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COMMENTS ON "ADIABATIC MODIFICATIONS TO PLASMA TURBULENCE THEORY"

BY

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PLASMA PHYSICS
LABORATORY

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Comments on "Adiabatic modifications to plasma turbulence theory"

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Catto introduced in Ref. 1 an interesting and plausible modification of the usual "resonance-broadening" prescription[2] for obtaining the nonlinear dielectric function. He argued reasonably that one should employ that prescription only for the nonadiabatic response, and that one should treat the adiabatic response essentially exactly. However, Misguich, in a recent Comment[3] on Catto's work,[1,4] found an apparent divergence in a form for the renormalized dielectric which he argued was equivalent to Catto's. Misguich was thus led to conclude that, at least for stationary turbulence, Catto's form was suspect, and that a more intricate renormalization might have to be used to obtain a sensible, convergent result.

I wish to argue that this conclusion is incorrect, at least for the reasons Misguich gives. My goal is not to criticize Misguich, whose work is detailed and instructive, but to exemplify some subtleties of renormalization. It is adequate to discuss the electrostatic, stationary, Gaussian-Markov approximation with constant diffusion coefficient D and Maxwellian background distribution <f>. In the resonance-broadening approximation, the dielectric ε then takes the form

$$\varepsilon(k,\omega) \approx 1 + \sum_s (k\lambda_D)^{-2} \int dv\,d\bar{v}\; U_{k,\omega}(v;\bar{v})\, ik\bar{v} \langle f(\bar{v})\rangle \;, \tag{1}$$

where

$$U_{k,\omega}(v;\bar{v}) \equiv \int_0^\infty d\tau\, \exp\left[i(\omega-kv)\tau - k^2 D\tau^3/3\right] P_v(v-\bar{v}-ikD\tau^2,\tau) \;, \tag{2}$$

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$$P_v(z,\tau) \equiv (4\pi D\tau)^{-1/2}\exp(-z^2/4D\tau) \;, \tag{3}$$

and $\lambda_D^2 \equiv T/4\pi n e_s^2$. To simplify (1), it is convenient to change variables from $\{v,\bar{v}\}$ to $\{u \equiv v-\bar{v}-ikD\tau^2,\bar{v}\}$ so that

$$I \equiv \int_{-\infty}^{\infty}dv\int_{-\infty}^{\infty}d\bar{v}\;\exp(-ikv\tau)P_v(v-\bar{v}-ikD\tau^2)ik\bar{v}\langle f(\bar{v})\rangle$$

$$= \exp(k^2D\tau^3)\int_{-\infty}^{\infty}d\bar{v}\;\exp(-ik\bar{v}\tau)ik\bar{v}\langle f(\bar{v})\rangle\int_{-\infty-ik^2D\tau}^{\infty-ik^2D\tau}du\;\exp(-iku\tau)P_v(u) \;. \tag{4}$$

A standard application of Cauchy's theorem enables one to shift the u contour upward onto the real axis, giving rise to

$$\int_{-\infty}^{\infty}du\;\exp(-iku\tau)P_v(u) = \exp(-k^2D\tau^3) \;. \tag{5}$$

This factor cancels with the first term in (4), whereupon

$$I = -\frac{\partial}{\partial\tau}\int d\bar{v}\;\exp(-ik\bar{v}\tau)\langle f(\bar{v})\rangle \;. \tag{6}$$

Then, upon inserting (6) into (1) and integrating by parts in τ, one finds

$$\varepsilon(k,\omega) = 1 + \sum(k\lambda_D)^{-2} + \sum(k\lambda_D)^{-2}\int d\bar{v}\;\langle f(\bar{v})\rangle$$

$$\times\int_0^{\infty}d\tau(i\omega-k^2D\tau^2)\exp\left[i(\omega-k\bar{v})\tau-k^2D\tau^3/3\right] \;. \tag{7}$$

Catto makes subsidiary approximations which lead him to neglect the $k^2D\tau^2$ term. The result,

$$\varepsilon \simeq 1 + \sum (k\lambda_D)^{-2} + \sum i\omega (k\lambda_D)^{-2} \int d\bar{v}\ \langle f(\bar{v})\rangle \int_0^\infty d\tau\ \exp\left[i(\omega - k\bar{v})\tau - k^2 D\tau^3/3\right] , \tag{8}$$

is the one which Misguich discusses.

Misguich claims to derive the convergent form (8) from a form [his Eq. (10)] which is apparently divergent. There are two facets to the resolution of this paradox: Misguich's derivation of (8) from his Eq. (10) is not consistent, and his divergent form itself does not follow from Eq. (1), but from an expression [Misguich's Eq. (5)] which does not seem to be consistent with the definition of the dielectric.[5,6] In an attempt to include non-Markovian corrections, Misguich and Balescu[7] argue that (1) should be replaced by

$$\varepsilon(k,\omega;t) = 1 - i\sum \frac{\omega_p^2}{k^2} \int_0^\infty d\tau\ e^{i\omega\tau} \int dv d\bar{v}\ U_k(v,\tau;\bar{v}) \frac{\partial}{\partial \bar{v}} \langle f(\bar{v},t-\tau)\rangle . \tag{9a}$$

They write

$$\langle f(\bar{v},t-\tau)\rangle = \int dv' U_{k=0}(\bar{v},-\tau;v')\langle f(v',t)\rangle \tag{9b}$$

and then take $\langle f(v',t)\rangle$ to be stationary. However, the Eulerian function $\langle f(\bar{v},t-\tau)\rangle$ is no less stationary than $\langle f(\bar{v},t)\rangle$. The dielectric describes the result of probing the system after the turbulent state is set up. Since both Misguich and Balescu as well as I assume stationarity, $\langle f\rangle$ is unchanging before the probe is applied and (9) is incorrect.

Nevertheless, Misguich proceeds from (9). One has

$$U_{k=0}(v,-\tau;\bar{v}) \simeq P_v(v-\bar{v},-\tau) . \tag{10}$$

Misguich, in a separate, also inconsistent approximation, now passes (9b) with (10) through the $\partial/\partial\bar{v}$ operator, after which it partially cancels with $P_v(v-\bar{v}-ikD\tau^2)$ in such a way that $P_v(v-\bar{v}-ikD\tau^2)$ is effectively replaced in (2) by $\Delta(v-\bar{v}-ikD\tau^2)$, where $\Delta(z)$ is the Dirac delta function analytically continued from real to complex values of z, e.g.,

$$\Delta(z) = \lim_{\sigma\to 0_+} (2\pi\sigma^2)^{-1/2} \exp(-z^2/2\sigma^2) \ . \qquad (11)$$

The manipulations leading up to (5) still hold; however, because of the delta function approximation to P_v, (5) is replaced by unity, the term $\exp(k^2D\tau^3)$ in (4) is not cancelled, and in (2) a net factor of $\exp(2k^2D\tau^3/3)$ remains. The resulting time integral is divergent, as Misguich notes.

In an attempt to circumvent the divergence and obtain Catto's result, Misguich (effectively) returns to the form (1) (with P_v still replaced by Δ) and integrates over $\bar{v}$:

$$\varepsilon = 1 - \sum (k\lambda_D)^{-2} \int_{-\infty}^{\infty} dv \int_0^{\infty} d\tau \, \exp\left[i(\omega-kv)\tau - k^2D\tau^3/3\right]$$

$$\times (ikv - k^2D\tau^2)\langle f(v-ikD\tau^2)\rangle \ . \qquad (12)$$

This result can be justified by Cauchy's theorem. It is obviously equivalent to the divergent result discussed above, as a change of variables to $v' \equiv v-ikD\tau^2$ reveals. However, Misguich now neglects the factor of $-ikD\tau^2$ inside (but not outside) $\langle f\rangle$, arguing inconsistently that the action of the propagator on $\langle f\rangle$ results in "higher order contributions" in D. The result,

$$\varepsilon \simeq 1 + \sum (k\lambda_D)^{-2} \int dv\, \langle f(v) \rangle$$

$$\times \int_0^\infty d\tau\, \exp\left[i(\omega - kv)\tau - k^2 D \tau^3/3\right](ikv - k^2 D \tau^2)\ , \qquad (13)$$

is convergent. It is equivalent to Catto's result and, upon integration by parts in τ, to (8). Thus, since the divergence Misguich discusses is spurious, Catto's result remains reasonable.

Other aspects of the nonlinear dielectric are discussed in Ref. 8. Recent work has attempted to systematically justify approximations similar to Catto's; Refs. 5 and 6 contain many references.

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