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CONVECTIVES MODES IN RECTANGULAR ENCLOSURES

BY

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In a convective fluid the onset of turbulence is size dependent[1]. In large containers, which is the only case to be discussed here, spatial disorder in the rolls pattern induces turbulence[2]. Though ordered parallel rolls is the prefered mode of flow in rectangular enclosures in the convective stationary regime near the threshold instability, others flow pattern can be destabilized in a non stationary regime.

In order to investigate these possible modes of flow, we derived a linear partial differential equation for the dependence $w(x,y)$ of the vertical velocity u_z :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + a_o^2\right)^2 = \frac{\lambda}{9} w \qquad (1.a)$$

with the following lateral boundary conditions :

$$w = \frac{\partial w}{\partial x} = 0 \text{ at } x = \pm L \text{ and } w = \frac{\partial w}{\partial y} = 0 \text{ at } y = \pm M. \qquad (1.b)$$

In (1.a,b), $\lambda \equiv R - R_o$ and R_o, a_o are respectively the critical Rayleigh number and wavenumber for the unbounded layer, 2L and 2M are respectively the width and the length of the rectangle and $M \geq L$. In the derivation of (1.a) assumption has been made of a free upper and lower boundary condition so that :

$$u_z = \sin \pi z \; w(x,y)$$

When $\lambda << 1$ equation (1.a) asymptotically fits the usual sixth order differential equation for u_z.[3,4]

Equation (1.a,b) is not solvable by means of analytical methods, so that we looked at numerical solutions. The rectangular region is overlaid with a square lattice having mesh length h and the differential equation (1) is replaced by a difference equation. Since the solutions of (1) have symmetrical properties :

$$w(-x,y) = \varepsilon_x\, w(x,y) \quad \text{and} \quad w(x,-y) = \varepsilon_y\, w(x,y) \quad \text{with } \varepsilon_x, \varepsilon_y = \pm 1,$$

the computational domain is limited to the quadrant defined by :

$$0 \leq x \leq L \quad , \quad 0 \leq y \leq M \; .$$

The coordinate of the lattice points are :

$$x_i = (i - \tfrac{1}{2})h \quad , \quad y_j = (j - \tfrac{1}{2}) \quad \text{with} \quad i = 1,\ldots,N_1$$
$$j = 1,\ldots,N_2$$

The choice of the difference schemes and more details about the calculation are given in Ref. 5.

Introducing the notations :

$$w_r = w(x_i,y_j) \qquad \text{with} \quad r = j + (i-1) \quad r = 1,\ldots,N = N_1 \times N_2,$$

one gets instead of (1) a set of N simultaneous equations :

$$A_{rs}\, w_r = \frac{\lambda}{9}\, w_r \qquad (2)$$

where A is a N × N matrix. Since N is large the eigenvalue problem (2) is solved numerically.

Results

Several values of the ratio N_1/N_2 and of h have been considered. The common feature (except for $N_1 = N_2$) is that the fundamental mode always corresponds to a configuration in form of rolls parallel to the shorter side of the rectangle. On the opposite the configurations associated to the excited modes are size dependent. Let us examine the different situations :

-$N_1 = 11$, $N_2 = 33$ and $a_o h = \frac{\pi}{2}$; there are 35 (even solution) or 36 (odd solution) x-rolls in the fundamental. Up to the fifteenth, the

the excited modes are either x-rolls (1 ≤ n ≤ 6 and n = 11,12) or y-rolls (7 ≤ n ≤ 10, 13 ≤ n ≤ 15) with modulated amplitude along Oy (Fig.1) or along Ox. These pecular states with an envelope node or more in the bulk are known to be unstable[3].

– $N_1 = 10$, $N_2 = 38$ and $a_o h = \frac{\pi}{4}$; the fundamental (even and odd solutions) consists in 19 or 18 x-rolls. The new feature is that the excited modes exhibit dislocations (Fig.2). This is also true when $N_1 = 8$, $N_2 = 48$ (Fig. 3). Our numerical results are in agreement[7] with recent experiments and theory[8] suggesting that dislocations may grow spontaneously in cellular flow making them turbulent in some sense.

– $N_1 = 13$, $N_2 = 26$ and $a_o h = \frac{\pi}{4}$; the primary solution consists in 13 or 12 rolls. When λ increases a "bimodal regime" with interacting orthogonal rolls is possible (Fig.4).

Though a non linear anslysis remains to be done, our results confirm the influence of sidewalls on the convective pattern selection.

References.

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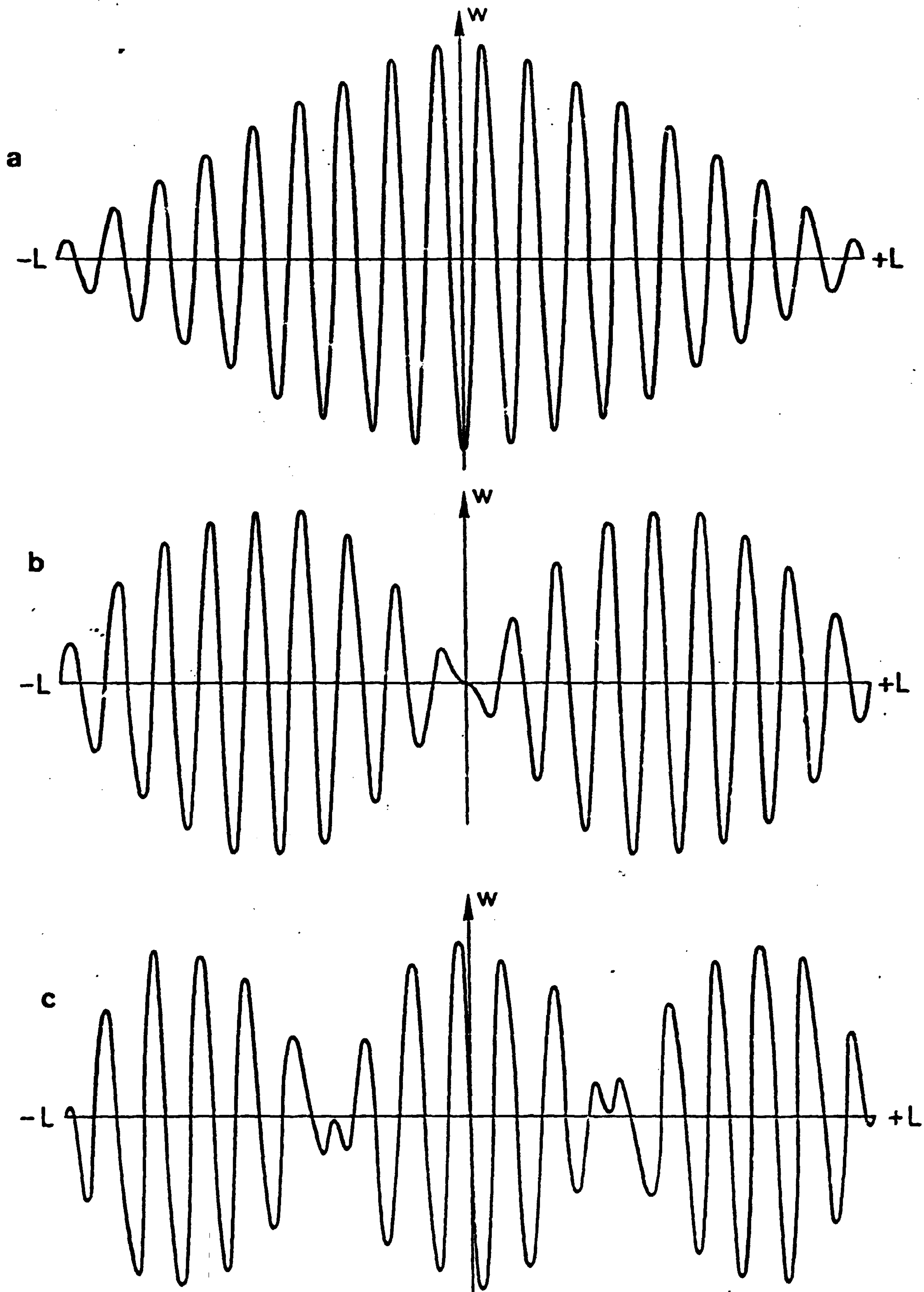


Fig.1. Profile of the velocity function $w(x,y)$ versus y for $x = \frac{h}{2}$ for the fundamental (a), the fourth mode (b) and the seventeenth (c) in the rectangle $N_1 = 11$, $N_2 = 33$

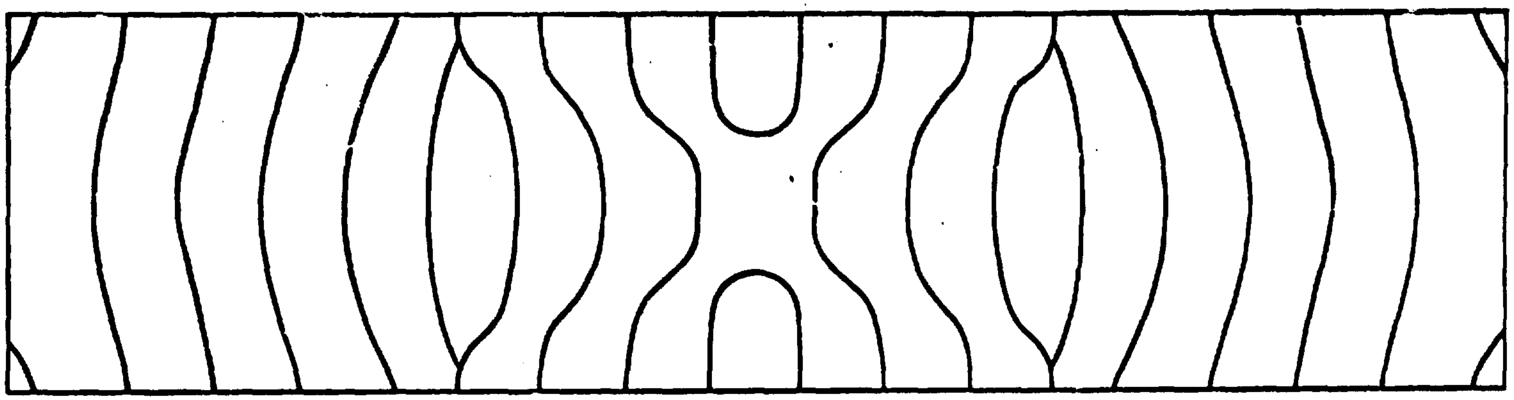


Fig.2. We have represented the nodal lines (w=0) of the configuration associated to the fifth mode ($N_1 = 10, N_2 = 38$).

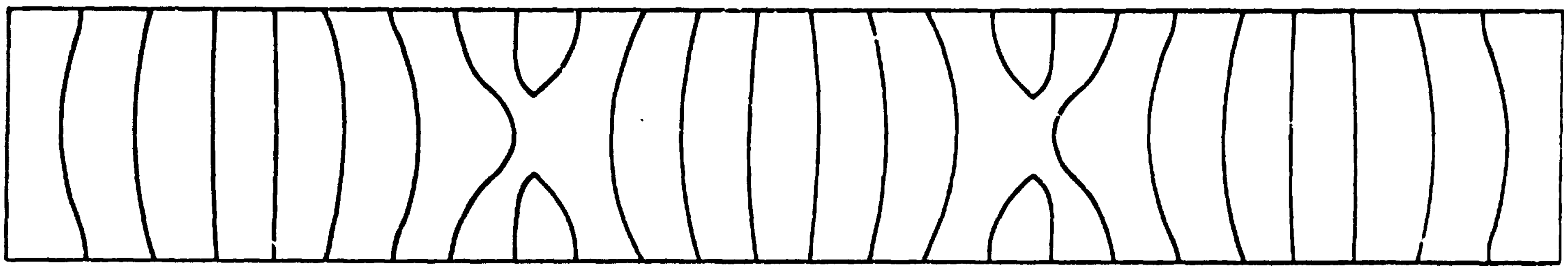
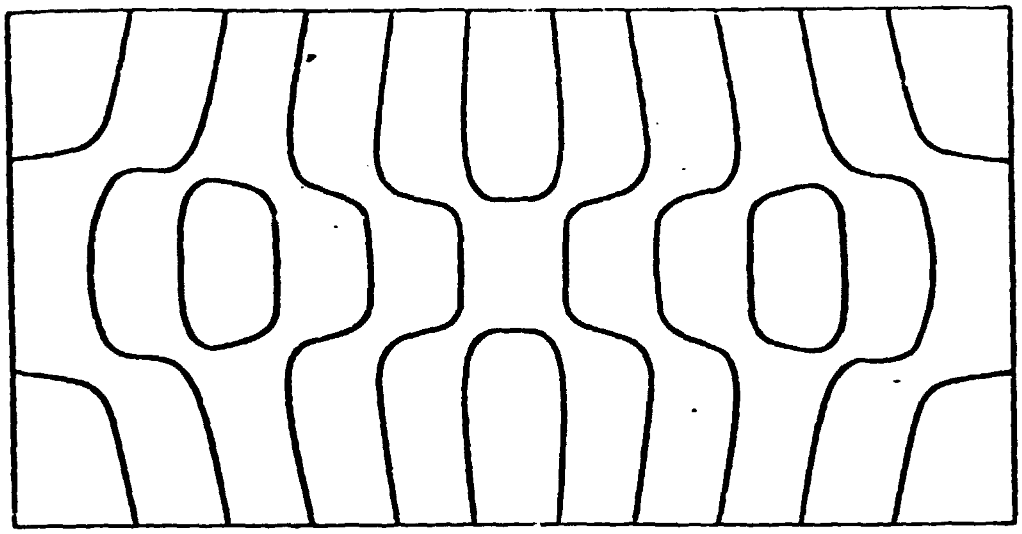


Fig.3. Nodal lines of the configuration associated to the seventh mode ($N_1 = 8, N_2 = 48$).

a



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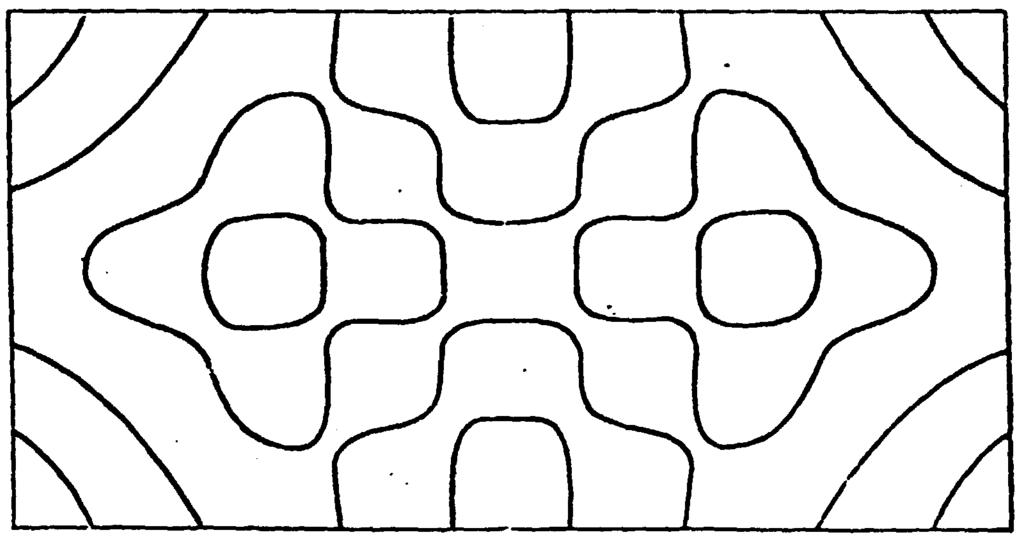

Fig.4 Nodal lines of the configurations associated to the fifth and tenth modes ($N_1 = 13$, $N_2 = 26$).