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NON-RELATIVISTIC BOTTONIUM IN THE PHYSICAL VACUUM OF QCD. MASS OF THE 1P LEVEL

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Abstract

Certain matrix elements of the nonrelativistic imaginary time propagator, $\exp(-H\tau)$, contributed by S and P waves are calculated for a heavy quarkonium. Effects of interaction of the quarkonium with nonperturbative fluctuations of the gluonic field in the physical vacuum of QCD are taken into account as well as those due to the short-distance Coulomb like gluon exchange. The results for the S wave are found to be in agreement with data on Υ and Υ' resonances, and it is shown that the Υ mass is definitely below the bare quark threshold: $2m_b - M_\Upsilon = 130\pm50$ MeV. Mass of the 1P level is calculated: $M_{1P} - M_\Upsilon = 370\pm30$ MeV and its annihilation rate is discussed.

It has been demonstrated [1-3] that properties of lowest hadronic states in channels with fixed flavor and spin-parity quantum numbers can be treated within the short-distance QCD. This is done by considering certain vacuum amplitudes induced by local quark operators at short distances and by studying leading nonperturbative contributions to the amplitudes which show up when distances get larger. For a heavy quarkonium such contributions appear due to interaction of quarks with nonperturbative fluctuations of gluonic field in the physical vacuum of QCD, the leading term being proportional to the following vacuum expectation value [1,3,4]

$$\eta = \langle 0 | \frac{\pi \alpha_s}{18} G^a_{\mu\nu}(0) G^a_{\mu\nu}(0) | 0 \rangle \tag{1}$$

The product of the gluonic field strength operators is defined in such a way that it vanishes when averaged over the perturbation theory vacuum. A general framework for considering nonperturbative effects in nonrelativistic dynamics of heavy quarkonium has been proposed in Ref. 4 in terms of the nonrelativistic Green function at short distances and energies below the bare quarks threshold. The approach is based on the multipole expansion of interaction of the quarkonium with the gluonic field of vacuum fluctuations, the dipole term being the leading one in the colorless sector at short separations between the quark and the antiquark. (A similar expansion has been used by Gottfried [5] and Peskin [6] in a somewhat different context).

This method has been applied [7] to an analysis of the Υ resonances which has provided with a preliminary estimate of the bare b-quark mass. In this note the analysis of Ref. 7 is improved by a more accurate calculation of the nonperturbative terms and is also extended to the P wave states of the $b\bar{b}$ quarkonium (bottonium, b-onium). As a result it is shown that the $\Upsilon(9.46)$ resonance is definitely below the bare quark threshold: $2m_b - M_\Upsilon = 130 \pm 50$ MeV, as distinct from the case of charmonium where the J/ψ resonance is well above the corresponding bare quark threshold [2] : $M_\psi - 2m_c \approx 1.4$ GeV. Also the mass splitting between the 1P and the 1S (Υ) levels in the b-onium is calculated: $M_{1P} - M_\Upsilon = 370 \pm 30$ MeV. (Our analysis is to the leading nonrelativistic order, so that spin dependent effects are completely neglected. The latter give rise to fine structure splittings between the 1P states which are expected [8] to be of order of a few MeV). This note is aimed to phenomenological analysis so that we skip most of theoretical background concerning calculations. The latter can be partly found in Refs. 4,7,6 and partly is quite new and will be published elsewhere.

The central role in our analysis is played by the following quantities

$$F_s(\tau) = \ln \left\{ \frac{\ldots \tau^{3/2}}{m^3} K(\vec{x}, \vec{y}, \tau) \Big|_{\vec{x}, \vec{y} = 0} \right\} \quad (2)$$

and

$$F_b(\tau) = \ln\left\{\frac{16\pi^{3/2}}{3m^5}\left[\frac{\partial}{\partial x_i}\frac{\partial}{\partial y_i}K(\vec{x},\vec{y},\tau)\right]_{\vec{x},\vec{y}=0}\right\} \quad (3)$$

Here

$$K(\vec{x},\vec{y},\tau) = \langle \vec{x} \,|\, e^{-H\tau} \,|\, \vec{y}\rangle \quad (4)$$

is the evolution operator (propagator) of the system in the imaginary (Euclidean) time τ. $\vec{x}\,(\vec{y})$ denotes the relative coordinate between the quark and the antiquark, e.g. $\vec{x} = \vec{x}_Q - \vec{x}_{\bar{Q}}$. The Green function $(H-E)^{-1}$ considered in Ref. 4 is related to the propagator K by the Laplace transform with respect to the τ variable. The Hamiltonian H besides the kinetic energy includes both the Coulomb like interaction in the quarkonium due to the short distance gluon exchange and the interaction of the quarkonium with the gluon field of the vacuum fluctuations.

The quantity $\exp[F_b(\tau)]$ is analogous to the one considered in Ref. 7 and in the nonrelativistic limit $m\tau \gg 1$ can be expressed in terms of the contribution of the $b\bar{b}$ states R_b to the ratio R measured in the e^+e^- annihilation [7]:

$$\frac{1}{9\sqrt{\pi}\,m^2 Q_b^2}\int R_b(s)\exp\left[\frac{4m^2-s}{4m^2}\,m\tau\right]ds = \left(1 - \frac{16}{3\pi}\alpha_s(m)\right)e^{F_b(\tau)} \quad (5)$$

where $Q_b = -1/3$ is the electric charge of the b-quark and m is its mass. It can be also noted that in the nonrelativistic limit $F_b(\tau)$ is contributed by the S wave states and $F_P(\tau)$ is determined by the P wave ones.

The relevance of the quantities $F_{S,P}(\tau)$ to a study of lowest S and P states of quarkonium is caused by the simple fact that for large τ only contributions of ground states survive:

$$\left.\begin{aligned} F_S(\tau) &= \ln C_S - E_{1S}\,\tau \\ F_P(\tau) &= \ln C_P - E_{1P}\,\tau \end{aligned}\right\} \quad \tau \to \infty . \tag{6}$$

Here the energies $E_{1S,1P}$ of the ground levels are counted from the bare quark threshold $2m$. The constants $C_{S,P}$ can be related in terms of a potential model to corresponding wavefunctions at the origin:

$$C_S = \frac{2\sqrt{\pi}}{m^3}\,|R_{1S}(0)|^2 , \qquad C_P = \frac{12\sqrt{\pi}}{m^5}\,|R'_{1P}(0)|^2 ,$$

or directly to the annihilation rates

$$\Gamma(\Upsilon \to e^+e^-) = \left(1 - \frac{16}{3\pi}\alpha_s(m)\right)\alpha^2 Q_b^2\, m\, C_S / 2\sqrt{\pi}$$

$$\Gamma(1^3P_0 \to \gamma\gamma) = 9\alpha^2 Q_b^4\, m\, C_P / 4\sqrt{\pi} \tag{7}$$

(a correction $\sim \alpha_s(m)$ to the latter width is disregarded here). Eqs. (7) are more general than a potential model since they can be derived [2] from dispersion relations for the electromagnetic vacuum polarization (cf. eq. (5)) and for the amplitude of the scattering $\gamma\gamma \to \gamma\gamma$.

By comparing contributions of the Υ, Υ' and

Υ'' resonances to the integral in eq. (5) one can readily see that it is saturated with an accuracy better than 5 % by only the Υ resonance when $m\tau \gtrsim 20$, which gives an estimate of values of τ at which the asymptotic behavior (6) sets on. It is clear that the larger is τ the larger are the relevant distances, so that the question arises whether it is possible by approaching the problem from the short distance side to calculate the quantities $F_{s,p}(\tau)$ in the region of τ where the asymptotic behavior (6) is already seen. It will be seen from what follows that for the b-onium the answer is affirmative. However this requires to take into account the leading nonperturbative contribution due to the v.e.v. (1) and to sum up all the Coulomb like terms of the form $(\alpha_s \sqrt{m\tau})^n$.

The physical reason for this becomes clear when one consideres the propagator $K(\vec{x},\vec{y},\tau)$ as given by integration of $\exp(-\int_0^\tau H\,dt)$ over paths $\vec{z}(t)$ with the boundary conditions $\vec{z}(0)=\vec{y}$ and $\vec{z}(\tau)=\vec{x}$. From a simple dimensional analysis it follows that for infinitesimal $|\vec{x}|$ and $|\vec{y}|$ the path integral is dominated by trajectories with characteristic values of $|\vec{z}(t)|$ of order $\sqrt{\tau/m}$. This means that the separation of quarks in space is $(m\tau)^{1/2}$ times smaller than the time duration of the paths τ. Therefore even for rather large τ the relevant distances are short provided that m is also large enough. Thus one can describe the interaction of the system with the gluonic fluctuations by first terms of the multipole expansion. Besides that the interaction of quarks with each other can be described at such distances by the short-distance gluon exchange which is reduced in the nonrelativistic limit to the Coulomb like interaction potential

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r}P_o + \frac{1}{6}\frac{\alpha_s}{r}P_8 \qquad (8)$$

where P_o and P_8 are projectors for the color singlet and color octet states of the quark pair respectively, and the effective coupling constant α_s should be taken at the relevent distances of order $\sqrt{\tau/m}$. The parameter for the interaction (8) for large $m\tau$ is not α_s but rather

$$\int_0^{\tau} V\,dt \sim \alpha_s\,\tau/(\tau/m)^{1/2} = \alpha_s\sqrt{m\tau}$$

so that for $m\tau \gg 1$ one should account for the Coulomb like interaccion explicitly.

After these preliminary remarks we write down the expansion for the operator $\mathcal{K}(\tau) = exp(-H\tau)$ in the colorless sector (which is equivalent to the expansion for the Green function obtained in Ref. 4):

$$\mathcal{K}(\tau) = \mathcal{K}_o(\tau) - \\ - \eta\int_0^{\tau}d\tau'\int_0^{\tau'}d\tau''\,\mathcal{K}_o(\tau-\tau')r_i\,\tilde{\mathcal{K}}_o(\tau'-\tau'')r_i\,\mathcal{K}_o(\tau'') + \\ + \ldots \qquad (9)$$

Here $\mathcal{K}_o$ and $\tilde{\mathcal{K}}_o$ are the propagators of Coulomb like systems with interaction described by respectively the first and the second terms in eq. (8) (note also that the products in the integrand in eq. (9) are operator ones). The three dots in eq. (9) refer to contributions of gluonic operators with dimension $d > 4$.

The first term in the expansion (9) is given by the propagation of a colorless system with the Coulomb like interaction, while the second one describes two dipole interactions at the moments τ'' and τ' with integration over τ'' and τ'. (The term with one interaction which is linear in the gluonic field drops out when averaged over vacuum). Note that between τ'' and τ' the quark system is a color octet.

In the second term in the expansion (9) the gluonic operators are taken at the same moment of time. This is justified if effective values of the interval $\tau' - \tau''$ are smaller than the characteristic time duration of the fluctuations. This approximation can be tested by considering the first nonvanishing term which appears due to the difference ($\tau' - \tau''$). This term is proportional [4] to

$$\begin{aligned} &\langle (\tau' - \tau'')^2 \rangle \langle 0 | \pi \alpha_s (D^2 G) G | 0 \rangle \sim \\ &\sim \langle (\tau' - \tau'')^2 \rangle \langle 0 | (\pi \alpha_s)^{3/2} f^{abc} G^a_{\mu\nu} G^b_{\nu\sigma} G^c_{\sigma\mu} | 0 \rangle \end{aligned} \qquad (10)$$

The latter v.e.v. has been extimated in Ref. 1, and we use this rough estimate in our analysis. As a result it can be argued that the contribution of this kind to $F_s(\tau)$ is less than ~ 0.3 of that given by the term with when $m\tau \lesssim 45$ (in the b-onium). (This is mainly because the effective value $\langle (\tau' - \tau'')^2 \rangle$ contains a small numerical factor (1/15) when compared to τ^2). As to the quantity $F_p(\tau)$ prior to the moment when the contribution of the type (10) becomes essential that of the d = 8 operators comes into play. The latter correction is roughly equal to the contribution of the term with η squared, and becomes noticeable in the b-onium for $m\tau \gtrsim 35$.

We proceed now to a more quantitative analysis of $F_{S,P}(\tau)$ taking into account the two first terms in the expansion (9). The results of calculation of $F_{S,P}(\tau)$ with the help of eq. (9) are as follows

$$F_S(\tau) = \ln\left[\Phi_S(\gamma)\right] - \frac{3}{2}\ln(m\tau) - \frac{1}{4}\frac{\eta}{m^4}(m\tau)^3\frac{X_S(\gamma)}{\Phi_S(\gamma)} + \dots \qquad (11)$$

$$F_P(\tau) = \ln\left[\Phi_P(\gamma)\right] - \frac{5}{2}\ln(m\tau) - \frac{3}{4}\frac{\eta}{m^4}(m\tau)^3\frac{X_P(\gamma)}{\Phi_P(\gamma)} + \dots \qquad (12)$$

where $\gamma = \frac{2}{3}\alpha_s\sqrt{m\tau}$ is the "Coulomb" parameter. The functions $\Phi_{S,P}(\gamma)$ and $X_{S,P}(\gamma)$ describe the effects of the Coulomb like short-distance gluon exchange. They are normalized to unity at $\gamma = 0$. The explicit expressions for the functions $\Phi_{S,P}(\gamma)$ are the following:

$$\Phi_S(\gamma) = 1 + 2\sqrt{\pi}\,\gamma + \frac{2\pi^2}{3}\gamma^2 + 4\sqrt{\pi}\sum_{n=1}^{\infty}\left(\frac{\gamma}{n}\right)^3 e^{\frac{\gamma^2}{n^2}}\left[1+\mathrm{erf}\left(\frac{\gamma}{n}\right)\right], \qquad (13)$$

$$\Phi_P(\gamma) = 1 + \frac{4\sqrt{\pi}}{3}\gamma + \frac{2}{3}\left(1+\frac{\pi^2}{3}\right)\gamma^2 + \frac{4\sqrt{\pi}}{3}\gamma^3 + \frac{4}{9}\pi^2\left(1-\frac{\pi^2}{15}\right)\gamma^4 + \frac{8\sqrt{\pi}}{3}\gamma^5\sum_{n=2}^{\infty}\left(\frac{1}{n^3}-\frac{1}{n^5}\right)e^{\frac{\gamma^2}{n^2}}\left[1+\mathrm{erf}\left(\frac{\gamma}{n}\right)\right] \qquad (14)$$

where $\mathrm{erf}(x) = (2/\sqrt{\pi})\int_0^x e^{-t^2}dt$. (The function (13) has been calculated in Ref. 7). The functions $X_{S,P}$ are also found in a form of rapidly convergent series, containing $\mathrm{erf}(\gamma/n)$. However the corresponding expressions are rather cumbersome. For example $X_S(\gamma)$ is given by the following one

$$
\begin{aligned}
\mathcal{X}_S(\gamma) = 4\sqrt{\pi}\ \Bigg\{ \sum_{n=0}^{\infty} (n+1)(n+2)(n+3) \Bigg\{ & \frac{1}{9n+16}\Big[\varphi_2\Big(\frac{\gamma}{n}\Big) - \\
& - \varphi_1\Big(\frac{\gamma}{n}\Big)\Big(1 + \frac{n}{12}\Big(25 - \frac{6}{9n+16}\Big)\Big)\Big] + \frac{16}{9n+17}\Big[\varphi_2\Big(\frac{\gamma}{n+1}\Big) - \\
& - \varphi_1\Big(\frac{\gamma}{n+1}\Big)\Big(1 + \frac{n+1}{6}\Big(5 - \frac{3}{9n+17}\Big)\Big)\Big] + \\
& + \frac{4}{n+2}\Big[\varphi_2\Big(\frac{\gamma}{n+2}\Big) - \frac{17}{18}\varphi_1\Big(\frac{\gamma}{n+2}\Big)\Big] + \\
& + \frac{16}{9n+19}\Big[\varphi_2\Big(\frac{\gamma}{n+3}\Big) - \varphi_1\Big(\frac{\gamma}{n+3}\Big)\Big(1 - \frac{n+3}{6}\Big(5 + \frac{3}{9n+19}\Big)\Big)\Big] \\
& + \frac{1}{9n+20}\Big[\varphi_2\Big(\frac{\gamma}{n+4}\Big) - \varphi_1\Big(\frac{\gamma}{n+4}\Big)\Big(1 - \frac{n+4}{12}\Big(25 + \frac{6}{9n+20}\Big)\Big)\Big]\Bigg\} \\
& + \frac{4}{81}\sum_{n=2}^{\infty} \frac{n^2-1}{(81n^2-1)(81n^2-4)}\,\varphi_1\Big(-\frac{\gamma}{8n}\Big) \Bigg\}
\end{aligned}
$$

where $\varphi_1(x) = x^{-3}\left[e^{x^2}(1+erf(x)) - 1\right] - (2/\sqrt{\pi})x^{-2} - x^{-1}$ and $\varphi_2(x) = \left[e^{x^2}(1+erf(x)) - 1\right]x^{-1} - \varphi_1(x)$. The expression for $\mathcal{X}_P(\gamma)$ is still more complicated. For values of γ essential in our analysis: $\gamma \leq 1.5$, the functions $\mathcal{X}_{S,P}(\gamma)$ can be approximated up to a better than 10% accuracy as follows

$$
\frac{\mathcal{X}_S(\gamma)}{\Phi_S(\gamma)} \simeq e^{-0.80\gamma} \quad ; \quad \frac{\mathcal{X}_P(\gamma)}{\Phi_P(\gamma)} \simeq e^{-0.72\gamma} \qquad (15)
$$

Thus from eqs. (11) - (15) one can derive formulas convenient for practical computation:

$$F_S(\tau) = 2\sqrt{\pi}\,\gamma + 0.297\,\gamma^2 + 0.0466\,\gamma^3 + 0.0063\,\gamma^4 + \ldots$$
$$- \frac{3}{2}\ln(m\tau) - \frac{1}{4}\frac{\eta}{m^4}(m\tau)^3 e^{-0.80\gamma} + \ldots, \qquad (16)$$

$$F_P(\tau) = \frac{4\sqrt{\pi}}{3}\gamma + 0.067\,\gamma^2 + 0.0042\,\gamma^3 + \ldots \qquad (17)$$
$$- \frac{5}{2}\ln(m\tau) - \frac{3}{4}\frac{\eta}{m^4}(m\tau)^3 e^{-0.72\gamma} + \ldots$$

From the values of coefficients in these expressions one sees that for $\gamma \lesssim 1.5$ it is sufficient to retain few first terms of the expansion in powers of γ.

Let us discuss now corrections to eqs. (16) and (17) and the relevant value of the coupling constant α_s. There are several small parameters involved : $(m\tau)^{-1}$, α_s, and v.e.v.'s of operators of dimension $d > 4$. Accordingly there are relativistic corrections $\sim (m\tau)^{-1}$, further perturbative terms of order α_s^2 and $\alpha_s v \sim \alpha_s/\sqrt{m\tau}$ and also subsequent nonperturbative corrections. The latter as stated above become essential for $m\tau \gtrsim 45$ in the S wave and for $m\tau \gtrsim 35$ in the P wave. In the region $20 \lesssim m\tau \lesssim 50$ the characteristic spacelike separations $\sqrt{m\tau}/m$ are of the order of $[(0.7 - 1)\ \mathrm{GeV}]^{-1}$ and the effective coupling constant should be taken somewhere in this region (the exact value connot be determined unless terms of order α_s^2 are evaluated). From the previous analysis [1,2] (see also Ref. 9) it can be inferred that at such distances $\alpha_s \simeq 0.3$. In our analysis the value of α_s is strongly correlated with the rate $\Gamma(\Upsilon \to e^+e^-)$. Variation of

α_s in the limits 0.3 ± 0.03 corresponds to $\Gamma(\Upsilon \to e^+e^-)$ $= 1.15 \pm 0.20$ KeV (see below, eq. (19)), which is practically the experimental value of the rate. It also looks like that with a value of α_s outside the region 0.3 ± 0.05 it is impossible to reproduce the e^+e^- width of the Υ. Thus one can think that the neglected perturbative terms amount to less than $\sim \alpha_s^2 \sim 0.1$ corrections to the quantities $F_S(\tau)$ and $F_P(\tau)$ in the region $20 \lesssim m\tau \lesssim 50$. In this connection it is necessary to outline that corrections of the orders α_s and $\alpha_s^2 \sqrt{m\tau}$ are abscent unless the coupling constant is taken at distances exponentially different from those characteristic of the problem. If the normalization point for α_s is taken properly corrections of order $\alpha_s(m)$ appear multiplicatively only when the quantities $F'_{S,P}$ are related to widths (cf. eqs. (5) and (7)).

The magnitude of the v.e.v. (1) has been estimated[1,3] from the analysis of charmonium: $\eta \simeq 0.7 \times 10^{-2}\ \mathrm{GeV}^4$ up to a factor of ~ 1.5. This value of η would also fit experimental data on Υ resonances in the sense of eq. (5). However a slightly better linearity of the τ-dependence of $F_S(\tau)$ in the region $m\tau \gtrsim 30$ (when calculated form eq. (16)) is reproduced with $\eta = 1.0 \times 10^{-2}\ \mathrm{GeV}^4$. Therefore we shall stick to this latter value of η which is also compatible with the previous work[1,3].

In Fig. 1 are shown plots of the quantities F_S and $F_P + 4$ versus $m\tau$ as calculated from eqs. (16) and (17) with $\alpha_s = 0.3$ and $\eta = 1.0 \times 10^{-2}$ GeV^4. (Four units are added to $F_P(\tau)$ to show it at the same plot with $F_S(\tau)$). The curves are drawn in the regions of $m\tau$ where the neglected terms are believed to be small enough as discussed above. It is seen from the figure that the correction due to the v.e.v. (1) is quite essential for reproducing the linear τ-dependence (especially for F_P) in the appropriate region of $m\tau$. At maximal values of $m\tau$ considered this correction give contribution about 0.2 to F_S (at $m\tau \simeq 50$) and about 0.3 to F_P (at $m\tau \simeq 38$). It is also seen that the τ-dependence of F_S and F_P is given by practically straight lines starting from $m\tau \simeq 25$ and $m\tau \simeq 28$ respectively. The slopes of these straight lines correspond to

$$E_{1S} \equiv M_\Upsilon - 2m \simeq -130 \text{ MeV}$$

and

$$E_{1P} - E_{1S} \equiv M_{1P} - M_\Upsilon \simeq 370 \text{ MeV}$$

and the positions of these lines (the values of their continuation at $m\tau = 0$) correspond to the following values of the constants $C_{S,P}$ in eqs. (6): $C_S \simeq 0.19$, $C_P \simeq 0.010$. According to eqs. (7) this gives

$$\Gamma(\Upsilon \to e^+e^-) \simeq 1.15 \text{ KeV}$$
$$\Gamma(1^3P_0 \to \gamma\gamma) \simeq 42 \text{ eV} \qquad (18)$$

If α_S and η are varied in the limits $\alpha_S = 0.30 \pm 0.03$ and $\eta = (1.0 \pm 0.3) \times 10^{-2}$ GeV^4 these results vary approximately as follows:

$$M_\Upsilon - 2m = -130 \pm 50 \text{ MeV},$$
$$M_{1P} - M_\Upsilon = 370 \pm 30 \text{ MeV}, \qquad (19)$$

$$\Gamma(\Upsilon \to e^+e^-) = 1.15 \pm 0.20 \text{ KeV},$$

$$\Gamma(1^3P_0 \to \gamma\gamma) = 42 \pm 5 \text{ eV}.$$

The latter rate does not seem to be of an imediate interest. However, it is believed [10,11, 2] to be related to the total hadronic width:

$$\Gamma(1^3P_0 \to 2g \to hadrons) \simeq 18 \frac{\alpha_s^2(m)}{\alpha^2} \Gamma(1^3P_0 \to \gamma\gamma) \simeq 360 \text{ KeV}.$$

The values of α_s and η if determined from data are not independent. Thus uncertainties in these parameters result in fact in a somewhat smaller (approximately by a factor of 2) errors in the results than those indicated in eq. (19). However keeping in mind uncertainties due to neglected in eqs. (16), (17) small terms it seems reasonable to leave the result as indicated in eq. (19).

The result $M_{1P} - M_{\Upsilon} = 370 \pm 30$ MeV contained in eq. (19) is correlated through the values of α_s and η with the width $\Gamma(\Upsilon \to e^+e^-)$, and this prediction for the 1P state mass can be insisted on unless the experimental data on $\Gamma(\Upsilon \to e^+e^-)$ are shifted from the region indicated in eq. (19).

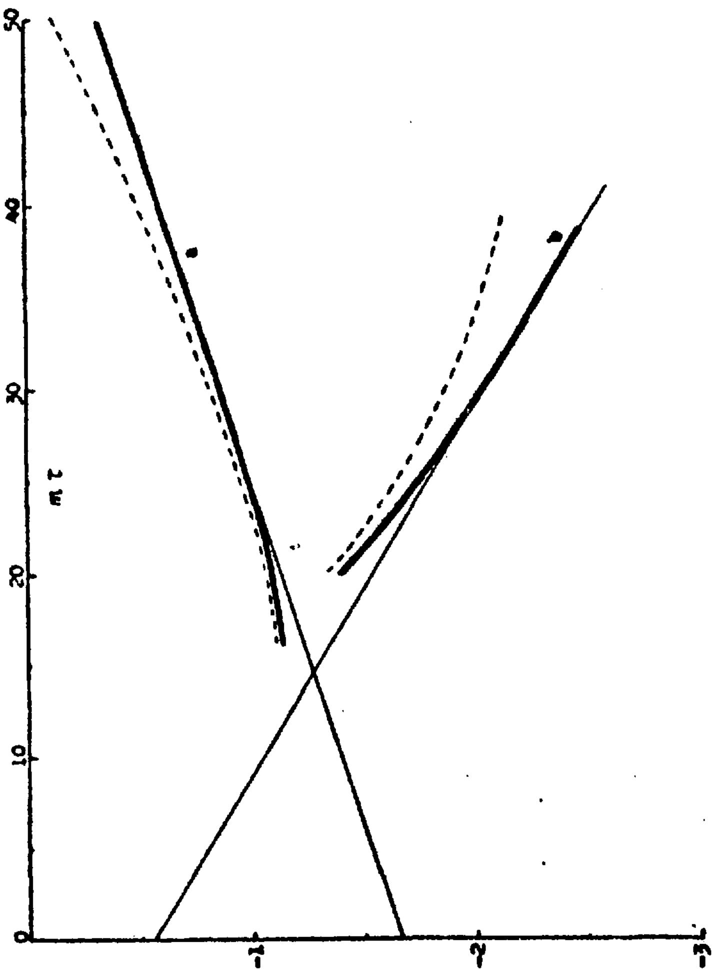

Fig. 1. The quantities $F_S(\tau)$ (a) and $F_P(\tau)+4$ (b) calculated from eqs. (16) and (17) with $\alpha_s = 0.3$ and $\gamma = 1.0 \times 10^{-2}$ GeV^4 (heavy lines). The dashed lines are the same with $\gamma = 0$. The thin straight lines indicated the asymptotic behavior (6) of $F_{S,P}(\tau)$ at large τ.

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