_\sigma^- - \partial_\sigma W_\rho^-) \quad , \tag{4}$$

and

$$a = \frac{\alpha \sin\theta_c}{4\sqrt{2}\,\pi \sin\theta_W} \tag{5}$$

in the Weinberg-Salam theory of weak interaction.[3] Equation (5) includes the color factor of 3.

The anomaly term[3] in (2) has been obtained from the quark diagrams in Fig. 1 in analogy to that for $\pi^0 \to \gamma\gamma$.[1] The divergence of the axial vector term $\partial^\alpha A_\alpha^-$ in (1) leads to zero in the matrix element (1), by Sutherland's theorem.[2] Introducing the coupling of the W with the leptonic current,[3] it can easily be shown that the vector decay amplitude is given by

$$M_V^{SR}(K^+ \to \gamma \ell^+ \nu) = i \frac{G_F e \sin\theta_c}{\sqrt{2}} \mathcal{V} \epsilon_{\lambda\mu\alpha\beta} k^\alpha p^\beta \epsilon^\lambda \bar{u}_\nu \gamma^\mu (1-\gamma_5) v_\ell \quad , \tag{6}$$

where the vector form factor $\mathcal{V}$ is

$$\mathcal{V} = \frac{1}{4\pi^2} \frac{1}{F_K} \quad , \tag{7}$$

or numerically ($F_K = 148$ MeV[9])

$$m_K \mathcal{V} = 0.084 \quad . \tag{8}$$

The matrix element analogous to (1) for the weak axial-vector current is

$$\langle \gamma(k,\epsilon) | A_\mu^+(0) | K^+(p) \rangle = -i \frac{1}{(2\pi)^{\frac{3}{2}}} \frac{1}{\sqrt{2\omega_\gamma}} e \epsilon^\lambda M_{\lambda\mu}(\nu, p^2) \quad , \tag{9}$$

where

$$M_{\lambda\mu}(\nu, p^2) = \int d^4x \, e^{ikx} \langle 0 | T(V_\lambda^{em}(x) A_\mu^+(0)) | K^+(p) \rangle \quad , \tag{10}$$

with $\nu = (k\, p)$ and V_λ^{em} the electromagnetic current.

The calculation of (10) has been carried out following the method developed for the axial-vector decay amplitude for $\pi^+ \to \gamma \ell^+ \nu$ within the chiral $SU(2) \times SU(2)$ symmetry.[5] A generalization to the chiral $SU(3) \times SU(3)$ is straightforward, and we simply state the result:

$$M_{\lambda\mu}(\nu, 0) = F_K g_{\lambda\mu} + \mathcal{G}(\nu g_{\lambda\mu} - p_\lambda k_\mu) + \mathcal{O}(\nu^2) \quad , \tag{11}$$

where the axial vector form factor G is given by

$$G = \frac{1}{F_K} \int dm^2 \frac{\rho_A(m^2) - \rho_V(m^2)}{m^4} + \tfrac{1}{3} F_K \langle r^2 \rangle_K \quad . \tag{12}$$

Here, $\langle r^2 \rangle_K$ is the mean squared charge radius of the charged kaon, and ρ_A, ρ_V are the axial-vector and vector two-point spectral functions, respectively. In the process of calculation, terms proportional to m_K^2 have been dropped. The uncertainties introduced thereby in the axial-vector amplitude are expected to be of the order of $m_K^2/m_Q^2 \approx 0.15$. This is to be compared with the uncertainties of the order of $m_\pi^2/m_{A_1}^2 \approx 0.02$ for $\pi^+ \to \gamma e^+ \nu$ (see Ref.[5]).

As is expected, the amplitude that results from the constant term, $F_K g_{\lambda\mu}$, in (11) is the same as that which follows from the modification of the PCAC relation in the presence of the electromagnetic potential (see Fig. 2(a)), i.e.

$$A_\mu^+(x) = -F_K(\partial_\mu - i\, e\, A_\mu(x))K^+(x) \quad . \tag{13}$$

It can be shown that this amplitude cancels with a term in the inner bremsstrahlung amplitude corresponding to the emission of the photon by the outgoing ℓ^+ (see Fig. 2(c)).

The axial-vector decay amplitude for the structure radiation, analogous to (6), is now given by

$$M_S^{SR}(K^+ \to \gamma \ell^+ \nu) = \frac{G_F\, e \sin\theta_c}{\sqrt{2}} G(p_\lambda k_\mu - \nu g_{\lambda\mu}) \epsilon^\lambda \bar{u}_\nu \gamma^\mu (1-\gamma_5) v_\ell \quad . \tag{14}$$

We evaluate (12) in the vector-dominance approximation, where[4]

$$\int dm^2 \frac{\rho_V(m^2)}{m^4} = \tfrac{1}{3} F_K^2 \langle r^2 \rangle_K \quad , \tag{15}$$

and with $\rho_A(m^2)$ saturated with the stangeness-carrying axial-vector meson Q:

$$\rho_A(m^2) \cong G_Q^2\,\delta(m^2 - m_Q^2) \tag{16}$$

with[12]

$$G_Q^2 \approx G_{K^*}^2 \approx 2m_{K^*}^2 F_K^2 \quad . \tag{17}$$

This makes use of the (approximately experimentally satisfied) relation

$$m_Q \approx \sqrt{2}\, m_{K^*} \quad , \tag{18}$$

and leads to ($m_{K^*} = 892$ MeV)

$$G \approx \tfrac{1}{2}\,\frac{F_K}{m_{K^*}^2} = 0.046\; m_K^{-1} \quad . \tag{19}$$

Combining (8) and (19) we find

$$m_K(v + G) \cong 0.13 \quad , \tag{20}$$

$$\gamma \equiv G/v \approx 0.55 \quad . \tag{21}$$

These compare quite favorably with the experimental results[6]

$$m_K\,|v + G| = 0.153 \pm 0.011 \quad , \tag{22}$$

as well as <u>exclusion</u> of the region

$$-1.8 < \gamma < -0.54 \quad . \tag{23}$$

Thus we see that kaon PCAC, current algebra, and the axial-vector anomaly seem to provide an adequate description of the decay $K\to\gamma e\nu$, perhaps at least as good as for the analogous decay $\pi\to\gamma e\nu$, where theoretically[5]

$$\gamma_\pi = 0.58 \quad , \tag{24}$$

as compared to the experimental value[13]

$$\gamma_\pi = 0.44 \pm 0.12 \quad \text{or} \quad -2.23 \pm 0.12 \quad . \tag{25}$$

Note that because Ref.[5] used CVC they could not predict the sign of γ_π, while the axial-vector anomaly yields an unambiguous sign.

Finally, for the sake of completeness we write down the inner bremsstrahlung amplitude (see Fig. 2(b) and (c)):

$$M^{IB}(K^+\to\gamma\ell^+\nu) = -m_\ell F_K \frac{G_F\, e \sin\theta_c}{\sqrt{2}}\, \bar{u}_\nu(\ell')(1+\gamma_5)$$

$$\times\left[\frac{(p\epsilon)}{(pk)} - \frac{(\ell\epsilon)}{(\ell k)} + \frac{(\gamma k)(\gamma\epsilon)}{2(\ell k)}\right] v_\ell(\ell) \quad . \tag{26}$$

This is manifestly gauge invariant, as are M_V^{SR} and M_A^{SR} individually. The total decay amplitude is the sum $M_V^{SR}+M_A^{SR}+M^{IB}$. Because of the appearance of m_ℓ in (26), M^{IB} is negligible for $K\to\gamma e\nu$ but dominant for $K\to\gamma\mu\nu$.

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Footnotes

1. See Ref.[1]. It was only with the advent of the concept of color that the coincidence between nucleon and quark models in this regard was recognized.

2. "Structure radiation" is distinguished from "inner bremsstrahlung" in which the photon is radiated from the external kaon or electron.

3. The non-renormalization of the anomaly in quantum electrodynamics was shown in Ref.[7]. The same for non-Abelian gauge theories was proved in Ref.[8].

4. Relation (15) is consistent with recent determinations of the charged kaon charge radius. See Refs.[10] and [11].

References

1. J. Schwinger, Phys. Rev. 82 (1951) 664; S. L. Adler, Phys. Rev. 177 (1969) 2426; J. Bell and R. Jackiw, Nuovo Cimento 60A (1969) 47; S. L. Adler, Lectures on Elementary Particles and Quantum Field Theory, ed. S. Deser, M. Grisaru, and H. Pendleton, MIT Press, Cambridge, Mass. (1970), Vol. 1, p.1; R. Jackiw, Lectures in Current Algebra and Applications, ed. S. B. Treiman, R. Jackiw, and D. J. Gross, Princeton University Press, Princeton, New Jersey (1972), p.97.
2. D. Sutherland, Phys. Lett. 23 (1966) 384; M. Veltman, Proc. Roy. Soc. A301 (1967) 107; S. L. Adler, Phys. Rev. B137 (1965) 1022; R. A. Brandt and G. Preparata, Ann. Phys. (N.Y.) 61 (1970) 119.
3. S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264; 27 (1971) 1688; A. Salam, Elementary Particle Theory, ed. N. Svartholm, Almquist, Stockholm (1968), p.367.
4. D. Yu. Bardin and E. A. Ivanov, Fiz. Elem. Chastits At. Yadra 7 (1976) 726 [English transl.: Sov. J. Part. Nucl. 7 (1976) 286].
5. T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 19 (1967) 859.
6. J. Heintze et al., Nucl. Phys. B149 (1979) 365.
7. S. L. Adler and W. A. Bardeen, Phys. Rev. 182 (1969) 1517.
8. W. A. Bardeen, Proc. XVI International Conference on High Energy Physics, Batavia, (1972), Vol. 2, p.295.
9. L. M. Chounet, J. M. Gaillard, and M. K. Gaillard, Phys. Rep. 4C (1972) 201.
10. E. Tsyganov, Proc. XIX International Conference on High Energy Physics, Tokyo (1978), p.315.
11. E. B. Dalley et al., Phys. Rev. Lett. 45 (1980) 232.

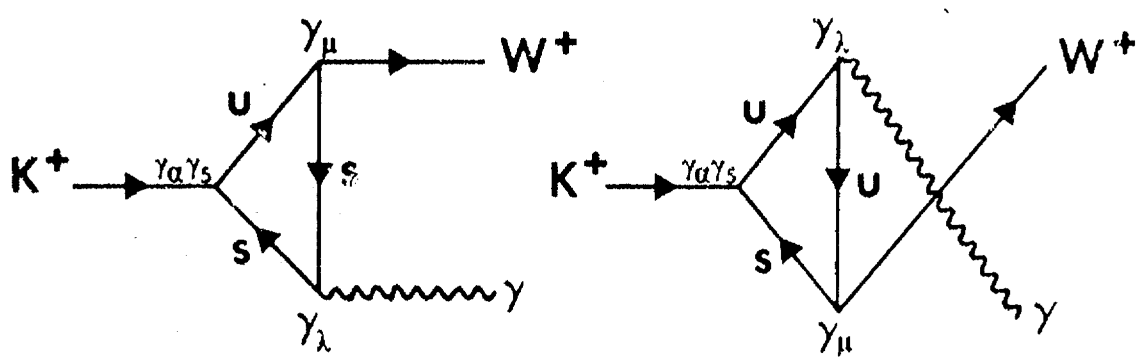
12. T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 18 (1967) 761.

13. D. A. Ortendahl, Lawrence Berkeley Laboratory Report No. LBL-5305 (unpublished); A. Stetz et al., Nucl. Phys. B138 (1978) 185.

Figure Captions

Fig. 1 Feynman diagrams for the anomaly.

Fig. 2 Feynman diagrams for the inner bremsstrahlung.



γ_μ
W$^+$
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Fig. 1

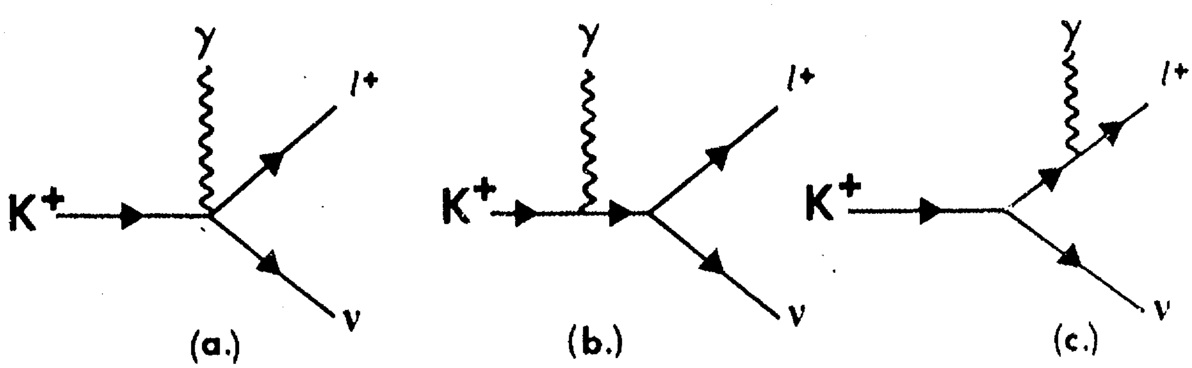

Fig. 2