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THE PHOTON STRUCTURE FUNCTION

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Deep inelastic scattering provides us with a unique probe into the structure of elementary particles. Experiments have revealed the existance of quarks and gluons as the pointlike constituents of hadrons. Almost ten years ago, Brodsky et al.[1] proposed that e^+e^- colliding beam experiments could be used to study the deep inelastic structure of photons by observing those processes which proceed through two photon annihilations. The photon is a particularly interesting particle due to its two component nature. In many reactions, the photon has hadronic character which may be understood through vector meson dominance ideas. However, the photon also has a pointlike component which can be observed in hard scattering processes. As emphasized by Brodsky et al.,[1] this pointlike component results in a dominant two photon annihilation cross section at sufficiently high energy.

The photon structure function is studied in e^+e^- colliding beam experiments by observing the electron (positron) which has been scattered at large angles, large Q^2, from the virtual photon associated with the unobserved positron (electron) which is scattered at small angles, $p^2 \backsim 0$. In these "single tag" events, the virtual target photon is almost real due to the kinematics of the two photon annihilation process. Experimental results for these processes are currently being obtained at PETRA, and future experiments at PETRA, PEP, and perhaps LEP should provide us with an accurate picture of the photon structure function.

In this talk I will review our current theoretical understanding of the photon structure function. As an illustration of the pointlike component, the parton model will be briefly discussed. However, the systematic study of the photon structure function will

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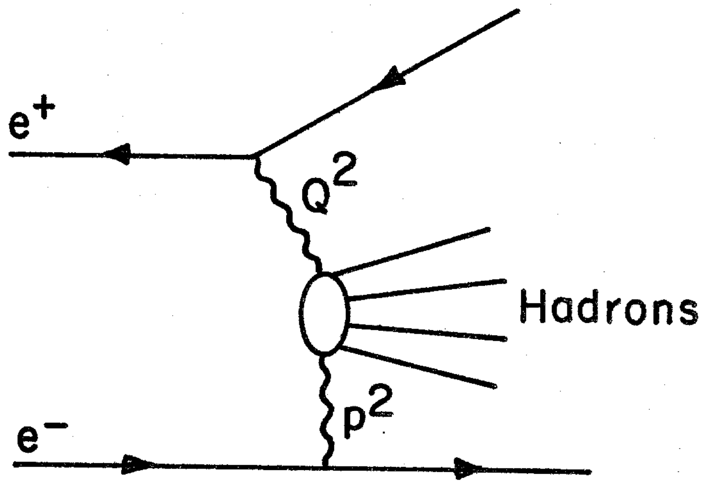

Fig. 1. Two photon processes.

be presented through the framework of the operator product expansion. Perturbative QCD is used as the theoretical basis for the calculation of leading contributions to the operator product expansion. The influence of higher order QCD effects on these results will be discussed. I will also briefly discuss recent results for the polarized structure functions.

The parton model is based on a free quark picture where the quarks have standard pointlike couplings to the photon. The parton model calculation[2] of the photon structure function is obtained from the cross section for $e^+e^- \rightarrow e^+e^-q^+q^-$ computed in tree approximations. The unpolarized structure functions are given in standard notation by

$$F_2^{\gamma}(x, Q^2) = \alpha^2 \cdot 12 \cdot f \cdot \langle e^4 \rangle \cdot \left\{ x \cdot [x^2 + (1 - x)^2] \cdot \log\left(\frac{Q^2}{m_q^2} \, \frac{1 - x}{x} \right) - x + 8x^2(1 - x) \right\}$$

$$F_2^{\gamma}(x, Q^2) = \alpha^2 \cdot 48 \cdot f \cdot \langle e^4 \rangle \cdot \{x^2(1 - x)\} \qquad (1)$$

where f is the number of quark flavors, α is the fine structure constant, and m_q is the quark mass. The pointlike nature of this reaction results in a large cross section, especially at high Q^2, and in an x distribution which is stiffer than in hadronic reactions. A special feature of this reaction is its sensitivity to the fourth power of the quark charge.

The parton model is expected to be only qualitatively correct as the quarks are not free but have a nontrivial pointlike dynamics. As in the case of deep inelastic scattering on hadron targets, a systematic analysis of the photon structure function is provided through the use of the operator product expansion. The general structure of this analysis was presented by Witten[3] along with the leading order QCD predictions for the structure functions.

The operator product formalism makes direct predictions for asymptotic behavior of moments of the structure functions, M_n^γ,

$$M_n^\gamma = \int dx\, x^{n-2} F_n^\gamma(x, Q^2) \to \sum_k C_n^k(Q^2) A_n^k \quad . \tag{2}$$

$\{C_n(Q^2)\}$ are the coefficient functions of the twist two operators appearing in the operator product expansion of two electromagnetic currents,

$$T(J_\mu(q) J_\nu(-q)) = \sum C_n^k(Q^2) K_{\mu\nu\alpha_1\cdots\alpha_n}(q) \cdot O^k_{\alpha_1\cdots\alpha_n}(0) \tag{3}$$

where $\{K(q)\}$ are the appropriate kinematic tensors and $\{O_n(0)\}$ are the spin n, twist two operators. $\{A_n\}$ are the reduced matrix elements of these operators for the photon target,

$$A_n^k = \langle\gamma\| O_n^k(0) \|\gamma\rangle \quad . \tag{4}$$

The important observation of Witten is that we must include twist two photon operators in addition to the usual hadronic operators. Since we are calculating to lowest order in QED, the reduced matrix elements of the photon operators are trivial, $A_n^\gamma = 1$. These additional contributions may be identified with the pointlike component of the photon.

In quantum chromodynamics, the coefficient functions are all calculable using renormalization group techniques and QCD perturbation theory. In the correct hadronic basis,[4] the solution may be presented in terms of an asymptotic expansion in the running coupling constant, $\bar{g}^2(Q^2)$. The coefficient functions for the hadronic operators are given by

$$C_n^h(Q^2) \to (\bar{g}^2)^{\gamma^o_{hn}/2\beta_o}(1 + O(\bar{g}^2)) \tag{5}$$

where $\gamma_{hn}^{o}/2\beta_o$ are the logarithmic anomalous dimensions of the various hadronic operators. The coefficient functions for the photon operator are given by

$$C_n^{\gamma}(Q^2) \rightarrow \frac{a_n}{\bar{g}^2} + b_n + \ldots \qquad . \qquad (6)$$

Using these expansions, the asymptotic behavior for the moments of the photon structure function is determined by

$$M_n^{\gamma}(Q^2) = \frac{a_n}{\bar{g}^2} + b_n + \ldots$$

$$+ \sum_h A_n^h (\bar{g}^2)^{\gamma_{hn}^{o}/2\beta_o} (1 + \ldots) \qquad . \qquad (7)$$

We note that all of the hadronic anomalous dimensions are non-negative, $\gamma_{hn}^{o}/2\beta_o \geq 0$. Because of the asymptotic freedom of QCD, the running coupling constant, $\bar{g}^2(Q^2)$, must vanish at large Q^2

$$\bar{g}^2(Q^2) \rightarrow 16\pi^2/\beta_o \log (Q^2/\Lambda^2) \qquad . \qquad (8)$$

This asymptotic behavior implies that the dominant contribution to the moments at high Q^2 comes from the pointlike component of the photon.

Using Eq. (8), the asymptotic forms of the moments are given by

$$M_n^{\gamma}(Q^2) \rightarrow A_n \log (Q^2/\Lambda^2) + \ldots \qquad . \qquad (9)$$

Although this Q^2 dependence is the same form as that found in the parton model in Eq. (1), the coefficient, A_n, as calculated by Witten,[3] differs from the parton model result. These leading order expressions are compared in Figure 2. The softening of the x distribution observed in Figure 2 may be directly related to the fact that the quarks in QCD can emit gluons with pointlike couplings.

These leading order results have been subsequently derived using a variety of alternative calculation methods including leading log sums[5] and mass sensitive renormalization group techniques.[6] A more intuitive approach[7] makes use of the Altarelli-Parisi evolution equations. Another method makes use of asymptotic field theory[8] to obtain these results.

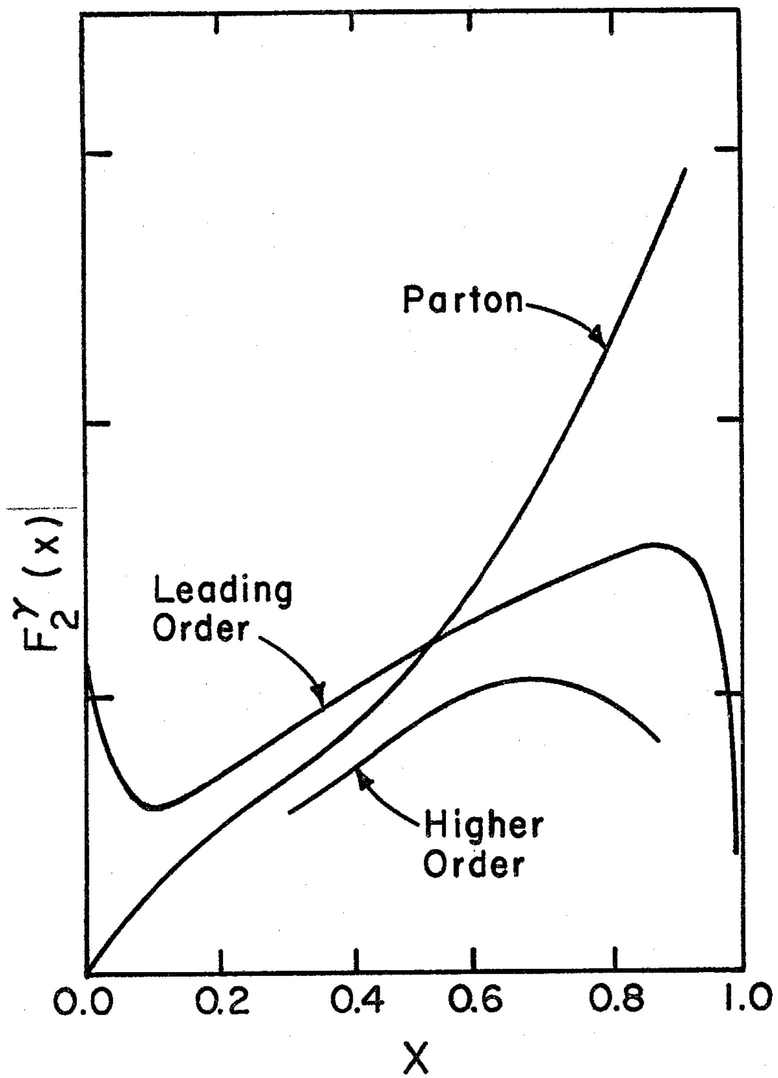

Fig. 2. The photon structure function.

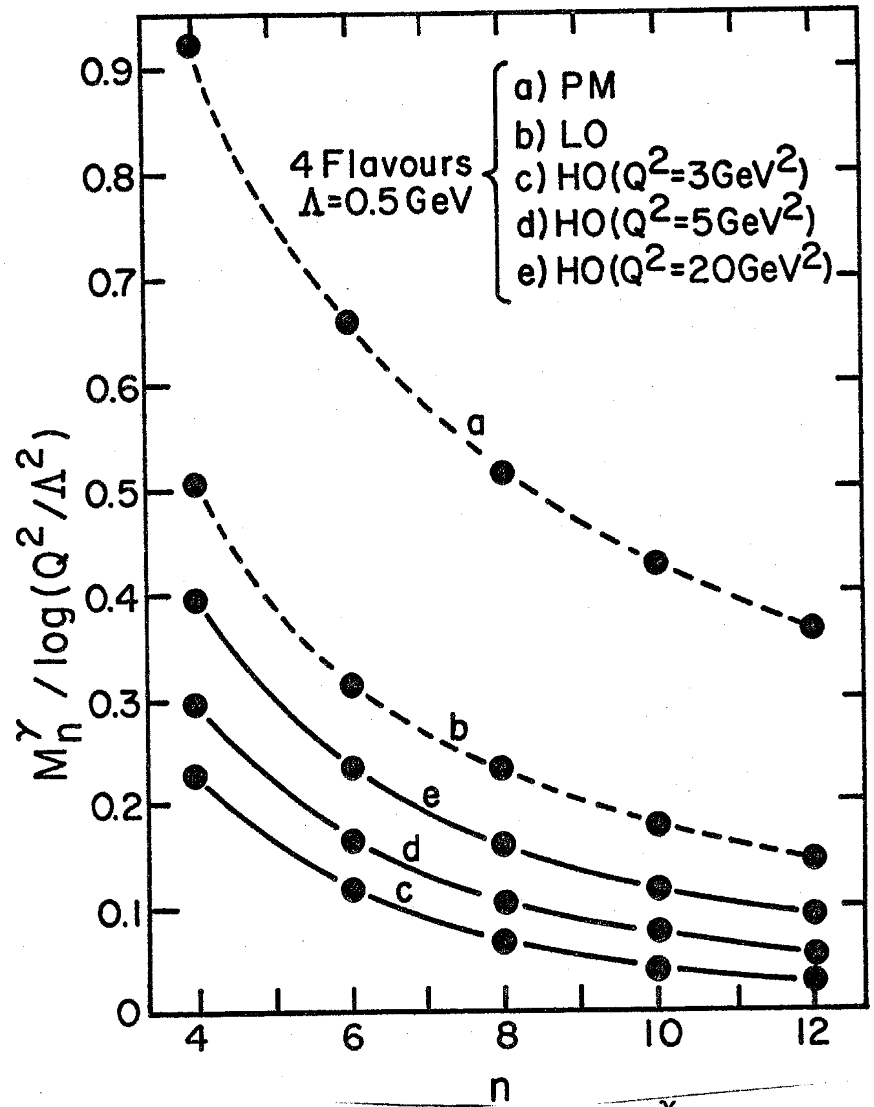

Fig. 3. Moments of F_2^γ.

We now turn to a study of the effects of higher orders in the QCD calculation of the photon structure function. Although the pointlike components of Eq. (7) (a_n, b_n,...) are calculable in perturbative QCD, the determination of the constants A_n^h require the knowledge of the photon matrix elements of hadronic operators which cannot be computed with present methods. However in next order, the pointlike contributions, b_n, continue to asymptotically dominate the hadronic components except in the case of the second moment, n = 2. For n > 2, the logarithmic anomalous dimension, $\gamma_n/2\beta_o$, is positive which implies that the hadronic components should vanish as $\bar{g}^2 \to 0$ relative to the constant, b_n. For the singlet second moment, the hadronic anomalous dimension vanishes, which implies a mixing of the photon operator with the hadronic stress tensor. In this case, the pointlike component cannot be separated from the hadronic component. The calculation of the higher order pointlike components for n > 2 was somewhat involved and required knowledge of every QCD constant computed in perturbative QCD. The details of this calculation are presented in Ref. 9.

The effect of the higher order terms is illustrated in Figure 3. The inclusion of these terms causes a further suppression of the photon structure function particularly at large x as seen in the higher order curve in Figure 2. As Q^2 becomes large, the higher order curves approach the leading order result. As noted in Ref. 9, the higher order terms cannot be absorbed by simply modifying the QCD scale, Λ^2, as the shape of the x distribution is changed and reflects a further softening.

Due to mixing, only the higher moments, $n \geq 4$, were computed in Reference 9. This restriction may be avoided, in part, by separating the structure function into its valence and sea components.[10] The valence component gets contributions from graphs such as shown in Figure 4a while the sea component comes from those of Figure 4b. These components differ in their dependence on the quark charges and may be written in obvious notation as

$$F^\gamma(x, Q^2) = \langle e^4 \rangle F_v{}^\gamma(x, Q^2) + \langle e^2 \rangle^2 F_s{}^\gamma(x, Q^2) \quad . \qquad (10)$$

As emphasized by Duke and Owens,[10] this separation is significant as the two distributions have much different character. As shown in Figure 5, leading order QCD predicts a large valence component with a stiff x distribution. The sea component is much smaller and is soft.

When higher order corrections are considered, Duke and Owens find that the valence contribution to b_n is completely calculable. Only the sea contribution to b_2 is ambiguous due to mixing with the hadronic components. Duke and Owens also estimate the hadronic component using a simple vector dominance model and find the QCD

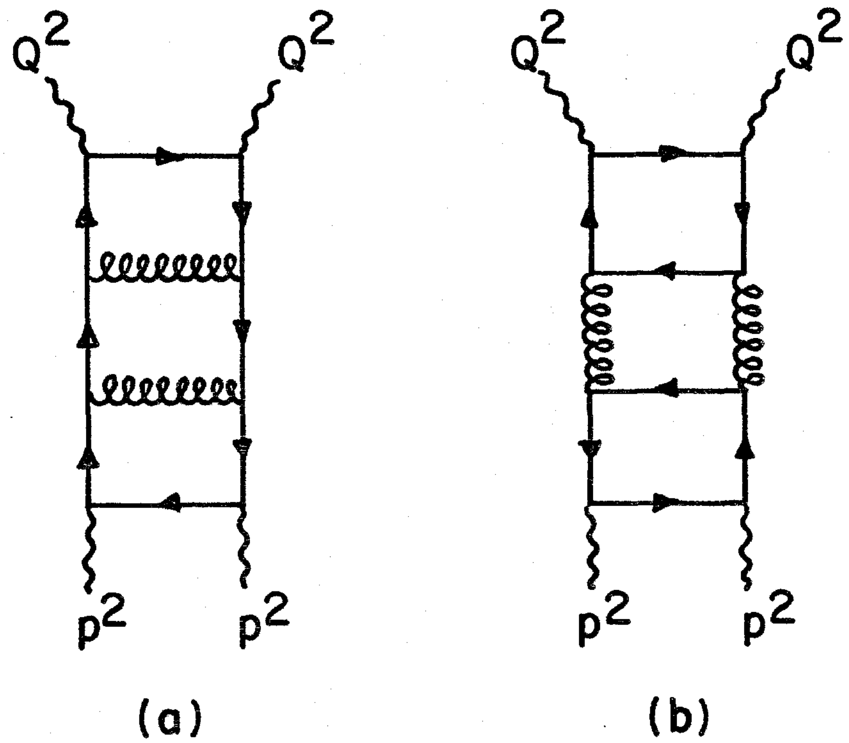

Fig. 4a. Valence. b. Sea.

predictions of the photon structure function as shown in Figure 6.

We conclude that these methods provide stable QCD predictions for the photon structure functions for moderate values of x, $.3 < x < .9$. Duke and Owens find bad behavior (negative $F_2(x)$) for small values of x which seem to be related to large, negative higher order contributions to the sea distribution. The predictions also become unreliable at large x. This may be due to a kinematic problem associated with an improper treatment of the phase space boundaries ($K_\perp^2 \sim Q^2(1 - x)$) in using the usual moments.[11] Another aspect of this problem which may affect the ability to make precise predictions concerns the role of quark masses. Hill and Ross[6] find that quark masses lead to slow scaling particularly in the case of the charm quark. Hopefully, as experimental information becomes available we may be able to determine which if any of these effects are important in our attempt to make a precise confrontation between theory and experiment.

I would also like to mention some recent work involving polarized structure functions. If we look only at "single tag" events where one of the photons is nearly on shell, there are actually four structure functions to be measured. With unpolarized beams, only F_2 and F_L are measured and the predictions of QCD for these quantities

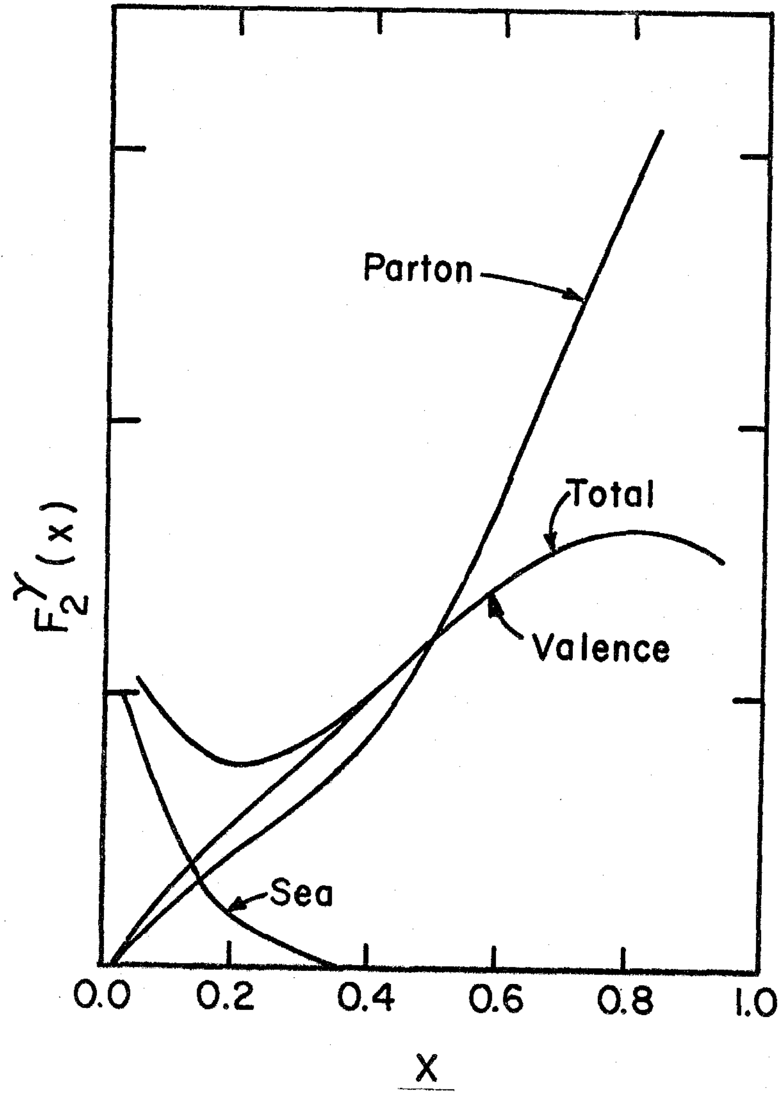

Fig. 5. Valence-Sea.

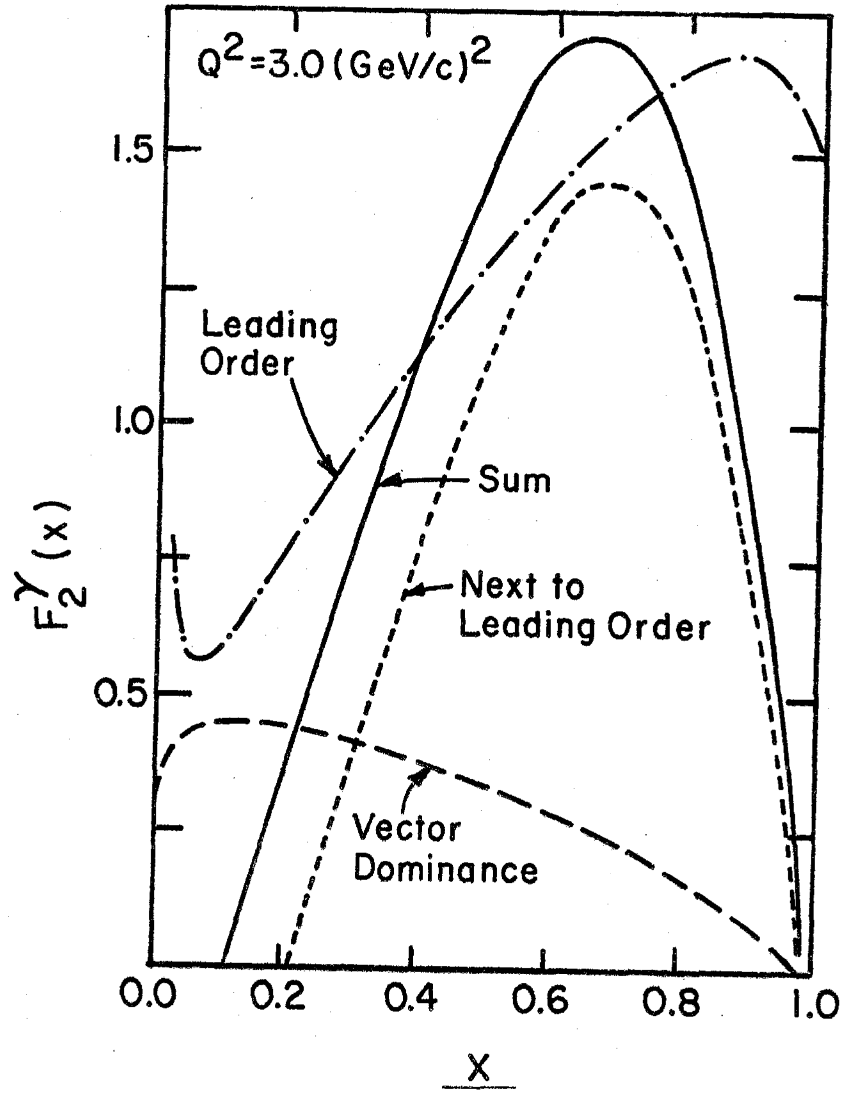

Fig. 6. Higher order.

are discussed above. With polarized beams, we can measure two more structure functions, conventionally written as W_3 and W_4. Several authors[12] have discussed these polarized structure functions. They conclude that W_3 scales and gives the parton result while W_4 increases as log (Q^2/Λ^2) and is affected by the QCD corrections[4] in a manner similar to the F_2 structure function. Finally double deep processes where both virtual photons are at large Q^2 have also been examined in QCD[14] with the parton model result as the asymptotic behavior. Unfortunately, it is improbable that these last predictions will ever be subjected to experimental test.

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