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Mode-Conversion Induced Tearing Effects in a Plasma Neutral Sheet

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Abstract

In this note we identify a new collisionless dissipation mechanism which can drive tearing modes in a plasma neutral sheet. The new mechanism relies on the presence of a background cold plasma which leads to mode conversion into a continuous spectrum of cold plasma waves.

In recent years, extensive theoretical work has been done on tearing modes in a collisionless plasma neutral sheet [Sagdeev 1979; Galeev 1978; Schindler 1974; Coppi et al. 1966]; such studies are believed to be relevant to observations of the dynamics of earth's geomagnetic tail. It is well known that tearing modes involve a breakup of the current sheet in the magnetically neutral layer into periodic current filaments. This necessitates a change in the magnetic field line topology and can occur only in the presence of a dissipative mechanism, which breaks ideal-MHD constraints. In a collisionless plasma, the dissipation is typically assumed to be due to wave-particle interactions viz. electron and ion Landau damping effects. In this paper we demonstrate the existence of a new collisionless dissipation mechanism which may contribute to tearing effects. This dissipation relies on the simultaneous presence of a cold background plasma in the hot plasma neutral sheet; the tearing mode equations now contain cold plasma resonances (associated with finite perpendicular inertia of cold plasma particles) which lead to mode-conversion into other damped and/or propagating waves. We show that for a cold plasma density comparable to hot plasma density, the contribution from mode-conversion effects can very significantly modify tearing mode growth rates.

We consider the well-known equilibrium for a hot plasma neutral sheet [see, e.g., Coppi et al. 1966]. The particle distribution functions are given by

$$f_{\alpha o} = \frac{n_o \exp(-m_\alpha v_{o\alpha}^2/2T_\alpha)}{(2\pi m_\alpha T_\alpha)^{3/2}} \exp\{-\frac{m_\alpha}{T_\alpha}[\frac{v^2}{2} - v_{o\alpha}(v_z + \frac{e}{m_\alpha} A_{oz})]\} \quad . \tag{1}$$

Where $A_{oz}(x)$ is the only non-zero component of vector potential. This produces a self-consistent magnetic configuration with $n_\alpha(x) = n_o \operatorname{sech}^2(x/L)$,

$J_z = -en_e(x)v_{oe}(1 + T_i/T_e)$, $B_{yo} = \frac{d}{dx} A_{oz} = B_o \tanh x/L$, $B_o^2 = 8\pi n_o(T_e + T_i)$ and $L^2 = (c^2/v_{oe}^2)[T_e^2/2\pi n_o e^2(T_e + T_i)]$; the electrostatic potential is zero by virtue of the choice of a frame where $v_{oe}/T_e + v_{oi}/T_i = 0$. We assume, in addition, the simultaneous presence of a cold collisionless plasma background with a density N_o particles cm^{-3}.

We now consider a tearing perturbation of this equilibrium. The perturbed quantities go as $g(x) \exp(iky-i\omega t)$. We ignore the displacement current and combine the various Maxwell equations to derive the equation for perturbed electric field $\underset{\sim}{E}$ viz.

$$\nabla(\nabla\cdot\underset{\sim}{E}) - \nabla^2\underset{\sim}{E} = \frac{4\pi}{c^2} i\omega\underset{\sim}{J} . \qquad (2)$$

The current density perturbation $\underset{\sim}{J}$ has a hot plasma contribution $\underset{\sim}{J}_h$, and a cold plasma contribution $\underset{\sim}{J}_c$. $\underset{\sim}{J}_h$ is given by

$$\underset{\sim}{J}_h = \sum_\alpha e_\alpha \int \underset{\sim}{v} f_\alpha \, d^3v , \qquad (3)$$

where f_α satisfies the linearized Vlasov equation; the Vlasov equation may be solved by integration along the unperturbed trojectories giving formally

$$f_\alpha = - \frac{e_\alpha}{T_\alpha} f_{\alpha o}\left\{i \frac{v_{\alpha o}}{\omega} E_z(\underset{\sim}{r},t) - \int_{-\infty}^{t} dt' \, \underset{\sim}{E}[\underset{\sim}{r}(t'),t']\cdot\underset{\sim}{v}(t')\right\} . \qquad (4)$$

The first term is the adiabatic contribution. The second term gives the non-adiabatic effects associated with wave-particle interactions; this term gives the collisionless dissipation mechanism responsible for the growth of conventional tearing modes. Using Eq. (3) and Eq. (4) one may write

$$\underset{\sim}{J}_h = - \sum_\alpha \left[i \frac{n_\alpha e_\alpha^2}{T_\alpha} v_{o\alpha}^2 \frac{E_z}{\omega} \hat{z} + \frac{n_\alpha e_\alpha^2}{m_\alpha} \frac{\underset{\sim}{E}}{\omega} \left(\frac{i\omega}{kv_{th\alpha}}\right) Z \left(\frac{\omega}{kv_{th\alpha}}\right) \right]$$

$$= \sigma_a E_z \hat{z} + \sigma_{na} \underset{\sim}{E} \ , \qquad (5)$$

where $v_{th\alpha}^2 = 2T_\alpha/m_\alpha$, $Z(\zeta)$ is the well-known plasma dispersion function (Fried and Conte 1961) and we assumed $m_\alpha v_{o\alpha}^2/T_\alpha < 1$. The derivation of the non-adiabatic term assumes (a) that $\underset{\sim}{E}\left(x(t')\right) \approx \underset{\sim}{E}\left(x(t)\right)$ may be treated locally and taken out of the t' integration and (b) that the term is finite only for $|x| < (\rho_\alpha L)^{1/2}$ [where $\rho_\alpha = v_{th\alpha}/(e_\alpha B_o/m_\alpha c)$] - a region in which particle orbits may be treated as essentially unmagnetized. The cold plasma response may be readily written down using cold plasma theory [Stix 1962]. Thus we have

$$\underset{\sim}{J}_{c\perp} = \sigma_1 \underset{\sim}{E} + \sigma_2 \hat{\underset{\sim}{y}} \times \underset{\sim}{E}$$

$$\underset{\sim}{J}_{c\parallel} = \hat{y}\, \sigma_3 E_y \ . \qquad (6)$$

Where $\perp$ and $\parallel$ refer to directions perpendicular and parallel to the field lines and

$$\sigma_1 = \sum_\alpha \frac{N_\alpha e_\alpha^2}{m_\alpha} \frac{(-i\omega)}{\Omega_\alpha^2 - \omega^2} \ , \quad \sigma_2 = - \sum_\alpha \frac{N_\alpha e_\alpha^2}{m_\alpha} \frac{\Omega_\alpha}{\Omega_\alpha^2 - \omega^2}$$

$$\sigma_3 = \sum_\alpha i \frac{N_\alpha e_\alpha^2}{m_\alpha \omega} \ , \qquad (7)$$

N_α being the density of cold species α . Substituting for the components of $\underset{\sim}{J}$ in Eq. (2) and eliminating E_x we get the two equations for E_y, E_z:

$$\left(\frac{d^2}{dx^2} - k^2 + \Sigma_a\right) E_z + \left(\Sigma_{na} + \Sigma_1 - \frac{\Sigma_2^2}{k^2 - \Sigma_1 - \Sigma_{na}}\right) E_z = \frac{-ik\Sigma_2}{k^2 - \Sigma_1 - \Sigma_{na}} \left(\frac{dE_y}{dx}\right) \qquad (8a)$$

$$\left(\frac{d^2}{dx^2} + \Sigma_3 + \Sigma_{na}\right)E_y - \frac{d}{dx}\left(\frac{k^2}{k^2-\Sigma_1-\Sigma_{na}} \frac{dE_y}{dx}\right) = ik \frac{d}{dx}\left(\frac{\Sigma_2 E_z}{k^2-\Sigma_1-\Sigma_{na}}\right) , \tag{8b}$$

where $\Sigma \equiv \frac{4\pi i\omega}{c^2}\sigma$. The full fourth order set can be used to study mode conversion process in detail. For our purposes of derivation of tearing mode growth rates, it is sufficient to work in the limit of infinite ∥ conductivity σ_3 . Physically, this corresponds to restricting our attention to wavelengths long compared to the collisionless skin depth c/ω_{pe} i.e., $ck/\omega_{pe} << 1$. This point should be emphasized. The infinite conductivity approximation is only meant to throw away correction terms of order c^2k^2/ω_{pe}^2. If the σ_3 for cold plasma were truly infinite, a thin region of $x \sim (T_c/m_\alpha\Omega_\alpha^2)^{1/4} L^{1/2}$ where the cold plasma particles are unmagnetized would shield all induction fields and prevent reconnection; as it is, the contribution to γ from electron inertia in this region is finite, but negligibly small. Equation (8b) shows that in this case $E_y(x) \to 0$ and then Eq. (8a) with right side equal to zero describes the tearing mode of interest. We rewrite it as

$$\left(\frac{d^2}{dx^2} - k^2 + \Sigma_a\right) E_z = -\left(\Sigma_{na} + \Sigma_1 - \frac{\Sigma_2^2}{k^2-\Sigma_1-\Sigma_{na}}\right) E_z, \tag{9}$$

where

$$\Sigma_a = \sum_\alpha \frac{4\pi e_\alpha^2}{T_\alpha} n_\alpha(x) \frac{v_{0\alpha}^2}{c^2} , \quad \Sigma_{na} = \sum_\alpha \frac{4\pi e_\alpha^2 n_\alpha}{m_\alpha c^2} \frac{\omega}{kv_{th\alpha}} Z\left(\frac{\omega}{kv_{th\alpha}}\right) , \tag{10}$$

$$\Sigma_1 = \sum_\alpha \frac{4\pi N_\alpha e_\alpha^2}{m_\alpha c^2} \frac{\omega^2}{\Omega_\alpha^2 - \omega^2} , \quad \Sigma_2 = \sum_\alpha \frac{4\pi N_\alpha e_\alpha^2}{m_\alpha c^2} \frac{(-i\omega)\Omega_\alpha}{\Omega_\alpha^2 - \omega^2} .$$

It is to be emphasized that the above expression for the hot plasma non-adiabatic contribution to the current, Σ_{na}, is derived under the assumption of unmagnetized particle orbits and is, as such, valid only for $|x| < (\rho_\alpha L)^{1/2}$; $\Sigma_{na} = 0$ for $|x| > (\rho_\alpha L)^{1/2}$. All terms independent of $\omega = i\gamma$ are written on the left side of Eq. (9); these will be treated as zeroth order terms. The terms proportional to ω are perturbations and written on the right side. We solve the eigenvalue problem posed by Eq. (9) and the boundary condition of well-behavedness at $x = \pm\infty$, by using standard quantum-mechanical perturbation theory. To the zeroth order we take $\omega = 0$, $k^2L^2 = 1$ and use the equilibrium conditions following Eq. (1) to get the equation

$$\left(L^2 \frac{d^2}{dx^2} - 1 + 2 \, \mathrm{Sech}^2 \frac{x}{L}\right) E_{zo} = 0 \,,$$

with the solution $E_{zo} = (1/2L)^{1/2}$ Sech (x/L). Treating $(1 - k^2L^2)$ as a perturbation, to the next order, we get the eigen-value condition

$$1 - k^2L^2 = \frac{L}{2} \int_{-\infty}^{\infty} \left(- \Sigma_{na} - \Sigma_1 + \frac{\Sigma_2^2}{k^2 - \Sigma_1 - \Sigma_{na}}\right) \mathrm{Sech}^2 \frac{x}{L} \, dx \,. \qquad (11)$$

Note that the various terms of Σ_{na} integral are finite only within $|x| < (\rho_\alpha L)^{1/2}$. For $\omega/kv_{th\alpha} \ll 1$, $Z(\omega/kv_{th\alpha}) \to i\pi^{1/2}$ and we get (for $\omega \equiv i\gamma$)

$$- \frac{L}{2} \int_{-\infty}^{\infty} \Sigma_{na} \, \mathrm{Sech}^2 \frac{x}{L} \, dx = \sum_\alpha \pi^{1/2} \frac{\omega_{p\alpha}^2 L^2}{c^2} \frac{\gamma L}{v_{th\alpha}} \left(\frac{\rho_\alpha}{L}\right)^{1/2} . \qquad (12)$$

The second and third terms on right side of Eq. (11) are the new terms arising out of the cold plasma response. A look at Eq. (10) shows the existence of cyclotron resonances at the points $\Omega_\alpha(x) = \omega$; similarly the regions $k^2 - \Sigma_1(x) = 0$ correspond to electromagnetic ion and electron cyclotron resonances. Integration around these resonances give non-adiabatic contributions (proportional to γ); thus, this is a source of dissipation in the equations (i.e., $\int \underline{J}\cdot\underline{E}\, dx \neq 0$ because of these terms). Physically, the time-dependence of tearing mode perturbations excites a continuous spectrum of normal modes in an inner layer, which then phase mix away. Such mode-conversion induced damping has been extensively studied for Alfvén waves [Hasegawa and Chen 1974; Tataronis and Grossman 1973], plasma waves [Barston 1964], surface waves [Baldwin and Ignat 1969] and even waves on field-reversed ion layers [Gerver and Sudan 1979]. Assuming γ is small, we can obtain simple expressions for the above integrals. In this case we may expand $\Omega_\alpha(x) \approx \Omega_{\alpha 0}(x/L)$ and treat N_α as constant and Sech x/L = 1. The resulting definite integrals are of the form $\int_{-\infty}^{\infty} dx\,/(x^2 + a^2) = \pi/a$ and its simple generalizations. We thus find

$$-\frac{L}{2}\int_{-\infty}^{\infty} \Sigma_1 \,\mathrm{Sech}^2 \frac{x}{L}\, dx \approx \frac{\pi}{2}\sum_\alpha \frac{\omega_{p\alpha c}^2 L^2}{c^2}\frac{\gamma}{\Omega_\alpha}\,, \qquad (13a)$$

and

$$-\frac{L}{2}\int_{-\infty}^{\infty} \frac{\Sigma_2^2}{k^2 - \Sigma_1 - \Sigma_{na}} \,\mathrm{Sech}^2 \frac{x}{L}\, dx \approx \frac{\pi}{2}\frac{\omega_{pic}^4 L^4}{c^4}\frac{\gamma}{\Omega_i}\left[1 + \left(1 + \frac{\omega_{pic}^2 L^2}{c^2}\right)^{1/2}\right]^{-1}, \qquad (13b)$$

where $\omega_{p\alpha c} = (4\pi N_\alpha e_\alpha^2/m_\alpha)^{1/2}$ is the plasma frequency for the cold species and in deriving Eq. (13b) we have neglected contributions from electron resonances since they are down by $(m_e/m_i)^{3/2}$ in magnitude. Combining Eqs. (11) through (13) we find

$$\frac{\gamma}{(1-k^2L^2)} = \{\frac{\omega_{pe}^2 L^2}{c^2}\frac{\pi^{1/2}}{\Omega_e}[(\frac{L\Omega_e}{v_{the}})^{1/2} + \frac{\pi^{1/2}}{2}\frac{N_o}{n_o}]$$

$$+ \frac{\pi}{2}\frac{\omega_{pic}^2 L^2}{c^2\Omega_{ic}}[1 + \frac{\omega_{pic}^2 L^2/c^2}{1+(1+\omega_{pic}^2 L^2/c^2)^{1/2}}]\}^{-1} . \qquad (14)$$

In the absence of cold plasma, we get the standard result [Coppi et al. 1966] (for $kL \lesssim 1$)

$$\gamma = \frac{c^2}{\omega_{pe}^2 L^2}(\frac{v_{the}}{\Omega_e L})^{1/2}\Omega_e = \frac{v_{the}}{L}(\frac{\rho_e}{L})^{3/2}. \qquad (15)$$

The contribution from cold plasma resonances becomes comparable to that from wave-particle interactions of hot plasma when $N_o/n_o \sim (\rho_{ci}/L)^{1/3} (M_{ci}/m_e)^{1/6} \sim O(1)$, where ρ_{ci} is the Larmor radius of the cold ion species at the warm plasma temperature $T_e \sim T_i$. For cold plasma densities in excess of the above, the non-adiabatic response to tearing perturbation is dominated by cold plasma resonances and the growth rates are considerably smaller.

In conclusion, we have identified a new collisionless plasma dissipation mechanism which can drive tearing modes. It is related to mode conversion into a continuous spectrum of normal modes at cold plasma resonances and can significantly modify tearing mode growth rates. In the above discussion we have assumed that the equilibrium field is only in y-direction. It is well known that a small B_{xo} at $x = o$ magnetizes the electrons so that they do not contribute to non-adiabatic response [Schindler 1974]; in the conventional theory the growth is then entirely due to ion wave-particle interactions. The mode conversion problem with $B_{xo} \neq 0$ is a genuine fourth order differential equation problem and will be treated in a separate publication. Preliminary estimates indicate that the non-adiabatic response due to cold plasma resonances continues to operate as long as $\gamma \gtrsim (eB_{xo}/M_i c)$. This could make the tearing mode driven by mode-conversion processes more important for nonlinear reconnection phenomena. Finally, it should be pointed out that the use of cold plasma response of the background plasma is valid only if $T_c/T_i \ll (M_i^3/M_{ci}^2 m_e)^{1/2} (\rho_i/L)^3$.

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